Large Deformation Response of Additively- Manufactured FCC Metamaterials: From Octet Truss Lattices Towards Continuous Shell Mesostructures

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Abstract. Starting with the statically-determinate solid octet truss lattice, the large strain compression response of different metamaterial architectures is analyzed through finite element analysis and compression experiments on additively-manufactured stainless steel specimens. Simulation results clearly demonstrate that for a relative density of 20% the conventional solid octet meso-lattice has a lower specific energy absorption capability than periodic shell structures. The analysis of the closed-packed hollow sphere assembly subject to hydrostatic compression reveals that the constituent hollow spheres initially transform into rhombic dodecahedra, which results in significant strengthening at the macroscopic level. Unlike the octet trusses with solid struts, hollow octet truss structures remain stable for hydrostatic, confined and uniaxial compression at the lowest relative density considered. Its strength, in particular at small plastic strains, is substantially improved when substituting the geometric hollow truss joints for hollow spheres. The continuous shell mesostructure defined by the Hybrid hollow Truss Sphere (HTS) and the hollow octet truss assembly exhibited the highest strength and energy absorption potential at 20% relative density for the high and low strain-hardening materials considered, respectively. Moreover, it is found that the HTS metamaterial exhibits the highest relative Young's and bulk moduli among all four architectures considered. It is also shown computationally that hollow rhombic dodecahedral mesostructures could provide nearly twice the strength and energy absorption of the conventional octet truss.

Keywords: Octet truss lattice, specific energy absorption, low density metamaterials, finite element analysis, hollow dodecahedron

1. Introduction

The macroscopic properties of most metallic materials is determined by their polycrystalline microstructure. Cellular materials also feature a so-called mesostructure which strongly influences their mechanical response. The mesostructure is defined through the geometric details that describe the exact shape and volume fraction of the porous phase of a cellular solid. For example, honeycomb materials feature a hexagonal thin-walled mesostructure. Recent progress in UV lithographic technologies such as the self-propagating polymer waveguide technique, projection micro stereolithography, and two-photon direct laser writing, as well as in additive manufacturing with metals such as selective laser melting now makes it possible to fabricate cellular solids with almost any type of mesostructure and at almost any length scale above 10^{-8} m. About two decades ago, foams with cell sizes at the millimeter level were the only man-made cellular solids with nearly isotropic mechanical properties (e.g. Gibson and Ashby, 1997).

When subject to compressive loading, foam mesostructures collapse which results in the well-documented progressive crushing response of foams and honeycombs (e.g. Santosa and Wierzbicki (1998), Hanssen et al. (2000), Gong et al. (2005), Hong et al. (2008), Jang and Kyriakides (2009)). At the macroscopic level, the crushing response of foams is characterized by a stress plateau before the material gains substantial strength during the densification phase. The engineering stress plateau is a desirable feature when using foams as an energy absorption material, but from a weight point of view, the progressive crushing mechanism is very inefficient: inside the crushed regions, the plastic deformation is highly localized at the mesoscopic level (e.g. plastic hinges lines form in the cell walls), which implies that the cell wall material away from the localization zones (e.g. away from the plastic hinge lines) contributes only little to the overall energy absorption of the foam structure.

After a decades of research on foams, it has been shown that stretching-dominated truss mesostructures provide significantly higher stiffness and strength than foams (e.g. Deshpande et al., 2001a, Evans et al., 2001). Wang and McDowell (2005) demonstrated that the yield surface of bending-dominated cell structures is curved, while a planar faceted yield surface is observed for stretch-dominated mesostructures. This result is

also supported by numerical simulations of Alkhader and Vural (2010) who observed a fully smooth yield surface for random Voronoi honeycomb structures. In engineering practice, lattice structures can be built from three-dimensionally formed perforated sheets (e.g. Sypeck and Wadley (2002), Wadley et al. (2003)). Alternatively, truss lattice metamaterials with solid struts can be readily obtained through additive manufacturing (e.g. Yan et al. (2012), Gumruk and Mines (2013), Stankovic et al. (2015), Tancogne-Dejean et al. (2016a)). The smallest achievable feature size of state-of-the-art selective laser melting with metals is still of the order of about $30\mu m$. Metallic structures with finer structural features can be fabricated by depositing metallic layers onto free-standing polymer scaffolds through electroless-plating, atomic layer deposition or RF sputtering. After removal of the polymeric support, hollow shell-like structures with nanometric wall thicknesses are obtained. It is noted that the same process also applies to making hollow ceramic nanostructures. The fabrication of porous solids with mesostructural features in the submicrometer range is mostly explored to leverage size effects in the solid base constituent. For example, Schaedler et al. (2011) produced a lattice scaffold with strut lengths of a few millimeters using a self-propagating photopolymer waveguide technique. Using electroless plating, they deposited thin nickel films with an average grain size of a few nanometers. Multiphoton lithography has been employed by Jang et al. (2013) and Montemayor et al. (2014) to produce solid polymer lattice templates with a few microns long struts. Jang et al. (2013) applied about $75nm$ thick ceramic layers onto the octet truss scaffold, thereby producing hollow TiN octet truss metamaterials with a base constituent strength of $1.7GPa$. Bauer et al. (2014) employed 3D laser lithography along with atomic layer deposition to create cellular polymer mesostructures with $10nm$ to $200nm$ thick alumina coatings that exceeded the compressive strength of foams at effective densities below $1g/cm^3$.

With all these advanced micro- and nanofabrication techniques emerging, it is worth revisiting the choice of mesostructural architectures in addition to achieving the exceptional effective material properties through leveraging size effects. Han et al. (2015) argued that continuous thin shells are stretching-dominated structures, thereby providing a valuable alternative to octet truss lattice material. They produced nickel shell mesostructures with wall thicknesses ranging from 0.2 to $56\mu m$ through

electroless plating of polymer scaffolds. In their compression experiments, the periodic shell mesostructures with smooth curvature variations exhibited density dependent strength and modulus scaling relationships that are close to the hollow octet truss lattice. Aside from the unmatched manufacturing flexibility of additive processes, it must also be mentioned that additively-manufactured structures usually come with building defects, entrained porosity and residual stresses at the meso- and microscopic levels (e.g. Mercelis and Kruth (2006), Wu et al. (2014)). At the same time, the grain structures and initial dislocation densities in additively-manufactured structures are often very different from conventional products such as sheets made from the same alloy (e.g. Tancogne-Dejean et al, 2016b).

In the present paper, we explore the mechanical behavior of cellular solids with Face- Centered Cubic (FCC) symmetry. In particular, we consider the octet truss lattice with solid and hollow trusses, hollow sphere assemblies and hybrid truss-sphere structures. The large deformation hydrostatic and confined compression responses of these mesostructures is analyzed for relative densities ranging from 0.2 to 0.6, and for high and low strain hardening base materials. Uniaxial compression experiments are performed on porous stainless steel 316L mesostructures to validate our non-linear finite element models and to demonstrate the superior weight specific energy absorption capabilities of continuous shell mesostructures over the conventional octet truss lattice at 20% relative density.

