Analysis and Design of a Nonlinear Stiffness and Damping System with a Scissor-Like Structure

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Abstract

An n-layer Scissor-Like Structured (SLS) vibration isolation platform is studied in this paper, focusing on the analysis and design of nonlinear stiffness, friction forces and damping characteristics for an advantageous vibration isolation performance. The system nonlinear stiffness and damping characteristics are theoretically investigated by considering the influence incurred by different structural parameters, friction forces and link inertia. Since stiffness and damping properties are both asymmetrical nonlinear functions, and Coulomb friction is piecewise nonlinear function, Perturbation Method (PM) and Average Method (AM) are applied together to achieve better solutions. The vibration isolation performance of the SLS platform is compared with known quasi-zero-stiffness vibration isolators in the literature, and a typical application case study as a vehicle seat suspension is also conducted, subjected to different load masses, and base excitations. The results show that much better vibration isolation performance and loading capacity can be easily achieved with the SLS platform by designing structural parameters, and the scissor-like structure provides a very powerful, practical and passive solution to design and realization of beneficial nonlinear stiffness and damping characteristics in vibration control.

Key words: Nonlinear stiffness and damping; Structural nonlinearity; Vibration isolation

1. Introduction

Vibration isolation is extensively applied in various engineering practice from vehicle ride comfort improvement [1], protection of high-precision machinery [2], to space launch or on-orbit vibration control [3]. Performance requirements for vibration isolation or suppression usually include several aspects, which are isolation effect (or transmissibility), loading capacity, energy cost, and complexity of implementation. Active control can achieve much better isolation performance but cost energy [3]-[4]. Semi-active control methods would be much better in saving energy. But similar to active control, special control units are usually needed in semi-active control which often increase isolator weight and development complexity.

It is particularly noticed that, some passive vibration control methods in the literature are designed by employing various structure properties and thus better vibration isolation performance is achieved. For example, a vibration isolation structure whose linear stiffness is close to zero called quasi-zero-stiffness vibration isolator (QZS-VI) is investigated in [5]-[13]. The QZS-VI can obtain a high static stiffness but small dynamic stiffness, which results in low natural frequency and small static displacement. Because the linear stiffness of the QZS-VI can be designed close to zero by properly choosing structural parameters, it is potentially advantageous in many practical applications [6]-[7]. However, obvious disadvantages of the class of QZS-VIs are the easiness to lose stability, low loading capability and potential bifurcation effect at equilibrium due to negative linear stiffness [8], [12]-[13]. The other noticeable vibration system is designed by using a one-layer truss structure which is applied in seat suspension systems [14]-[16]. Considering that the most sensitive frequency range in vertical vibration for seated human body is in 4 to 9 Hz [17]-[18], the

authors use additionally an air-spring and hydraulic shock-absorber to provide load force and vibration isolation from base excitation. Moreover, an active controller is designed in [14]-[15] with the same structure to improve ride comfort and vibro-isolation properties for different loading mass. The results show that this truss structure can obviously improve vibro-isolation performance of seat suspension.

Some recent studies indicate that nonlinear stiffness and damping are very beneficial in vibration control for both low and high frequencies [19]-[24,39]. For example, the research results in [25-27] show that MR dampers and pneumatic springs are very versatile in providing advantageous and adjustable damping and stiffness characteristics for vibration control. Considering that many practical systems have inherent nonlinearity, a fundamental question is whether it is possible to exploit those inherent nonlinearities in structures or control units to realize much better vibration isolation performance, without much energy cost or with only passive elements. The QZS-VIs mentioned before are good examples to this aim, but their limitations should be overcome for a much better engineering resolution.

Therefore, an n-layer scissor-like structured (SLS) platform is studied to explore new design of nonlinear vibration isolation systems by employing beneficial nonlinearity in stiffness and damping characteristics incurred by structures. In engineering practice, various structures of special characteristics or functions may be designed for different purposes. Scissor-like-elements have been used as self-deployable structures in the literature manipulated with active control [29, 30]. Due to nonlinear geometric relations within pantographic structures, a homogenized model is studied for the external contact action at extremities of thin-walled twisted tubes [30-31], which can overcome the limitation of classical continuum mechanics in microstructure. The SLS platform has been

proposed and studied recently in [38, 28]. Only some preliminary modeling and simulation results are presented in [38], and in [28] system modelling is conducted without considering mass of connecting rods and corresponding friction forces.

In this study, for the n-layer SLS vibration isolation system, the Coulomb friction [32-33] and mass of connecting links are all carefully considered in system modeling and performance analysis. Different from the results in [28], the influence incurred by joint or contract frictions and link inertia on system equivalent mass, stiffness and damping of the SLS platform are theoretically analyzed. Some preliminary experimental analysis is also conducted to verify the excellent vibration isolation performance of the platform. It is shown that (a) the inherent nonlinear stiffness and damping characteristics of the SLS system can achieve superior vibration isolation using only pure linear and passive elements in the system with a simple and flexible installation structure; (b) the friction and inertia effects can all be minimized by choosing appropriately structure parameters; (c) the SLS platform can achieve much better quasi-zero stiffness property, without those disadvantages of the existing QZS-VI systems (such as strong nonlinearity and poor loading capacity) and drawbacks of the seat suspension system with active control in the literature.

The paper is organized as follows. The schematic structure of an n-layer SLS platform is introduced in Section 2 and mathematical modeling is conducted in Section 3. Then, the nonlinear stiffness, equivalent friction force and damping properties under different structural parameters are studied in Section 4. An effective solving method is also provided in this paper which is combined the Perturbation Method (PM) and Average Method (AM) [34-36] in Section 5 and the self-locking phenomenon induced by the Coulomb friction is analyzed. Comparisons and application studies of the SLS platform are discussed in Section 6. Some experimental results for the response of the

platform under random impact excitation are shown in Section 7. A conclusion is drawn thereafter.

2. The n-layer SLS platform

Fig. 1 A 3-layer SLS platform

Consider the platform with n -layer scissor-like structure in this section. Fig. 1 is the diagram of a 3-layer SLS system. Each layer of the structure is constructed by connecting rods and corresponding rotating joints. The mass of the connecting rods is taken into consideration but the mass of corresponding rotating joints is ignored since the mass of joints is often much smaller than rods. The supporting joints in the left bottom and top layer can be freely sliding along pre-designed horizontal tracks. Obviously, the properties of contacting surface on these sliding tracks and in rotating joints can be designed for different damping properties. Potentially, some linear springs can be used as indicated to achieve the stiffness of the system.

Because the platform is symmetric, it can be simplified as a plane problem as shown in Fig. 2, which demonstrates the front view of the n -layer platform. The mass of the isolation object is denoted by M_1 . The connecting rods have the same length denoted by $2l$ and the assembly angle with respect to the horizon line is represented by θ . More practically, the stiffness of the spring in

the first layer in horizontal direction is supposed to satisfy a property $f=k_l(\cdot)$. The friction coefficients of rotational motion and horizontal motion are set as c_1 and c_2 , respectively. Additional dampers in y and x direction are considered shown in Fig. 2. Two linear dampers, i.e. Damper 1 and Damper 2, are assembled in horizontal and vertical vibration directions, which are much simpler to implement than some complex nonlinear control methods using feedback signals. Damper 1 only affects the vibration in vertical direction and its effect is the same as the case assembling it in the center of the platform. Damper 2 is assembled in the bottom layer to generate nonlinear damping. All the structural parameters are listed in Table 1 in Appendix.

Fig. 2 Flat view of n -layer SLS platform model with two dampers

The absolute motion of the isolation object M_1 is denoted by y , the base excitation is z , ϕ is the rotation motion of each connecting rod, and x is the horizontal motion of each connecting joint. The positive direction of the motion y is upward as shown in Fig. 2. All the motion variables of the platform are listed in Table 2 in Appendix. The relative motion between the isolation object and the

base is $\hat{y}=y-z$. The geometrical relations among rotation motion φ , horizontal motion x and relative motion $\hat{y}$ are shown in Fig. 3.

