Modeling of thermal and mechanical behavior of a magnesium alloy AZ31 during electrically-assisted micro-tension

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Abstract

Many researchers have used a material response function termed "electroplasticity" to account for the mechanical behavior of metals subjected to electric current during plastic deformation. However, other researchers claimed that the electrically-assisted (EA) deformation behavior of metals could be successfully characterized using thermal-mechanical constitutive models without the need for electroplasticity theories. In order to examine the controversial mechanisms and determine which dominates the flow stress behavior under EA forming, this work established a flow stress model including the effects of strain hardening, rate hardening, thermal softening, solute-dislocation interaction and electron wind, where the latter three effects were assumed to contribute to the stress drop due to electric current. Additionally, an analytic thermal model was also established to capture the temperature variations during EA tension based on the energy balance between the heat generation due to Joule heating and the heat losses due to conduction and convection. Also, the evolutions of strain rate and strain at specimen center were incorporated into both models to capture the effects of diffuse necking on thermal and mechanical behaviors during EA tension. Uniaxial micro-tension tests were conducted on AZ31 magnesium alloy specimens subjected to continuous electricity with various current densities to verify the proposed models. Results show that the thermal and mechanical models can effectively predict the thermal and mechanical behaviors of the AZ31 magnesium alloy at various current densities in EA micro-tension, respectively. The modeling results also demonstrate that Joule heating is the major factor to affect the deformation behavior under micro-tension subjected to continuous electricity.

Keywords: electrically-assisted, micro-tension, electroplastic effect, magnesium alloy,

micro-forming

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1. Introduction

Recently, there has been growing interest in the experimental and modeling study of the forming limit of metals during micro/meso-scale plastic deformation (Ran and Fu, 2014; Ran et al., 2013). It was found that the formability decreased with the reduction of thickness-to-grain-size ratio due to the so-called size effect (Fu and Chan, 2011; Xu et al., 2015b). This undesirable effect on formability would be more severe for hard-to-form lightweight materials, especially magnesium alloys. Due to very few active slip systems in hexagonal close packed (HCP) crystal structure, plastic deformation of magnesium and its alloys is usually accommodated by both slip and twinning. As a result, magnesium and its alloys show very limited formability caused by significant flow asymmetry and heterogeneity, which is significantly affected by temperature, strain rate, strain path and micro-texture (Khan et al., 2011; Lévesque et al., 2010; Neil and Agnew, 2009). An increase in forming temperature can greatly reduce the degree of asymmetry and anisotropy (Jain and Agnew, 2007; Piao et al., 2012), which is still the most effective way to enhance the formability of magnesium alloys so far, like superplastic deformation (Babu et al., 2014; Watanabe et al., 2001). However, in most traditional processes including micro/meso-scale plastic forming, materials should be heated up to recrystallization temperatures by conduction/convection which is energy consuming as well as time consuming. Therefore, a more economical and ecological technique which can also overcome forming difficulties for hard-to-form metals is desirable.

Previous research has shown that the mechanical behavior of a metal can be modified by passing high densities of electric current through the metal during plastic deformation, i.e., EA

forming. The EA forming has been considered as an alternative to hot/warm forming (Perkins et al., 2007) since it can overcome various forming difficulties. Typical advantages associated with the EA forming include improved formability (Salandro et al., 2010), reduced flow stress (Perkins et al., 2007) and decreased springback (Green et al., 2009), etc, for high strength or hard-to-form materials at relatively lower temperatures.

It is necessary to understand the mechanism behind the current-induced effects so that it can be modeled and effectively applied within metal forming industries. Trotskii and Likhman (1963) first found a significant electron-induced stress drop and an improvement in ductility when electron irradiating Zn single crystals during plastic deformation. This current-induced effect on plastic flow was termed as the electroplastic effect (EPE), which was then attributed to an interaction between drift electrons and elastic fields of dislocations by Troitskii (1969). Afterwards, the influences of the drift electrons were extended by Sprecher et al. (1986) to other material parameters such as the vibration frequency of dislocation segments, the obstacle strength and the stacking fault energy. Note that most of the earlier studies were focused on the electron-dislocation interaction by means of reducing Joule heating and increasing the action of drift electrons using liquid nitrogen cooling or short-duration high-current pulses, which were reviewed in Kir'yanchev et al. (1983). As a result, classical theories were mainly established in terms of the electron-dislocation interaction including: (i) drift electrons can exert a "wind" on dislocations, i.e., electron wind, depending on the difference between the drift electron velocity and the dislocation velocity (Roshchutkin et al., 1979; Sprecher et al., 1986); (ii) drift electrons can enhance dislocation slipping velocity by influence on internal local stress field (Li and Yu, 2009); (iii) drift electrons can facilitate dislocation depinning by the magnetic field induced by current (Molotskii and Fleurov, 1995).

However, some of recent experimental studies showed that the above non-thermal EPE may be

insignificant when continuous electricity was applied. For example, Kinsey et al. (2013) applied a high strain rate (i.e., $\sim 10^3 \text{ s}^{-1}$) in the EA tension of 304SS and Ti-6Al-4V using a Kolsky bar with DC current supply (at current densities of 60 - 180 A/mm²), and found no EPE occurred during the tests and flow stress was strongly temperature dependent. Jordan and Kinsey (2015) also did not observe the EPE during the EA three-point bending of brass sheets and it was found that the variations in the bending force were caused by temperature effects. Magargee et al. (2013b) cooled the Joule heating temperature down to the room temperature during the EA tension of commercially pure (CP) titanium sheets using forced air, and found that the measured flow stress was consistent with that in the room temperature, which led them conclude that Joule heating dominated the plastic flow in EA forming, while the EPE was negligible.

More recently, the current-induced mechanism was extended for EA forming using pulsed current. Kim et al. (2014) observed the microstructure of aluminum alloy before and after necking in tension with 0.5 s duration, 110 A/mm² current pulses, and found that the microstructure before necking was similar to that of non-EA samples, while the one after necking showed recrystallization due to the dominant effect of Joule heating. Moreover, the comparison among the degrees of X-ray diffraction (XRD) line broadening for initial, nonpulsed, heat treated and pulsed samples at the strain of 0.075 demonstrated that the pulsed sample included the lowest dislocation density and indicated that current could induce annealing without exceeding the critical annealing temperature of the aluminum alloy. Similar results were also observed using electron backscatter diffraction (EBSD) by Roh et al. (2014) for aluminum alloy samples in pulsed EA tension, and they also concluded that the EA deformation behavior cannot be simply explained by Joule heating since dynamic strain aging (DSA) (Kabirian et al., 2014) may influence on the stress-strain curves. Magargee (2014) claimed that favorable conditions for DSA existed in pulsed EA tension since the

sharp temperature increase caused by Joule heating and the enhancement of the strain rate could improve the mobility of solute atoms. Therefore, it is still unclear whether the EA deformation is exclusive to thermally activated plastic flow resulting from the temperature rise due to Joule heating, or combined plastic flow also taking account of the direct electron-dislocation interaction and DSA.

Due to the considerable debating regarding the existence of the EPE, modeling of the plastic flow during EA forming is a challenging task and the reported models vary from case to case. Sprecher et al. (1986) experimentally found that the passage of electric current pulses during the tension of metals could enhance the rate of deformation. Then the strain rate enhancement associated with stress drops were modeled based on different current-induced effects, which found that skin, pinch and magnetostrictive effects were of less important, and elastic strains caused by the thermal expansion due to Joule heating only made a partial contribution to the enhancement of the strain rate. In order to account for a significant contribution to the enhanced rate of plastic flow due to the direct effect of drift electrons on dislocation motion, a rate-controlled model was proposed based on the thermally-activated plastic flow concept. However, it is questionable to use the thermally activated rate equation to model the EPE at the presence of Joule heating. Additionally, the calculated proportionality constant between the electron wind stress and the current density, i.e., the electron force coefficient, is very small with the order of 10^{-12} N mm/A. Salandro (2012) introduced a new material property, i.e., EPE coefficient, to divide the input electrical power into two parts based on energy conservation (one for plastic flow and the other for Joule heating), which was then incorporated into empirical material models to successfully capture the deformation behavior during EA compression and EA bending. However, it should be noted that the EPE coefficient is empirical and varies with time and current density, which is not suitable for general applications.

Some researchers questioned whether the EPE mechanisms were necessary to explain the

observed data in EA forming and proposed material models without using the theories associated with the EPE. Magargee et al. (2013b) modified the Hollomon model and the Johnson–Cook model using empirical temperature-dependent parameters, and they showed that the thermal-mechanical models can effectively predict the mechanical behavior of CP titanium during EA tension and compression tests. However, a relationship between Joule heating temperature and strain was used based on the EA tests of CP titanium, which may not be effectively used in other cases. Ng et al. (2014) also developed an analytical model only based on Joule heating and successfully predicted the channel depth during EA micro-rolling for surface texturing of Ti-6Al-4V and AA3003-H14. However, the temperature was calculated based on the adiabatic Joule heating conditions, which may overestimate the Joule heating effect.

The objective of this work is to understand and model how a continuous current affects the temperature response and flow stress behavior of a magnesium alloy AZ31 during EA micro-tension. An analytic solution of the temperature response is obtained based on the energy balance between the heat generation due to Joule heating and the heat losses due to conduction and convection. A combination of power-law hardening behavior, kinetics of dynamic recrystallization (DRX), semi-physical constitutive relation of DSA (Rusinek and Rodríguez-Martínez, 2009) and classical electron wind theory is used to model the flow stresses at various current densities in EA micro-tension, which is possible to divide the contributions due to Joule heating, solute-dislocation interaction and electron wind. It should be noted that the proposed thermal and mechanical models are implemented using the parameters obtained from the literature and linked by using the calculated temperatures in the thermal model during the implementation of the mechanical model, which would be more general and effective for various cases of EA forming.

2. Modeling of Joule heating

In order to establish the temperature model due to Joule heating, two assumptions which are schematically shown in Fig. 1 need to be made:

(i) The temperature distributions in the X (width) and Z (thickness) directions are neglected since the sizes in this two directions are relatively small in this study, and the variation of the temperature along the Y (length) direction in the gauge area follows a parabolic function of Y coordinates. This assumption would be justified by the empirical data obtained in the context of the study.

(ii) Since the first order calculation of potential heat loss due to heat convection reveals its value being about one order of magnitude larger than that from heat radiation over the temperature range in this study. Therefore, in this work, heat loss due to heat radiation is neglected. Note that this factor is recommended to be considered in future work when the temperature of specimens is higher than ~ 820 °C. The heating due to plastic work is also neglected since we focus on quasi-static tensile tests with the strain rate of 0.01 s^{-1} in our study. Thus, the heat is dissipated through conduction and convection.