2. Materials

When discussing the mechanical response of a metamaterial, it is useful to distinguish among three length scales: (1) the macroscopic level, where the porous material is considered as a homogeneous solid, (2) the mesoscopic level, where the porous material is considered as a two phase material composed of a mesostructure made from a homogenous solid base material and a fluid phase (pores), and (3) the microstructural level, where heterogeneities (e.g. polycrystalline grain structure or microporosity) within the base material are considered.

2.1. Base materials

2.1.1 Materials considered in the computational analysis

The mechanical stability of plastically-deforming structures is strongly affected by the strain hardening of the base material after first yield. We anticipate that a higher strain hardening capacity of the base material will favorably affect the stability of the mesostructures considered. We therefore carry out all our simulations for two distinct base materials that represent two rather extreme cases (among engineering materials) of high strain hardening and low strain hardening (nearly perfectly plastic) at small and moderate strains:

1. High strain Hardening (HH) base material: the assumed uniaxial stress-strain response for the HH material is shown in Fig. 1a. The relationship between the von Mises equivalent stress $\bar{\sigma}$ and the work-conjugate equivalent plastic strain $\bar{\varepsilon}_p$ is described through a Swift law

$$\bar{\sigma} = A(\bar{\varepsilon}_p + \varepsilon_0)^n . \quad (1)$$

Based on manufacturer data for selectively laser molten stainless steel 316L, the Swift parameters $A = 842.5 \text{ MPa}$, $\varepsilon_0 = 0.0364$ and $n = 0.18$ are used. The elastic behavior of the HH material is characterized through a Young's modulus of $E = 200 \text{ GPa}$ and a Poisson's ratio of $\nu = 0.3$.

2. Low strain Hardening (LH) base material: the uniaxial stress-strain response for the LH material is represented through a piecewise-linear, monotonically increasing function (Fig. 1b), while its elastic response is described through the constants $E = 2 \text{ GPa}$ and $\nu = 0.3$. The assumed response has been chosen from experiments on a PE-like polymer which is used for stereolithography.

Both the HH and LH materials are modeled using the standard finite strain plasticity model of the Abaqus material library (Simulia, 2015), which assumes a von Mises yield function, associated plastic flow and isotropic strain hardening.

2.1.2 Material used in the experiments

All specimens tested are made from stainless steel 316L powder using Selective Laser Melting (SLM)¹. As shown by Tancogne-Dejean et al. (2016a), substantial base material property gradients may prevail within SLM made structures due to locally different temperature histories. As a first approximation, it is expected that the average base material response in the experiments is similar to the HH material behavior assumed in the simulations.

2.2. Meta-material architectures

We limit our attention to four mesostructures of FCC symmetry. Here, the term “FCC symmetry” is employed to emphasize that the unit-cells of all the structures considered exhibit the same rotational symmetries as face-centered cubic lattices, i.e. octahedral symmetry with 24 rotational symmetries and invariance through 48 reflection/rotation combinations. In other terms, if one considers the group of affine transformations of space through which the set of vertices (nodes) of the FCC lattice is invariant, the only structures considered are those that are unchanged through these transformations. The primitive cell of the FCC geometries is a tetrahedron ,

$$= \{ (x, y, z) \in \mathbb{R}^3 / \begin{matrix} x \geq 0 \\ y \geq 0 \\ z \geq 0 \\ \sqrt{2} + \sqrt{2} + 2 \geq 0 \end{matrix} \} , \quad (2)$$

with denoting the closest neighbor distance in the FCC lattice. In particular, we investigate the mechanical behavior of the following FCC mesostructures:

1. Solid Octet Truss (SOT) lattice: The SOT is obtained by connecting the closest neighbors in an FCC lattice through cylindrical solid struts (Fig. 2a). As shown by Deshpande et al. (2001b), the relative density ρ , i.e. the ratio

¹ An attempt has also been made to manufacture polymer specimens through stereolithography. However, we abandoned this approach due to the high number of support structures required to stabilize numerous over-hanging surfaces during the manufacturing process.

of the density of the porous solid and the solid base material, is controlled by the strut diameter-to-length ratio D / L ,

$$\bar{\rho}_{SOT} = \frac{3\pi}{\sqrt{2}} \left[\left(\frac{D}{L} \right)^2 - C \left(\frac{D}{L} \right)^3 \right] \quad (3)$$

where the cubic correction coefficient $C = 13.6$ has been identified by Tancogne-Dejean et al. (2016) from fitting Eq. (3) to CAD calculations.

2. Hollow Sphere Assembly (HSA): this configuration is obtained by placing hollow spheres at the vertices of the FCC lattice (Fig. 2b). The outer diameter of the spheres is equal to the closest neighbor distance L . The relative density of the HSA material is then controlled by the sphere wall thickness-to-diameter ratio t / L . In our simulations and experiments, hollow cylindrical joints are included between adjacent spheres, with outer diameter $L/5$ and of wall thickness t . These joints ensure the cohesion of the assembly while being compatible with the use of powder-bed and liquid-bath additive manufacturing techniques (as the non-molten material inside the spheres can still be extracted through these joints). Neglecting the mass contribution of the joints between adjacent spheres, the theoretical estimate of the relative density reads

$$\bar{\rho}_{HSA} = \frac{3\pi}{\sqrt{2}} \left[\left(\frac{t}{L} \right)^2 - C \left(\frac{t}{L} \right)^3 \right] \quad (4)$$

In addition to these two extreme cases of solid struts and hollow spheres, we consider two intermediate configurations:

3. Hollow Octet Truss (HOT) lattice: when substituting the solid struts of the SOT by hollow struts, the HOT configuration is obtained (Fig. 2c). In addition to the D / L ratio, the relative density of the HOT lattice depends on the inner-to-outer strut diameter ratio d / D ,

$$\bar{\rho}_{HOT} = \frac{3\pi}{\sqrt{2}} \left[\left(\frac{D}{L} \right)^2 - \left(\frac{d}{D} \right)^2 \right] - 13.6 \left[\left(\frac{D}{L} \right)^3 - \left(\frac{d}{D} \right)^3 \right]. \quad (5)$$

Equation (5) is readily obtained when subtracting an SOT of strut diameter d from an SOT of strut diameter D .

4. Hybrid Truss-Sphere (HTS) assembly: Hollow spheres are placed at the vertices of the FCC lattice and connected by hollow struts (Fig. 2d). The

relative density of the HTS is then controlled by four ratios: (i) the normalized strut outer diameter t/L , (ii) the strut inner to outer diameter ratio t_i/t , (iii) the normalized sphere outer diameter s/L , and (iv) the sphere thickness to the strut thickness ratio $s/t =$

$(-)/(-)$. For the particular case of $t/L = 2/3$, $t_i/t = 1/\sqrt{2}$ and $s/t = 1$, the theoretical estimate of the relative density reads

$$\bar{\rho}_{HTS} \cong 1.975 \left(\frac{t}{L} \right) + 57.7 \left(\frac{t}{L} \right)^2 - 446.9 \left(\frac{t}{L} \right)^3 \quad (6)$$

when neglecting terms of higher order in t/L .