Fig. 3 The motion of one connecting rod in the first layer and the geometrical relations among φ , x and $\hat{y}$

The horizontal motion x and rotation motion of each rod φ are shown in Fig. 3. From Fig.2 and Fig.3, it can be seen that the n-layer SLS platform is one-degree-of-freedom structure because the rotational motion φ and horizontal motion x can be expressed by relative motion $\hat{y}$. The nonlinear geometrical relations among φ , x and $\hat{y}$ can be referred to Ref. [28], i.e.,

$$x = 2l \cos \theta - 2\sqrt{l^2 - \left(l \sin \theta + \frac{\hat{y}}{2n}\right)^2}$$

Thus this system is 1-degree-of-freedom dynamic system. Because the excitation is in vertical direction, to show the isolation effect of the system, the absolute motion of the isolation object y is chosen as the generalized coordinate. Considering the mass of connecting rods, the equation of motion of the system can be obtained by the Lagrange principle.

3. Modeling of the n-layer SLS platform

The kinetic energy of connecting rods is made up of translational energy and rotational energy. The connecting rods are supposed to be made from uniform materials with the center of mass

located at the geometric center. The rotational velocity around the mass center is denoted by $\dot{\varphi}$; the translational velocity of any connecting rod contains horizontal and vertical velocity which is given by $\sqrt{\left(\frac{\dot{x}}{2}\right)^2 + \left(\frac{2i-1}{2n}\dot{y}\right)^2}$ ($i=1, 2, \dots, n$). In each layer, there are two connecting rods, thus the sum number of the rods using in this platform is $2n$. Therefore, the kinetic energy T of the system can be written as

$$T = \frac{1}{2}M_1\dot{y}^2 + 2\sum_{i=1}^n \frac{1}{2}M_2 \left[\left(\frac{\dot{x}}{2}\right)^2 + \left(\frac{2i-1}{2n}\dot{y}\right)^2 \right] + 2n \cdot \frac{1}{2}J_0\dot{\varphi}^2 \quad (1)$$

Then, the kinetic energy of the system can be written as

$$T = \frac{1}{2}M_1\dot{y}^2 + nM_2\left(\frac{\dot{x}}{2}\right)^2 + M_2\sum_{i=1}^n \left(\frac{2i-1}{2n}\dot{y}\right)^2 + nJ_0\dot{\varphi}^2 \quad (2)$$

where J_0 is the moment of inertia of the connecting rod, M_1 is the mass of isolation object and M_2 is the mass of connecting rod. The potential energy V of the system is the elastic energy of the spring in the first layer in the x direction, which can be expressed as

$$V = \frac{1}{2}k_l x^2 \quad (3)$$

where k_l is the stiffness of spring.

The friction forces of rotational motion φ of each connecting joint and horizontal motion x of left connecting joint in bottom and top layer are represented as $F_{f\varphi}$ and F_{fx} . Considering more generalized situation, the friction model of horizontal tracks and rotational joints are Coulomb friction, so the friction forces are as

$$F_{f\varphi} = \mu N_\varphi \operatorname{sgn}(\dot{\varphi}) = c_1 \operatorname{sgn}(\dot{\varphi}) \quad (4)$$

$$F_{fx} = \mu N_x \operatorname{sgn}(\dot{x}) = c_2 \operatorname{sgn}(\dot{x}) \quad (5)$$

where μ is the friction resistance coefficient, N_φ the tightening force (or positive pressure) of joints and N_x the positive pressure, c_1 and c_2 are friction forces in φ and x direction, respectively. The

value of the friction resistance coefficient μ is dependent on the material and contact surface which can be referred to engineering handbook of materials. The damping forces of Damper 1 and Damper 2 are defined as F_{dy} and F_{dx} which are linear functions of relative vertical velocity $\dot{\hat{y}}$ and horizontal velocity $\dot{\hat{x}}$ as

$$F_{dy} = \alpha \dot{\hat{y}} \quad (6)$$

$$F_{dx} = \beta \dot{\hat{x}} \quad (7)$$

So, the generalized force Q is defined as

$$Q = \sum \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial y} = -2F_{fx} \cdot \frac{\partial x}{\partial y} - 6nF_{f\varphi} \cdot \frac{\partial \varphi}{\partial y} - F_{dy} \cdot \frac{\partial \hat{y}}{\partial y} - F_{dx} \cdot \frac{\partial \hat{x}}{\partial y} \quad (8)$$

Therefore, the Lagrange principle is written as

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = -6nc_1 \operatorname{sgn}(\dot{\varphi}) \frac{d\varphi}{dy} - 2c_2 \operatorname{sgn}(\dot{x}) \frac{dx}{dy} - \alpha \dot{\hat{y}} - \beta \dot{\hat{x}} \frac{dx}{dy} \quad (9)$$

From (9), it can be seen that the modeling process is effective even the more complex friction model is considered because Lagrange principle has its generalization in physical modeling. Substituting kinetic energy (2), potential energy (3) and transport motions into (9), the dynamic equation of the n-layer SLS platform can be obtained as

$$M_1 \ddot{y} + f_M \ddot{\hat{y}} + k_l x \frac{dx}{dy} + 6nc_1 \operatorname{sgn}(\dot{\varphi}) \frac{d\varphi}{dy} + 2c_2 \operatorname{sgn}(\dot{x}) \frac{dx}{dy} + \alpha \dot{\hat{y}} + \beta \dot{\hat{x}} \frac{dx}{dy} = 0 \quad (10)$$

where $\dot{\varphi} = (d\varphi/d\hat{y}) \cdot \dot{\hat{y}}$, $\dot{x} = (dx/d\hat{y}) \cdot \dot{\hat{y}}$, $f_M \ddot{\hat{y}}$ is equivalent mass of connecting rods of the platform. The expression of f_M is as

$$f_M = nM_2 + \frac{J_0 n}{2(4l^2 n^2 \cos^2 \theta - 4ln \sin \theta \hat{y} - \hat{y}^2)} + \frac{M_2 (2ln \sin \theta + \hat{y}) [8l^3 n^3 \cos^2 \theta \sin \theta + 2l^2 n^2 (-1 + 3 \cos 2\theta) \hat{y} - (6ln \sin \theta + \hat{y}) \hat{y}^2]}{2n(4l^2 n^2 \cos^2 \theta - 4ln \sin \theta \hat{y} - \hat{y}^2)^2} \quad (11)$$

Considering that the motion of the isolation platform is within about several centimeters and

the coefficients in the inertia force are much smaller than stiffness or friction coefficients, in order to simplify the dynamic equation, the first order Taylor series expansion is used for the equivalent mass. Expanding f_M , the first order Taylor series of f_M is defined as M_f , which is

$$M_f = \frac{M_2 \tan^2 \theta}{2n} + \frac{J_0 \sec^2 \theta}{8l^2 n} + nM_2 \quad (12)$$

where J_0 is as

$$J_0 = \frac{1}{12} M_2 (2l)^2 = \frac{M_2 l^2}{3} \quad (13)$$

Substituting M_f , J_0 , x and φ into (10), because $d\varphi/d\hat{y} > 0$ and $dx/d\hat{y} > 0$ it can be obtained that

$$\begin{aligned} M_1 \ddot{y} + M_2 \left(n + \frac{\tan^2 \theta}{2n} + \frac{\sec^2 \theta}{24n} \right) \ddot{\hat{y}} + \frac{k_l \left(l \sin \theta + \frac{\hat{y}}{2n} \right) \left(2l \cos \theta - 2\sqrt{l^2 - \left(l \sin \theta + \frac{\hat{y}}{2n} \right)^2} \right)}{n \sqrt{l^2 - \left(l \sin \theta + \frac{\hat{y}}{2n} \right)^2}} \\ + \alpha \dot{\hat{y}} + \frac{\beta \left(l \sin \theta + \frac{\hat{y}}{2n} \right)^2 \dot{\hat{y}}}{n^2 \left[l^2 - \left(l \sin \theta + \frac{\hat{y}}{2n} \right)^2 \right]} + \frac{[6n^2 c_1 + 2c_2 (2ln \sin \theta + \hat{y})] \operatorname{sgn}(\dot{\hat{y}})}{n \sqrt{4l^2 n^2 - (2ln \sin \theta + \hat{y})^2}} = 0 \end{aligned} \quad (14)$$

