The Elastic-Plastic Transition of Metals

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ABSTRACT

Uniaxial stress-strain curves are known to exhibit significant curvature and hysteresis even in the nominally elastic regime, i.e. before the standard yield stress is attained. In order to probe the nature of this behavior, hundreds of high-precision loading-unloading-loading tensile tests were performed using 26 commercial sheet alloys exhibiting a wide range of strength, ductility and crystal structure. Corresponding analysis shows that:

1. There is no significant linear elastic region, that is, the proportional limit is 0 MPa. While the first increment of deformation shows a stress-strain slope equal to Young's modulus, progressive deviations of slope start immediately.
2. The shape of the transitional stress-strain curve can be represented by a simple one-parameter equation representing the "modulus reduction rate." It captures ~80% of the measured variation and can be determined from a single test. This approach reduces the error inherent in standard Young's modulus or chord modulus approximations by a factor of 3 to 6.
3. A "Universal Law" having no independently-determined parameters, i.e. no testing or fitting required, was developed. It captures ~90% of the variation represented by the one-parameter representation for the materials tested.

The practical and theoretical implications of these results are discussed. On the practical side, the results provide an immediate path to improving applied constitutive models in the transitional regime. An example of an application and results is provided. On the theoretical side, the consistency of the effect for a wide range of metals suggests answers to questions about the governing deformation mechanisms.

Keywords: steel, Young's modulus, microplastic transition, QPE, elastic-plastic transition.

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INTRODUCTION

This paper addresses is referred to as the “transition” region of deformation. For initial loading, this region extends roughly between zero stress and the yield stress. As will be discussed below, this region is known to be at least in part nonlinear, such that any attempt to extract a Young’s modulus from mechanical tests results in lower values than the one measured physically, i.e. by sound velocities. In spite of the known nonlinearity, most treatments of this transition phenomenon have considered it, or treated it, as a linear response with reduce stress-strain slope.

Linear Treatments

The chord modulus, E_{chord} , measured as the slope of a mechanical stress-strain curve in a uniaxial tension test up to yield (or similarly, by unloading) has long been known to be lower than the physically-measured Young’s modulus¹. Furthermore, the chord modulus decreases following plastic straining (Lems, 1963). Several forms describing the reduction of chord modulus with prestrain have been presented, including: bilinear laws (Morestin et al., 1996b; Zang et al., 2007), three-stage laws (Cleveland and Ghosh, 2002; Luo and Ghosh, 2003), polynomial laws (Yu, 2009; Zavattieri et al., 2009), sinusoidal functions (Zang et al., 2006), and power laws (Li et al., 2002). An exponential form was presented by Yoshida et al., (2002):

$$E_{\text{chord}} = E_0 - (E_0 - E_a)[1 - \exp(-\zeta \varepsilon_p)] \quad (1)$$

where ε_p is plastic pre-strain strain, E_{chord} is the slope of a straight line drawn through the two stress-strain endpoints for a nominally elastic loading or unloading excursion, E_0 and E_a represent the chord modulus for unstrained and highly strained materials, respectively, and ζ is a material constant.

The reduced-modulus phenomenon received little attention until Morestin and Boivin (1996a) showed that using the chord modulus enabled more accurate springback predictions than using standard handbook values. The typical reductions reported for the chord modulus range from 10% to 30% for a variety of metals (Morestin and Boivin, 1996a; Yoshida et al., 2002; Cleveland and Ghosh, 2002; Agnew and Weertman, 1998).

Nonlinear Treatments

Although typically treated in FE applications as linear elastic behavior with reduced slope² (Fei and Hodgson, 2006; Iwata et al., 2004), the observed transitional stress-strain response is clearly

¹ Linear elasticity in crystalline solids arises from the nature of interatomic bonding. Quantitatively, the physically measured Young’s modulus is proportional to the curvature of the atomic bonding vs. interatomic distance curve at the equilibrium interatomic distance. See, for example [Seitz, 1940].

² The majority of applied sheet forming simulations use handbook values of Young’s modulus; a small subset of those intended for springback application might use a measured chord modulus; virtually no applied simulations use a complex constitutive law that reproduces the nonlinearity of loading and unloading under nominally elastic conditions.

revealed as nonlinear whenever sufficient testing precision is used. Furthermore, a load/unload cycle exhibits a hysteresis loop that produces no net strain but dissipates work (Sun and Wagoner, 2011). The nonlinearity, whenever considered in applied problems, has been modeled in an elastic-only framework, even though such an approach is inconsistent with the observed hysteresis.

Exponential forms describing stress-strain and slope vs. strain are most-commonly proposed (Yoshida et al., 2002; Zang et al., 2013; Kim et al., 2013; Lee et al., 2013, Zhu et al., 2010, Sun and Wagoner, 2011). These are inspired in part by the discovery of a scalable, universal atomic binding relationship (Rose et al., 1983) that can be differentiated to obtain a stress-strain relationship as follows:

$$U(a) = \Delta U \times U^*(a^*) = -\Delta U (1 + a^*) \exp(-a^*) \quad (2)$$

where U is the total energy as a function of interatomic distance a or scaled interatomic distance a^* , and ΔU is the equilibrium binding energy. Differentiation and conversion to strain ε obtains (Argon, 1996) an equivalent relationship for the slope of stress-strain in a tensile test (tangent modulus, E_T):

$$E_T(\varepsilon) = E_0 (1 - \alpha \varepsilon) \exp(-\alpha \varepsilon) \quad (3)$$

where E_0 is the initial slope and α is a constant related to a in Eq. 2.

Bond stretching, such as is represented by Eqs. 2 and 3 is purely elastic, reversible, and energy preserving. This can be easily seen because the total energy is a state function of interatomic distance, with no path dependence. Therefore, if atomic bond stretching were the source of the nonlinearity observed in mechanical testing, there would be no hysteresis. Instead, a concave-down loading path would be traced by a concave-down unloading path.

Despite this inconsistency, Eq. 3 inspired a simplified equation to describe the observed modulus phenomenon (Zhu et al., 2010):

$$E_T(\varepsilon) = E_0 \exp(-A\varepsilon) \quad (4)$$

where A is a constant.

In practical cases, there is little difference between Eqs. 3 and 4, as will be shown later. In order to reproduce the observed hysteresis, Zhu et al. (2010) replaced the strain term in Eq. 4 by a generalized internal variable that is to be reset to zero at each stress path reversal in order to construct a 1-D hypoelastic model. Therefore, the transitional loading and unloading paths in a tensile test are identical when considered from the zero stress/strain point (loading) and from the unloading point (unloading). A loading/unloading cycle is reversible in the sense that the deformation is removed, but the loop is energy dissipating as indicated by the area contained within the loop.

Concurrent with the Zhu hypo-elastic developments, Sun and Wagoner (2011) proposed, developed, and implemented a three-regime general constitutive model with a linear elastic region, a classically plastic region, and a transitional region called QPE (for Quasi-Plastic-Elastic). The slope of the transitional stress-strain behavior following an assumed linear elastic region³ was represented as follows:

$$E_T = E_0 - A_1 [1 - \exp(-A_2 |d\varepsilon|)] \quad (5)$$

where E_0 was taken to be the Young's modulus corresponding to atomic bond-stretching, and $|d\varepsilon|$ was defined such that it was reset upon the onset of transitions. In particular, for a tensile test unloading leg, $|d\varepsilon|$ represents the negative of the increment of strain following the linear unloading portion⁴. As in Zhu's work, the loading and unloading paths were assumed to be symmetrical in terms of $|d\sigma|$ vs. $|d\varepsilon|$.

The QPE model reduced the error of springback FE simulations by factors of 2 to 6 as compared to using a chord modulus or a handbook modulus. This demonstrated the importance of incorporating nonlinearity in the transitional stress-strain response, similar to Morestin and Boivin's work (1996a) on the need to use a chord modulus instead of a handbook modulus. This conclusion was broadened to non-springback applications (automotive structure collapse) by Gandhi and Wagoner (2013) and Chen et al. (2016).