A solid sphere-based assembly is not considered due to its high relative density ($\rho = \pi / 3\sqrt{2} \cong 0.74$). It is worth noting that the SOT features a single continuous porous phase while all other architectures (HSA, HOT and HTS) each feature two separate porous phases.

3. Finite element analysis

Three basic types of simulations are performed to investigate the mechanical behavior of porous solids:

- (1) Estimation of the macroscopic elastic moduli: Implicit small-strain analysis of the unit cell subject to uniaxial tension, hydrostatic tension and pure shear loading (using Abaqus/Standard solver);
- (2) Determination of the large strain response: Explicit finite-strain analysis of the unit cell subject to hydrostatic and confined compression (using Abaqus/Explicit solver);
- (3) Determination of specific energy absorption: Explicit finite-strain analysis of cubic specimens comprised of 27 unit cells subject to uniaxial compression (using Abaqus/Explicit solver);

3.1. Unit cell models

An automated meshing scheme has been developed to generate solid element meshes of the respective FCC unit cells. Starting from a mesh of the primitive cell (Fig. 3), the full unit cell mesh is generated through a series of mirror operations (Fig. 2):

- SOT unit cell meshes are created for three relative densities, $\bar{\rho} = \{0.2, 0.4, 0.6\}$. The SOT meshes feature three first-order elements along the radial strut direction. Maintaining an element aspect ratio close to one, the respective total number of elements per unit cell are 57,600, 37,440 and 28,800.
- HSA unit cell meshes are created for three relative densities, $\bar{\rho} = \{0.2, 0.4, 0.6\}$. The HSA meshes feature eight first-order elements along the sphere thickness direction. Maintaining an element aspect ratio close to one, the respective total number of elements per unit cell are 276,864, 104,256 and 66,240.
- HOT and HTS unit cell meshes are created for a relative density of $\bar{\rho} = 0.2$ only. A minimum of three elements through thickness is used in the respective meshes. For element aspect ratios close to unity, this results in a total number of elements per unit cell of 165,888 and 266,304, respectively.

Periodic displacement boundary conditions are created by imposing kinematic constraints on the matching nodes of each pair of parallel unit cell boundary surfaces; the difference in displacement between each pair of corresponding nodes is equal to the displacement of a master node. By prescribing either the displacement or the force (if a single node per pair of boundary surfaces is used) history at the master nodes, one can control the macroscopic deformation gradient $\mathbf{F}$ or the Piola stress $\mathbf{P}$, respectively.

Implicit analyses are performed to compute the macroscopic elasticity constants. To estimate the macroscopic Young's modulus E , the shear modulus G and the bulk modulus K , three distinct sets of boundary conditions are applied. With $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ defining a Cartesian coordinate frame that is aligned with the unit cell edges, the respective boundary conditions read

$$\text{Uniaxial tension: } \begin{cases} F_{11} = 1 + \varepsilon \\ P = 0 \\ \forall (i, j) \neq (1, 1) \end{cases} \quad (7)$$

$$\text{Simple shear: } \mathbf{F}_S = \varepsilon_{\max} \mathbf{e}_1 \otimes \mathbf{e}_2 \quad (8)$$

$$\text{Hydrostatic: } \mathbf{F}_H = \frac{\varepsilon_{\max}}{3} (\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{e}_3 \otimes \mathbf{e}_3) \quad (9)$$

Note that due to the FCC symmetry of the unit cells, the spherical deformation gradient (9) generates a hydrostatic stress response at the macroscopic level. In the elastic simulations, the magnitude of the applied deformation gradients is limited to small strains by setting $\varepsilon_{\max} = 10^{-4}$.

In the explicit finite-strain simulations, we define self-contact with a friction coefficient of 0.2 within the unit cells. The gradient $\mathbf{F}_H$ is also applied to determine the large deformation response under hydrostatic compression, while the macroscopic deformation gradient

$$\mathbf{F}_{CC} = -\varepsilon_{\max} \mathbf{e}_1 \otimes \mathbf{e}_1 \quad (10)$$

is applied to assess the material response to confined compression. In the large deformation simulations, $\varepsilon_{\max}$ is chosen such that about half of the unit cell porosity is absorbed at the end of the simulation. Mass scaling is used to solve the quasi-static boundary value problems with about 200,000 stable explicit time steps. The reported macroscopic stresses are computed from the work-conjugate forces at the master nodes. Note that we always report true strains and stresses when showing unit cell simulation results.

3.2. Uniaxial compression specimens

The periodic boundary conditions imposed onto the unit cell models are potentially confining as far as the localization under uniaxial compression is concerned. We therefore generated uniaxial compression specimens comprising multiple unit cells. With regards to the limitations of the high performance computing infrastructure, cubic specimens comprised of $3 \times 3 \times 3 = 27$ unit cells are generated based on the unit cell

meshes for a relative density of $\rho = 0.2$. Note that the largest specimen model included about 7.5 million solid elements.

The virtual specimen is placed between two parallel flat surfaces (analytical rigid surface). A friction coefficient of 0.2 is assumed between the specimen and the rigid surfaces. The lower surface is kept stationary, while the upper surface is moved downward at a constant velocity. Using automatic mass scaling, the base material density is adjusted such that a maximum macroscopic compressive engineering strain of 0.6 is reached after about 500,000 time steps. As in the explicit unit cell simulations, self-contact with a friction coefficient of 0.2 is defined within the cellular structures.

3.3. Simulation results

3.3.1 Elastic moduli

Given the octahedral symmetry of the FCC structures, and assuming an isotropic linear elastic base material, the unit cells have a cubic elastic response at the macroscopic level. Denoting the stress and strain tensors in the principal material frame (which is aligned with the edges of the unit cells), the linear elastic stress-strain relationship reads

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{pmatrix} = \begin{pmatrix} C_1 & C_2 & C_2 & 0 & 0 & 0 \\ C_2 & C_1 & C_2 & 0 & 0 & 0 \\ C_2 & C_2 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_3 \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{pmatrix}. \quad (11)$$

The three elastic moduli $\{C_1, C_2, C_3\}$ are related to the Young's modulus E , the shear modulus G , and the bulk modulus K through the identities

$$(12)$$

Recall that we created models of different relative densities for the SOT and HSA configurations only. Consequently, the relative modulus versus relative density plots are only provided for these two materials (Figs 4a to 4c). It is worth noting that the

curves are not in hierarchical order. For example, the Young's modulus of the SOT is lower than that of the HSA material at low relative densities, while the shear and bulk moduli of the SOT are always higher irrespective of the relative density. The differences are most pronounced at a relative density of 0.6, where the SOT shear and bulk moduli are about 60% higher. The comparison of the moduli of all four mesostructures (Fig. 4d) at a relative density of $\rho = 0.2$ shows that the HTS structure exhibits the highest Young's and bulk moduli, while its shear modulus is 30% lower than that of the shear stiffest material (SOT). Overall, the HOT architecture exhibits the lowest elastic properties, in particular its shear modulus is 40% lower than that of the SOT. The widest spread in moduli among different structures is observed for hydrostatic loading. Here, the bulk modulus of the HSA is only half that of the HTS material.