Note that all nonlinear terms in (14) can be simplified by Taylor series expansion. If each term is expanded by the third order Taylor series, and also use the relation between relative motion and absolute motion (i.e., $y = \hat{y} + z$) into (14), a simplified dynamic equation can be obtained as

$$F_M \ddot{\hat{y}} + F_k + F_{c1} + F_{c2} = -M_1 \ddot{z} \quad (15)$$

where F_M , F_k , F_{c1} and F_{c2} stand for equivalent mass, stiffness property, friction force and damping properties of the SLS platform. The expressions of F_M , F_k , F_{c1} and F_{c2} are as follows

$$F_M = M_1 + M_f = M_1 + M_2 \left(n + \frac{\tan^2 \theta}{2n} + \frac{\sec^2 \theta}{24n} \right) \quad (16)$$

$$F_k = f_k \hat{y} = (k_1 + k_2 \hat{y} + k_3 \hat{y}^2) \hat{y} \\ = \left[\frac{k_l \tan^2 \theta}{n^2} + \frac{3k_l \tan \theta \sec^3 \theta}{4ln^3} \hat{y} + \frac{k_l (3 - 2 \cos 2\theta) \sec^6 \theta}{8l^2 n^4} \hat{y}^2 \right] \hat{y} \quad (17)$$

$$F_{c1} = f_{c1} \operatorname{sgn}(\dot{\hat{y}}) = (\xi_1 + \xi_2 \hat{y} + \xi_3 \hat{y}^2) \operatorname{sgn}(\dot{\hat{y}}) \\ = \left[\frac{3nc_1 \sec \theta + 2lc_2 \tan \theta}{ln} + \frac{3nc_1 \sec^2 \theta \tan \theta + 2c_2 l \sec^3 \theta}{2l^2 n^2} \hat{y} \right. \\ \left. + \frac{3nc_1 \sec^5 \theta (2 - \cos 2\theta) + 6c_2 l \sec^4 \theta \tan \theta}{8l^3 n^3} \hat{y}^2 \right] \operatorname{sgn}(\dot{\hat{y}}) \quad (18)$$

$$F_{c2} = f_{c2} \dot{\hat{y}} = (\xi_4 + \xi_5 \hat{y} + \xi_6 \hat{y}^2) \dot{\hat{y}} \\ = \left(\alpha + \frac{\beta \tan^2 \theta}{n^2} + \frac{\beta \sec^3 \theta \tan \theta}{ln^3} \hat{y} + \frac{\beta \sec^6 \theta (5 - 3 \cos 2\theta)}{4l^2 n^4} \hat{y}^2 \right) \dot{\hat{y}} \quad (19)$$

where f_k equivalent stiffness coefficient, f_{c1} equivalent friction force, f_{c2} equivalent damping coefficient, k_1, k_2, k_3 the linear and nonlinear stiffness coefficients; ξ_1, ξ_2, ξ_3 the friction coefficients; and ξ_4, ξ_5, ξ_6 the linear and nonlinear damping coefficients.

Fig. 4 Comparison of stiffness and damping properties between original equation and Taylor expansion

Fig 4 shows the comparison of stiffness property and damping coefficients in (14) and (17-19) as $n=2$, $\theta=\pi/4$, $l=0.5$, $c_1=1$, $c_2=2$, $\alpha=2$, $\beta=4$. It can be seen that for reasonable values of structural parameters, the Taylor expansion of third order has sufficient accuracy to describe the stiffness and damping properties of the original dynamics equation. The nonlinearity and value of the original equation are reflected by the third expansion Taylor series. Because of the Taylor expansion, the theoretical results of the SLS system can be obtained by nonlinear theory for differential equation with high accuracy. Therefore, the dynamic equation can be written as

$$F_M \ddot{y} + k_1 \hat{y} + k_2 \hat{y}^2 + k_3 \hat{y}^3 + \left(\xi_1 + \xi_2 \hat{y} + \xi_3 \hat{y}^2 \right) \text{sgn}(\dot{y}) + \left(\xi_4 + \xi_5 \hat{y} + \xi_6 \hat{y}^2 \right) \dot{y} = -M_1 \ddot{z} \quad (20)$$

where z is the base excitation, which can be periodic excitation, impact load, multi-periodic excitation and random excitation. The mathematical model in this paper is different from Ref. [28] because the dynamic equation (20) contains the effect of the mass and inertia of connecting rods, the equivalent friction force due to Coulomb friction in links, and the overall equivalent damping property of linear dampers.

4. Equivalent mass, stiffness and damping

From (17-19), the values of the equivalent mass, stiffness and damping coefficients are affected by the structural parameters k_l , θ , n , l , c_1 and c_2 . When materials and springs in the platform system are chosen, the values of k_l , c_1 , c_2 are constant. Compared with these parameters, structural parameters θ , n and l can be adjusted more easily. Therefore, the coefficients of equivalent mass F_M , equivalent stiffness F_k , friction force F_{c1} and equivalent damping F_{c2} can be easily adjusted for better response by choosing different values for θ , n and l .

The values of structural parameters are according engineering practice. For instance, in the

following section, in order to show the equivalent mass, stiffness and damping properties and friction force of the SLS platform and its isolation effect for excitation clearly, if there is no description of the values of structural parameters, the default values are as $k_l=1000$, $c_1=0.1$, $c_2=0.2$, $n=2$, $\theta=\pi/4$, $l=0.5$, $\alpha=0$, $\beta=0$. For the cases in application if the SLS platform as an isolator and seat suspension system, the values of structural parameters are set according to the limitation of size and better isolation effectiveness.

4.1 Equivalent mass F_M

From (14), it shows that the value of equivalent mass of the platform is function of θ and n . The mass of connecting rods M_2 can be designed to much smaller than the mass of platform and isolation object M_1 in order to reduce the potential influence. If the value of M_1 and M_2 are fixed as $M_1=10\text{kg}$ and $M_2=0.1\text{kg}$, the equivalent mass for different θ and n is shown in Fig. 5.

Fig. 5 The value of equivalent mass F_M for different θ and n . (a) For different assembly angle θ , (b) for different number of layer n .

From Fig. 5 (a), the increase of θ induces the increase of equivalent mass. From Fig. 5 (b), for the cases $\theta=\pi/6$, $\theta=\pi/4$ and $\theta=\pi/3$, the equivalent mass increases as n increases. But for the case $\theta=7\pi/18$, the equivalent mass for $n=2$ is smaller than $n=1$. Then, the equivalent mass monotonously

increases with the value of n larger than 3. As a brief summary, in the range of $n \in [1, 5]$ and $\theta \in [0.2, 1.2]$ (rad), the equivalent mass F_M can be approximately considered as a constant value $M_1 + nM_2$. Therefore, structural parameters have very small effect on the equivalent mass of the platform, and the mass of connecting rods can be ignored for heavy isolation objects in practice.

4.2 Equivalent stiffness property

A. Linear stiffness coefficient k_1

Eq.(19) shows the equivalent stiffness property F_k of the platform. The coefficients k_1 , k_2 and k_3 affect the resonant frequency, vibration offset and nonlinearity of the system. Since the dominant resonant frequency of a vibration system is mainly determined by the value of linear stiffness, it can be obtained for the platform as

$$\omega_r = \sqrt{\frac{k_1}{F_M}} = \frac{\tan \theta}{n} \sqrt{\frac{k_l}{F_M}} \approx \frac{\tan \theta}{n} \sqrt{\frac{k_l}{M_1 + M_2 \left(n + \frac{\tan^2 \theta}{2n} + \frac{\sec^2 \theta}{24n} \right)}} \quad (21)$$

With $k_l=1000$, $M_1=10\text{kg}$, $M_2=0.1\text{kg}$ and different values of θ and n , the linear stiffness coefficient k_1 is shown in Fig. 6 and the resonant frequency ω_r is shown in Fig. 7.

Fig. 6 Linear stiffness coefficient k_1 (a) for different assembly angel θ , (b) for different number of layer n .

Fig. 7 Resonant frequency ω_r for (a) different assembly angle θ , (b) different layer number n

From Fig. 6, as the value of θ increases or n decreases, the value of the linear stiffness coefficient k_1 increases. For a mass-spring-damper vibration system, as the spring is chosen, the stiffness cannot be changed. However, for the n -layer SLS platform, although the spring is chosen with $k_f=1000$, the linear stiffness coefficient k_1 can still change from a very small value (near to zero) to about 2000 by adjusting θ and n .