Sun and Wagoner (2011) also found that, to first order, the transitional response is strain-rate independent, with little difference between single-cycle vs. multi-cycle tests. Intriguingly, the hysteresis behavior of two steels having the same strength but markedly different microstructures⁵ was identical. These results suggest an unsuspected common aspect to the underlying mechanism among diverse alloys in terms of modulus nonlinearity and a first-order dependence on strength. The current work pursues the extent and nature of this commonality.

Chen et al. (2016) found extended commonality for 12 steels in 4 widely-varied grades. Figures 1 summarize their results, first in terms of loop width⁶ (Fig. 1a) and then in terms of initial slope, final slope, and chord modulus⁷ (Fig. 1b). Recent, but less-directly related literature has focused on modifications of QPE theory for HCP alloys (Lee et al., 2013), comparing the role of the

³ In Sun and Wagoner (2011), the assumed linear elastic region was defined (visually) as a fixed stress range equal to 0.3 times the yield stress (as defined by the unloading stress or 0.2% offset on initial loading). As introduced later in this paper, the current probes this simple assumption will be probed by optimizing the extent of both the linear and nonlinear regions.

⁴ Sun and Wagoner graphically selected a transitional stress between linear and nonlinear portions as being 1/3 the value of the total unloading stress. Probing the accuracy of this convention is one object of the current work, as presented below.

⁵ The softer DP780 steel was strain hardened until it reached the same flow stress as the DP980 exhibited initially. Therefore the strength of the DP780 was derived from dislocation multiplication in a softer composite structure (less martensite) of ferrite and martensite as compared to the harder structure (more martensite) of the DP980.

⁶ Several definitions of loop width were examined by Chen et al. (2016), with no essential difference from the simplest one, defined as the largest distance between the reloading and unloading strain for any value of stress.

⁷ The initial slope was defined by Chen et al. (2016) over the first third of the total stress difference during loading or unloading; the final slope was defined over the final quarter the total stress difference during loading or unloading.

modulus representation on springback (Lee et al., 2015; Zang et al., 2014) , the relationship of nonlinearity to single crystal properties (Hama et al., 2015) and twinning or phase transformation (Pavlina et al., 2015; Ghaffari Tari et al., 2013; Kim et al., 2013), various elastic-plastic equations to capture unloading behavior (Soldatos and Polymenakos, 2015; Mendiguren et al., 2013), behavior following reversals in bending (Zang et al., 2014), shear (Choi et al., 2015), and tension (Cullen and Kokolis, 2013).

(a)

(b)

Figure 1: Summary of mechanical unloading-reloading behavior for 12 steels in 4 grades: (a) variation of hysteresis loop width, δ , as a function of unloading stress, σ_u , and (b) slopes at various stages of unloading. The results are adapted from Chen et al. (2016). The error bars represent single standard deviations between measurements and the trend lines provided. E_{initial} is the best-fit slope through the first third of the unloading stress range; E_{final} is the best-fit slope through the last quarter of the unloading range.

Mechanisms Governing the Transition Region

Note: This sub-section is motivated in part by the results from the current work. As such, it would normally appear in a Discussion section. However, during peer-review it was requested to move this to an early point in this paper to facilitate reader comprehension of the results, which are surprising and novel. Particularly surprising is the disclosure of a “Universal Law” that governs the nonlinearity of the transition region in metals, independent of crystal structure (bcc, fcc, hcp) and even the combination of deformation mechanisms (slip, twinning, phase transformation).

Most of the mechanisms proposed to account for the so-called “modulus effect” are inconsistent with the behavior revealed by the current results. The simplest proposed mechanism is second-order (nonlinear) elasticity via atomic bond stretching (Powell and Skove, 1982). However, the nonlinearity of bond stretching for metals has been shown (Sun and Wagoner, 2011) to be smaller by orders of magnitude than the effect quantified in the current work. Furthermore, any such mechanism would be energy preserving, contrary to every result in the current work.

Twinning/detwinning, another proposed mechanism (Caceres et al., 2003; Hama et al., 2013; Wang et al., 2013), is rare in non-HCP metals, i.e. most of the materials tested here. That is, if twinning were required to produce significant transitional nonlinearity, most the alloys here would not exhibit the phenomenon. Furthermore, the quantitative commonality of behavior between twinning and non-twinning materials embodied in the Universal Law suggests that the primary role of twinning is as an obstacle to slip. That is, the role of twinning shows up as a strengthening mechanism with a role equivalent to other hardening mechanisms.

Textural changes are known to affect elastic constants (Agnew and Weertman, 1998; Wang et al., 2013), but the effect occurs very gradually with plastic strain as compared with the current results, and would be expected to lead to increased slopes for some materials, which has not been observed. Furthermore, “texture effects” refers to a bond-stretching mechanism - it would not show hysteresis (energy dissipation) upon unloading and loading as is observed for all materials tested here.

Material damage (Kachanov, 1999; Chaboche, 1987; Yeh and Cheng, 2003; Lin et al., 2005; Halilovic et al., 2009; Murakami, 2012) is never measured in sufficient volume fractions in commercial alloys at normal tensile strains to produce the magnitude of the observed effects (Sun and Wagoner, 2011). Furthermore, damage would not induce energy absorption (hysteresis) for cycles of unloading and loading.

There are two proposed mechanisms that are consistent with the current results. They are variations that invoke local plastic effects, as follows:

- 1) continuum elastic-plastic response of an inhomogeneous medium including internal and residual stresses (Zang et al., 2006; Agnew and Weertman, 1998; Wang et al., 2013; Govik et al., 2014), and
- 2) dislocation motion (Mott, 1952; Friedel, 1953), more particularly a pile-up and relaxation process (Ledbetter et al., 1988; Morestin and Boivin, 1996a; Cleveland and Ghosh, 2002; Luo and Ghosh, 2003; Caceres et al., 2003; Benito et al., 2005; Perez et al., 2005; Zavattieri et al., 2009; Kim et al., 2013; Lim et al., 2011; Lee et al., 2010).

These mechanisms, without distinction between the two classes, have been referred to in the literature as “micro-plastic” (Cleveland and Ghosh, 2002) or “quasi-plastic-elastic,” QPE (Sun and Wagoner, 2011). They produce behavior consistent with the observed characteristics: reversibility, nonlinearity, hysteresis / energy dissipation, approximate rate independence, and approximate repeated cycle independence. While both of these mechanisms are plausible and undoubtedly represent real effects that operate in inhomogeneous metals and alloys (i.e. all

alloys of structural interest), new and as-yet unpublished work suggests that the second mechanism, dislocation pile-up and relaxation, dominates.

Figure 1 represents the ongoing research⁸ comparing three simulation approaches based on an RVE for a DP980 steel similar to the one introduced and used in the current work. The RVE and the constituent properties were derived from independent characterization, not back-fitting to any macroscopic behavior. All three of the simulation approaches: Continuum, Crystal Plasticity, and Crystal Plasticity with Dislocation Interactions, incorporate Mechanism 1 (residual stress/mechanical inhomogeneity). Only the Crystal Plasticity with Dislocation Interactions incorporates Mechanism 2, generalized pile-up and relaxation effects that occur by discrete dislocation interactions.

Therefore, in reading and assessing the current work, it is suggested that a dislocation pile-up and relaxation mechanism be provisionally considered as the dominant mechanism potentially accounting for the curious behavior that is reported here. It produces behavior that is contrary to what is normally taught and learned in terms of pure elastic and plastic deformation separated by an abrupt transition.