According to the first law of thermodynamics, the capacitively stored heat rate P_{stored} in the sample gauge area should be the sum of the entering heat rate P_{Joule} due to Joule heating and the heat loss rate due to conduction P_{cond} and convection P_{conv} , as shown in Fig. 1, that is,

$$P_{stored} = P_{Joule} + P_{cond} + P_{conv} \quad (1)$$

Neglecting the variations in the width x_0 and the thickness z_0 due to the small sample sizes, the thermal model could be simplified into a one-dimensional case. Therefore, the items in Eq. (1) can be expressed as follows,

$$P_{stored} = c \rho_m x_0 z_0 \Delta y \frac{\partial T}{\partial t} \quad (2)$$

$$P_{Joule} = \rho_e x_0 z_0 \Delta y J^2 \quad (3)$$

$$P_{cond} = -kx_0z_0 \left| \frac{\partial T}{\partial y} \right| \quad (4)$$

$$P_{conv} = -2h(x_0 + z_0)\Delta y(T - T_R) \quad (5)$$

Note that, c , ρ_m in Eq. (2), ρ_e in Eq. (3), k in Eq. (4) and h in Eq. (5) are the specific heat capacity, the mass density, the electrical resistivity, the thermal conductivity and the heat transfer coefficient under natural convection of the studied material, respectively, J the current density, T the relative temperature, T_R the room temperature, t the time.

Since a rather small elastic strain occurs at the onset of turning on electricity (Magargee, 2014), we assume that no plastic strain occurs when the sample is held stationary while electric current density of J_{sta} is passing through. Based on the assumption (i), i.e., the parabolic temperature distribution, the temperature profile in the Y direction can be expressed as,

$$T_{sta}(t, y) = f_{sta}(t)(-a_{sta}^*y^2 + 1) + T_R \quad (6)$$

at time t , where y is the coordinate relative to the center of the gauge area, a_{sta}^* is associated with the thermal gradient and defined as thermal gradient factor, $f_{sta}(t)$ is the function of temperature rise due to electric current (compared to T_R) at the center with time.

Substituting Eq. (6) into Eqs. (1) - (5), we have,

$$f'_{sta}(t) + M(y)f_{sta}(t) = N(J_{sta}, y) \quad (7)$$

where the coefficient functions are,

$$M(y) = \frac{2k|-a_{sta}^*y|}{c\rho_m\Delta y(-a_{sta}^*y^2 + 1)} + \frac{2h(x_0 + z_0)}{c\rho_mx_0z_0} \quad (8)$$

$$N(J_{sta}, y) = \frac{\rho_e J_{sta}^2}{c\rho_m(-a_{sta}^*y^2 + 1)} \quad (9)$$

Solving the first order linear differential equation, i.e., Eq. (7), the general expression of $f_{sta}(t)$ is,

$$f_{sta}(t) = \frac{N(J_{sta}, y)}{M(y)} [1 - \exp(-M(y)t)] \quad (10)$$

Using the boundary conditions, i.e., $f_{sta}(t) = 0$ at $t = 0$ and $y = \Delta y/2 (\Delta y \rightarrow 0)$ at the center, Eq. (10) can be simplified as,

$$f_{sta}(t) = \frac{N(J_{sta}, 0)}{M(0)} [1 - \exp(-M(0)t)] \quad (11)$$

where the coefficients are,

$$M(0) = \frac{ka_{sta}^*}{c\rho_m} + \frac{2h(x_0 + z_0)}{c\rho_m x_0 z_0} \quad (12)$$

$$N(J_{sta}, 0) = \frac{\rho_e J_{sta}^2}{c\rho_m} \quad (13)$$

Then, the maximum temperature at time t during the stationary Joule heating test is,

$$T_{sta}^{max}(t) = \frac{N(J_{sta}, 0)}{M(0)} [1 - \exp(-M(0)t)] + T_R \quad (14)$$

According to Eq. (12) - Eq. (14), it is evident that the maximum temperature during EA tests without deformation not only depends on the thermal and electrical properties of material, but also relies on the process parameters such as the current density J_{sta} and the sample geometry size x_0 and z_0 . It should be noted that all the parameters in the above equations can be determined using data in literature, except for the thermal gradient factor a_{sta}^* . We assume that the magnitude of the quadratic term coefficient of Eq. (6) is $a_{sta} = a_{sta}^* f_{sta}(t)$. It is rational to believe that a_{sta} is a function of current density, which would be justified in the following context of the study. Also, it is reasonable to have two boundary conditions regarding their relationship, i.e., $a_{sta}|_{J_{sta}=0} = 0$ and

$\left. \frac{\partial a_{sta}}{\partial J_{sta}} \right|_{J_{sta}=0} = 0$. Thus, we can assume a_{sta}^* as,

$$a_{sta}^* = \frac{a_{sta}}{f_{sta}(t)} = \frac{c_1 J_{sta}^2}{f_{sta}(t)} \quad (15)$$

where c_1 is a constant determined by stationary Joule heating tests in the context of the study.

If plastic strain takes place while passing through electric current, we first assume that the axial

strain $\bar{\varepsilon}$ is uniform over the entire gauge area. Similar to Eq. (6), the temperature distribution in EA tension can be obtained as,

$$T(t, y) = f(t, \bar{\varepsilon}) \left(-a^* y^2 + 1 \right) + T_R \quad (16)$$

where a^* is the thermal gradient factor at time t in EA tension. We assume that $f(t, \bar{\varepsilon})$ also follows the same formula as Eq. (11), i.e.,

$$f(t, \bar{\varepsilon}) = \frac{N(J_{sta}, \bar{\varepsilon})}{M(\bar{\varepsilon})} \left[1 - \exp(-M(\bar{\varepsilon})t) \right] \quad (17)$$

The difference is that the coefficients are functions of plastic strain. Assuming that homogeneous deformation occurs in the gauge area and no diffuse necking initiates prior to fracture, the actual current density J passing through specimens would be in inverse proportion to the transient cross-section area in the gauge length, that is,

$$J = J_{sta} \frac{A_0}{A} = J_{sta} \frac{y}{y_0} = J_{sta} \exp(\bar{\varepsilon}) \quad (18)$$

where A and A_0 are the cross-section areas at transient and initial time, respectively, y_0 and y are the initial and transient gauge lengths of specimens, respectively. Meanwhile, taking account of the plastic strains in the X and Z directions, we have,

$$x = x_0 \exp(\nu_x \bar{\varepsilon}) \quad (19)$$

$$z = z_0 \exp(\nu_z \bar{\varepsilon}) \quad (20)$$

where ν_x and ν_z are the Poisson ratio s in the width and thickness directions, respectively. For magnesium alloys, ν_x is normally different from ν_z due to their anisotropic behaviors. In this model, the material is assumed to be isotropic with $\nu_x = \nu_z = \nu$, and the value is -0.5 for uniaxial tension under the incompressible assumption, i.e., $\nu_x + \nu_z + 1 = 0$. According to Eq. (15) and Eqs. (18) - (20), the coefficient functions in Eq. (17) can be expressed as,

$$N(J_{sta}, \bar{\varepsilon}) = N(J_{sta}, 0) \exp(2\bar{\varepsilon}) = \frac{\rho_e J_{sta}^2}{c \rho_m} \exp(2\bar{\varepsilon}) \quad (21)$$

$$M(\bar{\varepsilon}) = \frac{ka^*}{c \rho_m} + \frac{2h}{c \rho_m} \left\{ \frac{x_0 \exp(\nu_x \bar{\varepsilon}) + z_0 \exp(\nu_z \bar{\varepsilon})}{x_0 z_0 \exp[(\nu_x + \nu_z) \bar{\varepsilon}]} \right\} = \frac{kc_1 J_{sta}^2}{c \rho_m f(\bar{\varepsilon})} \exp(2\bar{\varepsilon}) + \frac{2h}{c \rho_m} \left(\frac{x_0 + z_0}{x_0 z_0} \right) \exp(-\nu \bar{\varepsilon}) \quad (22)$$

Therefore, the temperature at the sample center during EA micro-tension is,

$$T(t, \bar{\varepsilon}) = \frac{N(J_{sta}, 0)}{M(\bar{\varepsilon})} \exp(2\bar{\varepsilon}) [1 - \exp(-M(\bar{\varepsilon})t)] + T_R \quad (23)$$

According to Eq. (23), the saturation temperature T^* at center can be obtained as,

$$T^* = T(\infty) = \frac{N(J_{sta}, \bar{\varepsilon})}{M(\bar{\varepsilon})} + T_R = J_{sta}^2 \frac{x_0 z_0}{2h(x_0 + z_0)} (\rho_e - kc_1) \exp[(\nu + 2)\bar{\varepsilon}] + T_R \quad (24)$$

Since all the parameters in Eq. (24) should vary little or be constant at saturation except for the electrical resistivity and the strain, we can take the derivative of Eq. (24) with respect to time as,

$$\dot{T}^* = J_{sta}^2 \frac{x_0 z_0}{2h(x_0 + z_0)} \exp[(\nu + 2)\bar{\varepsilon}] \{ \dot{\rho}_e + (\nu + 2)(\rho_e - kc_1) \dot{\bar{\varepsilon}} \} \quad (25)$$

As it can be seen in Eq. (25), the Joule heating rate during EA tension depends on the plastic strain, the plastic strain rate, the electrical resistivity and the electrical resistivity rate under a given initial current density, initial geometry size and material. It should be also noted that the positive contribution to the saturated temperature rate due to the electrical resistivity rate and the strain rate increases with plastic deformation. This can be used to account for the sharp temperature rise near the end of EA tension tests.

The saturation temperature rate is assumed to be proportional to the square of current density, i.e.,

$$\dot{T}^* = c_2 J^2 = c_2 J_{sta}^2 \exp(2\bar{\varepsilon}) \quad (26)$$

where c_2 is a constant determined by the regression analysis of experimental data obtained in this study. Substituting Eq. (26) into Eq. (25), we have,

$$\dot{\rho}_e + (\nu + 2) \dot{\bar{\varepsilon}} \rho_e = (\nu + 2) kc_1 \dot{\bar{\varepsilon}} + \frac{2c_2 h (x_0 + z_0)}{x_0 z_0} \exp(-\nu \bar{\varepsilon}) \quad (27)$$

In order to determine the evolution of electrical resistivity with time and strain, we assume that the strain rate is uniform over specimen gauge area, i.e., $\bar{\varepsilon} = \dot{\varepsilon}t$. Therefore, we can solve Eq. (27) as,

$$\rho_e = kc_1\dot{\varepsilon}\left\{1 - \exp\left[-(\nu + 2)\dot{\varepsilon}t\right]\right\} + \frac{c_2h(x_0 + z_0)}{\dot{\varepsilon}x_0z_0}\left\{\exp(-\nu\dot{\varepsilon}t) - \exp\left[-(\nu + 2)\dot{\varepsilon}t\right]\right\} + \rho_e^{sta} \quad (28)$$

where ρ_e^{sta} is the electrical resistivity at saturation under stationary Joule heating. According to Eq. (28), the electrical resistivity during EA tension should be associated with strain, strain rate for a given initial geometry size, heat transfer condition and material. Also, it is obviously observed that the electrical resistivity increases with plastic deformation, indicating that Joule heating temperature should increase faster and faster. This would be justified by the experimental data in this study.