3.3.2 Large strain response to hydrostatic compression

Simulations are performed for all four mesostructures {SOT, HSA, HOT, HTS} for a relative density of $\rho = 0.2$, while the mechanical response of the first two structures is also analyzed for $\rho = \{0.4, 0.6\}$. The simulations are stopped when the initial porous volume is reduced by 50%. In the sequel, we discuss selected simulation results in detail.

- *Solid Octet Truss (SOT) lattice*: Figures 5a and 5b show the computed macroscopic pressure versus volumetric strain relationships for the high strain hardening (HH) and low strain hardening (LH) base materials, respectively. In addition, selected contour plots of the equivalent plastic strain distribution at the mesostructural level are shown. The hydrostatic loading condition at the macroscopic level puts all struts under uniaxial compression. The stress state within the 'nodes', i.e. the joints connecting 12 struts, is almost perfect hydrostatic compression as long as the struts are straight. The plastic deformation is therefore confined to the struts as the von Mises base material does not deform plastically in the regions of locally hydrostatic stress states. For small strut length to diameter ratios (e.g. Figs. 5c and 5d), the struts remain straight even at large plastic strains. Slender struts buckle plastically (e.g. Figs. 5e and 5f). As a result, plastic hinges form within the struts and the nodes undergo a rigid body rotation

(so-called ‘twist mode’, see Tancogne-Dejean et al. (2016a)). The pressure versus volumetric strain curves are monotonically increasing for ‘stable’ mesostructures, while it exhibits a more or less pronounced peak for ‘unstable’ mesostructures that respond through strut buckling. The stability of the mesostructure is not only a function of the relative density; it also depends on the strain hardening capacity of the base material. For example, at a relative density of $\bar{\rho} = 0.4$, the computed response of the SOT made from the HH material is still stable (Fig. 5d), while that made from the LH material is unstable (Fig. 5e).

- *Hollow Sphere Assembly (HSA)*: Figure 6 summarizes the simulation results for the HSA. All macroscopic pressure-strain curves exhibit a low and a high strain hardening phase. In the low hardening phase, the geometry of the mesostructure changes dramatically. The hollow spheres are progressively transformed into a hollow dodecahedron (Fig. 6f). In the two-dimensional section views shown in Figs. 6c to 6e, this mesostructural evolution corresponds to the transformation of a circular central pore to a square pore. The contact area between two neighboring spheres increases throughout this bending-dominated transformation. Once all spheres surfaces are in contact, the high hardening phase sets in. For the $\bar{\rho} = 0.2$ ($\bar{\rho} = 0.6$) structures, this change occurs at a volumetric strain of about 0.23 (0.11). As compared to the SOT material, the response of the HSA is initially much softer, but it quickly exceeds the macroscopic deformation resistance of the SOT as soon as the high hardening phase sets in.
- *Hollow Octet Truss (HOT) lattice*: As compared to the SOT, the strut cross sections of the HOT lattices have a significantly higher moment of inertia for the same relative density. As a result, we did not observe any buckling when subjecting the HOT unit cells to hydrostatic loading. The macroscopic pressure versus volumetric strain curves for all HOT lattices (high and low strain hardening base materials) are monotonically increasing for a relative density of $\bar{\rho} = 0.2$ (Figs. 7a and 7b). As opposed to the highly pressure-resistant solid nodes of the SOT, the hollow nodes of the HOT mesostructure are weak. Plastic deformation initiates early at the nodes. As a consequence, the macroscopic pressure resistance of the HOT material is even lower than that of the SOT material at small volumetric strains (Figs. 7a and 7b). However, due to its

positive strain hardening at the macroscopic level, it can carry higher pressures than the unstable SOT lattice of the same weight at large strains.

- *Hybrid hollow Truss-Sphere Assembly (HTS)*: The HTS material architecture is very similar to that of the HOT material. The main modification is an increase of the size of the nodes, which leads to both an overall reduction in surface area (and corresponding increase in wall thickness) and a reduction of the angle sharpness. The former explains this structure's higher elastic properties (Fig. 4d), while the smoother node geometry reduces plastic strain concentration at the nodes, providing a significantly higher pressure resistance at small volumetric strains (for a same relative density of $\bar{\rho} = 0.2$). The large deformation response of the HTS mesostructure remains stable irrespective of the strain hardening capacity of the base material (Figs. 7e and 7f) which results in a monotonically increasing macroscopic pressure-strain response.

3.3.3 Large strain response to confined compression

The hydrostatic loading condition is special in the sense that the struts of octet trusses (SOT and HOT) are all subject to axial compression only. In other words, the mechanical loading includes the symmetries of the mesostructures. We therefore repeated our simulations for confined compression, i.e. a loading condition where the material is compressed along a first direction while the macroscopic deformation must be zero along the other two orthogonal directions. Despite the change in the loading condition, most of the previously reported deformation mechanisms for hydrostatic compression are also observed for confined compression. However, the following differences are worth noting:

- *Solid Octet Truss (SOT)*: for all relative densities and base materials considered, the nodes of the SOT deform elastically only and do not contribute to the plastic dissipation (Fig. 8). In the case of the high density structure ($\bar{\rho} = 0.6$), the sturdy struts are primarily sheared (Fig. 8c). For lower relative densities, the increased strut slenderness shifts the strut deformation mode from shear-dominated (Figs. 8c and 8d) to bending-dominated (Figs. 8e and 8f). Due to the evolution of the mesostructural geometry (strut rotation with respect to the loading direction), the

overall load carrying capacity is reduced. Base material strain hardening can partially compensate for this geometric softening. The HH-SOT therefore still exhibits macroscopically positive strain hardening, while the LH-SOT material is already at the limit to macroscopic softening. The latter is observed for both base materials at a relative density of $\rho = 0.2$. The LH-SOT responds in a twist-mode, while the bending deformation localizes in one half of the unit cell for the HH-SOT.

- *Hollow Sphere Assembly (HSA)*: As for the hydrostatic loading, the macroscopic stress-strain response of the HSA material is characterized through two positive hardening phases that are related to the initial growth of contact area between neighboring spheres (Figs. 9a and 9b). As illustrated in Fig. 9e, the shape of the spheres still transforms towards a dodecahedron, but with non-congruent faces due to the deviatoric loading. The plastic deformation is highly localized in the vicinity of the joints between spheres.
- *Hollow Octet Truss (HOT)*: The high second moment of the hollow cross-sections is also advantageous under confined compression, while the hollow nodes are flattened (Figs. 10c and 10d). At the macroscopic level, all HOT structures exhibit a positive strain hardening response (Figs. 10a and 10b).
- *Hybrid Truss-Sphere Assembly (HTS)*: Its deformation response is very similar to that of the HOT material except that the flattening of the nodes is less pronounced while the node walls respond by bending (Figs. 10e and 10f). At the macroscopic level, the HTS exhibits positive strain hardening for all densities and base materials considered. It is also the strongest up to an axial strain of about -0.2, beyond which the HOT reaches similar strength, while the hardened HSA material surpasses both for axial compressive strains greater than about 0.3.