⁸ Please note that Figure 1 is not offered as a peer-reviewed research result to be relied upon for anything other than as identifying a potential mechanism that might account for the results appearing later in this paper. It is hoped that this identification will assist in understanding the results presented in this paper. Introducing this figure in sufficient detail for other purposes is beyond the scope and reasonable length limitation of this paper. It will appear in a forthcoming publication.

Figure 2: Comparison of the measured evolution of the tangent modulus with FE simulations based on alternate constitutive models.

Objectives and Approach of the Current Work

The current work addresses questions raised by the Sun and Wagoner (2011) and Chen et al. (2016) with nonlinear analysis applied to a larger range of alloys exhibiting various crystal structures and myriad hardening mechanisms. Three principal questions correspond to the three divisions of this paper:

1. What is the nature of the elastic plastic transition? That is, what is the limit of the linear elastic response and what is the start of the nonlinear response?
2. Can the transitional behavior be represented by a single, simple nonlinear equation? On what material parameters does it depend?
3. Do the nonlinear equations for various alloys have common features that can be exploited to improve constitutive models?

In order to limit the volume of testing for a large number of alloys, several simplifying principles based on the results from Sun and Wagoner (2011) and Chen et al. (2016) were adopted. In most cases, when practicable, these principles were verified using the new data, as will be discussed below. The starting simplifications are as follows:

- The nonlinear loading and unloading legs are the same within experimental scatter, but the experimental scatter is less for unloading, so unloading legs can be used exclusively.
- There is little in-plane anisotropy for transitional nonlinearity, so testing in one direction, the rolling direction is sufficient.
- The transitional behavior is nearly independent of strain rate, multiple cycles, and pre-strain, thus in most cases only a single pre-strain is sufficient for analysis.

EXPERIMENTAL PROCEDURES

Materials

Twenty-nine sheet alloys, for which the pertinent properties are listed in Table 1 and as shown graphically in Fig. 3, were assembled from various sources. Table 1 shows that their properties differ greatly: by a factor of ~6x in tensile strength (213-1199 MPa) and ~10x in total elongation (4.8-49.2%). The details of their chemistries and processing are not relevant to the current work except to note that they are typical sheet forming alloys, not acquired or selected according to any criterion.

Three of the alloys shown in Table 1 were found unsuitable for detailed stress-strain measurement because they exhibited serrated flow, not only in the typical tensile large-strain regime, but also during transitional stress-strain measurements. The three alloys, BH 340, Al 6016-0, and Al 7075-T6, were excluded from further analysis in this paper, leaving 26 alloys. These excluded alloys are shown in Table 1 in red/*italics*.

The physical Young's moduli presented in Table 1, E_{US} and E_{RF} were measured using ultrasonic (US) and resonant frequency (RF) methods, respectively (Chen et al., 2016). E_{US} is measured in the sheet-thickness direction while E_{RF} is measured in the appropriate rolling direction. The average signed difference between the two measures for all 29 alloys is 1% (2 GPa) and the averaged absolute difference is 3% (5 GPa); insignificant differences for the current purposes. With these results in mind, physical Young's modulus values of two types are used in the remainder of this paper, depending on the intended purpose: 1) E_{US} values (as presented in Table 1), which exhibit slightly lower test-to-test scatter than E_{RF} values and are available for all of the materials, or 2) "handbook" values as follows: 208 GPa for steels, 70 GPa for aluminum alloys, 46 GPa for magnesium alloys and 110 GPa for brass and titanium. For the 12 core

steels⁹, the average E_{US} is 208 ± 3 GPa, for all 21 steels tested here it is 205 ± 3 GPa, in general agreement with the handbook value that was adopted.

Table 1: The mechanical properties of tested material samples, rolling direction

Material	t (mm)	σ_{YS} (MPa)	σ_{UTS} (MPa)	e_u (%)	e_t (%) [*]	ρ (kg/m ³)	E_{US} (GPa)	E_{RF} (GPa)
DP980-1	1.65	660	962±4	10.2	16.8	7821	211±0	204±0
DP980-2	1.00	576	960±0	10.2	17.8	7727	216±0	222±0
DP980-3	1.38	790	1036±5	7.6	12.4	7709	213±0	203±0
DP600-1	1.36	390	650±5	15.7	25.3	7767	210±0	196±0
DP600-2	1.63	417	693±2	15.7	25.7	7750	208±0	211±0
DP600-3	1.22	394	636±3	17.6	27.3	7894	215±0	217±0
HSLA-1	1.42	415	453±4	20.4	31.1	7724	209±1	200±0
HSLA-2	1.23	362	485±5	17.3	26.7	7681	211±0	202±0
HSLA-3	1.50	382	449±2	18.7	31.0	7734	210±0	201±0
IF-1	1.20	147	278±1	28.6	50.2	7764	200±0	203±0
IF-2	1.18	134	300±1	28.7	47.2	7745	196±1	201±0
IF-3	1.50	150	278±1	26.4	50.5	7742	198±0	211±0
<i>BH 340</i>	<i>0.70</i>	<i>300</i>	<i>343</i>	<i>25.6</i>	<i>28.0</i>	<i>7710</i>	<i>196</i>	<i>194</i>
DP 780	1.15	555	850	13.8	22.5	7780	202	201
TRIP 780	1.42	489	804	20.4	27.4	7712	200	203
TWIP	1.40	543	977	49.2	51.0	7461	203	198
CP 1180	1.20	933	1199	4.8	9.1	7729	205	201
SS 301	0.76	273	829	70	78.0	7721	200	197
SS 304	1.15	275	676	28.7	31.7	7671	203	195

⁹ The first 12 alloys in Table 1 are “core steels” that were tested by Chen et al. (2016). They represent examples from 3 suppliers in each of 4 grades with the -1, -2, or -3 designation referring to supplier identity.

SS 316	0.88	260	606	53.4	62.7	7786	195	192
Al 7075	0.87	478	526	9.4	10.4	2747	71	69
Al 6016	1.11	130	241	19	21.2	2700	70	67
Al 5754	2.01	100	213	20.8	23.2	2670	70	69
Brass 260	0.82	327	406	25	28.0	8525	112	110
Mg AZ 31	4.00	143	250	12.1	13.1	1780	46	46
Mg AZ 61	4.00	162	285	7.5	7.7	1800	46	46
Ti CP Grade 2	1.82	350	464	12	36.0	4426	114	112
QP 980	1.38	830	1013	11	19.0	7853	200	203
TBF 980	1.38	820	1020	6.7	16.4	7872	218	225

Key: Tensile properties were determined at the strain rate of 0.0005/s, with a stroke rate of 0.04 mm/s.

Alloy naming conventions are as follows. If a number is present for a non-stainless steel, it refers to the minimum ultimate tensile strength for the grade. Stainless steels, aluminum alloys, brass and titanium are designated by their standard grade number. Non-stainless steel shorthand is as follows: DP (dual phase), HSLA (high-strength low-alloy), IF (interstitial free), BH (bake hardenable), TRIP (transformation induced plasticity), TWIP (twinning induced plasticity), CP (complex phase), TBF (TRIP-aided bainitic ferrite), QP (quench and partition).

t - sheet thickness

σ_{YS} - 0.2% offset yield strength;

σ_{UTS} - ultimate tensile strength;

e_u - uniform elongation (engineering strain at maximum tensile load);

e_t -total elongation (total engineering strain to fracture);

ρ - density.

E_{US} : ultrasonic-measured Young's modulus

E_{RF} : resonant frequency-measured Young's modulus

$\pm$: indicate one-half of the standard deviation for repeated tests.

The hardening and deformation mechanisms among the alloys vary widely:

- IF contains less than 0.005% of active alloying; it is nearly as soft as pure iron.
- HSLA has a carbon content of 0.05%-0.25% and micro-alloying to increase its strength, primarily by grain refinement.