In order to determine ρ_e^{sta} , we need to consider the special case that both plastic strain and plastic strain rate are zero, then Eq. (27) can be simplified as,

$$\dot{\rho}_e^{sta} = \frac{2c_2h(x_0 + z_0)}{x_0z_0} \quad (29)$$

It can be seen that the electrical resistivity rate under stationary conditions is a small constant depending on initial geometry size, heat convection condition and material property. By integrating Eq. (29), we can have the stationary electrical resistivity as,

$$\rho_e^{sta} = \frac{2c_2h(x_0 + z_0)}{x_0z_0}t + \rho_e^{ref} \quad (30)$$

where ρ_e^{ref} is a reference electrical resistivity which is temperature dependent and can be calculated within a time scale around 0.5 s using an iterative algorithm since Joule heating temperatures tend to be saturated in ~ 0.5 s. This would be justified by the experimental data obtained in the context of the study. It can be seen from Eq. (30) that the stationary electrical resistivity slightly increases with time, which may be the reason for the reported observations that the saturation temperature slightly increased with time during stationary Joule heating tests and the

increase rate was relatively consistent for a given current density and material (Salandro, 2012).

Note that Eq. (28) and Eq. (30) should be implemented using their incremental forms, i.e.,

$$\rho_e(t + \Delta t) - \rho_e(t) = kc_1 \dot{\bar{\varepsilon}} \left\{ 1 - \exp \left[-(\nu + 2) \dot{\bar{\varepsilon}} \Delta t \right] \right\} + \frac{c_2 h (x_0 + z_0)}{\dot{\bar{\varepsilon}} x_0 z_0} \left\{ \exp(-\nu \dot{\bar{\varepsilon}} \Delta t) - \exp \left[-(\nu + 2) \dot{\bar{\varepsilon}} \Delta t \right] \right\} \quad (31)$$

$$\rho_e^{sta}(t + \Delta t) - \rho_e^{sta}(t) = \frac{2c_2 h (x_0 + z_0)}{x_0 z_0} \Delta t \quad (32)$$

The last aspect that should be considered in the Joule heating model is that EA plastic deformation is usually nonuniform due to diffuse necking near specimen center. Equation (23) cannot capture the maximum Joule heating behavior of EA tension due to the diffuse necking. Therefore, the uniform strain $\bar{\varepsilon}$ in the model should be replaced by the transient strain at specimen center to take account of the diffuse necking. The relationship between the strain at center and the uniform strain would be given in the following context in this study. All coefficients used in this model are listed in Table 1.

Table 1. Material coefficients for thermal model in EA micro-tension tests of magnesium alloy AZ31.

Property	Symbol	Units	Value	Reference
Mass density	ρ_m	g/cm ³	1.77	Magargee (2014)
Room temperature	T_R	°C	25	Magargee (2014)
Convective heat transfer coefficient	h	W m ⁻² °C ⁻¹	13.2	Causone et al. (2009)
Reference electrical resistivity	ρ_e^{ref}	nΩ m	Temperature dependent *	Avedesian and Baker (1999)
Specific heat capacity	c	J g ⁻¹ °C ⁻¹	Temperature dependent *	Lee et al. (2013)
Thermal conductivity	k	W m ⁻¹ °C ⁻¹	Temperature dependent *	Lee et al. (2013)

Notes:

* The reference electrical resistivity, the specific heat capacity and the thermal conductivity are obtained by fit the data taken from the above mentioned literatures, respectively, i.e.,

$$\rho_e^{ref} = 0.161T + 87.38 \quad (33)$$

$$c = 4.846 \times 10^{-9} T^3 - 3.171 \times 10^{-6} T^2 + 7.574 \times 10^{-4} T + 0.989 \quad (34)$$

$$k = -1.39 \times 10^{-4} T^2 + 0.0962T + 82.234 \quad (35)$$

where the relative temperature is used in this Section.

3. Modeling of flow stress

The deformation modes of magnesium alloys include three individual slip systems and one twin system, i.e., basal slip, prismatic slip, pyramidal slip and extension twin, which exhibit various critical resolved shear stresses (CRSS) due to the varying natures of dislocation-dislocation interaction (Bertin et al., 2014; Oppedal et al., 2012), solute-dislocation interaction (Bhattacharyya et al., 2016; Raeisinha et al., 2011), grain-boundary-dislocation interaction (Bhattacharyya et al., 2016; Wang and Choo, 2014), thermal activation (Bhattacharyya et al., 2016; Couret and Caillard, 1985; Raeisinha et al., 2011) and rate sensitivity (Bhattacharyya et al., 2016; Obara et al., 1973) on these slip/twinning planes. Besides, the activities of the above deformation modes are quite sensitive to the angle between load direction and initial crystallographic texture (Liu and Wei, 2014; Wang and Choo, 2014). Therefore, a general constitutive modeling of the deformation behavior of magnesium alloys is quite challenging since they exhibit anisotropy, asymmetry, temperature-dependent and rate-dependent behavior. Due to these complex deformation modes, material models in terms of continuum approaches generally treat HCP metals as anisotropic rather than isotropic materials, which exhibit different yield functions (Cazacu et al., 2006) and hardening rules (Li et al., 2010) under different strain paths. For example, Kim et al. (2013) proposed a temperature-dependent elasto-plastic constitutive model using various hardening rules and yield functions based on three deformation modes, i.e., twinning, untwinning and slip, to describe the temperature-dependent asymmetric cyclic behavior of magnesium alloy sheets. Recently, more

popular models proposed for magnesium alloys are based on crystal plasticity because it is a physically based approach taking account of most of the above interaction behaviors. Moreover, it can be incorporated within various continuum approaches and criteria, which helps to broaden its application field like forming limit prediction (Lévesque et al., 2010; Neil and Agnew, 2009), texture evolution (Liu and Wei, 2014). In spite of these and various modeling approaches presented for magnesium alloys, an appropriate modeling of the EA flow stress is still difficult since there may exist differences of the deformation behaviors between EA tension and conventional tension and the EPE remains controversial.

As a result, it is necessary to analyze the EA flow behavior prior to modeling. Previous experimental studies have shown that an obvious diffuse necking is usually observed throughout various testing metals due to nonuniform temperature distribution during EA uniaxial tension (Salandro et al., 2009; Salandro et al., 2010). Considering the symmetric configuration of EA uniaxial tension, most plastic strain would be located in the vicinity of specimen center during diffuse necking. Thus, the corresponding plastic strain rate would be larger than that in the remaining gauge area. Due to the evolution of the nonuniform Joule heating temperature distribution, the plastic strain rate at the center would increase faster than that in the rest area as plastic deformation proceeds. Since AZ31 has been reported to exhibit positive strain rate sensitivity (PSRS) at room and elevated temperatures, especially at high temperatures and low strain rates (Khan et al., 2011), it is rational to assume that the EA micro-tension behavior in our study should be influenced by three aspects, i.e., the strain hardening effect, the thermal softening effect and the strain rate strengthening effect. The first one is caused by the creation of new immobile dislocations during plastic deformation. The second one is associated with the dynamic recovery (DRV) / DRX (Liu et al., 2008; Qin et al., 2010; Quan, 2013) which weakens dislocation multiplication intensity

through thermally activated dislocation annihilation. The third one is linked to the PSRS due to material anelasticity at elevated temperatures (Dreyer et al., 2010; Polesak et al., 2009). Since it is reported that the passage of electric current during plastic deformation could culminate in conditions conducive for DSA (Magargee, 2014; Roh et al., 2014) which is more pronounced during localized plastic deformation (Benallal et al., 2006), another factor that should be taken into account regarding the EA flow is the negative strain rate dependence of flow stress caused by DSA. This effect could decrease the flow stress in terms of weaker obstacles along dislocation slip path since the time available for diffusion of solute atoms to the dislocations temporarily arrested at the obstacles is shortened due to the increasing strain rate with plastic deformation (i.e., solute declustering effect).

Other mechanisms associated with the EA flow behavior include thermal expansion due to Joule heating, some side effects of current (i.e., skin, pinch, and magnetostrictive effects), EPE due to direct electron-dislocation interaction (i.e., electron wind). In this model, the side effects of current would be neglected due to their small contributions in most cases (Sprecher et al., 1986). Since the initial temperature rise due to Joule heating is very sharp, most stress drop due to the thermal expansion effect would occur at the onset of turning on electricity, which has been demonstrated in the pulsed current case (Magargee, 2014). However, we only focus on the flow stress behavior after the stabilization of the Joule heating temperature (i.e., only small temperature rise) under the application of continuous current, which makes the neglect of the thermal expansion effect possible in this model. Another neglected factor that should be mentioned here is the asymmetry and anisotropy of magnesium alloys. The reasons include: (i) The degree of the asymmetry and anisotropy of AZ31 tends to decrease as temperature increases due to the increasing activity of non-basal slip systems (Agnew and Duygulu, 2005; Jain and Agnew, 2007); (ii) Texture

evolution may be modest during EA uniaxial tension according to the EBSD examinations by Kim et al. (2014); (iii) The deformation twinning may be precluded since McNeal et al. (2009) reported that AZ31 showed no significant signs of twinning while undergoing EA uniaxial tension. (iv) The normal anisotropy (r-value) measured in Section 5.1.3 tends to be ~ 1 under the strain path in this study, although the annealed AZ31 sample is likely to include texture (Yi et al., 2010).