3.3.4 Large strain response to uniaxial compression

The uniaxial compression simulations are performed to estimate the specific energy absorption capacity of the porous materials. To account for possible far field buckling instabilities, these simulations are performed using cubic virtual specimens composed of $3 \times 3 \times 3$ unit cells (close to limit of computational feasibility). Figure 11 summarizes the results for uniaxial compression for all materials. The results are reported in terms

of the average axial engineering stress and strain. Except for the SOT materials, all other materials exhibit a monotonically increasing engineering stress-strain response under uniaxial compression. Due to the possible lack of macroscopic homogeneity of the deformation fields in the compression specimens, the reported axial strains must be interpreted as average strains or normalized axial displacements.

The oscillations in the stress-strain curve for the SOT material are characteristic for cellular materials such as metallic foams that exhibit true strain softening followed by important hardening during densification. The instability related to the force peak causes the localization of deformation in narrow bands. The width of the localization bands equates to the height of a single unit cell. The equivalent plastic strain contour plots extracted from simulations confirm this mesostructural deformation mechanism (Fig. 11c). The other mesostructures remain stable under uniaxial compression. The deformation is therefore uniformly distributed at the macroscopic level. For example, the contour plot of the equivalent plastic strain for the HOT material (Fig. 11d) is periodic at the mesoscopic level which corresponds to a homogeneous field at the macroscopic level. The advantage of the HTS over the HOT at small strains is less pronounced under uniaxial compression than for hydrostatic compression. For the HH material at 5% engineering strain, the HTS macroscopic deformation resistance is only 15% higher than that of the HOT (20% for LH, Fig. 11), against 20% (30% for LH) for confined compression (Fig. 10a) and 40% (60% for LH) for hydrostatic compression.

The specific energy absorption is defined as the ratio of the mechanical work performed under uniaxial compression up to an engineering strain of -0.5 and the mass,

$$SEA = \frac{1}{\rho} \int_0^{-0.5} \sigma d\varepsilon. \quad (13)$$

We find = 10.6, 14.5, 15.1, and 16.3 J / g for the SOT, HSA, HOT and HTS materials made from the high strain hardening steel alloy. For the low strain hardening polymer, the corresponding values are = 6.15, 8.8, 9.5, and 9.3 J / g for the respective meta-materials.

4. Experiments

The main purpose of the experiments is to support the conclusions drawn from our computational study. Instead of using experiments as a primary means to investigate the deformation response of additively-manufactured metamaterials, the experiments are just performed here to validate the main qualitative features.

4.1. Specimens

Cubic compression specimens are manufactured for all four material architectures for a target relative density of $\rho = 0.2$. All cubic specimens are composed of $5 \times 5 \times 5$ unit cells. The specimen edge length is 47.1mm which corresponds to a unit cell edge length of 9.43 , and an inter-nodal distance of 6.67 . Characteristic geometric features of the meso-structures are as follows:

- strut diameter $t = 1.3$ for the SOT material,
- sphere wall thickness $t = 340\mu\text{m}$ for the HSA material,
- outer and inner strut diameters $= 2.17$ and $= 1.53$ for the HOT material (i.e. a wall thickness of $= 320$),
wall thickness $t = 350\mu\text{m}$, sphere outer diameter 5.15 , and an outer
 strut diameter of $= 2.39$ for the HTS material.

The specimens tested are made from stainless steel 316L using Selective Laser Melting (SLM). All specimens except for the SOT are built with layers parallel to two of the cube faces. After manufacturing, all specimens are cleaned in an ultra-sound bath. Figure 12 provides an 3D view of representative specimens along with detailed side views of the respective unit cells.

Even though the overall linear specimen dimensions are met with an uncertainty of less than 0.2% , we observe systematic deviations in the specimen weight. The measured relative density of the SOT specimens is only 0.19 , while it is 0.21 for all other specimens. The slightly higher weight of the specimens with two distinct porous phases is tentatively attributed to a combination of residual non-molten powder within the structure, and additional material attached to the overhanging, down-facing surfaces of the structures. The powder bed supports the structure during its layer-by-layer build-

up. Inevitably some of the supporting powder is partially molten. As a result, we observe residual powder at downward facing cell walls (Figs. 13a and 13b), while no traces of partially molten powder are seen on upward facing walls (Fig. 13c). To facilitate the interpretation of the traces seen on the photographs (Figs. 13a-13c), the corresponding depths profiles are provided in the subsequent row of figures (Figs. 13d-13f). In the side view (Fig. 13b), the residual powder is only seen on the downward facing walls. Traces of some excess powder in the HTS structure are shown in Fig. 13g. We also observed some strut diameter variations in the SOT structure (Fig. 13h) and hollow struts of the HOT structure that are obstructed by excess powder (Fig. 13i), while in some cases some thin-walls collapsed (Fig. 13c).

4.2. Experimental procedures

4.2.1 Young's modulus measurements

A series of non-destructive compression experiments is performed to measure the effective Young's moduli of the metamaterials. Given that the wave propagation response of the present metamaterials is not known yet, we take a conventional static testing approach instead of measuring wave propagation speeds to deduce the Young's modulus. The specimen is placed between two polished steel platens of a hydraulic universal testing machine (Model 8800, Instron, USA). It is subsequently pre-loaded up to an axial stress of about 9MPa to ensure good contact between the specimen and the loading platens. A sinusoidal force loading history of a frequency of 0.1Hz is then applied with an axial stress amplitude of $\pm 6\text{MPa}$. Two cameras are employed to monitor the deformation of two opposing specimen sides (front and back sides). The corresponding strain response is then measured using virtual extensometers. As illustrated in Fig. 14a, the virtual extensometers are positioned around the central row of unit cells, where the possibly confining effect due to the friction at the specimen boundaries is the lowest. The weighted average of all virtual extensometers is reported as the axial strain, thereby reducing experimental uncertainty related to the non-parallelism of the top and bottom loading platens. The camera system took images at a framerate of 5Hz with 1500 pixels along the vertical frame direction. With the cameras (2448x2048 pixels, Model GS3-U3-51S5M, PointGrey) positioned at a distance of

about $0.45m$, the size of one pixel is $30\mu m$. This corresponds to 300 *pixels* along one unit cell length. For a stress increment of $12MPa$, we expect a relative displacement of at least $9.43mm \times 12 / (0.2 \times 200000) \cong 2.8\mu m$, i.e. about 0.1*pixels* which is still in the range of the subpixel resolution of the VIC2D algorithm. To approach the optimal measurement accuracy of the DIC code, a speckle pattern with an average black speckle size of about $60\mu m$ is applied. Preliminary experiments using an optical microscope (mounted horizontally at a distance of less than $25mm$) were not successful due to significant artificial strains related to the out-of-plane displacements (due to the Poisson's effect).