From Fig. 7, it can be seen that for vibration isolation, the resonant frequency of the platform can be reduced by decreasing the assembly angle θ or increasing the layer n for better isolation effect without active control device. In [17]-[18], it is shown that most dangerous frequency range of a seated human body is between 25 and 56.5 rad/s. Note that an isolation system is usually effective for vibration frequency ω larger than $\sqrt{2}\omega_r$. Therefore, to apply the SLS platform to vehicle seat suspension, the resonant frequency of the SLS platform can be designed as smaller than $25/\sqrt{2}=17.68\text{rad/s}$. It is very easy to achieve this performance.

B. Equivalent stiffness force F_k

The equivalent stiffness coefficient f_k in (17) determines the stiffness of the system. For different structural parameters, the stiffness force F_k is shown in Fig. 8, where the motion $\hat{y}$ is chosen in range $[-0.2, 0.2]$ (m) which is sufficient for practical application.

Fig. 8 Equivalent stiffness force F_k for (a) different n as $\theta=\pi/4$ and $l=0.5$; (b) different θ as $n=2$ and $l=0.5$; (c) different l as $\theta=\pi/4$ and $n=2$.

Fig. 8 shows that the equivalent stiffness is always nonlinear with different values of structural parameters, and tends to be stronger nonlinearity with smaller n , larger θ , and shorter l . However, bigger n , smaller θ , and longer l lead to smaller equivalent stiffness, which is helpful for vibration isolation. When the relative motion $\hat{y}$ is smaller, the rod length l has relatively smaller effect on equivalent stiffness. Because the linear and nonlinear stiffness coefficients are dependent on the structural parameters, the stiffness property can be adjusted to achieve desired resonant frequency and nonlinearity for different applications.

4.3 Equivalent friction force F_{c1}

The equivalent friction force F_{c1} is defined in (18), where the three friction coefficients ξ_1 , ξ_2 and ξ_3 are dependent on structural parameters θ , n and l . The parameters c_1 and c_2 are constant since they are determined by the material of connecting rods and joints. The values of friction force

coefficients ξ_1 , ξ_2 and ξ_3 are discussed for different θ , n and l to illustrate the friction force of the platform. Considering general smooth metal materials such as steel or aluminum, the friction coefficients set as for example $c_1=0.1$, $c_2=0.2$, the values of ξ_1 , ξ_2 and ξ_3 are shown in Fig. 9.

Fig. 9 Equivalent friction force for different θ , n and l . Coefficient ξ_1 (a) for different θ and (b) for different l ; Coefficient ξ_2 (c) for different θ and (d) for different l ; coefficient ξ_3 (e) for different θ and (f) for different l .

From Fig. 9, the values of ξ_1 , ξ_2 and ξ_3 increase as θ increases or n and l decreases, and they share very similar increasing trend. However, the changing rates of the three coefficients are different. For example, for $n=2$, as θ increases from 0.2 to 1.2 rad, the friction force coefficient ξ_1 increases to about 2 while nonlinear friction force coefficient ξ_2 can reach 10 and ξ_3 can reach more than 50. From (18), it can be seen that when the assembly angle θ is in range of $[0.2, 1.2]$ (rad), the number of layer n is in $[1, 4]$ and l is in $[0.2, 1]$ (m), the value of ξ_1 is from 0.5 to 3; the value of ξ_2

is from 0.033 to 10; the value of ξ_3 is from 0.012 to 50.

From (18), the friction force F_{c1} is a piecewise function of velocity $\dot{y}$ but a nonlinear function of motion y . For considering Coulomb friction in links, the friction force F_{c1} has effect on stiffness of the SLS platform although it is the piecewise function of velocity. The value of friction force F_{c1} is shown in Fig. 10.

Fig. 10 Friction force F_{c1} for (a) different n ; (b) different θ ; (c) different l

From Fig. 10, for larger n , l or smaller θ , the friction force F_{c1} is smaller and closer to constant while for decreasing n and l or increasing θ , F_{c1} is nonlinear function of motion y and its direction is determined by the direction of the relative velocity $\dot{y}$. It shows that the effect of the friction force induced by Coulomb friction in the links is totally different from the viscous friction considered in Ref. [28]. Although the friction force of the system is energy consumption induced by the contact surface and its direction is determined by the direction of the relative velocity, the value of the

equivalent friction force is dependent on the relative motion the system. For larger friction force F_{c1} , it is more difficult for the platform turn into the vibration state which means that the platform system is too rigid for excitation. Therefore, to reduce the value of friction force F_{c1} , one method is to smooth the surface of links to reduce the value of c_1 and c_2 and the other one is to increase the value of n and l or to decrease the value of θ .

4.4 Equivalent damping property F_{c2}

Damping property has large influence on the amplitude of vibration peak. In this SLS platform, dampers can also be applied in different directions to tune the equivalent damping of the system. Two dampers in the vertical direction and horizontal tracks are linear functions of $\dot{y}$ and $\dot{x}$ while the equivalent damping property is nonlinear function in (15). Fig. 11 is the equivalent damping coefficient f_{c2} for different structural parameters as $\alpha=1$ and $\beta=2$.

Fig. 11 Equivalent damping coefficient f_{c2} for (a) different n as $\theta=\pi/4$ and $l=0.5$; (b) different θ as $n=2$ and $l=0.5$; (c) different l as $\theta=\pi/4$ and $n=2$.

Fig. 11 is the equivalent damping coefficient of the system for different structural parameters. It can be seen that smaller n and l , larger θ can increase the value of damping. The equivalent damping coefficient is not symmetrical and increases quickly with larger positive motion $\hat{y}$. For isolating periodic excitation, increasing the damping can reduce the value of resonant peak. However, the damping coefficient should be smaller for high frequency vibration. Fig. 11 shows that a larger motion $\hat{y}$, corresponding to low frequency vibration, produces a larger damping coefficient, while a smaller $\hat{y}$, corresponding to high frequency vibration, a smaller damping. This demonstrates the advantage of this nonlinear damping property of the platform. Although it is shown that larger θ and smaller l increase the value of nonlinear damping coefficients more quickly than the linear damping coefficient, reducing the value of l is the best choice for improving isolation performance of the system, because a smaller l could induce larger nonlinear damping coefficient while it has little effect on the linear damping coefficient.

From the expressions of damping coefficients in (19), α only influences the linear damping coefficient ξ_4 while the value of β can change the values of the three damping coefficients ξ_4 , ξ_5 and ξ_6 . Fig. 14 is the values of damping coefficients for different β .

Fig. 12 Damping coefficient (a) ξ_4 ; (b) ξ_5 ; (c) ξ_6 , for different strength β in Damper 2 as $\alpha=0$, $\theta=\pi/4$ and $l=0.5$.

From Fig. 12, Damper2 in x direction changes the three damping coefficient while the changing rates are different. If increasing β which is linear damper in x direction, the value of nonlinear damping coefficient ξ_6 raises more quickly than damping coefficient ξ_4 and ξ_5 . This would be beneficial in vibration control, since it is known that smaller linear damping coefficient and larger nonlinear damping coefficients can improve vibration isolation performance [20].

It can be concluded that the changing rates of these three damping coefficients vary for different structural parameters, similar to the stiffness property of the system. Reducing n , l or increasing θ , β in the SLS system can make the nonlinear damping coefficients to change more quickly than linear damping coefficient which is helpful for the improvement of isolation effectiveness. The equivalent damping F_{c2} is a linear function of velocity $\dot{\hat{y}}$ but a nonlinear function of motion $\hat{y}$.