- DP steels are composite-like alloys composed of a matrix of soft ferrite and large islands of hard martensite phase, thus introducing large stress concentrators.
- TRIP steel is composed of retained austenite, bainite and martensite in a ferrite matrix, with transformation of austenite to martensite during deformation.
- TWIP steel typically contains Mn to stabilize the austenitic structure and allow for twinning.
- CP steel contains the ferrite/bainite matrix, small amounts of martensite and retained austenite with extremely fine grain size.
- QP steel has a microstructure comprising ~30% of ferrite and ~10% of carbon-enriched stabilized austenite and ~60% of martensite.
- TBF steel has a fine microstructure of retained austenite in a bainitic ferrite matrix.
- The stainless steels used here are nominally austenitic ones that can undergo martensitic transformation to various degrees when deformed.
- The magnesium alloys have HCP crystal structure, with the designation referring to the weight percent of aluminum and zinc.
- Brass 260 (cartridge brass, 70/30 Cu/Zn) is a high-strain-hardening, high-ductility FCC alloy.
- CP Ti, Grade 2 is a titanium alloy has an HCP crystal structure.

Some of the tested steels were zinc coated. By acid stripping of the coatings, the average thickness was found to be $1.3\% \pm 0.2\%$ of the overall sheet thickness. In view of these minimal effects relative to the purpose of the current work, no adjustments to nominal area and thickness were undertaken to account for coating thickness.

Mechanical Testing

Standard ASTM E-8 room-temperature tensile tests were performed in the rolling direction at a nominal strain rate of 10^{-3} /s (the end displacement rate was controlled at 0.08 mm/s). Testing was done using an MTS 810 tensile frame, equipped with a 100 kN MTS Model 318.10 force transducer (MTS, 2006). For the 12 core steels, two or three duplicate tests were carried out to estimate the reproducibility of results.

Strain measurements were taken using an Epsilon Class A¹⁰ extensometer (3452-0200-030-ST), 2” gage, +30%/-10% extension range. The high-precision extensometer has the highest resolution and accuracy commercially available, which is essential for differentiating very small strain differences, a fraction of the linear elastic strains. The tests were displacement-controlled. Data was collected at a sampling frequency of 10 Hz up to 0.005 strain, and at 1 Hz subsequently.

Similar tests aimed at transitions were performed with 5 loading-unloading-loading (LUL) cycles at engineering pre-strains of 0.02, 0.04, 0.06, 0.08, and 0.10 where accessible in view of the ductility limits, using an automated control program for this purpose. Data was collected at a sampling frequency of 10 Hz in the approximate elastic regions, i.e. at the start of the test, as in the standard tensile tests, and from $\epsilon_r - 0.005$ to $\epsilon_r + 0.005$, (where ϵ_r is the engineering strain at

¹⁰ This model of extensometer has Class A resolution and accuracy in the range of extensions used in the current work.

which the displacement rate is reversed), and at 1 Hz outside of those regions. The control program was set to reverse the displacement direction (to a compressive direction) at a fixed displacement corresponding to the desired engineering strain. LUL tests for the 12 core steels were repeated at least twice to estimate the test-to-test scatter.

Figure 3: Standard tensile test results for 29 alloys

Unless otherwise stated, unloading data for a nominal prestrain of 8% were used for all of the analysis presented in this paper. The data points recorded during unloading were reduced to $(\Delta\epsilon, \Delta\sigma)$ values defined relative to the first unloading point recorded after the reversal of the specimen end displacement rate: (ϵ_u, σ_u) . More explicitly, $\Delta\epsilon = \epsilon_{\text{measured}} - \epsilon_u$ and $\Delta\sigma_u = \Delta\sigma = \sigma_{\text{measured}} - \sigma_u$. This procedure largely eliminates extensometer and load cell calibration concerns when comparing multiple tests and materials.

Each tangent slope point, E_T ($\equiv d\Delta\sigma/d\Delta\epsilon$), was calculated from the least-squares slope of a line fit to 6 adjacent $(\Delta\epsilon, \Delta\sigma)$ data points, with the value of $\Delta\epsilon$ corresponding to E_T taken as the mean value of the 6 $\Delta\epsilon$ values in that interval. The interval for the next E_T data point used 6 data

points that overlapped the previous interval by 3 $\Delta\sigma(\Delta\varepsilon)$ data points. Several alternate procedures were considered for the conversion of stress-strain data to a slope-strain basis, but this one gave a reasonable combination of resolution and scatter for most experiments.

RESULTS & ANALYSIS 1: FUNDAMENTAL NATURE AND ITS REPRESENTATION

The limit of the linear elastic response and the start of the nonlinear response, can be addressed using the QPE equation representing the nonlinear portion of the response, Eq. 5¹¹. The 12 core steels were used because the testing for these alloys was more extensive: hundreds of tensile tests were performed to establish test-to-test variation, to verify load-unload symmetry and to verify multi-cycle independence.

In order to integrate the linear and non-linear response ranges in the treatment, a critical transition strain separating the linear and nonlinear portions, ε_c , was added as an adjustable parameter, bringing to 4 such parameters, E_0 , A_1 , A_2 , and ε_c , as follows:

$$E(\Delta\varepsilon) = \begin{cases} E_0 & \text{for } \Delta\varepsilon \leq \varepsilon_c \\ E_0 - A_1 \left[1 - \exp(-A_2(\Delta\varepsilon - \varepsilon_c)) \right] & \text{for } \Delta\varepsilon \geq \varepsilon_c \end{cases} \quad (6)$$

where $\Delta\varepsilon$ represents the absolute value of the increment of strain from the strain rate reversal, i.e. the unloading strain.

Integration produces the stress form of Eq. 6, as follows:

$$\Delta\sigma(\Delta\varepsilon) = \begin{cases} E_0\Delta\varepsilon & \text{for } \Delta\varepsilon \leq \varepsilon_c \\ (E_0 - A_1)(\Delta\varepsilon - \varepsilon_c) + \frac{A_1}{A_2} \left\{ 1 - \exp[-A_2(\Delta\varepsilon - \varepsilon_c)] \right\} + E_0\varepsilon_c & \text{for } \Delta\varepsilon \geq \varepsilon_c \end{cases} \quad (7)$$

where $\Delta\sigma$ is the absolute value of the stress change from the unloading stress.

When all 4 parameters of Eqs. 6 and 7 are adjustable and thus subject to optimization, the representation is denoted as “QPE-4.” “QPE” refers to the use of Eq. 5 to represent the shape of the nonlinear region of the stress-strain response and “-4” refers to the 4 adjustable parameters. Using a similar naming convention, “QPE-3” sets ε_c equal to zero. That is, QPE-3 has no region of linear behavior; the nonlinearity begins at the reversal point or upon initial loading. QPE-2 additionally sets E_0 to the handbook Young’s modulus, 208 GPa for steels, or to a specifically-measured Young’s modulus, thus leaving only 2 remaining adjustable parameters: A_1 and A_2 .

¹¹ The initial slope E_0 is first treated as an adjustable parameter. The results are later shown to be consistent with E_0 representing the independently-measured Young’s modulus, E .

A Matlab program (Matlab 8.4) was developed to optimize the parameters of Eq. 7¹² and thus minimize the least-squares error relative to unloading stress data. A second program, “Origin” (Origin 8 SR3) was later used because it offers improved tolerances (10^{-9} MPa vs. 10^{-6} MPa for MatLab) that showed a small benefit in terms of the consistency and precision of the optimized parameters, the standard errors of fit $\langle\sigma\rangle$, and the correlation coefficients. However, none of the results or conclusions changed significantly. For all multi-parametric equations, several widely-varied trial coefficient sets were used to start the process in order to insure that the optimal coefficients were obtained.

The process and results are illustrated in Figs. 4. In Fig. 4a, QPE-4, QPE-3, and QPE-2 fits to DP 980-1 data are compared for an unloading leg. Standard errors of fit, $\langle\sigma\rangle$, and adjusted multiple correlation coefficients¹³, R^2 , are presented. All three equations represent the data “perfectly,” that is, the values of R^2 for all three are 1.000¹⁴. This is also reflected in the standard errors of fit, which are smaller than the test-to-test variation.