4.2.2 Large deformation response

The compression experiments are performed on a screw-driven universal testing machine (Model RME250, Zwick) equipped with two flat loading platens and a 250kN load cell. No grease or lubricant are applied to the polished steel platens. Two high resolution digital cameras (2448x2048 pixels, Model GS3-U3-51S5M, PointGrey) equipped with lenses (Model AF-X M100 Pro D, Tokina) are employed to monitor two orthogonal specimen surfaces at a spatial resolution of about $30\mu m / pixel$. A constant crosshead velocity of $6mm / min$ is applied (which equates to a constant engineering strain rate of about $2.1 \times 10^{-3} / s$ for specimens with homogeneous macroscopic strain fields). Different from the simulations which were all stopped at a strain of -0.6, we continued the experiments until the maximum machine loading capacity of 250kN had been reached. Average engineering stresses are obtained from the force measurements, while average engineering strains are deduced from the top platen displacement measurement after correction for a machine stiffness of $100 / .$

The main purpose of the optical system is to monitor the deformation mode, but we also make use of the natural contrast in the images to estimate surface strains through planar Digital Image Correlation (DIC) with the software VIC2D (Correlated Solutions). Given the depth to the monitored surfaces, only the points located on the closest surface (on the foreground) are extracted and used for DIC. In the DIC algorithm, Gaussian weights are used over a window of 13 pixels in size and one measure every 5 pixels. Strains are calculated using Gaussian derivative estimators of

size 5. In all experimental figures, we show the effective surface strain. Contour bars are omitted as the sole purpose of the DIC results is to highlight regions of severe plastic deformation. Higher quality images and 3D correlation would be needed to draw quantitative conclusions from the DIC in the large deformation experiments.

4.3. Experimental results

4.3.1 Elastic response

After determining the strain history from the vertical extensometers, we verified that it has the same frequency as the applied stress history. Furthermore, the applied stress and measured strain histories are found to be in phase, i.e. possible viscous effects are negligible at a frequency of 0.1Hz . In other words, we may use the relation

$$E = \frac{d\sigma}{d\varepsilon} \quad (14)$$

to determine the Young's modulus. Due to the noise observed on the strain measurements, sinusoidal signals are fitted onto both force and strain signals. The obtained moduli for the four meta-materials are depicted as red solid dots in Fig. 14b. The SOT material exhibited with 7.1GPa the lowest modulus. The measured moduli of the HSA and HOT materials are about 9GPa , while the highest modulus of 10.4GPa is observed for the Hybrid Truss-Sphere (HTS) structure. Experiments were performed on one specimen per mesostructure only.

4.3.2 Large deformation response

Figures 15 to 18 summarize the engineering stress-strain curves obtained from uniaxial compression experiments on all four metamaterials at a relative density of 0.2. These figures also include the photographs (with superposed surface strain fields) that show the front and side view of the specimens at different stages of loading. For each of the four geometries, two tests are performed to verify the repeatability of the reported experimental results. The main experimental observations are:

- For the SOT material, an oscillating stress-strain response is observed at the macroscopic level (see red and blue solid curves in Fig. 15). After deforming in a linear-elastic manner up to a stress of about 20MPa , the stress-strain curve becomes non-linear, reaching a first peak in stress at about 35MPa . The deformation fields are still macroscopically uniform at this point, i.e. the surface strain variations are in phase with the periodicity of the mesostructure. After reaching a first peak in stress, a diagonal localization band forms of the width of approximately one unit cell (stage ①, Fig. 15). A minimum stress of 21MPa is reached at an engineering strain of about 0.17. A second peak of 33MPa is then observed at a strain of about 0.22. Thereafter, a second localization band becomes apparent (stage ②, Fig. 15). After the second peak, the stress-strain curve exhibits shallow oscillations in stress prior to the densification at a strain of 0.6.
- The first two localization bands in the SOT material are not adjacent to each other. Instead they seem to have formed independently, triggered by imperfections at the top and bottom boundaries, respectively (stages ① and ②, Fig. 15). They are also not flat. On the front plane, they are composed of segments that are either aligned with the (1,1,1) or the (1,0,0) directions of the FCC structure. After the formation of the first two bands of localization, the truss structure is progressively crushed, i.e. the zones of localization broaden through the collapse of the cells that are located at the interface towards the regions of intact mesostructure (stage ③).
- During crushing, the mesoscopic deformation mode is clearly bending-dominated. In the struts within the interface zones between crushed and intact regions, plastic hinges form near the nodes. The latter are supported by the struts in the intact regions in a way that their rotation is prevented.
- The experimentally-measured macroscopic engineering stress-strain curve for the HOT material (Fig. 16) is monotonically increasing. The surface strain fields preserve the periodicity of the mesostructure at all stages of deformation, with no localization visible at the macroscopic level. The only feature that is reminiscent of a cellular solid is the steep increase in hardening rate due to densification, at an engineering strain of about 0.5. The deformed

configurations of the HOT material show the change of the unit-cell geometry, which involves the rotation of the struts and the flattening of the nodes.

- The response of the HTS assembly (Fig. 17) is very similar to that of the HOT material, i.e. the macroscopic stress-strain response is monotonically increasing and the deformation fields respect the periodicity of the mesostructure. Mesoscopic features of the deformation response include a concentration of plastic deformation at the spherical nodes (see green zones at stage ① in Fig. 17) which are compacted by folding.
- The macroscopic stress-strain response of the HSA (Fig. 18) is also monotonically increasing. However, the specimen responds in a global shear mode. This can be clearly seen in the side views of the deformed configurations in Fig. 18. Due to the shearing, the normal stress distributions along the specimen boundaries also become non-uniform at the macroscopic level. For example, the compressive stress at the bottom left corner of the specimen (in the side views) is expected to be higher than the normal stress at the top left corner. As a result, the surface strain fields in the front view are graded, with the largest compressive strains near the bottom platen. This gradient effect is potentially amplified in the surface strain field due to artificial strains related to the change in camera to surface distance during shearing.

4.4. Comparison of experiments with simulations

Figure 14b shows the comparison of the Young's modulus estimates obtained from experiments and finite element simulations. The experimental results are systematically lower than the simulation results, with a maximum difference of 18% for the HSA structure. However, the same trends are observed in the experiments and simulations, with the SOT and HTS structures being the softest and the stiffest, respectively.