5. Displacement transmissibility

The dynamic equation (20) of the SLS platform is a piecewise function as Coulomb friction in links is considered. Also, because of the squared terms and cubic terms in (20), the zero shift of vibration cannot be ignored. So a basic Perturbation Method (PM) and Average Method (AM) are combined to solve this dynamic equation to obtain an accuracy description. Firstly, the dimensionless transform is given by

$$\begin{aligned} \omega_0 &= \sqrt{\frac{k_l}{F_M}}, t' = \omega_0 t, \gamma_1 = \frac{k_1}{k_l}, \gamma_2 = \frac{k_2}{k_l}, \gamma_3 = \frac{k_3}{k_l}, \zeta_1 = \frac{\xi_1}{k_l}, \zeta_2 = \frac{\xi_2}{k_l}, \zeta_3 = \frac{\xi_3}{k_l}, \\ \zeta_4 &= \frac{\xi_4}{\sqrt{F_M k_l}}, \zeta_5 = \frac{\xi_5}{\sqrt{F_M k_l}}, \zeta_6 = \frac{\xi_6}{\sqrt{F_M k_l}}, \Omega = \frac{\omega}{\omega_0} \end{aligned} \quad (22)$$

Then, the dimensionless dynamic equation of the SLS platform is

$$\begin{aligned} \hat{y}'' + \gamma_1 \hat{y} + \gamma_2 \hat{y}^2 + \gamma_3 \hat{y}^3 + (\zeta_1 + \zeta_2 \hat{y} + \zeta_3 \hat{y}^2) \text{sgn}(\hat{y}') \\ + (\zeta_4 + \zeta_5 \hat{y} + \zeta_6 \hat{y}^2) \hat{y}' = \frac{M_1}{F_M} z_0 \Omega^2 \cos(\Omega t') \end{aligned} \quad (23)$$

where $(\cdot)' = d(\cdot)/dt'$. Utilizing PM process, the periodic form of solution of the relative motion $\hat{y}$ can be written as

$$\hat{y} = y_0 + y_1(t) = y_0 + a \cos(\Omega t' + \phi) = y_0 + a \cos(\psi) \quad (24)$$

Substituting the periodic solution (24) into (23), the expression of (23) is

$$\begin{aligned} y_1'' + \gamma_1(y_0 + y_1) + \gamma_2(y_0 + y_1)^2 + \gamma_3(y_0 + y_1)^3 + [\zeta_1 + \zeta_2(y_0 + y_1) + \zeta_3(y_0 + y_1)^2] \text{sgn}(y_1') \\ + [\zeta_4 + \zeta_5(y_0 + y_1) + \zeta_6(y_0 + y_1)^2] y_1' = \frac{M_1 z_0 \Omega^2}{F_M} \cos(\Omega t') \end{aligned} \quad (25)$$

And then introducing a tuning parameters σ to reflect the difference of natural frequency and excitation frequency as $\gamma_1 = \Omega^2 - \sigma$. Substituting solution (24) in to (25) and after linearization of (25), the equations of y_0 and $y(t)$ are as

$$\gamma_1 y_0 + \gamma_2 y_0^2 + \frac{1}{2} \gamma_2 a^2 + \gamma_3 y_0^3 + \frac{3}{2} \gamma_3 a^2 - \left(\zeta_1 + \zeta_2 y_0 + \zeta_3 y_0^2 + \frac{\zeta_3 a^2}{2} \right) \frac{\sqrt{a^2}}{a} = 0 \quad (26)$$

and

$$\begin{aligned} y_1'' + \Omega^2 y_1 = \sigma y_1 - 2\gamma_2 y_0 y_1 - 3\gamma_3 y_0^2 y_1 - \gamma_3 y_1^3 \\ - (\zeta_1 + \zeta_2 y_1 + 2\zeta_3 y_0 y_1) \text{sgn}(y_1') - (\zeta_4 + \zeta_5 y_1 + 2\zeta_6 y_0 y_1) y_1' + \frac{M_1 z_0 \Omega^2}{F_M} \cos(\psi - \phi) \end{aligned} \quad (27)$$

For piecewise friction in (27), utilizing AM, the amplitude and phase of the first order vibration are as

$$\begin{aligned} \dot{a} = & -\frac{1}{2\pi\Omega} \int_0^{2\pi} \left[(\sigma - 2\gamma_2 y_0 - 3\gamma_3 y_0^2 - \gamma_3 a^2 \cos^2 \psi) a \cos \psi + \frac{M_1 z_0 \Omega^2}{F_M} \cos(\psi - \phi) \right] \sin \psi d\psi \\ & - \frac{1}{2\pi\Omega} \int_0^{2\pi} (-\zeta_4 - \zeta_5 a \cos \psi - 2\zeta_6 a^2 \cos^2 \psi) (-a\Omega \sin \psi) \sin \psi d\psi \\ & - \frac{1}{2\pi\Omega} \int_0^\pi (-1) (-\zeta_1 - \zeta_2 a \cos \psi - 2\zeta_3 y_0 a \cos \psi) \sin \psi d\psi \\ & - \frac{1}{2\pi\Omega} \int_\pi^{2\pi} (-\zeta_1 - \zeta_2 a \cos \psi - 2\zeta_3 y_0 a \cos \psi) \sin \psi d\psi \end{aligned} \quad (28)$$

$$\begin{aligned}
a\dot{\phi} = & \frac{1}{2\pi\Omega} \int_0^{2\pi} \left[(\sigma - 2\gamma_2 y_0 - 3\gamma_3 y_0^2 - \gamma_3 a^2 \cos^2 \psi) a \cos \psi + \frac{M_1 z_0 \Omega^2}{F_M} \cos(\psi - \phi) \right] \cos \psi d\psi \\
& + \frac{1}{2\pi\Omega} \int_0^{2\pi} (-\zeta_4 - \zeta_5 a \cos \psi - 2\zeta_6 a^2 \cos^2 \psi) (-a\Omega \sin \psi) \cos \psi d\psi \\
& + \frac{1}{2\pi\Omega} \int_0^\pi (-1) (-\zeta_1 - \zeta_2 a \cos \psi - 2\zeta_3 y_0 a \cos \psi) \cos \psi d\psi \\
& + \frac{1}{2\pi\Omega} \int_\pi^{2\pi} (-\zeta_1 - \zeta_2 a \cos \psi - 2\zeta_3 y_0 a \cos \psi) \cos \psi d\psi
\end{aligned} \tag{29}$$

Therefore, from (28) and (29) and the condition $\sin^2 \phi + \cos^2 \phi = 1$, amplitude-frequency equation of y_1 can be obtained as

$$\left[\frac{F_M (16\zeta_1 + 4a\pi\zeta_4\Omega + a^3\pi\zeta_6\Omega)}{4M_1\pi z_0\Omega^2} \right]^2 + \left[\frac{aF_M (8y_0\gamma_2 + 3\gamma_3 a^2 + 12\gamma_3 y_0 - 4\sigma)}{4z_0 M_1 \Omega^2} \right]^2 = 1 \tag{30}$$

From (26) and (30), the zero shift y_0 and amplitude a of the relative motion can be obtained. So the displacement transmissibility T_d is defined as

$$T_d = \frac{\sqrt{a^2 + 2az_0 \cos \phi + z_0^2}}{z_0} = \sqrt{\frac{a^2}{z_0^2} + \frac{a^2 F_M (8y_0\gamma_2 + 3\gamma_3 a^2 + 12\gamma_3 y_0 - 4\sigma)}{2z_0^2 M_1 \Omega^2}} + 1 \tag{31}$$

As an example, for $M_1=10$, $M_2=0.1$, $n=2$, $\theta=\pi/4$, $l=0.5$, $k_l=1000$, $c_1=0.1$, $c_2=0.2$, $\alpha=0$, $\beta=0$, the coefficients in (20) are as $F_M=10.23$, $k_1=250$, $k_2=530.3$, $k_3=750$, $\xi_1=1.05$, $\xi_2=0.883$, $\xi_3=1.15$, $\xi_4=\xi_5=\xi_6=0$, so the coefficients in dimensionless equation (25) are as $\gamma_1=0.25$, $\gamma_2=0.5303$, $\gamma_3=0.75$, $\zeta_1=0.00105$, $\zeta_2=0.000883$, $\zeta_3=0.00115$ and $\zeta_4=\zeta_5=\zeta_6=0$. The displacement transmissibility T_d for periodic base excitation is shown in Fig. 13.

Fig. 13 Displacement transmissibility T_d for periodic base excitation Ω from theoretical analysis (Solid Lines) and numerical simulation (Dots)

Fig. 13 is the displacement transmissibility T_d for different excitation frequency Ω . The solid line obtained by theoretical analysis is close to the dots by numerical simulation, so the theoretical solution by PM and AM can describe the response of the vibration system with high accuracy. From Fig. 13, it can be seen that there exists rigid body motion phenomenon. In the low-frequency band, the displacement transmissibility always equals to 1. The motion SLS plat is rigid body for low-frequency excitation because the friction of the links in the system is in the form of Coulomb friction. When the excitation force is less than the static friction of the links, the motion of the SLS platform cannot in vibration motion.