The QPE-4 and QPE-3 fits shown in Fig. 4a are identical: that is, the optimal value of ε_c in QPE-4 is zero, all other parameters being identical. Therefore, there is no linear region of stress-strain behavior observed for DP 980-1.

Comparing QPE-2 and QPE-3 fits in Fig. 4a shows no significant improvement achievable by allowing the initial slope to differ from the handbook value. Therefore, the conclusion of Chen et al. (2016) that the initial slope following a strain rate reversal is equal to the measured physical Young’s modulus is confirmed by the current analysis.

Figure 4a also shows that any of the nonlinear fits improves on the two typical engineering approaches practiced today: 1) adopting a handbook physical modulus value or 2) adopting a chord modulus value, i.e. an adjusted, constant slope passing through the end points of the unloading path. The improvement of the nonlinear treatments over these, respectively, is by a factor of ~ 10 and ~ 30 (the standard errors of fit are 4-5MPa vs. 62MPa or 180 MPa). Put another way, using a chord modulus reduces the error inherent in adopting a physical Young’s modulus by approximately a factor of 3 while using any of the nonlinear fits reduces the error by a factor of 30. Clearly the transition phenomenon cannot be profitably viewed as a reduced

¹² All fitting was performed by minimizing the standard error of fit, $\langle\sigma\rangle$, which is proper in view of the experimentally measured quantities. The optimized $\Delta\sigma(\Delta\varepsilon)$ function can be differentiated to obtain a corresponding tangent slope function $E_T(\Delta\varepsilon)$ which is more visually revealing when compared with experimental slopes. It should be noted that the fits to E_T are not precisely optimal, nonetheless they are nearly so in view of the optimized fits to the corresponding direct $\Delta\sigma(\Delta\varepsilon)$ data.

¹³ The adjusted multiple correlation coefficient (also called the coefficient of multiple determination), “ R^2 ” or “R-square,” is a standard statistical quantity measuring how successful a fit is in explaining the variation of the data. When the equation represents an adjustable constant term, R^2 varies between 0-1, with 1 representing a perfect reproduction of the data to the equation (no deviation between equation and data) and 0 representing no systematic reproduction of the data variation (only random correlation). An R^2 value of 0.75 means that means that the fit explains 75% of the total variation in the data about the average. The following link contains a succinct summary: <http://www.mathworks.com/help/curvefit/evaluating-goodness-of-fit.html>

¹⁴ R^2 values have an observed tolerance of ~ 0.001 from MatLab, ~ 0.0001 for Origin. Regardless, the distinctions among representing data with equations having 99.9% vs. 99.99% accuracy have little practical consequence such that in the remainder of this paper only 3 significant digits are reported unless there is a particular need for more precision.

Young's modulus but must be treated as a curve. This nature is inherently contrary to the linear nature of atomic bond stretching for metals. (See Discussion section also.)

Figure 4b represents exactly the same data and fits as shown in Fig. 4a, that is, the fit parameters minimize $\langle\sigma\rangle$, but not necessarily $\langle E_T\rangle$. Plotting of slopes rather than stress values visually reveals the features of the data and equations more sensitively than Fig. 4a can. For example, it can be readily seen that the initial unloading modulus is approximately equal to the physically-measured Young's modulus E_{US} , and, that there is no suggestion of a region of constant slope at the beginning of unloading corresponding to the presumed linear elastic response over some finite stress range. Furthermore, the standard deviations and R^2 values are more meaningful for slope comparisons. They show clearly that all of the equations reproduce the data as well as or better than repeated tests do, with R^2 values of 0.882 to 0.906 for equations vs. 0.886 for data. The standard Young's modulus and chord modulus representations provide no useful variation of slope as reflected in the non-positive R^2 values. The standard deviations of those representations can be improved by factors of ~ 8 and ~ 4 respectively by using any of the nonlinear equations.

(a)

(b)

Figures 4: QPE-2, QPE-3 and QPE-4 fits compared with experimental data for a typical steel, DP 980-1 (a) $\Delta\sigma$ vs. $\Delta\epsilon$ (b) E_T vs $\Delta\epsilon$.

Analyses for each of the 12 core steels similar to that for DP 980-1 give the following average results, which are presented below in order of descending $\langle E_T \rangle$. The fits below the line have better correlation to the data than the reproducibility of the data itself, which is shown in *italics*:

Representation	$\langle \sigma \rangle$ MPa (R^2)	$\langle E_T \rangle$ GPa (R^2)
E_{US} :	81 MPa (0.816)	48 GPa (-5.816 $\rightarrow$ 0) ¹⁵
E_{chord} :	25 MPa (0.982)	21 GPa (0)
<i>Test-to-test scatter:</i>	<i>10 MPa (0.992)</i>	<i>11 GPa (0.646)</i>
QPE-2	3 MPa (1.000)	8 GPa (0.799)
QPE-3:	2 MPa (1.000)	7 GPa (0.849)
QPE-4:	2 MPa (1.000)	7 GPa (0.825)

¹⁵ Negative values of R^2 have no significance beyond indicating no correlation, the same as for $R^2=0$. Therefore, subsequent reports of negative R^2 values are shown simply as "0" (zero).

Therefore, the following conclusions apply to all 12 of the core steels as well as to DP980-1 specifically:

- 1) Any of the nonlinear equations greatly improve, by approximately an order of magnitude, on using a physically-measured modulus or a chord modulus representation of the transitional response.
- 2) There is no linear elastic region suggested by any of the 12 steel results¹⁶, that is, the proportional limit¹⁷ is 0 MPa.
- 3) The initial slope of unloading is equal to the Young's modulus, but this slope rapidly decreases with strain.

In terms of the last conclusion, the average initial slope for the 12 core steels (taken from the first six measurement as shown in Fig. 4b) is 203 ± 6 GPa, which is equal to both the E_{handbook} value of 208 ± 3 GPa and the E_0 value from the QPE-3 equation: 205 ± 16 GPa.

Although not shown here for reasons of space limitations, reloading cycles were compared in detail with the corresponding unloading ones. For DP980-1, the average standard deviation between loading and unloading cycles for two repeated tests is 6 MPa, as compared with the test-to-test scatter of 18 MPa and 11 MPa for unloading and loading legs, respectively. The average of standard deviations between loading and unloading cycles for all 12 core steels is 7 MPa, as compared with average test-to-test scatters of 10 MPa for both unloading and loading, respectively. Therefore, the unloading and reloading responses differ insignificantly, confirming the similar conclusion of Chen et. al (2016), and justifying the exclusive use of more-reproducible unloading cycles in the current work. Thus, all conclusions in this paper apply equally for loading and unloading paths.

RESULTS & ANALYSIS 2: A SIMPLIFIED 1-PARAMETER REPRESENTATION

In view of the foregoing results and analysis, it is clear that 2 of the original 4 adjustable parameters in the original QPE equation can be set independently or eliminated without loss of accuracy: E_0 can be taken as the physical Young's modulus (consistent with Sun and Wagoner, 2011, and Chen et al., 2016) and ϵ_c is identically zero. Two fit parameters remain in the resulting QPE-2 form: A_1 and A_2 . However, it is preferable for application purposes to adopt an even simpler 1-parameter representation of the transitional stress-strain behavior, if such a representation can be shown to be sufficiently accurate.

¹⁶ To be more precise: the QPE-4 fits for 8 of the 12 steels produced $\epsilon_c=0$, i.e. exactly no linear region for the optimal fit even with the fourth parameter being freely adjustable. For the other 4 steels, very small ϵ_c values were indicated for QPE-4 fits, but there was no significant improvement in adding ϵ_c as an adjustable parameter: each of the standard errors of fit differed by less than 0.1 MPa between QPE-3 and QPE-4 and the R^2 values differed less than 0.0001, both of these differences being vanishingly small.