For large deformations, the experiments confirm the main qualitative conclusions drawn from our computational study. In both the simulations and experiments, the deformation in the SOT structures localizes at the macroscopic level when subject to uniaxial compression, while all other structures exhibit a macroscopically homogeneous deformation mode. Differences are the 3D nature of the crushed regions (localization bands) in the SOT experiments (Fig. 15) which reduce to a planar band of

growing width in the simulations (Fig. 11c). Boundary imperfections clearly affected the formation of the first two localization bands in the experiments. Larger specimens would be needed to reduce the effect of boundary imperfections on the crushing of the SOT structure. Different from the simulations (Fig. 11f), the HSA structure goes into a shear mode in the experiments which is also attributed to imperfections. It is striking in both the simulation and experimental results that the deformation fields in the HOT and HTS structures follow the periodicity of the mesostructure at all stages of deformation. The comparison of the surface deformation fields (e.g. compare Fig. 11d with configuration ② in Fig. 16 for the HOT structure, and Fig. 11e with the configuration

② in Fig. 17) shows qualitative agreement of the simulations and experiments with regards to the strain concentration patterns at the mesoscopic level. It is also worth noting that the macroscopic stress-strain curves lie perfectly on top of each other for repeat experiments.

The experimentally-measured stress-strain curves (colored solid curves) also agree reasonably well with the numerical predictions (dashed back curves). The quantitative agreement is surprisingly good given that the hardening response of the base material had just been estimated based on rudimentary material data provided by the manufacturer. The simulations systematically overestimate the stress level for the HOT and HTS configurations by up to 12% (see Figs. 16 and 17). Even smaller differences are observed for the HSA (Fig. 18), where the simulation curve meanders around the measured stress-strain curves. Aside from the uncertainty in the base material behavior, quantitative differences are due to the assumption of a spatially-uniform base material property distribution, the absence of imperfections in the simulations, and the difference in the size of the compression specimens. Recall that the simulation models included only $3 \times 3 \times 3 = 27$ unit cells while the experiments comprised $5 \times 5 \times 5 = 125$ unit cells.

The influence of the specimen size on the macroscopic stress-strain response is expected to be most pronounced for the SOT structure because of the localization of deformation at a length scale that is set by the unit cell size. We therefore repeated the simulations for the SOT structure using a FE model with 5^3 unit cells. The 3^3 -simulation (thin dashed curve) overestimates the initial peak stress, while an improved estimate is obtained with the 5^3 -model (thick dashed curve). Knowing the localization mechanism in the SOT structure, the width of the first peak is expected to be monotonically related to the ratio of the unit cell size and specimen height. Consequently, the macroscopic

strain (or normalized displacement, which is the more appropriate term in the case of the SOT specimens) at the first valley is smaller and closer to the experiment for the 5^3 -model than the 3^3 -simulation. The minimum stress in the first stress valley is significantly lower in the experiments (21MPa versus 30MPa in the simulations). This is attributed to the difference in the mesostructural deformation mode (inside the localization bands). In the experiments, the unit cells predominantly respond in a shear localization mode (see Figs. 15 and 19b), while a normal localization mode is observed in the simulations (see Figs. 11c and 19c). Kinematic constraints due to the small specimen size and non-planarity of the initial bands of localization prevent the progressive crushing of the SOT structure in a shear mode only. As a result, the initially pronounced quantitative differences between the simulations and the experiments vanish at large macroscopic strains. Densification also sets in earlier in the simulations than in the experiments which is consistent with the reported differences in the mesostructural deformation modes.

4.5. Discussion

The numerical simulations and the experimental results elucidate the poor mechanical performance of the conventional octet truss architecture at large macroscopic strains. The initial small-strain strength of the SOT is among the highest of the 20%-relative density metamaterials considered, but its mesostructure collapses in the plastic range. This behavior is observed for hydrostatic compression, confined compression and uniaxial compression. Overall, the large strain response of the SOT material at a relative density of 20% is still reminiscent of that of conventional cellular solids such as metallic foams or low density honeycombs. (e.g. Jang and Kyriakides, 2009). The response of the SOT structure becomes stable at larger relative densities. At the mesoscopic level, the 3D connections of the SOT struts (nodes) are usually subject to stress states close to hydrostatic loading. In the case of incompressible base materials (such as most metals), the nodes will thus not deform plastically which is a major drawback of the SOT structure as far as weight efficient energy absorption is concerned.

The other three metamaterials exhibit a stable mesostructural response within the range of relative densities considered. All structures feature FCC symmetry, but the mesostructural configuration of the HSA, HOT and HTS materials is fundamentally

different from that of the SOT material. The first three fall into the class of continuous shell structures. As compared to foams, their continuity order is still low, i.e. the surface curvatures are still discontinuous. However, despite their discontinuous surface curvature, the low order shell structures exhibit already a significantly higher stability under compression than the solid octet truss lattice. The simulation results also show that the plastic deformation concentrates near sharp angles (i.e. at curvature discontinuities) within the mesostructures, which is particularly inefficient from a specific energy absorption point of view.

As compared to the SOT material, the hollow sphere assembly (HSA) exhibits almost the opposite qualitative features: its small strain strength is low, but due to the progressive increase in internal contact within its compressed mesostructure, the HSA shows a macroscopically stable large deformation response that is accompanied by high macroscopic strain hardening under hydrostatic and confined compression. Since the high hardening commences after transforming the spheres into dodecahedra, we also repeated the unit cell simulations for $\rho = 0.2$ assuming a Rhombic Dodecahedron (RDH) as initial mesostructure. As shown by the dashed lines in Figs. 7a, 7b, 12a and 12b, the dodecahedron shell mesostructure is by far the strongest. The estimated small strain strength of the RDH is at least 75% higher than that of the other metamaterials. At large strains the RDH is prone to cell wall buckling which reduces its load carrying capacity, in particular in case of the low strain hardening base material. However, the overall stress level still remains high which makes it an attractive alternative to the metamaterial architectures studied in the previous sections. Due to unresolved issues encountered during the additive manufacturing of RDH specimens, the experimental confirmation of the exceptional mechanical performance of the dodecahedron shell mesostructure is deferred to future research.

It is reemphasized that results from compression experiments on cellular solids are potentially specimen size dependent. In the hypothetical case of zero friction between the specimen the loading platens, a free lateral boundary is less constraining than another column of unit cells, i.e. the formation of macroscopic localization bands is more prone to occur in small specimens. In the case of non-zero friction, the specimen cannot expand freely in the lateral direction and lateral compressive stresses are expected to build up. This friction boundary effect leads to the macroscopic barreling

which we observed in our experiments. The friction effect is expected to stabilize the specimen response. The comparison of the simulation results for 3x3x3 and 5x5x5 SOT specimens suggest that the specimen size effect is not significant for the SOT structures, while the experimental demonstration for all structures is deferred to future research.

The development of analytical models is also deferred to future research. Possible starting points are Gurson's (1977) approximate solution of the plastic response of a thick-walled hollow sphere. It will provide a valuable basis for developing an analytical model for the Hollow Sphere Assembly (HSA). There exists also an approximate analytical expression for the macroscopic yield surface of the SOT material (Mohr, 2005) which could also be extended to HOT materials.