For the case in Fig. 13, it can be seen that the inflection point occurs as $\Omega \approx 0.25$. From the dimensionless dynamic equation (23), the value of frequency Ω_c at the inflection point can be estimated by the comparison of friction force and excitation force. The amplitude of vibration of the relative motion equals to zero as $a=0$ for low-frequency excitation. So the critical value of frequency Ω_c can be obtained by substituting $a=0$ into (26) as

$$\left(\frac{4F_M \zeta_1}{M_1 \pi z_0 \Omega_c^2} \right)^2 = 1 \quad (32)$$

So, the value of Ω_c is as

$$\Omega_c = \sqrt{\frac{4F_M \zeta_1}{\pi M_1 z_0}} = \sqrt{\frac{4F_M \xi_1}{\pi M_1 z_0 k_l}} = \sqrt{\frac{4F_M (3nc_1 \sec \theta + 2lc_2 \tan \theta)}{\pi M_1 z_0 k_l n}} \quad (33)$$

From (33), when $\Omega \in [0, \Omega_c]$, the SLS platform is rigid motion while it is in vibration when $\Omega \in [\Omega_c, \infty]$. For the case shown in Fig. 13, the value of $\Omega_c = 0.261$ which is the accuracy value of the inflection point in Fig. 13. It can be seen that for larger friction force c_1 and c_2 , the value of Ω_c is increasing. Although the friction of links in the system has damping effect, it is dangerous for the occurrence of self-locking phenomenon for a vibration system in rigid motion especially for high-frequency excitation. So, to avoid the rigid motion of the SLS platform, the best way is to reduce the value of Coulomb friction in links in this system.

6. Performance comparison with existing isolators

The equivalent stiffness property F_k , friction force F_{c1} and damping property F_{c2} of the SLS platform are shown in (17-19), and they are all nonlinear functions of displacement. In the analysis above, it is indicated that all these values of coefficients $k_1, k_2, k_3, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ and ξ_6 can be adjusted by different structural parameters θ, n and l easily. Although it is shown that these values are not independent, the stiffness force F_k can achieve different requirements for vibration isolation or suppression. The friction force F_{c1} in (18) can be reduced not only by processing of materials, but also adjusting the changeable structural parameters n, θ and l . Very similar cases can be seen for the equivalent damping property of the platform F_{c2} in (19), which is not only a function of the velocity but also a nonlinear function of displacement and can also be designed easily for different vibration control objectives.

Therefore, by adjusting structural parameters θ , n and l , and designing damping coefficient α and β , the stiffness and damping properties of the system can be freely tuned for much better vibration isolation performance.

6.1 Performance comparison with quasi-zero-stiffness vibration isolator (QZS-VI)

As plenty of pervious researches shown, the quasi-zero-stiffness vibration isolators (QZS-VIs) can realize excellent isolation effect [5-13], due to a very low resonant frequency that can be achieved. From [5], the dynamic equation of a QZS-VI can be modeled as a Duffing oscillator with base excitation. The stiffness property of a QZS-VI is as $k_{q1}(\cdot) + k_{q2}(\cdot)^3$ where k_{q1} is linear stiffness and k_{q2} is the cubic stiffness. Due to pre-deformation of horizontal springs, the linear stiffness k_{q1} can be designed very small. In [5], the values k_{q1} and k_{q2} of QZS-VI are designed as $k_{q1} = k_l[1 - 2(1 - \lambda)/\lambda]$ and $k_{q2} = k_l(1 - \lambda^2)/\lambda^3$ where k_l is the stiffness of springs and λ is the ratio between spring length with pre-deformation and original length. When the pre-deformation of horizontal springs is increased, the value of λ is smaller, while changing the value of λ results in adjustment of the stiffness property of the QZS-VI system. In this section, it will be shown that the stiffness property of the SLS platform can easily be designed as good as the stiffness of QZS-VI system by simply tuning structural parameters. The structural parameters of the QZS-VI are shown in Table 3 in Appendix.

To compare the vibration isolation performance between the SLS platform and QZS-VIs, consider the parameter values of the QZS-VIs by using the same spring as $k_l = 1000$. The mass of isolation object is given as $M_1 = 10\text{kg}$. The dynamic equation of the QZS-VIs is given with these parameters as

$$M_1 \ddot{\hat{y}} + k_{q1} \hat{y} + k_{q2} \hat{y}^3 + c \dot{\hat{y}} = -M_1 \ddot{z} \quad (34)$$

where $z=z_0\cos\alpha$, $k_{q1}=k_l[1-2(1-\lambda)/\lambda]$, $k_{q2}=k_l(1-\lambda^2)/\lambda^3$, $k_l=1000$ and $c=5$. To compared the isolation effect such as value of resonant frequency and resonant peak, introducing the dimensionless transform for (34) similar as (22), the dimensionless parameters are as

$$\omega_0 = \sqrt{\frac{k_l}{M_1}}, t' = \omega_0 t, \gamma_{q1} = \frac{k_{q1}}{k_l}, \gamma_{q2} = \frac{k_{q2}}{k_l}, \zeta = \frac{c}{\sqrt{M_1 k_l}}, \Omega = \frac{\omega}{\omega_0} \quad (35)$$

For (35), the dimensionless dynamic equation of QZS-VI is as

$$\hat{y}'' + \gamma_{q1}\hat{y} + \gamma_{q2}\hat{y}^3 + \zeta\hat{y}' = \Omega^2 z_0 \cos(\Omega t') \quad (36)$$

For comparison, a typical mass-spring-damper vibration isolator (MSD-VI) of similar parameter setting is

$$M_1 \ddot{\hat{y}} + k_l \hat{y} + c \dot{\hat{y}} = -M_1 \ddot{z} \quad (37)$$

Utilizing the same dimensionless transform as (37), the dimensionless dynamic equation of MSD-VI is as

$$\hat{y}'' + \hat{y} + \zeta \hat{y}' = \Omega^2 z_0 \cos(\Omega t') \quad (38)$$

For the three isolation system, same main components such as spring and passive damper are used as $k_l=1000$ and $c=5$. For QZS-VI system, to improve isolation effect, the structural parameter λ is set as $\lambda=0.7$ and then its stiffness property is as $142.9\hat{y}+1486.9\hat{y}^3$. To compared with QZS-VI and MSD-VI, for SLS platform, the structural parameters are as $\theta=1$ rad, $n=4$, $l=0.5$ m, $c_1=0.05$ and $c_2=0.1$. Then the equivalent mass of the SLS-VI is $F_M=10.443$ and the stiffness property is as $F_k=151.6\hat{y}+231.4\hat{y}^2+300.86\hat{y}^3$; the friction force is as $F_{c1}=(0.63315+0.47937\hat{y}+0.577602\hat{y}^2)\cdot\text{sgn}(\dot{\hat{y}})$ for $\alpha=0$ and $\beta=0$. Firstly, the stiffness properties of the three isolators are shown in Fig. 14.

Fig. 14 Equivalent stiffness force for three systems

In Fig. 14, the equivalent stiffness property of the SLS platform is very close to or even better than that of the QZS-VI, both of which behave much softer than the MSD-VI. The comparison of displacement transmissibility is shown in Fig. 15 with $z_0=0.02\text{m}$.

Fig. 15 comparison of displacement transmissibility of MSD-VI, QZS-VI system and SLS-VI with same spring component

From Fig. 15, it can be seen that without any extra dampers or semi-active/active controllers, the dynamic performance of SLS-VI can be much better than the other isolators due to its special stiffness properties above (smaller resonance frequency, lower peak value and more stability

property at the peak frequency). For adjusting the structural parameters of the SLS-VI, its isolation effect is improved because the vibration amplitude is suppressed obviously compared with the other two isolators in a wider frequency band. In Fig. 15, the comparison between the SLS-VI (Dots) and QZS-VI (Dashed Lines) is conducted with the linear stiffness coefficients of these two isolators adjusted to the same value (more general than Ref. [28]). It is found that the SLS-VI and QZS-VI have the same resonant frequency which is smaller than the MSD-VI, but the isolation effectiveness of SLS-VI is still better than the QZS-VI. In addition, although the SLS system can be in rigid motion induced by the friction of links which is adverse for vibration isolation, the value of resonant peak is reduced because the nonlinearity in the equivalent friction force F_{c1} has the damping effect for the system.