Based on the above analysis, the EA flow stress should be modeled by taking account of the combined behaviors of the viscoplastic flow resulting from temperature rise and strain rate enhancement due to nonuniform Joule heating, the solute-dislocation interaction due to DSA and the direct electron-dislocation interaction due to electron wind. Rusinek and Rodríguez-Martínez (2009) have proposed a thermo-viscoplastic material model with negative strain rate sensitivity (NSRS) based on the Rusinek-Klepaczko (R-K) constitutive relation (Rusinek and Klepaczko, 2001) at a wide range of strain rates and temperatures. This extended R-K model is deduced from an additive decomposition of the total stress, i.e.,

$$\sigma(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T) = \lambda \left(\sigma_{sh}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T) + \sigma_{ta}(\dot{\bar{\varepsilon}}^p, T) + \sigma_{ns}(\dot{\bar{\varepsilon}}^p, T) \right) \quad (36)$$

where σ_{sh} , σ_{ta} and σ_{ns} represent the flow stress components due to the strain hardening, the thermally activated softening and the NSRS, respectively, which are functions of plastic strain $\bar{\varepsilon}^p$, plastic strain rate $\dot{\bar{\varepsilon}}^p$ and temperature T . In our case, we neglect the elastic strain since it is much smaller than the plastic strain. Also, the evolution of the Young's modulus ratio λ with temperature is neglected since it changes little over our temperature range. Similarly, the EA flow stress σ_{EA} could be formulated based on the extended R-K model if we add a new term in Eq.(36) to take account of the contribution of the electron wind σ_{ew} as well, that is,

$$\sigma_{EA}(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T, J) = \sigma_{sh}(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T) + \sigma_{ta}(\dot{\bar{\varepsilon}}, T) + \sigma_{ns}(\dot{\bar{\varepsilon}}, T) + \sigma_{ew}(J) \quad (37)$$

where the first two terms represent the combined contributions to the EA flow stress due to the

strain hardening effect, the rate hardening effect as well as the thermal softening effect, and the last two terms are negative representing the contributions to the stress drop due to the DSA and the electron wind, respectively. These stress components would be elaborated in the following sections in turn.

3.1 Strain hardening and thermally activated components

The procedure for the calibration of the first two terms in Eq. (37) is quite complicated which needs to determine many parameters based on a vast amount of experimental data collected over a wide range of temperatures and strain rates (Rusinek and Klepaczko, 2001; Rusinek and Rodríguez-Martínez, 2009; Rusinek et al., 2010). Unfortunately, there is still no work dealt with the calibration of the R-K constitutive relation of magnesium alloys. Therefore, in this part we use a more straightforward and phenomenological approach to capture the strain hardening and the thermally activated components of the flow stress of a magnesium alloy AZ31 in EA uniaxial tension.

Typical flow curves of magnesium alloys during cold and hot deformation are schematically illustrated in Fig. 2. At medium temperatures, the shape of the flow curve is bounded under a saturated stress of σ_{sat} due to the counterbalance between the strain hardening and the DRV. At a critical strain ε_c , dynamically recrystallized grains start to appear at the original grain boundaries (Xia et al., 2013), afterwards, more and more newly grown grains are added in the process, causing more softening. Once an equilibrium between the dislocation accumulation and the DRX is reached at a certain amount of strain ε_p (i.e. peak strain), a plateau with a peak stress of σ_p will occur on the flow curve. The steep stress drop follows since the dislocations are annihilated in large numbers by the grain boundary migrations, and further material flow will enter into a steady state (denoted by σ_{ss}) because a new equilibrium is obtained.

In view of the above flow behaviors, many researchers (Qin et al., 2010; Sheng and Shivpuri, 2006; Shin et al., 2014) used piecewise functions to model the strain hardening and DRV/DRX behaviors of magnesium alloys at elevated temperatures. We assume the combination of the first two terms in Eq. (37) $\sigma^*(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$ satisfies two-stage equations as follows,

$$\sigma^*(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T) = \begin{cases} K(\dot{\bar{\varepsilon}}, T) \bar{\varepsilon}^{n(\dot{\bar{\varepsilon}}, T)} & \bar{\varepsilon} \leq \varepsilon_c \\ K(\dot{\bar{\varepsilon}}, T) \bar{\varepsilon}^{n(\dot{\bar{\varepsilon}}, T)} - D(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T) & \bar{\varepsilon} > \varepsilon_c \end{cases} \quad (38)$$

where the strength coefficient $K(\dot{\bar{\varepsilon}}, T)$ and the strain hardening exponent $n(\dot{\bar{\varepsilon}}, T)$ depend on the strain rate $\dot{\bar{\varepsilon}}$ and the temperature T , $D(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$ is the strain softening term controlled by DRV / DRX, also depending on the strain rate $\dot{\bar{\varepsilon}}$ and the temperature T . The kinetics of DRX is commonly described in terms of the DRX volume fraction X_{DRX} which can be expressed by a modified Avrami equation (Serajzadeh and Taheri, 2003) as,

$$X_{DRX} = 1 - \exp \left[-\beta_d \left(\frac{\bar{\varepsilon} - \varepsilon_c}{\varepsilon_p} \right)^{n_d} \right] \quad \bar{\varepsilon} \geq \varepsilon_c \quad (39)$$

where β_d and n_d are DRX parameters and $\varepsilon_c \approx 0.8\varepsilon_p$ (Serajzadeh and Taheri, 2003). n_d was found to be independent of strain rate and temperature with a value of 1.6, while β_d could be expressed as a function of the strain rate and temperature (Qin et al., 2010), i.e.,

$$\beta_d = 6.92 \times 10^{-2} \dot{\bar{\varepsilon}}^{0.0704} \exp \left(\frac{12070}{RT} \right) \quad (40)$$

where R is the universal gas constant. Additionally, by the assumption of pure DRX, Qin et al. (2010) proposed that the strain softening item, i.e., $D(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T)$, after the critical strain can be given by,

$$D(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, T) = (\sigma_{sat} - \sigma_{ss}) X_{DRX} \quad (41)$$

where $\sigma_{sat} = 1.2\sigma_p$ and $\sigma_{ss} = 0.62\sigma_p$ directly obtained from Qin et al. (2010).

It should be noted that all the parameters in Eq. (38) depend on the temperature and the strain

rate. In order to capture the combined effect of T and $\dot{\varepsilon}$ on the thermally activated flow behavior, the Zener-Hollomon parameter Z based on the Arrhenius equation is usually used, which is expressed as follows (Neil and Agnew, 2009; Qin et al., 2010; Takuda et al., 1998),

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{RT}\right) \quad (42)$$

where Q is the activation energy for deformation. It is reported that the peak stress σ_p is a function of the Zener-Hollomon parameter Z , that is,

$$Z = A_z \left[\sinh(\sigma_p) \right]^{n_z} \quad (43)$$

In order to quantify the constants A_z and n_z in Eq. (43) in all stress ranges, experimental data obtained from the warm / hot deformation of magnesium alloys in literatures (Balík et al., 2007; Barnett, 2007; Doege and Dröder, 2001; Jiang et al., 2006; Li et al., 2012; Liu et al., 2008; Lukáč et al., 2011; Qin et al., 2010; Quan, 2013; Takuda et al., 2005; Zhou et al., 2005) are utilized to perform regression analysis. As shown in Fig. 3, the following expression can give a good fit to the data,

$$\log(\sinh(\sigma_p)) = 9.94 \log(Z) - 64.64 \quad (44)$$

The strain rate and temperature dependence of the strain hardening exponent can be described by a power-law form (Bayraktar et al., 1993),

$$n(\dot{\varepsilon}, T) = \kappa(T) \dot{\varepsilon}^{\alpha_n} \quad (45)$$

where α_n has an average value of 0.01 (Bayraktar et al., 1993) which is weakly affected by temperature, $\kappa(T)$ is a function of temperature, which could be determined using the regression analysis of experimental data at a constant strain rate. Thus, data from literatures (Barnett, 2007; Jiang et al., 2006; Liu et al., 2008; Shin et al., 2014; Takuda et al., 2005; Zhou et al., 2005) are collected as shown in Fig. 4. Note that all the data in Fig. 4 were obtained under a constant strain

rate of 0.01 s^{-1} which is consistent with the strain rate in this paper. Since Jiang et al. (2006) found that at a given strain rate, the strain hardening exponent of a magnesium alloy increased with temperature below $200 \text{ }^{\circ}\text{C}$ but decreased above it due to the decreasing amount of twinning, we assume that $n(\dot{\varepsilon}, T)$ approximately follows a second order function of temperature at a given strain rate. Particularly for the strain hardening exponent collected at 0.01 s^{-1} in Fig. 4, the fit curve can be approximated using the following equation,

$$n_{0.01}(T) = -2.173 \times 10^{-6} T^2 + 0.00177 T - 0.20638 \quad (46)$$

Then, using the boundary condition, i.e., $n(\dot{\varepsilon}, T) = n_{0.01}(T)$ at $\dot{\varepsilon} = 0.01 \text{ s}^{-1}$, we could have $\kappa(T)$ as,

$$\kappa(T) = -2.28 \times 10^{-6} T^2 + 1.85 \times 10^{-3} T - 0.216 \quad (47)$$

For power-law hardening materials, it is known that the strain at the onset of necking is numerically equal to the strain hardening exponent (Chakrabarti et al., 1975; Kang et al., 2014; Ling, 1996) which is assumed to be the peak strain in our case, i.e., $\varepsilon_p \approx n(\dot{\varepsilon}, T)$. Therefore, we can approximate the strength coefficient in Eq. (38) as follows,

$$K_T(\dot{\varepsilon}, T) = \frac{\sigma_p(\dot{\varepsilon}, T)}{n(\dot{\varepsilon}, T)^{n(\dot{\varepsilon}, T)}} \quad (48)$$

3.2 Strain rate strengthening component

In Section 3.1, the strain rate $\dot{\varepsilon}$ reflects a kind of overall behavior in the entire gauge length which would be more applicable to uniform deformation. However, for EA deformation, the flow stress may be underestimated using the overall strain rate since diffuse necking would cause an accelerated strain rate in the vicinity of the center with respect to the surrounding gauge length of EA-tension sample. Therefore, it is necessary to determine the instantaneous strain rate at the center in the diffuse neck resulted from nonuniform Joule heating temperature.