5. Conclusions

The large deformation response of four different mesostructures of FCC symmetry are analyzed. The analysis includes the widely-used octet truss lattice material with solid and hollow trusses, a closed-packed hollow sphere assembly and a hybrid hollow truss sphere assembly. The unit cell simulations for hydrostatic compression reveal that the conventional octet truss lattice is prone to buckling failure for relative densities below about 0.3, with the exact transition density depending on the base material hardening capacity. As a result, the twisting mode, i.e. the rotation of the connecting nodes is observed at low relative densities due to the excessive plastic bending of individual struts. The mesostructures of the hollow sphere assembly turned out to remain stable for all relative densities considered (from 0.2 to 0.6), but it underwent substantial geometric changes. Individual spheres actually transformed into rhombic dodecahedra. This transformation maximized the contact surface between the constituent spheres which is accompanied by step increase in the hardening rate at the macroscopic level.

Both the simulations and experiments on stainless steel metamaterials made through selective laser melting demonstrated that the solid octet truss lattice material is the weakest among all 20%-relative density configurations considered. Its specific energy absorption capacity is 35% lower than that of the hybrid hollow truss sphere assembly. It is also shown computationally that the hollow dodecahedron shell

mesostructure provides the highest specific small-strain strength and energy absorption capability among all metamaterials considered. This leads to the tentative conclusion that periodic shell structures provide superior strength and energy absorption capacity for the same weight.

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Figures

Figure 1. Assumed stress-strain response of the (a) high strain hardening (steel-like) base material, and (b) the low strain hardening (polymer-like) base materials.

Figure 2. Composition of metamaterial architectures: (a) solid octet truss, (b) hybrid sphere assembly, (c) hollow octet truss, and (d) hybrid truss sphere assembly.

Figure 3: Details of the building blocks of the meshes used to discretize the geometries. Each unit-cell model consists in 96 rotated and translated copies of the shown details, combined with 96 copies of its mirror mesh.

a) SOT b) HSA c) HOT d) HTS.

Figure 4. Comparison of the computed macroscopic elastic properties: (a) Relative Young's modulus, (b) relative shear modulus, and (c) relative bulk modulus of the Solid Octet Truss (SOT) and the Hollow Sphere Assembly (HSA) as a function of the relative density. Each open symbol highlights a simulation result, while the dashed lines indicate the trends. (d) Comparison of the moduli for all four metamaterial architectures at a relative density of 0.2.

Figure 5. Solid Octet Truss (SOT) subject to hydrostatic compression: Stress-strain response for the (a) high, and (b) low strain hardening base material. (c)-(f) selected deformed configurations with equivalent plastic strain contour; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b).

Figure 6. Hollow Sphere Assembly (HSA) subject to hydrostatic compression: Stress-strain response for the (a) high, and (b) low strain hardening base material. (c)-(f) selected deformed configurations with equivalent plastic strain contour; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b). (f) presents the view of an individual sphere.

Figure 7. Metamaterials of 20% relative density subject to hydrostatic compression: Stress-strain responses for the (a) high, and (b) low strain hardening base materials. Selected deformed configurations with equivalent plastic strain contour for the (c)-(d) Hollow Octet Truss (HOT), and (e)-(f) Hybrid Truss-Sphere (HTS) assembly; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b).

Figure 8. Solid Octet Truss (SOT) subject to confined compression: Stress-strain response for the (a) high, and (b) low strain hardening base material. (c)-(f) selected deformed configurations with equivalent plastic strain contour; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b).

Figure 9. Hollow Sphere Assembly (HSA) subject to confined compression: Stress-strain response for the (a) high, and (b) low strain hardening base material. (c)-(f) selected deformed configurations with equivalent plastic strain contour; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b). (f) presents the view of an individual sphere.

Figure 10. Metamaterials of 20% relative density subject to confined compression: Stress-strain responses for the (a) high, and (b) low strain hardening base materials. Selected deformed configurations with equivalent plastic strain contour for the (c)-(d) Hollow Octet Truss (HOT), and (e)-(f) Hybrid Truss-Sphere (HTS) assembly; the macroscopic stress and strain corresponding to each subplot have been highlighted by an open dot in (a) and (b).

Figure 11. Metamaterial specimens of 20% relative density subject to uniaxial compression: Stress-strain responses for the (a) high, and (b) low strain hardening base materials. (c-f) face views of 3^3 -cell simulations with equivalent plastic strain contours. For clarity purposes, in the case of the SOT only the front surface is color-coded. The pictures correspond to the markers on (a).

Figure 12. Stainless steel compression specimens comprised of 5x5x5 unit cells fabricated through selective laser melting: (a) solid octet truss, (b) hybrid sphere assembly, (c) hollow octet truss, and (d) hybrid truss sphere assembly.

Figure 13. Printing defects on SLM-produced specimens. a-c) Surface finish and defects on HSA structure on a) bottom face b) side face c) top face. d-f) Surface defects with elevation measurements on HSA structure on d) bottom face e) side face f) bottom face. g) Detail of excess molten powder on an overhanging, down-facing surface of the HTS structure. h) Detail of uneven cross-section on a strut of the SOT structure. i) Angled view of the top face of the HOT structure: evidence of some obstructed channels.

Figure 14. Results of experimental measurement of Young's modulus. (a) Face-view of a SOT specimen with virtual extensometers. (b) Comparison of numerical estimates and experimentally measured values of Young's modulus for the four metamaterials considered.

Figure 15. Uniaxial compression response of SOT specimens: Engineering stress-strain curve as obtained from experiments (red and blue solid curves) and numerical simulations (dashed lines). The photographs in the left column show the front view of the deforming specimen. The corresponding side views are shown in the right column. The color indicates the deformation intensity as obtained from planar DIC.

Figure 16. Uniaxial compression response of HOT specimens: Engineering stress-strain curve as obtained from experiments (red and blue solid curves) and numerical simulations (dashed line). The photographs in the left column show the front view of the deforming specimen. The corresponding side views are shown in the right column. The color indicates the deformation intensity as obtained from planar DIC.

Figure 17. Uniaxial compression response of HTS specimens: Engineering stress-strain curve as obtained from experiments (red and blue solid curves) and numerical simulations (dashed line). The photographs in the left column show the front view of the deforming specimen. The corresponding side views are shown in the right column. The color indicates the deformation intensity as obtained from planar DIC.

Figure 18. Uniaxial compression response of HSA specimens: Engineering stress-strain curve as obtained from experiments (red and blue solid curves) and numerical simulations (dashed line). The photographs in the left column show the front view of the deforming specimen. The corresponding side views are shown in the right column. The color indicates the deformation intensity as obtained from planar DIC.

Figure 19. Schematic illustration of possible deformation modes of the Solid Octet Truss (SOT) structure when subject to uniaxial compression: (a) stable response with uniformly distributed macroscopic strains throughout the entire specimen, (b) normal strain localization band, and (c) shear localization band.