¹⁷ However, as noted in detail by Sun and Wagoner (2011), the presence of initial nonlinearity does not correspond with irreversible deformation although it is energy absorbing (non-conservative).

Equation 4 was investigated to test the suitability of a single-parameter representation¹⁸. The corresponding stress form is as follows:

$$\Delta\sigma = \frac{E_0}{A} [1 - \exp(-A \Delta\varepsilon)] \quad (8)$$

where E_0 is taken to be the physically-measured Young's modulus (E_{US} here) and the only adjustable parameter is A , the modulus reduction rate. Eqs. 4 and 8 will be referred to together in shorthand as "1-P," i.e. a one-parameter description of the nonlinear transitional stress-strain function, where the modulus reduction rate, A , is optimized to reproduce the experimentally measured $\Delta\sigma$ - $\Delta\varepsilon$ response.

Fitting results for two typical steels with greatly differing strengths, DP980-1 and IF-1, are shown in Figs. 5. The curves and standard deviations compare three fitting equations: QPE-4 (Eq. 6), QPE-3 (Eq. 6), 1-P (Eq. 4), and Eq. 3. In the spirit of simplified application, E_0 is taken to be the handbook value of 208 GPa for each steel, rather than using a specifically measured value. The standard deviations and R^2 values are shown on Figs. 5, but the results can be summarized as follows: all of the nonlinear fits represent the data with better accuracy than the experimental scatter; adding parameters is not justified. Furthermore, the comparisons show that the 1-P (Eq. 4) and Eq. 3 representations are indistinguishable.

¹⁸ Note that a single-parameter nonlinear representation is similar in complexity or adjustability to the chord modulus representation, which also has a single adjustable parameter, the constant slope. The utility of the simplest nonlinear representation can thus be judged by comparing its accuracy to the linear chord modulus approach.

(a)

(b)

Figure 5: Comparison of fit equations to data for two typical steels: (a) DP 980-1, (b) IF-1.

A comparison of the 1-P equation with previous multi-parametric representations for the 12 core steels confirms the conclusions gleaned from the visual analysis of Figs. 5:

Representation	$\langle \sigma \rangle$ MPa (R^2)	$\langle E_T \rangle$ GPa (R^2)
$E_0=208$ GPa (handbook)	81 MPa (0.816)	48 GPa (0)
Chord modulus	25 MPa (0.982)	21 GPa (0)
<i>Test-to-test scatter:</i>	<i>10 MPa (0.992)</i>	<i>11 GPa (0.646)</i>
1-P (Eq. 4):	4 MPa (0.999)	9 GPa (0.726)
Eq. 3:	5 MPa (0.999)	9 GPa (0.722)
QPE-2 (Eqs 6, 7):	3 MPa (1.000)	8 GPa (0.799)
QPE-3 (Eqs 6, 7):	2 MPa (1.000)	7 GPa (0.849)
QPE-4 (Eqs 6, 7):	2 MPa (1.000)	7 GPa (0.825)

As expected, the 1-P form has a slightly higher standard error of fit than the multi-parameter forms, but all remain less than the test-to-test scatter of the data itself. The 1-P (Eq. 4) and Eq. 3

representations are indistinguishable for all 12 steels. Therefore, only the 1-P equation (Eq. 4), was retained for further work.

The same analysis was extended to all 26 materials, with results as follows:

$E_0=E_{\text{handbook}}$	72 MPa (0.849)	41 GPa (0)
Chord modulus:	25 MPa (0.980)	19 GPa (0)
<u>Test-to-test scatter¹⁹:</u>	<u>10 MPa (0.992)</u>	<u>11 GPa (0.646)</u>
1-P (Eq. 4):	5 MPa (0.999)	8 GPa (0.767)
QPE-2 (Eqs 6, 7):	4 MPa (0.999)	7 GPa (0.805)
QPE-3 (Eqs 6, 7):	2 MPa (1.000)	6 GPa (0.838)
QPE-4 (Eqs 6, 7):	2 MPa (1.000)	6 GPa (0.827)

The results are the same. In view of these comparisons, it is concluded that the simplest nonlinear representation, 1-P, greatly improves on standard handbook or chord modulus representations. The 1-P form represents the experimental data nearly as well, 91%-95% as well, as the more adjustable and complex forms of QPE-2, QPE-3, or QPE-4. In any case, all such equations represent the data itself within the test-to-test scatter. The 1-P representation is therefore sufficiently accurate and sensitive for the subsequent analysis, and is to be preferred as compared with the more flexible multi-parameter fits.

To better quantify the relative accuracy of the 1-P equation compared with physical and chord moduli, an “improvement ratio” was defined comparing the standard deviations between each standard representation and the 1-P equation. The average improvement ratios for 26 materials are as follows:

<u>Representation</u>	<u>Improvement Ratio</u>
$\langle\sigma\rangle_{\text{US}}/\langle\sigma\rangle_{\text{1P}}$	13.5
$\langle E_T\rangle_{\text{US}}/\langle E_T\rangle_{\text{1P}}$	4.9
$\langle\sigma\rangle_{\text{Chord}}/\langle\sigma\rangle_{\text{1P}}$	4.8
$\langle E_T\rangle_{\text{Chord}}/\langle E_T\rangle_{\text{1P}}$	2.3

The 1-P equation improves on a physically-measured Young’s modulus by a factor of 5 to 14 and on a mechanically-measured chord modulus by a factor of 2 to 5.

RESULTS & ANALYSIS 3: A PRACTICAL RULE FOR METALS

The simple 1-P representation offers the advantage of characterizing the entire transition region of deformation with a single, simply understood parameter. With this tool, the relationship of the transitional response among diverse alloys can be probed profitably.

A Rule for Steels

¹⁹ The quoted test-to-test scatter is from the 12 core steels only: the only ones for which multiple tests were performed. The results for the remaining alloys seem consistent with these numbers, but have not been measured directly.

Figure 6 shows for each of the 21 steels tested here the modulus reduction rate, A , as a function of material strength, that is, the flow stress just prior to unloading. The modulus reduction rate for steels depends to first order only on material strength (i.e. stress prior to unloading) - not on the diverse microstructure, not on diverse hardening or deformation mechanisms, not even on the crystal structure (bcc and fcc).

A single curve reproduces with reasonable fidelity the individual values of A , with an R^2 value of 0.710. That is, 71% of the variation in A is captured by using the independently determined “strength” without measuring anything during the stress-strain transition. The equation for the curve shown is

$$A = \frac{15000 \text{ MPa}^{3/4}}{\sigma_u^{3/4} \text{ MPa}^{3/4}} \quad (9)$$

where the two explicit fit parameters, $15000 \text{ MPa}^{3/4}$ and $3/4$, were rounded off from the precise optimized values with no significant increase of standard error of fit ($\langle A \rangle = 39$ for both).

Although not shown here, a similar optimization was also performed for the 12 core steels only; the same parameters (Eq. 9) showed an insignificant increase of standard error of fit relative to the optimized fit ($\langle A \rangle = 42$ vs. $\langle A \rangle = 39$).

Therefore, for at least this set of 21 diverse steels, Eq. 9 represents a single, explicit equation relating A to material strength (unloading stress, σ_u). Eq. 9, combined with Eq. 8 (see Eq. 10 for the explicit combined form, below), predicts and reproduces the approximate shape of the transitional stress-strain curve (70% reproduction quality) *with no adjustable parameters and therefore no need for specialized testing.*

Figure 6: The variation of modulus reduction rate A vs. unloading stresses for all 21 tested steels. Triangles indicate typical examples selected for further presentation; see Figs. 5. The two red triangles represent two steels selected for more detailed presentation: DP 980-1 and IF-1.