Based on the approach presented by Dreyer et al. (2010), the force exerted on the cross section of the neck should be consistent with that of the rest gauge area, i.e.,

$$\sigma_n A_n = \sigma_u A_u \quad (49)$$

where A_n and A_u are the cross-section areas in the neck and the uniform deformation area, respectively, σ_n and σ_u are their corresponding stresses. We define a characteristic proportion η with $\eta = A_{n0}/A_{u0}$ (A_{n0} and A_{u0} are their corresponding initial cross-section areas) that leads to necking and replace the cross-section areas in Eq. (49) using their corresponding $A = A_0 \exp(-\varepsilon)$, that is,

$$\sigma_n \eta \exp(-\varepsilon_n) = \sigma_u \exp(-\varepsilon_u) \quad (50)$$

where ε_n and ε_u are the instantaneous strains in the neck and the uniform deformation area, respectively. Based on the power-law strain rate hardening of flow stress, Eq.(50) can be rewritten as,

$$\left(\frac{\dot{\varepsilon}_n}{\dot{\varepsilon}_u} \right)^m = \frac{1}{\eta} \exp(\varepsilon_n - \varepsilon_u) \quad (51)$$

where $\dot{\varepsilon}_n$ and $\dot{\varepsilon}_u$ are the strain rates in the neck and the uniform deformation area, respectively, m is the strain rate sensitivity exponent. In order to determine the difference between ε_n and ε_u in Eq. (51), as shown in Fig. 5, we assume that the instantaneous strain distribution along gauge length $\varepsilon(t, y)$ follows a parabolic function intercepting the Y-axis at the top end y_{top} and the bottom end y_{bot} of the gauge length, i.e.,

$$\varepsilon(t, y) = a_\varepsilon(t)(y - y_{bot})(y - y_{top}) \quad (52)$$

where $a_\varepsilon(t)$ is a parameter representing the evolution of the strain at the center with time. It should be noted that the actual $\varepsilon(t, y)$ follows Gaussian distribution functions which would be justified by the experimental data obtained in the context of the study. In the modeling case, we use

the parabolic function assumption for the sake of obtaining simpler analytic solutions to our problem. We define a kind of overall strain over the entire gauge length $\bar{\varepsilon}$ which should produce a consistent displacement in Y direction u_y with that produced by the parabolic strain distribution instantaneously, i.e., the shielded areas as shown in Fig. 5. Therefore, we have,

$$u_y = \int_{y_{bot}}^{y_{top}} \bar{\varepsilon}(t) dy = \int_{y_{bot}}^{y_{top}} \varepsilon(t, y) dy = \int_{y_{bot}}^{y_{top}} a_{\varepsilon}(t) (y - y_{bot}) (y - y_{top}) dy \quad (53)$$

Solving Eq.(53), we have,

$$a_{\varepsilon}(t) = -\frac{6\bar{\varepsilon}(t)}{(y_{top} - y_{bot})^2} \quad (54)$$

Then, the maximum necking strain can be obtained at the center ($y=0$), i.e.,

$$\varepsilon_{center}(t) = a_{\varepsilon}(t) y_{bot} y_{top} = -\frac{6y_{bot} y_{top} \bar{\varepsilon}(t)}{(y_{top} - y_{bot})^2} \quad (55)$$

Assuming $\bar{\varepsilon} = \varepsilon_u$ and $\dot{\bar{\varepsilon}} = \dot{\varepsilon}_u$, the maximum strain rate at the center $\dot{\varepsilon}_{center}$ can be obtained by substituting Eq. (55) into Eq. (51), that is,

$$\dot{\varepsilon}_{center} = \dot{\bar{\varepsilon}} \exp \left\{ -\frac{\left[\frac{6y_{bot} y_{top}}{(y_{top} - y_{bot})^2} + 1 \right] \bar{\varepsilon} + \ln \eta}{m} \right\} \quad (56)$$

In our study, the area of interest in length in the gauge area is $y_{top} = 1$ mm and $y_{bot} = -1$ mm. Also, substituting $\eta = 0.983$ (Dreyer et al., 2010) and $\bar{\varepsilon} = \dot{\bar{\varepsilon}} t$ into Eq. (56), we have,

$$\dot{\varepsilon}_{center} = \dot{\bar{\varepsilon}} \exp \left(\frac{\dot{\bar{\varepsilon}} t + 0.0343}{2m} \right) \quad (57)$$

Note that the strain rate sensitivity exponent varies with temperature and their analytic relationship can be obtained by the regression analysis of the data from Dreyer et al. (2010) using a second order function, that is,

$$m(T) = 0.07 - 3.827 \times 10^{-4} T + 8.252 \times 10^{-7} T^2 \quad (58)$$

Afterwards, we would replace the overall strain rate $\dot{\epsilon}$, i.e., 0.01 s^{-1} in our study, in all the equations in Section 3.1 using the instantaneous strain rate $\dot{\epsilon}_{center}$ at the center in Eq. (57) to capture the strain rate strengthening effect during EA tension.

3.3 DSA and electron wind components

DSA is one of the prevailing mechanisms for the serrated flow of metals, which refers to the dynamical pinning / unpinning of gliding dislocations caused by the diffusion of solute atoms. This process highly depends on the difference between the waiting time of mobile dislocations and the diffusion time of solute atoms to the mobile dislocations. If the former is larger than the latter, it is more likely to produce unstable flow since fully locked mobile dislocations tend to collectively unlock under higher stresses, causing instantaneous stress drops. In contrast, mobile dislocations cannot be locked and smooth flow would occur. Therefore, the important characteristic of jerky flow behavior is that it should be bounded by a certain range of strain rates and temperatures since both of them affect the waiting time and the diffusion time (Picu, 2004), which needs to be considered in the modeling of DSA. However, many relevant models are either too phenomenological without taking account of the bounded conditions explicitly (Benallal et al., 2006; Lebyodkin et al., 1996) or too physical with many hard-to-determine parameters (Picu, 2004). Note that it is very important to take the bounded conditions into account for the modeling of DSA in our case since the strain rate and the temperature at specimen center resulted from electrical current vary with plastic deformation. Therefore, we use a semi-physical formulation proposed by Rusinek and Rodríguez-Martínez (2009) to simulate σ_{ns} in Eq. (37) since it explicitly includes the bounded conditions, i.e.,

$$\sigma_{ns}(\dot{\epsilon}_{center}, T) = \sigma_0^{ns} \cdot \left\langle \log \left(\frac{\dot{\epsilon}_{trans}}{\dot{\epsilon}_{center}} \right) \right\rangle \cdot \left\langle 1 - \beta_{ns} \left(\frac{T_m}{T} \right) \log \left(\frac{\dot{\epsilon}_{center}}{\dot{\epsilon}_{max}} \right) \right\rangle \quad (59)$$

where σ_0^{ns} and β_{ns} are the reference stress drop due to DSA and the reciprocity between strain rate and temperature, respectively, and their values are assumed to be consistent with those obtained based on aluminum alloys, i.e., 0.01 MPa and 36 (Rusinek and Rodríguez-Martínez, 2009), respectively. $\dot{\epsilon}_{trans}$, $\dot{\epsilon}_{max}$ and T_m are material parameters representing the transition strain rate between PSRS and NSRS, the maximum allowable strain rate and the melting point, respectively, and their values can be obtained based on magnesium alloys in our study. The angle bracket in Eq. (59) is the McCauley operator which means $\langle \bullet \rangle = \bullet$ if $\langle \bullet \rangle \geq 0$ and $\langle \bullet \rangle = 0$ if $\langle \bullet \rangle \leq 0$. It should be noted that the serrated flow behavior has been mainly reported for BCC or FCC metals like aluminum alloys (Benallal et al., 2006; Lebyodkin et al., 1996; Rusinek and Rodríguez-Martínez, 2009). To the authors' knowledge, fewer relevant results were found for magnesium alloys, most of which were obtained from Mg-RE alloys (Jiang et al., 2011; Stanford et al., 2010; Wu et al., 2012), while only one was associated with Mg-Al-Zn alloys (Corby et al., 2004). This indicates that Eq. (59) may overestimate the stress component caused by DSA since Mg-Al-Zn alloys normally exhibit PSRS.

The last stress component in Eq. (37) is associated with electron-dislocation interaction. Theoretical consideration of this effect is that drift electrons can exert a push or "wind" on dislocations to ease glide resistance since the former velocity is much larger than that of the latter. Based on the classical law of momentum transfer, many researchers (Roshchutkin et al., 1979; Sprecher et al., 1986) have established the relationship between the electron wind stress σ_{ew} and the current density J as ,

$$\sigma_{ew} = \frac{M_{Taylor}}{b} K_{ew} J \quad (60)$$

where K_{ew} is the electron force coefficient, M_{Taylor} the Taylor orientation factor and b the

Burgers vector.

To sum up, our flow stress model established in this section for EA micro-tension has taken five effects into account including strain hardening, strain rate hardening, DRV / DRX, DSA and electron wind, which renders the deep understanding of EPE possible. It should be noted that the temperature mentioned in this section is the absolute temperature obtained based on the calculated results of Eq. (23). The coefficients used in the flow stress model are summarized in Table 2.

Table 2. Material coefficients for mechanical model in EA micro-tension tests.

Property	Symbol	Units	Value	Reference
Activation energy for deformation	Q	kJ mol^{-1}	136	Takuda et al. (1998)
Universal gas constant	R	$\text{J mol}^{-1} \text{K}^{-1}$	8.31	Takuda et al. (1998)
Electron force coefficient	K_{ew}	N mm/A	2.4×10^{-12}	Sprecher et al. (1986)
Taylor orientation factor	M_{Taylor}	/	2.5	Shen et al. (2013)
Burgers vector	b	nm	0.321	Frost and Ashby (1982)
Transition strain rate	$\dot{\epsilon}_{trans}$	s^{-1}	1	Jiang et al. (2011)
Maximum allowable strain rate	$\dot{\epsilon}_{max}$	s^{-1}	10^3	Ishikawa et al. (2005)
Melting temperature	T_m	K	903	Magargee et al. (2013a)

4. Experimental setup

Uniaxial micro-tension tests were performed on AZ31 magnesium alloy specimens which were prepared at Harbin Institute of Technology. Specimens were cut into the dogbone geometry with a gauge length of $2.5 \pm 0.1 \text{ mm}$ and a gauge width of $1 \pm 0.1 \text{ mm}$ using electrical discharge machining (EDM). Prior to the tests, the specimens were annealed at $400 \text{ }^\circ\text{C}$ for 2 h. In order to avoid the effect of surface burning and oxidation, the samples were mechanically polished to a thickness of $\sim 0.2 \pm 0.05 \text{ mm}$ for micro-tension tests.

Figure 6 shows the schematic diagram of the EA testing system at Northwestern University. Specimens were mechanically loaded using a scanning electron microscope (SEM) loading stage

with a DC motor. The stage was used, however, not conducted within the SEM environment, but rather in the indoor air environment. The tensile force was measured by a load cell of 100 N capacity with a resolution of 0.01 N. A constant current passed through specimens from the upper gripper to the lower gripper in the duration of EA micro-tension tests in this work, and the electricity was supplied by a rectifier-based DC power supply with the peak output of 3600 W. Two ceramic washers and four Teflon-covered steel screws were used to electrically insulate the SEM loading stage and the sensors from the charged wedge-grips. A factory-calibrated infrared camera with the thermal resolution of 0.1 °C and the temperature range of -20 ~ 900 °C was used for real time measurements of temperatures on the backside surface of specimens during experiments. In order to avoid inaccurate measurements of the temperature due to the variable emissivity of shiny metallic surfaces, a high-temperature pure black paint (Zynolyte Hi-Temp Paint, Aervoe Industries Inc.) was sprayed on the back side of the specimens, forming a thin layer of coating with an uniform emissivity of ~1, and the emissivity value was calibrated using thermocouples. The noncontact digital image correlation (DIC) method was used to obtain overall and local strains in the gauge area of the samples since it is more appropriate and accurate for miniaturized samples in EA tension. In order to produce random speckle patterns with high contrast for digital image analysis, a thin coat of high-temperature white paint (Zynolyte Hi-Temp Paint, Aervoe Industries Inc.) was sprayed on the front-side surfaces of the specimens first, and then highly atomized black speckles were sprayed on the white paint layer through an airbrush with a very fine tip. Additionally, it was found that a pressure of around 40 psi resulted in the best atomization of the black speckles for the geometry size of the specimens tested in this study. During experiments, a high resolution CCD camera was used to capture the deformed images of the patterned specimens. The captured images were then DIC processed using Ncorr (Blaber et al., 2015) which has been demonstrated to be an

alternative to commercial 2D DIC packages (Ramesh and Ramji, 2014). In this study, the tensile tests were conducted with a fixed strain rate of 0.01 s^{-1} for all the specimens until fracture, and every test was run at least three times for repeatability. Another point that should be mentioned in this work is that the mechanical load occurred after the thermal equilibrium of the Joule heating (i.e., 5 - 10 s) since initial sharp increase of temperature due to Joule heating may have additional effects on the mechanical behavior of materials.