6.2 Performance comparison with a controlled pneumatic vehicle seat suspension

In this section, the SLS platform is applied as a vehicle seat suspension system and compared with an active seat suspension system using controlled pneumatic springs in [14-15]. The dynamic equation of the active seat suspension in [14-15] is given as

$$M_1 \ddot{\hat{y}} = F_a + F_d + F_f + M_1 g - M_1 \ddot{z} \quad (39)$$

where M_1 is the load mass, F_a the active control force given by $F_a = k\hat{y}$, F_d the damping force and F_f friction force. The structural parameters of the seat suspension system are listed in Table 4 in Appendix. In this case, the SLS platform is applied as a seat suspension assembled in vehicle which has requirement of size limitation, so the adjustable structural parameters are set in fixed ranges as $n \in [1, 3]$, $\theta \in [0.3, 1.2]$ (rad) and $l \in [0.1, 0.3]$ (m).

A. Better static feature

The seat suspension should have sufficient stiffness for different load and the static

deformation should be limited in a small range e.g. $[0, y_s]$ for a better operation position. For the active seat suspension system, the spring deformation for load mass M_1 is given by $y_s = M_1 g / k_l$. The SLS platform can be designed to have sufficient stiffness for different load mass but with a much smaller static deformation in seat position, if the following inequality is satisfied,

$$k_1 y_s + k_2 y_s^2 + k_3 y_s^3 > M_1 g \quad (40)$$

The layer number of the SLS platform can be chosen as $n=2$ for easy assembly and saving space. The structural parameters θ and l should make the SLS platform have better load capacity than the system with linear active control in [15]. Considering $k_l = 10000 \text{ N} \cdot \text{m}^{-1}$ and $M_1 = 200 \text{ kg}$ and Substituting $y_s = M_1 g / k_l$ into (40), the following inequality can be obtained

$$\frac{M_1 g \tan^2 \theta}{n^2} + \frac{3M_1^2 g^2 \tan \theta \sec^3 \theta}{4k_l l n^3} + \frac{M_1^3 g^3 (3 - 2 \cos 2\theta) \sec^6 \theta}{8k_l^2 l^2 n^4} > M_1 g \quad (41)$$

When (41) is satisfied, the SLS platform has better load capacity for the values of θ and l . Fig. 16 shows the range of θ and l for better static characteristic of the SLS platform for $n=2$.

Fig. 16 The value of θ and l to raise load capacity

From Fig. 16, for a mass M_1 less than 200kg, when θ and l are chosen in Region 1, (41) is satisfied which means that the load capacity of the SLS platform is better than the suspension

system with linear active control. For a pair of parameters θ and l in Region 1, the static deformation y_s for different mass for the active control system and SLS platform are compared in Fig. 17. It indicates clearly that the SLS platform can have a much smaller static deformation than the active control method in [15]. For Fig. 17, the structural parameters are as $n=2$, $\theta=1.1(\text{rad})$ and $l=0.2(\text{m})$. The stiffness property of the SLS platform is as $F_k=9650.7\hat{y}+98683\hat{y}^2+936653\hat{y}^3$.

Fig. 17 Static deformation in Reference [15] and SLS platform

B. Isolation effect for random excitation

Consider the seat suspension system subjected to different suspension masses and random excitation in this section. In the active control method, the active spring force and damping force are $F_a=k_l\hat{y}$ and $F_d=c\dot{\hat{y}}$ as in [15], and the values for k_l and c are set as the same as structural parameters in the SLS platform, i.e., $k=k_l$, $c=\alpha$. Two mass cases $M_1=55\text{kg}$ and $M_2=102\text{kg}$ are considered as in [15]. The structural parameters of the SLS platform are set as the case of $k_l=10000$, $\alpha=c=100$, $\beta=0$, $n=2$, $\theta=1.1\text{rad}$, $l=0.2\text{m}$.

The dynamic responses of the seat suspension system with the two methods are shown in Fig. 20, subjected to the same random excitation with frequency range $[0.5, 4]$ (Hz) and considering to

mass cases as in [15]. It can be seen clearly that the SLS platform behaves much better in both cases for the random excitation, although only pure passive control method is applied in the SLS platform while the method in [15] is an active control one.

Fig. 18 Power spectral densities of the seat suspension with active control device in [15] and the n-layer SLS platform for random base excitation; (a) power spectral density of response for $M_1=55\text{kg}$ and (b) its time series; (c) power spectral density of response for $M_1=102\text{kg}$ and (d) its time series.

When $k_l=10000$, $n=2$, $\theta=1.1$ rad, $l=0.2$ m, $c_1=0.1$, $c_2=0.2$, $\alpha=100$ and $\beta=0$, the stiffness force is as $F_k=9650.7\dot{y}+9.87\times 10^4\dot{y}^2+9.37\times 10^5\dot{y}^3$; the friction force is as $F_{c1}=(0.196+0.268\dot{y}+0.44\dot{y}^2)\cdot\text{sgn}(\dot{y})$; and the equivalent damping property is as $F_{c2}=100\dot{y}$.

Based on the discussion above, as a vehicle seat suspension system, the SLS platform can have a smaller static deformation in seat position and much better vibration isolation with respect to body-sensitive-frequency excitation, compared with the active control method in [14-15], although

it is a pure passive system. The SLS platform can also be much lighter in weight since it needs not any large pneumatic or hydraulic or active control units.

From the two cases of vibration isolation system and an application for a vehicle seat suspension system, it can be seen the improvement of isolation effect and other advantages of the SLS platform. The stiffness and damping properties of the SLS platform show that, (a) the equivalent stiffness of the SLS platform is nonlinear and obviously asymmetrical, which is much softer (close to zero) around equilibrium and dramatically increasing for larger vibration amplitude – very ideal for vibration control; while the active control method can only provide a constant stiffness property; (b) Similar cases can be seen for equivalent damping property; The damping coefficient of the SLS platform is a nonlinear function of displacement and asymmetrical with respect to the zero equilibrium, which is much larger than the active control method, smaller for small vibration amplitude (low frequency vibration) and much larger for large vibration amplitude (high frequency vibration) – very ideal for vibration suppression too.

7. Experiment analysis

Fig. 19 shows the prototype of the SLS platform. To reduce the effect of Coulomb friction forces in the system, the rollers are used in horizontal tracks and lubricated bearings are used in rotational joints (Fig. 19 (b)). Some structural parameters are used as $M_1=0.65$ kg, $F_M=0.745$ kg, $k_l=1400$ N·m⁻¹, $n=2$, $\theta=0.45$ rad, $l=0.1$ m and no semi-active or active devices are involved. In the preliminary experimental analysis, a random hammering excitation is provided to the base, and the dynamic responses of the platform are shown in Fig. 20.

Fig. 19 A prototype of the SLS platform: the prototype (a) and rollers in a horizontal track and bearings in rotational joints assembled in the system (b).

Fig. 20 The time-domain acceleration response signals (a) and corresponding FFTs (b) of the SLS platform (red dashed lines) and base excitation (black solid lines).

Fig. 20 indicates clearly the excellent vibration isolation performance of the SLS platform subject to a random base impact excitation over a very broad frequency range, and about 50% (or above) reduction in magnitude can be seen at most frequency components. Note that the system damping comes only from the friction of links or joints but demonstrates very excellent nonlinear damping characteristics (i.e., high damping at resonance peaks and low damping at high frequencies [19, 20, 24, 39]). For this prototype, the first resonant frequency can be estimated as

$$\omega_r = \sqrt{\frac{k_1}{F_M}} = \sqrt{\frac{k_l}{F_M} \frac{\tan \theta}{n}} \approx 10.77 \text{ rad}\cdot\text{s}^{-1} = 1.713 \text{ Hz}$$
 (very small system dynamical stiffness which would be even smaller with loading weight on the platform). Therefore, excellent isolation performance is

observed in the impact response. The impact response amplitude of the platform is actually smaller than the base at the resonant frequency as shown in Fig. 20 (b), and no self-locking phenomenon is observed due to joint frictions.