The combined form of Eqs. 8 and 9 that represents explicitly the transitional stress-strain behavior of all steels is as follows:

$$\Delta\sigma(MPa) = \frac{E_0(MPa) \sigma_u^{3/4}(MPa^{3/4})}{15000(MPa^{3/4})} \left[1 - \exp\left(-\frac{15000(MPa^{3/4})}{\sigma_u^{3/4}(MPa^{3/4})} \Delta\varepsilon \right) \right] \quad (10)$$

where E_0 and σ_u are readily-measured standard quantities that are determined independently of the transitional behavior itself.

Equation 10 provisionally represents a “Rule for Steels” or “Steel Rule” that can be used directly without additional testing to represent the transitional behavior of any steel. The equivalent equation in terms of slope is derived by combining Eqs. 9 and 4 or by taking the derivative of Eq. 10:

$$E(\text{GPa}) = E_0(\text{GPa}) \exp \left(- \frac{15000(\text{MPa}^{3/4})}{\sigma_u^{3/4}(\text{MPa}^{3/4})} \Delta \varepsilon \right) \quad (11)$$

Figure 7 shows a typical result comparing the quality of fit of the Steel Rule (no fit parameters) and the 1-P law (with 1 fit parameter fit to the data for each test in question). As can be seen from the curves themselves or by the standard deviations and R^2 values, there is only a modest benefit attained by performing a special test in terms of overall reproduction of the data, changing the quality of the prediction/reproduction from 82% to 89%. Conversely, the parameter-less Steel Rule reproduces the experimental data with a standard deviation of 12 GPa, a factor of 5 less than using a handbook Young's modulus ($\langle E_T \rangle = 64$ GPa). It even reduces the standard deviation as compared with the 1-parameter chord modulus representation, 12 GPa vs. 27 GPa, a reduction greater than 2. (The zero and negative values of R^2 for handbook and chord modulus representations indicate that they reproduce none of the observed variation of E_T .)

Figure 7: Typical 1-P and the Steel Rule fits compared with experimental data for a typical steel, DP980-1.

The conclusion from the DP 980-1 example is verified more widely by similar numerical results for all 21 of the steels shown in Fig. 6:

Representation	$\langle\sigma\rangle$ MPa (R^2)	$\langle E_T \rangle$ GPa (R^2)
$E_0 = E_{\text{handbook}}$	83 MPa (0.831)	48 GPa (0)
Chord modulus:	27 MPa (0.979)	22 GPa (0)
<i>Test-to-test scatter</i> ²⁰ :	10 MPa (0.992)	11 GPa (0.646)
Rule for Steels (Eqs 10, 11):	12 MPa (0.993)	11 GPa (0.722)
1-P (Eq 4), optimized A:	5 MPa (0.999)	9 GPa (0.771)

The Rule for Steels has a slightly larger average standard error of fit (but better R^2) than the test-to-test scatter. It is thus concluded that Eqs. 10 and 11 represent a Rule for Steels that can be used effectively, i.e. with significantly improved accuracy as compared with handbook or chord modulus values. These benefits are conferred *with no testing required*.

A Rule for Metals

The foregoing analysis is now extended to all 26 materials tested, i.e. including other base elements and other crystal structures. The initial 1-P analysis results are compared with Eqs. 10 and 11 in Table 2 and Figs. 8.

Table 2: Comparisons of 1-P and Metal Rule Results

Material	1-P (Eq. 4)			Metal Rule (Eq. 4)		
	A	$\langle\sigma\rangle$ (MPa)	$\langle E_T \rangle$ (GPa)	A	$\langle\sigma\rangle$ (MPa)	$\langle E_T \rangle$ (GPa)
DP980-1	97	5(1.000)	9(0.889)	83	28(0.995)	15(0.819)
DP980-2	89	10(0.999)	10(0.792)	84	11(0.989)	10(0.799)
DP980-3	78	6(1.000)	8(0.868)	79	8(1.000)	10(0.869)
DP600-1	156	7(0.998)	10(0.888)	115	27(0.981)	13(0.672)
DP600-2	142	5(0.999)	9(0.870)	111	21(0.989)	12(0.770)
DP600-3	127	5(0.999)	10(0.704)	115	9(0.998)	10(0.714)
HSLA-1	162	4(0.999)	9(0.499)	154	4(0.999)	9(0.624)
HSLA-2	131	3(0.999)	8(0.729)	142	4(0.999)	9(0.684)
HSLA-3	171	5(0.999)	10(0.293)	154	7(0.997)	8(0.568)
IF-1	322	3(0.999)	8(0.713)	231	9(0.988)	10(0.547)
IF-2	217	3(0.998)	9(0.840)	217	3(0.998)	9(0.860)
IF-3	333	2(0.999)	8(0.714)	236	9(0.987)	11(0.551)
DP 780	87	3(1.000)	7(0.901)	92	6(0.999)	7(0.896)
TR 780	125	7(0.999)	10(0.765)	107	15(0.995)	11(0.760)
TWIP	101	4(1.000)	8(0.899)	102	5(1.000)	8(0.908)
QP 980	66	7(0.999)	10(0.679)	78	19(0.996)	13(0.449)
TBF 980	48	12(1.000)	10(0.571)	80	38(0.989)	20(-0.155)

²⁰ The quoted test-to-test scatter is from the 12 core steels only: the only ones for which multiple tests were performed. The results for the remaining alloys seem consistent with these numbers, but have not been measured directly.

CP 1180	50	6(1.000)	9(0.740)	72	39(0.989)	18(0.066)
SS 301	202	6(0.998)	7(0.927)	150	17(0.983)	19(0.665)
SS 304	197	5(0.999)	7(0.952)	149	16(0.987)	17(0.700)
SS 316	148	6(0.998)	12(0.817)	156	6(0.997)	11(0.863)
Al 5754	57	1(1.000)	2(0.779)	95	5(0.992)	3(0.237)
Mg AZ31	53	2(0.999)	3(0.752)	50	3(0.999)	3(0.728)
Mg AZ61	43	3(0.999)	3(0.713)	46	3(0.998)	3(0.741)
Ti	56	3(1.000)	4(0.810)	76	11(0.994)	5(0.763)
Brass 260	105	4(0.999)	5(0.890)	88	8(0.995)	8(0.764)

Key: The E_0 values used in these fits are handbook values: 208 GPa for steels, 70 GPa for aluminum alloys, 46 GPa for magnesium alloys and 110 GPa for brass and titanium.

Figure 8a is identical to Fig. 6 except that points have been added for the nonferrous alloys using alternate point types. Note that the nonferrous alloys have lower A values than predicted by the Steel Rule. Closer examination of Fig. 8a reveals that the A values for these new alloys follow the same basic trend in terms of dependence on strength, but are also approximately proportional to the physical modulus for each alloy. For example, the A value for Brass 260 is approximately 100 whereas the A value for a steel of the same strength is approximately 200, for a ratio is 1:2, the same as the ratio of the physically measured Young's moduli for brass and steel: 110 GPa / 208 GPa.

In order to probe whether such a correction is consistent for the various tested alloys, an adjusted modulus reduction rate A' was defined from the experimental results for each alloy in terms of the modulus reduction rate A and the physical Young's Modulus:

$$A' = A \left(\frac{208 \text{ GPa}}{E_0 \text{ GPa}} \right) \quad (12)$$

where the handbook value of Young's modulus has been used here, consistent with the concept of applying the practical rules without special testing²¹. Adopting the adjusted modulus reduction rate A' for nonferrous alloys brings their results into reasonable agreement ($\langle A' \rangle = 64$) with the Steel Rule. The results shown in Figure 8b suggest that a "Rule for Metals" or a "Metal Rule" can be constructed taking the following form:

$$A = \frac{15000 \text{ (MPa)}^{3/4}}{\left[\sigma_u^{3/4} \text{ (MPa)} \right]} \left(\frac{E_0 \text{ GPa}}{208 \text{ GPa}} \right) \quad (13)$$

²¹ While handbook values have been used here in assessing the usefulness of the Steel Rule and Metal Rule, the accuracy of the rules can likely be improved if one measures or otherwise knows the actual Young's modulus (E) for a given alloy. Therefore, the rules are written in terms of a generic Young's modulus, E_0 , without specifying how that should be determined.