5. Results and discussion

This section is organized in the following manner: Section 5.1 presents the experimental results including Joule heating, thermal gradient factors, saturation temperature rate, diffuse necking and flow stresses at various current densities; Section 5.2 gives the verification and analysis of the thermal and mechanical models through the comparison of the measured and calculated data.

5.1 Experimental results

5.1.1 Characteristics of Joule heating

Much work has shown that the common observations associated with the Joule heating during EA forming include short heating time, increased energy efficiency and non-uniform heating. As shown in Fig. 7(a), typical temperature profiles along the X and Y directions present that the maximum temperature is located at the center of the specimen. Due to the spatial resolution of our infrared camera, $\sim 0.17 \text{ mm/pixel}$, which is of the same order of magnitude of the specimen width of 0.8 - 0.9 mm (0.1-0.2 mm smaller than the designed value in our study, i.e., 1mm, due to the machining error of EDM), among the 7 data points presented in Fig. 7(a), two data points are outside of sample, two data points are near the edges and three data points are near the center. Note that the data near the sample edges in the width direction should be excluded since they have a relatively low confidence level due to the fact that the black paint sprayed on the edges may easily

crack and peel off and that EDM could cause ablation surfaces at the edges during sample preparations. Therefore, if we concentrate the rest 3 data points in the sample, they are quite consistent, which matches with the assumption of the thermal model in Section 2 that the temperature distribution in the width direction can be neglected because our sample size is quite small, easy to maintain an uniform temperature. Figure 7(b) shows the variation in the maximum temperature in the duration of the micro-tension test of the sample corresponding to that in Fig. 7(a). As it can be seen, the heating rate is very high at the heating stage with the temperature reaching ~ 71 °C from the room temperature within 0.5 s. Afterwards, the temperature is saturated within a few seconds due to the equilibrium between Joule heating and heat losses. During the deformation stage, the heating rate gradually increased with increasing time since the cross-section area decreased while maintaining a constant current during deformation. At the end of the test, the temperature started to soar and then dropped quickly at the center because cracks initiated and then propagated quickly through the specimen.

The maximum temperature histories at various current densities before and after the initiation of deformation are shown in Fig. 8(a) and Fig. 8(b), respectively. Each case was measured at least 3 times and the error bars are also shown in the plots. As it can be seen, the maximum temperature nonlinearly increases with the current density, and the higher current density causes larger varying ranges and standard deviations, i.e., $160 \sim 240 \pm 40$ °C at the current density of 130 A/mm^2 . As a matter of fact, the geometry sizes of our samples were not perfectly consistent due to machining error, grinding error and polishing error during sample preparation. Also, the sample-grip contact position, contact area and contact force varied from test to test since it is impossible to consistently clamp miniaturized specimens using hands. These errors would in turn cause the variations of electrical contact resistances at the top and bottom ends of specimens, specimen electrical resistance,

cross-section area, heat transfer conditions, etc. No matter how many variables the test includes, the Joule heating temperature rise would be in the form of $\Delta T = \zeta J^2$ where ζ is a function that depends on all the above variables. For tests with the same current density, if we take derivatives of $\Delta T = \zeta J^2$, it is obvious that the deviation of Joule heating temperature resulted from the above variables would be magnified by the square of current density. Therefore, at the higher current density the smaller variations in geometry sizes and clamping conditions (contact area, contact force, etc.) would cause the larger fluctuations of the capacitively stored heat in specimens since the relationship between Joule heating temperature and current density is nonlinear. It is also found that the maximum temperatures measured under stationary conditions are slightly smaller than those under EA tension, which is due to the continuously increased actual current densities with plastic strain under tension.

The temperature evolutions with time are also plotted along the locations in the Y direction during the stabilizing and the deforming stages, as shown in Fig. 9. Note that, the temperature at every location follows the same trend as at the center, and the spatial distribution at every moment follows a parabolic function.

5.1.2 Thermal gradient factor and saturation temperature rate

In order to determine the thermal gradient factor a_{sta}^* in the Joule heating model, stationary Joule heating tests were conducted. Basically, no deformation occurred during the passage of electric current through the samples. The current densities were changed with an interval of 10A/mm², meanwhile, the temperature distribution in the gauge area of the samples after saturation were measured for each current density. Afterwards, the magnitude of the quadratic term coefficient a_{sta} was obtained by the regression analysis of the temperature distributions along the Y direction using parabolic functions, as shown in Fig. 10. Based on Eq. (15), we have,

$$a_{sta} = c_1 J_{sta}^2 \quad (61)$$

where c_1 is equal to 1.05×10^{-3} by the regression analysis of the data in Fig. 10.

Taking account of the effect of plastic strain on current density, i.e., Eq. (18), the thermal gradient factor in EA tension is,

$$a^* = \frac{c_1 J^2}{f(t, \varepsilon)} = \frac{c_1 J_{sta}^2 \exp(2\varepsilon)}{f(t, \varepsilon)} \quad (62)$$

Since the maximum Joule heating temperature slightly increases with time at the stabilizing stage and the increase rate is relatively consistent, as shown in Fig. 7(b), we could obtain the slope of Joule heating temperature versus time, i.e., saturation temperature rate, by regression analysis using linear functions. Therefore, the saturation temperature rates at various current densities were also calculated based on the experimental data obtained in the above stationary Joule heating tests, as shown in Fig. 11. Based on Eq. (26), we can express the saturation temperature rate $\dot{T}_{sta}^*$ in the stationary case as,

$$\dot{T}_{sta}^* = c_2 J_{sta}^2 \quad (63)$$

where c_2 is 1.3×10^{-4} determined by the regression analysis of the data in Fig. 11.

5.1.3 Characteristics of diffuse necking

Local strain distributions along specimen length direction can be obtained from the DIC measurement in this study. Figure 12 shows the variations of width strains ε_{xx} and length strains ε_{yy} at different locations in Y direction with the overall strains in the gauge area of a micro-tension specimen subjected to 130 A/mm². It can be seen that as plastic deformation proceeds, more and more plastic strain concentrates in the vicinity of the specimen center at higher overall strains, while relatively consistent strain is observed across the whole specimen length at lower overall strains, e.g., 0.01 ~ 0.024 and 0.05 ~ 0.3 length strains at 0.017 and 0.176 overall strains, respectively. Both

the width strain and the length strain follow the same trend that the diffuse necking area broadens with plastic deformation and their distribution curves approximately follow Gaussian functions which correlate with the temperature distributions in Fig. 9.

Figure 13 shows the evolution of the length strain at center ε_{center} with the overall strain $\bar{\varepsilon}$ at various current densities. It can be seen that at lower overall strains, ε_{center} linearly increases with $\bar{\varepsilon}$ showing line slopes of 1.36, 1.18 and 1.09 for 130 A/mm², 91.1 A/mm² and 58.3 A/mm², respectively, which is due to the increase of strain concentration at center with current density. It should be noted that the amounts of the above slopes are smaller than the predicted value, i.e., 1.5, using Eq. (55) since Eq. (55) is based on the assumption that no local strains occur at the top and the bottom ends of the gauge area during tension. At higher overall strains, the curves in Fig. 13 exhibit deviations to the linear relationships and bend upwards, indicating enhanced strain rates at center. We use a second order function with zero intercept to capture this behavior due to diffuse necking, i.e.,

$$\varepsilon_{center} = c_3 \bar{\varepsilon}^2 + c_4 \bar{\varepsilon} \quad (64)$$

where c_3 are equal to 1.91, 0.96 and 1.35 for 130 A/mm², 91.1 A/mm² and 58.3 A/mm², respectively, and c_4 are 1.22, 1.12 and 1.04, respectively, by the regression analysis of the data in Fig. 13. If differentiating both sides of Eq. (64) with respect to time, we have,

$$\dot{\varepsilon}_{center} = \dot{\bar{\varepsilon}} (2c_3 \bar{\varepsilon} + c_4) \quad (65)$$

Since $\dot{\bar{\varepsilon}}$ is 0.01 s⁻¹ in this study, $\dot{\varepsilon}_{center}$ linearly increases with the overall strain at a given current density. Thus, Eq. (57) may over-estimate the strain rate at center, especially at larger overall strains due to its exponential form.

As shown in Fig. 14, the magnitude of the ratio of the width strain to the length strain at center $\varepsilon_{center}^x / \varepsilon_{center}$, i.e., the Poisson ratio ν_x , nonlinearly increases with the overall strain, indicating

increasing width contraction in diffuse neck during EA tension. The ratios are relatively consistent at higher overall strains, e.g., -0.56, -0.51 and -0.49 for 130 A/mm², 91.1 A/mm² and 58.3 A/mm², respectively, which are almost identical with the Poisson ratio assumed in Eqs. (19) - (20). Since the ratio decreases with current density, diffuse necking is more severe and plastic deformation is more located in the neck area caused by more nonuniform Joule heating temperatures at the larger current density. In order to capture these behaviors of the Poisson ratio in EA tension, we use a logarithmic function as,

$$\nu_x = c_5 - c_6 \ln(\bar{\epsilon}) \quad (66)$$

where c_5 are equal to -0.74, -0.641 and -0.625 for 130 A/mm², 91.1 A/mm² and 58.3 A/mm², respectively, and c_6 are 0.084, 0.062 and 0.0613, respectively, by the regression analysis of the data in Fig. 14.