8. Conclusions and discussions

A scissor-like structure is investigated in this study for exploring beneficial nonlinear stiffness and damping characteristics in vibration control. Although different nonlinearity in stiffness and damping has been shown to be advantageous for vibration suppression or isolation, how to achieve a special beneficial nonlinear characteristic in vibration control via a passive structure or material design is far from development in the literature. Based on the results of this study, the following conclusions can be drawn:

- (a) The scissor-like structure is very flexible and versatile in vibration control. It can be designed as an independent vibration isolator, as an important damping or stiffness component, working in a pure passive manner, or integrated with the other semi-active components. The structure size is also flexible to be designed for different application situations.
- (b) Although the friction of contact links in the system has damping effect, the Coulomb friction could be reduced to avoid self-locking phenomenon and potential negative influence on system performance by surface treatment of links and/or usage of lubricants, rollers or bearings in links. The experiment in the section 7 verified the excellent vibration isolation performance of the SLS platform.
- (c) Due to nonlinear geometric relationship, the scissor-like structure is inherently nonlinear in its equivalent stiffness and damping characteristics, which can be freely designed by tuning

(sufficiently many) structural parameters. The nonlinearity both in stiffness and damping involves no strong nonlinear behavior such as chaos and bifurcation, compared with the QZS-VIs in the literature. The equivalent nonlinear stiffness can be designed to be close to zero but maintain sufficient static stiffness, and the equivalent damping is highly dependent on displacement of vibration motion. These are noticeable features of this SLS system.

- (d) The scissor-like structure actually provides a very useful, practical and passive solution to the design and realization of beneficial nonlinear stiffness and damping characteristics in vibration control, which can benefit to a broad spectrum of engineering practice.

It is noticed that pantographic structures are able to give remarkable mechanical performance, and classical continuum mechanics is however not applicable for studying materials having a pantographic microstructure [35-37]. The SLS structure presented in this study provides interesting perspectives to the investigation of dynamics of the second gradient continua involved in these studies. This will be investigated in our future study. Moreover, further studies will also focus on 3D or more vibration platform using the SLS, and novel applications in aeronautic engineering etc.

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Appendix

Table 1 The structural parameters of the n-layer SLS platform.

Symbol	Structural parameters	Unit
k_l	linear stiffness	$\text{N}\cdot\text{m}^{-1}$
M_1	mass of isolation object	kg
M_2	mass of connecting rod	kg
n	number of layer	
l	length of connecting rod	m
θ	assembly angle of connecting rod	rad
c_1	friction force of rotational motion	N
c_2	friction force of horizontal motion	N
f_M	equivalent mass of connecting rods	kg
F_M	equivalent mass of platform and connecting rods	kg
J_0	rotational inertia of connecting rods	$\text{kg}\cdot\text{m}^2$
F_{dy}	output force in y direction of Damper1	N
F_{dx}	output force in x direction of Damper2	N
F_{fx}	friction force in x direction	N
$F_{f\varphi}$	friction force in φ direction	N
F_k	equivalent stiffness force	N
F_{c1}	equivalent friction force	N
F_{c2}	equivalent damping force	N
f_{c1}	friction force	N
f_{c2}	damping coefficient	$\text{N}\cdot\text{s}\cdot\text{m}^{-1}$
α	damping coefficient of Damper1	$\text{N}\cdot\text{s}\cdot\text{m}^{-1}$
β	damping coefficient of Damper2	$\text{N}\cdot\text{s}\cdot\text{m}^{-1}$
k_1	linear stiffness coefficient	$\text{N}\cdot\text{m}^{-1}$
k_2	nonlinear stiffness coefficient	$\text{N}\cdot\text{m}^{-2}$
k_3	nonlinear stiffness coefficient	$\text{N}\cdot\text{m}^{-3}$
ξ_1	friction force coefficient	N
ξ_2	friction force coefficient	$\text{N}\cdot\text{m}^{-1}$
ξ_3	friction force coefficient	$\text{N}\cdot\text{m}^{-2}$
ξ_4	linear damping coefficient	$\text{N}\cdot\text{s}\cdot\text{m}^{-1}$
ξ_5	nonlinear damping coefficient	$\text{N}\cdot\text{s}\cdot\text{m}^{-2}$
ξ_6	nonlinear damping coefficient	$\text{N}\cdot\text{s}\cdot\text{m}^{-3}$

Table 2 The motion and variables of SLS platform

Symbol	Structural parameters	Unit
y	absolute motion of isolation object	m
z	base excitation	m
$\hat{y}$	relative motion of isolation object and base	m
φ	rotation transport motion	rad
x	horizontal transport motion	m
z_0	amplitude of base excitation z	m
ω	frequency of base excitation z	rad·s ⁻¹
y_1	amplitude of relative motion $\hat{y}$	
y_0	zero drift of relative motion $\hat{y}$	
$\square$	phase of relative motion $\hat{y}$	
T_d	displacement transmissibility	

Table 3 The structural parameters and motion in comparison of QZS-VI

Symbol	Structural parameters and motions	Unit
k_{q1}	linear stiffness coefficient of QZS-VI	N·m ⁻¹
k_{q2}	nonlinear stiffness coefficient of QZS-VI	N·m ⁻³
λ	ratio between spring length with pre-deformation and original length	m
c	damping coefficient in QZS-VI	N·s·m ⁻¹
T_d	displacement transmissibility	

Table 4 The structural parameters and motion in comparison of seat suspension

Symbol	Structural parameters and motions	Unit
M_1	first mass of isolation object	kg
k	active control strength of air-spring	N·m ⁻¹
F_a	linear active control force	N
F_d	linear damping force	N
F_f	friction force	N
c	damping in QZS-VI	N·s·m ⁻¹
$\hat{y}$	displacement of motion	m
y_s	static deformation	m

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Fig. 1 A 3-layer SLS platform

Fig. 2 Flat view of n-layer SLS platform model with two dampers

Fig. 3 The motion of one connecting rod in the first layer and the geometrical relations among φ , x and $\hat{y}$

Fig. 4 Comparison of stiffness and damping properties between original equation and Taylor expansion

Fig. 5 The value of equivalent mass F_M for different θ and n . (a) For different assembly angle θ , (b) for different number of layer n .

Fig. 6 Linear stiffness coefficient k_1 (a) for different assembly angle θ , (b) for different number of layer n .

Fig. 7 Resonant frequency ω_r for (a) different assembly angle θ , (b) different layer number n

Fig. 8 Equivalent stiffness force F_k for (a) different n as $\theta=\pi/4$ and $l=0.5$; (b) different θ as $n=2$ and

$l=0.5$; (c) different l as $\theta=\pi/4$ and $n=2$.

Fig. 9 Equivalent friction force for different θ , n and l . Coefficient ξ_1 (a) for different θ and (b) for different l ; Coefficient ξ_2 (c) for different θ and (d) for different l ; coefficient ξ_3 (e) for different θ and (f) for different l .

Fig. 10 Friction force F_{c1} for (a) different n ; (b) different θ ; (c) different l

Fig. 11 Equivalent damping coefficient f_{c2} for (a) different n as $\theta=\pi/4$ and $l=0.5$; (b) different θ as $n=2$ and $l=0.5$; (c) different l as $\theta=\pi/4$ and $n=2$.

Fig. 12 Damping coefficient (a) ξ_4 ; (b) ξ_5 ; (c) ξ_6 , for different strength β in Damper 2 as $\alpha=0$, $\theta=\pi/4$ and $l=0.5$.

Fig. 13 Displacement transmissibility T_d for periodic base excitation Ω from theoretical analysis (Solid Lines) and numerical simulation (Dots)

Fig. 14 Equivalent stiffness force for three systems

Fig. 15 comparison of displacement transmissibility of MSD-VI, QZS-VI system and SLS-VI with same spring component

Fig. 16 The value of θ and l to raise load capacity

Fig. 17 Static deformation in Reference [15] and SLS platform

Fig. 18 Power spectral densities of the seat suspension with active control device in [15] and the n-layer SLS platform for random base excitation; (a) power spectral density of response for $M_1=55\text{kg}$ and (b) its time series; (c) power spectral density of response for $M_1=102\text{kg}$ and (d) its time series.

Fig. 19 Experiment of the SLS platform; (a) is the prototype and (b) shows the horizontal track and joints.

Fig. 20 The responses and spectrums of SLS platform (Red dashed lines) and base excitation (Black solid lines); (a) is the time series of responses of the platform and base; (b) is the amplitude spectrum of the accelerometer of the platform and base obtained by direct Fourier transform.

Table 1 The structural parameters of the n-layer SLS platform.

Table 2 The motion and variables of SLS platform

Table 3 The structural parameters and motion in comparison of QZS-VI

Table 4 The structural parameters and motion in comparison of seat suspension