In order to characterize the anisotropy of the AZ31 sheet during EA tension in this study, the r-value, representing as a macroscopic indicator of the degree of normal anisotropy due to texture or preferred orientation, is used. Note that r-value measurements are normally performed on AZ31 samples strained at the limit of uniform elongation (Agnew and Duygulu, 2005). However, it is no longer valid to use uniform deformation assumption due to diffuse necking in EA tension. Therefore, the r-value r for EA micro-tension is calculated locally at specimen center based on plastic incompressibility, i.e.,

$$r = \frac{\epsilon_{center}^x}{\epsilon_{center}^z} = -\frac{\epsilon_{center}^x}{\epsilon_{center}^x + \epsilon_{center}^y} \quad (67)$$

where ϵ_{center}^x and ϵ_{center}^z are the instantaneous strains in the width and thickness directions at specimen center, respectively. By substituting DIC measurements into Eq. (67), the r-values at various overall strains for different current densities are obtained, as shown in Fig. 15. It can be

seen that all the EA-tension samples tend to strain in a nearly isotropic fashion at center, exhibiting the value of normal anisotropy ~ 1 . This indicates that the texture evolution in the vicinity of specimen center is subtle due to the non-basal slip of $\langle c+a \rangle$ dislocations (Agnew and Duygulu, 2005) activated by Joule heating. Due to the increase of diffuse necking with strain in sheet plane, the r -values slightly increase with overall strains for all the current densities. At 130 A/mm^2 , the decrease in r -value at high strain levels is perhaps due to the additional decrease in the degree of texture evolution caused by DRX. For the non-EA tension case, the r -values are also ~ 1 at ~ 0.02 and ~ 0.05 , further indicating that the anisotropy of AZ31 would not play a significant role in the flow behavior under the strain path in this study. Additionally, the r -value at ~ 0.08 decreases to ~ 0.8 perhaps because the localized thinning in thickness direction occurs at center, causing fracture ultimately.

5.1.4 Characteristics of flow stress

Since the conventional calculation method of true stress is defined based on the uniform deformation assumption, it is not valid to be used for EA tension due to diffuse necking. In our case, the true stress should exert on the minimum cross section through specimen center due to the maximum axial strain ε_{center} , which could be obtained as,

$$\sigma_{true} = \frac{F}{\min(xz)} = \frac{F}{x_0 z_0 \exp(\varepsilon_{center}^x + \varepsilon_{center}^z)} = \sigma_{eng} \exp(-\varepsilon_{center}^x - \varepsilon_{center}^z) \quad (68)$$

where F and σ_{eng} represent the force and the engineering stress, respectively. Taking account of the assumption of plastic incompressibility, we have $\varepsilon_{center}^x + \varepsilon_{center}^y + \varepsilon_{center}^z = 0$, then Eq. (68) can be expressed as,

$$\sigma_{true} = \sigma_{eng} \exp(\varepsilon_{center}^y) \quad (69)$$

Considering that the normal anisotropy at specimen center is subtle, as shown in Fig. 15, we

could easily calculate the true stress during EA tension using the instantaneous axial strain at the center obtained by DIC. It should be mentioned that we cannot obtain the true stress with absolute accuracy using Eq. (69) since the stress in the diffuse necking area changes from uniaxial state to multiaxial state and the strain distribution in the minimum section may not be uniform. In spite of these, Eq. (69) at least captures a first approximation of the true stress after diffuse necking (Aronofsky, 1951).

Based on Eq. (69), the true stresses under uniaxial micro-tension with various current densities are calculated at different overall strains, as shown in Fig. 16. It can be observed that the flow stress decreases with increasing current density. It is worth noting that strain softening occurs and the strain hardening range is narrowed with respect to the elongation at the higher current density, for example, the ratio of the peak strain to the elongation for 91.1 A/mm^2 is 0.84 compared to 0.36 for 130 A/mm^2 . This trend is consistent with the results in the warm/hot tension of magnesium alloys, i.e., a rapid decrease in strain hardening capacity with the increase of temperature (Agnew et al., 2008; Qin et al., 2010; Shin et al., 2014). Another characteristic of the EA flow behavior is that the fracture strain is larger for the higher current density case. However, the amount of the elongation is smaller than those presented in hot micro-tension of AZ31 at the same strain rate of 0.01 s^{-1} , i.e., ~ 0.24 for the EA micro-tension at $J = 130 \text{ A/mm}^2$ and ~ 1.0 for the isothermal tension with $T = 200 \text{ }^\circ\text{C}$ (Xu et al., 2013; Xu et al., 2015a). It is found that this is a common nature (Ross et al., 2007) of EA tension applied by continuous DC current since the cross-sectional area of the specimen continuously decreases under tension, causing a continuous increase of current density and in turn excessive heating to prematurely melt the specimen, especially at the location of diffuse necking. The flow stress reductions at different strains for various current densities are shown in Fig. 17. The amount of the reduction slightly increases with plastic strain which is due to the increasing current

density with strain. Careful observations of Fig. 17 give that the flow stress decreases faster with the increase of current density at higher strain, which is due to the increasing thermal softening effect at the higher strain.

5.2. Model results

5.2.1 Prediction of Joule heating temperatures

In order to verify the developed Joule heating model, an iterative algorithm was developed to repeatedly update the values of the coefficients and the maximum temperature at every time increment. Comparisons between the experimental and the predicted results are carried out. Figures 18(a), (b) and (c) show the measured and calculated maximum temperatures at various current densities under the stationary Joule heating condition, the uniform EA tension and the EA tension with diffuse necking, respectively. It can be observed that the proposed Joule heating model gives a good estimate of the maximum temperature for AZ31 magnesium alloy under the application of electric current. The predicted errors of the maximum temperature, i.e., $(T_{\max}^{\text{cal}} - T_{\max}^{\text{exp}}) / T_{\max}^{\text{exp}} \times 100\%$, for Figs. 18(a), (b) and (c) are shown in Figs. 18(e), (f) and (g), respectively. For the stationary case, the model predicts the temperature with less than 5% error, i.e., $< 5\text{ }^{\circ}\text{C}$, at 58.3 A/mm^2 and 91.1 A/mm^2 , and less than 15% error, i.e., $< 20\text{ }^{\circ}\text{C}$, is obtained at 130 A/mm^2 . This is probably because a smaller over-estimation of the electrical resistivity or underestimation of the thermal gradient factor would cause a larger over-estimation of the temperature at the higher current density due to their nonlinear relationship. Additionally, the variations of the errors with time are due to the underestimation of saturation temperature rates. For the EA tension case, both temperatures at 58.3 A/mm^2 and 91.1 A/mm^2 are also well estimated with their predicted errors decreasing with time, which is due to the inclusion of the evolution of electrical resistivity with time in the model, i.e., Eq. (31). It should be noted that at 130 A/mm^2 , the model cannot capture the upward bending of the

temperature history profile based on the uniform deformation assumption, showing the predicted error of around -14% at ~25 s. However, the inclusion of diffuse necking causes a better match with this behavior, i.e., -4% error. As shown in Fig. 13, both strain and strain rate at specimen center vary and increase with time during diffuse necking, especially near the end of tension tests, which leads to increasing electrical resistivity rate based on Eq. (31) and then the increasing rate of temperature. This behavior is more obvious at the larger current density due to the higher Joule heating rate, i.e., Eq. (26).

5.2.2 Prediction of flow stresses

An algorithm was also developed to solve the functions in the flow stress model. It should be noted that various combinations of temperature, strain rate and strain at center based on different assumptions as well as experimental data were assigned to the flow stress model instantaneously for the determination of the true stress at center at every overall strain level in the EA micro-tension. Compared to the measured flow stresses at various current densities, it can be seen in Fig. 19 that the proposed model can capture the strain-hardening and the strain softening behavior during the EA micro-tension of AZ31 magnesium alloy. The predicted errors, i.e., $(\sigma_{cal} - \sigma_{exp}) / \sigma_{exp} \times 100\%$, in Figs. 19(a), (b), (c) and (d) at various strain levels are shown in Figs. 19(e), (f), (g) and (h), respectively, for different current densities. Note that the model can well predict the flow stress at 0 A/mm² with the predicted errors falling in $\pm 1\%$, which indicates that the data obtained based on the conventional tension of magnesium alloys in literature can be effectively used to estimate the mechanical properties of AZ31 in the micro-tension in our study.

The EA flow stress behaviors in Fig. 19(a) are predicted using the calculated temperatures in Fig. 18(b) and the constant strain rate of 0.01 s⁻¹. The predictions in Figs. 19(b) and (c) are obtained based on the same calculated temperature in Fig. 18(c), but different strain rates, i.e., the strain rate

predicted by Eq. (57) and the experimental strain rate obtained by Eq. (65), respectively. The flow stresses in Fig. 19(d) are predicted using the same strain rate as that in Fig. 19(c), but measured temperatures are inputted in this case. As it can be seen, the amount of underestimation decreases approximately in order of the predicted errors in Fig. 19(e), Fig. 19(g), Fig. 19(h) and Fig 19(f) during strain hardening stages. The significant negative predicted errors in Fig. 19(e) at strain hardening stages are caused by the underestimations of strain and rate hardening effects due to uniform deformation assumption and the over-estimation of DSA, while the over-estimation of the strain softening behavior at 130 A/mm² is due to the temperature underestimation near the end of the test in Fig. 18(b). The degree of underestimation in Fig. 19(f) is the least among the above cases, especially for 58.3 A/mm² and 91.1 A/mm², because the rate strengthening effect and the strain hardening effect, i.e., Eq. (45), are enhanced as well as the solute declustering effect, i.e., Eq. (59), is reduced due to the over-estimation of the strain rate at center. This trend is more significant near the ends of the flow curves since the strain rate at center increases with plastic deformation in an exponential form, i.e., Eq. (57). Figure 19(g) shows that the model underestimates the flow stresses for all the current densities and the predicted errors decrease with overall strains during strain hardening stages. The same trend is also shown in Fig. 19(h), even though the experimental temperature, the experimental strain rate and strain at center are used in the model implementation. As a matter of fact, most of the underestimations among the above cases are caused by the over-estimation of DSA. Additionally, the reason for the largest underestimation at the onset of plasticity is that the stress drop due to DSA decreases with plastic deformation according to Eq. (59), while the stress increases due to strain hardening and rate hardening increase with plastic deformation, thus leaving the largest difference in early plastic region. In order to verify these, flow stresses calculated at various current densities with the exclusion of DSA component are shown in

Fig. 20(a). It is observed from its predicted errors, as shown in Fig. 20(b), that the underestimations in Fig. 19 are significantly eliminated, especially for the flow stresses near the onset of plasticity. It should be noted that the predicted errors at 58.3 A/mm² and 91.1 A/mm² are less than $\pm 5\%$ which is quite close to the experimental data. The relatively larger stresses predicted at the higher strain levels for both cases may be associated with the truth that the parameter A_z in Eq. (43) increases with strain (Slooff et al., 2007). However, in this study it is a constant fitted at peak strains, which would over-estimate the flow stresses near the ends of the stress-strain curves.

The prediction errors at 130 A/mm² are also minor, i.e., $-8.4\% \sim 2.5\%$, below the strain of ~ 0.1 , while the calculated flow stress above the strain starts to decrease faster than the experimental one, causing increasing negative predicted errors, e.g., -19.8% at 0.15. This is because all the parameters in Eqs. (39)-(41) of DRX model are obtained based on uniform deformation assumption, while the model is applied using the strain at center, i.e., Eq. (64), which is larger than the uniform strain due to diffuse necking, thus causing the over-estimation of DRX. However, the trend of the strain softening for 130 A/mm² can be effectively predicted using the DRX model, indicating that the strain softening during EA tension is probably dominated by DRX. In order to further confirm the occurrence of recrystallization, a transmission electron microscopy (TEM) sample was prepared near the fracture tip of an AZ31 tensile specimen subjected to 130 A/mm², as shown in Fig. 21(a), which was carefully ground to $\sim 50 \mu\text{m}$ first, then thinned for $\sim 20 \text{ h}$ at 4° irradiation angle using Gatan PIPS ion milling at 4.2 keV until the thickness for the TEM observation was $\sim 50 \text{ nm}$. Afterwards, a field emission transmission electron microscope (Tecnai G2 F30, FEI Instruments Co., Ltd., USA) operated at 300 kV was used for the observations of recrystalline grains. Figure 21(b) shows multiple equiaxed recrystallized grains with the size of $\sim 1 \mu\text{m}$ in this area. The average fraction of DRV/DRX falls in 50%-60% by estimating DRV/DRX areas in multiple TEM

micrographs examined at different locations, which is almost consistent with the result predicted by Eq. (39). Therefore, DRV / DRX indeed occurs during EA tension at 130 A/mm² and this observation has also been demonstrated using EBSD in Al5052-H32 subjected to EA tension (Kim et al., 2014).

In order to understand the actual mechanisms that cause the stress drop of AZ31 subjected to continuous electricity in uniaxial tension, the stress drops at 91.1 A/mm² due to the effects of the thermal softening, the rate hardening, the solute declustering and the electron wind are calculated. Particular steps are: (i) The stress drop due to the thermal softening effect is calculated using Eq. (38) at the fixing strain rate of 0.01 s⁻¹ to exclude the strain rate hardening effect as well as at the measured temperatures to avoid the predicted temperature errors; (ii) The enhanced strain rate at specimen center is calculated using Eq. (65) rather than Eq. (57) to avoid the over-estimation of rate hardening effect, which is then inputted into Eq. (38) to calculate the combined stress drop resulted from the thermal softening and the rate hardening. Afterwards, the exclusive contribution due to the rate hardening would be obtained by subtracting the result in step (i) from that in step (ii); (iii) Those caused by the solute declustering and the electron wind would be directly obtained using Eq. (59) and Eq. (60), respectively. The results are shown in Fig. 22 at various strains which exhibit that the thermal softening effect accounts for the largest proportion of the total stress drop (i.e., more than 80 MPa), while very small contributions are caused by the EPE including the solute declustering and the electron wind. The stress drop due to the solute declustering effect decreases with the overall strain, while the stress increase caused by the rate hardening effect shows an opposite trend because both strain rate and temperature increase with plastic deformation. Since the DSA of Mg-Al-Zn alloys has been seldom reported as well as the predicted errors in Fig. 19 are mainly caused by DSA, it is rational to infer that the stress drop due to the solute declustering effect

in Fig. 22 should be significantly over-estimated. Considering that the stress drop due to the electron wind is even smaller with $\sim 0.002\text{MPa}$, we could conclude that the EA deformation behavior of metals subjected to continuous electricity may be modeled without taking account of the nonthermal current-induced effects.

The contributions to the total stress drop, i.e., stress drop component / total stress drop, at 91.1 A/mm^2 due to the thermal softening effect, the rate hardening effect and the EPE are also calculated for all the strain levels in Fig. 22. The contributions are then averaged for the thermal softening effect, the rate hardening effect and the EPE, as shown in Fig. 23, which are compared with the results obtained from pulsed EA tension (Jones, 2012). It should be noted that the pulsed EA tension tests were also conducted on AZ31 and the electric pulse had a square wave shape with parameters of 40 A/mm^2 current density, 1 s pulse duration and 60 s pulse period (Jones, 2012). Both case show that the maximum contribution comes from the Joule heating effect, while 10% or even smaller results from the EPE. We neglect the contribution due to the thermal expansion in the continuous EA tension since the Joule heating temperature increases at a very small rate, i.e., $\sim 2\text{ }^\circ\text{C/s}$ in average (as shown in Fig. 7(b)), during the deforming stage of the continuous EA tension. However, in the pulsed EA tension, the thermal expansion causes a very significant contribution, i.e., $\sim 29.3\%$, which is mainly due to the sharp increase and decrease of the Joule heating temperature at the onset of power-on and power-off.

6. Conclusions

In this work, the proposed thermal and mechanical models have been verified by the uniaxial micro-tension tests of AZ31 specimens subjected to various current densities and provided useful insights about the current-induced mechanisms. The following conclusions are drawn from this research:

(1) The Joule heating temperature model was established based on the energy balance between the heat generation due to Joule heating and the heat losses due to conduction and convection. Also, the evolution of electrical resistivity with diffuse necking was incorporated into the model to effectively capture the upward bending behavior of Joule heating temperature history profile.

(2) The thermal model can effectively predict the measured Joule heating temperatures of magnesium alloy AZ31 during stationary Joule heating tests and EA micro-tension tests with less than 13 % error and 10 % error, respectively. The inclusion of diffuse necking in the model could cause a better match with the thermal behavior at 130 A/mm² than that based on uniform deformation assumption, especially near the end of EA tension tests.

(3) The flow stress model has taken five effects into account including thermal softening, strain hardening, rate hardening, solute-dislocation interaction and electron-dislocation interaction, which would be possible to capture the dominant mechanism behind the EA deformation behavior.

(4) The flow stress model underestimated the EA flow stresses of AZ31 specimens at various current densities, most of which were caused by the over-estimation of DSA. The largest underestimation was produced in the uniform case due to the additional underestimations of strain and rate hardening effects, while the case of predicted diffuse necking showed the least underestimation due to the over-estimations of strain and rate hardening effects. The flow stresses calculated at various current densities without the inclusion of DSA component exhibited the best match with experimental data, showing less than 9% predicted errors during strain hardening stages.

(5) The strain softening behavior at 130 A/mm² was over-estimated with less than -20% predicted errors due to the over-estimation of DRX using diffuse necking strains, but the trend can be effectively predicted based on DRX. The occurrence of DRX was further demonstrated by TEM observations on a fractured specimen subjected to 130 A/mm².

(6) The division of the calculated stress drops at 91.1 A/mm² during EA micro-tension of AZ31 shows that the thermal softening due to Joule heating dominates the EA flow stress reduction, i.e., more than 80 MPa. A comparison of the average contributions to the stress drop due to various current-induced effects further shows that an insignificant contribution results from the EPE in continuous current EA micro-tension of AZ31 specimens.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 51475124, the China Scholarship Council under Grant No. 201306120136, and the National Science Foundation of the United States under Grants No. CMMI-1100787 and DMR-1121262. The authors are also grateful to Guohua Fan of the Harbin Institute of Technology for TEM characterizations as well as Justin Blaber of the Georgia Institute of Technology for providing open source codes of Ncorr.

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Figure captions

Fig. 1. Schematic diagram for the assumptions of Joule heating model.

Fig. 2. Typical flow curves during cold and hot deformation of magnesium alloys.

Fig. 3. Relationship between Zener-Hollomon parameter and peak stress of magnesium alloys in log-log scale.

Fig. 4. Relationship between strain hardening exponent and temperature of magnesium alloys at the strain rate of 0.01 s^{-1} .

Fig. 5. Schematic illustration of instantaneous strain distribution along gauge length in diffuse neck during EA uniaxial tension.

Fig. 6. Schematic diagram of EA testing system.

Fig. 7. (a) Temperature distribution in the X and Y directions of a representative EA micro-tension specimen at a current density of 91.1 A/mm^2 , (b) Maximum temperature history of the EA micro-tension specimen.

Fig. 8. Variations of maximum temperatures with time for specimens under the application of various current densities (a) without uniaxial tension and (b) with uniaxial tension.

Fig. 9. Temperature evolutions with time in the Y direction of an EA test specimen during the stabilizing and the deforming stages at the current density of 91.1 A/mm^2 .

Fig. 10. Relationship between a_{sta} and J_{sta} in stationary Joule heating tests.

Fig. 11. Relationship between saturation temperature rate and current density in stationary Joule heating tests.

Fig. 12. Distributions of width strain and length strain in Y direction at various overall strains for an uniaxial tension specimen subjected to 130 A/mm^2 current density.

Fig. 13. Relationship between length strain at center and overall strain in gauge area for various current densities.

Fig. 14. Variations of the ratio of width strain to length strain at center with overall strain in gauge area at various current densities.

Fig. 15. Normal anisotropy at specimen center as a function of overall strain at various current densities during EA micro-tension of AZ31 sample.

Fig. 16. Variations of true stress at center with overall strain in gauge area during EA micro-tension of AZ31 specimens at various current densities.

Fig. 17. Stress drops at various overall strains for EA micro-tension specimens subjected to various current densities.

Fig. 18. Thermal model predictions of maximum temperatures of AZ31 specimens subjected to various current densities under (a) stationary Joule heating, (b) uniform EA tension and (c) EA tension with diffuse necking, showing predicted errors in (e), (f) and (g), respectively.

Fig. 19. Flow stress behaviors predicted by flow stress model at various current densities for EA micro-tension specimens of magnesium alloy AZ31 when inputting (a) calculated temperatures in Fig. 18(b) and constant strain rate of 0.01 s^{-1} , (b) calculated temperatures in Fig. 18(c) and predicted strain rate, (c) calculated temperatures in Fig. 18(c) and experimental strain rate, and (d) measured temperatures and experimental strain rate, and their corresponding predicted errors in (e), (f), (g) and (h), respectively.

Fig. 20. Flow stresses (a) and errors (b) predicted by flow stress model at various current densities under the exclusion of DSA component using measured temperatures and experimental strain rate at center.

Fig. 21. TEM examination of an AZ31 tensile sample subjected to 130 A/mm^2 , (a) TEM sample prepared from the tip of the fractured sample, (b) a typical TEM micrograph showing multiple recrystalline grains.

Fig. 22. Stress drops at various strains due to the effects of thermal softening, solute declustering, rate hardening and electron wind for EA micro-tension specimens subjected to continuous current density of 91.1 A/mm^2 .

Fig. 23. Comparison of the average contributions to stress drop of current-induced effects between continuous current EA tension and pulsed current EA tension (obtained by Jones (2012)) of AZ31.