This results for this form of a Metal Rule is plotted in Fig. 8b.

Equation 13 may be combined with Eqs. 8 and 4, respectively, to provide the explicit stress-strain and slope-strain transitional behavior, depending only upon the independently measured strength (σ_u) and the physical Young's modulus (E_0):

$$\Delta\sigma = \frac{208(GPa) [\sigma_u(MPa)]^{3/4}}{15000 (MPa)^{3/4}} \left[1 - \exp \left(\frac{-15000 (MPa)^{3/4} E_0 GPa}{[\sigma_u (MPa)]^{3/4} 208 GPa} \Delta\varepsilon \right) \right] \quad (14)$$

$$E = E_0 (GPa) \exp \left(- \frac{15000 (MPa)^{3/4} E_0 GPa}{[\sigma_u (MPa)]^{3/4} 208 GPa} \Delta\varepsilon \right) \quad (15)$$

The Rule for Metals appears to be an adequate and useful representation for a wide range of randomly-selected sheet alloys representing a wide range of mechanical properties, metallurgical mechanisms, and crystal structures. Its accuracy as compared with the custom-fitting of an optimized value for each of the 26 material tests can be assessed by considering the average standard errors of fit as follows:

<u>Representation</u>	<u><σ> MPa (R^2)</u>	<u><E_T> GPa (R^2)</u>
Physical E_0 :	72 MPa (0.849)	41 GPa (0)
Chord modulus:	25 MPa (0.980)	19 GPa (0)
<u>Test-to-test scatter²²:</u>	<u>10 MPa (0.992)</u>	<u>11 GPa (0.646)</u>
Metal Rule:	13 MPa (0.994)	11 GPa (0.648)
1-P (Eq 4):	5 MPa (0.999)	8 GPa (0.767)

It is thus provisionally concluded that Eqs. 14 and 15 represent a useful rule that can be used profitably, without specialized testing, as compared with handbook or chord modulus values. This zero-parameter representation exhibits little or no loss of fidelity on average as compared with test-to-test scatter, yet it improves greatly on existing representations in common use, i.e. handbook Young's modulus values or chord modulus values.

To better quantify the relative accuracy of the Metal Rule compared with physical and chord moduli, the average improvement ratios for 26 materials were determined as follows:

<u>Representation</u>	
< σ > _{US} / < σ > _{Metal Rule}	6.7
< E_T > _{US} / < E_T > _{Metal Rule}	3.8
< σ > _{Chord} / < σ > _{Metal Rule}	2.5

²² The quoted test-to-test scatter is from the 12 core steels only: the only ones for which multiple tests were performed. The results for the remaining alloys seem consistent with these numbers, but have not been measured directly.

$$\langle E_T \rangle_{\text{Chord}} / \langle E_T \rangle_{\text{Metal Rule}} \quad 1.8$$

The Metal Rule (with no adjustable parameters) improves on a physically-measured Young's modulus by a factor of 4 to 7 and on a mechanically-measured chord modulus by a factor of 2 to 3.

(a)

(b)

Figure 8: Addition of nonferrous alloys to probe the existence of a principle for all metals, measured modulus reduction rates, A, compared with: (a) the Rule for Steels, and (b) a Rule for Metals.

Applied Tests of the Metal Rule

The hysteresis loop width, δ , predicted by the Metal Rule can be derived from Eq. 14 as follows:

$$\delta = \frac{2}{A} \ln \left[1 - \frac{A}{E_0} \sigma_u + \frac{A}{E_0} \left(\sigma_u - \frac{E_0}{A} - \frac{\sigma_u}{\ln \left(1 - \frac{A \sigma_u}{E_0} \right)} \right) \right] - \frac{2}{A} \left[\frac{\sigma_u - \left(\sigma_u \frac{E_0}{A} - \frac{\sigma_u}{\ln \left(1 - \frac{A \sigma_u}{E_0} \right)} \right)}{\sigma_u} \right] \ln \left(1 - \frac{A}{E_0} \sigma_u \right) \quad (16)$$

Figure 1a (the first figure in this paper) represents a first test of the Metal Rule for the 12 core steels in terms of predicted loop width, even though that was not apparent when presented. The curve shown in Fig. 1a is in fact Eq. 16 (with $E_0 = 208$ GPa). Visual inspection shows no systematic deviation from the measured δ values.

Figure 9 reproduces Fig. 1b (solid lines) and compares them to the finite-range slopes defined in Chen et al. (2016) as derived from the Metal Rule (dashed lines). In order to obtain the explicit dashed lines, the tensile stress-strain data for this alloy was represented by an H/V strain-hardening equation (Sung et al., 2010, Smith et al., 2014). The predictions derived from the Metal Rule results are generally within the experimental scatter of the measurements.

Figure 9: Comparison of Metal Rule predicted slopes for DP980-1 with experimental results of Chen et al. (2016) for 12 core steels, including DP980-1.

A broader comparison appears in Figs 10, where the loop width, δ , for all 26 alloys tested here is compared with the corresponding Metal Rule predictions. Figure 10a is similar to Fig. 1a for the 12 core steels only. While exhibiting considerable scatter for the broader classes of materials, the trends and agreement are shared. It is worth noting that the representations of the transitional behavior in practical use today, i.e. a physically-measured Young's modulus or a custom-measured chord modulus, both exhibit loop widths of identically zero for all strengths and Young's moduli. That is, the values themselves on Figs. 1 or 9a represent the error of using the standard formulations.

Figure 10b removes the visual complexity of considering the large E_0 differences among the alloys by comparing the predicted and measured loop widths for specific alloys. There is little systematic deviation of the curve from the data and it clearly represents the data within the scatter of the data itself, with the possible exception of two steels, CP 1180 and TBF 980, which are known to have extremely fine microstructures.

In conclusion, while there is considerable scatter in terms of the highly-sensitive loop-width parameter, the Metal Rule represents the spectrum of behavior for 26 highly diverse alloys adequately, particularly as compared with traditional forms which predict zero loop width identically. In view of its applicability to the broad range of metals tested here, and of its basis with no adjustable parameters, it can provisionally be considered a “Universal Law for Metals”.

(a)

(b)

Figure 10: Summary of loop width (δ) measurements at various pre-strains compared with Metal Rule predictions: (a) as a function of unloading stress and Young's modulus, and (b) compared with Universal Law predicted values.

Example Application: Crush of Automotive Pillars

The current work was motivated by experiments performed in 2009 at the Toyota Research Institute (Toyota, 2009). Related FE analysis was carried out at Toyota (using LS-DYNA) and OSU (using Abaqus Standard). The work is summarized here in view of the results presented in this paper. Details of the experimental procedure and simulation specifications have been presented elsewhere (Gandhi and Wagoner, 2013, and Chen et al., 2016).

Sheet-formed structural pillars 600 mm long having two cross-sectional designs were formed and spot welded (4 from 0.8 mm - thick SPC 440²³ steel sheet. Two pillar designs differ only in terms of the cross-section configuration, Figs. 11, which in both cases is uniform along the length of the pillar. The final pillars were compression tested and the force-displacement curves were recorded, as shown as points in Figs. 12.

²³ SPC440 is a JIS designation for a Steel Plate Cold-rolled with an ultimate tensile strength of 440 MPa.

Figure 11: Schematic of cross-sectional geometry of Toyota pillars: (a) Flat design, (b) Jogged design.

The results from the standard elastic-plastic simulations, which are typical of standard industrial methods, are shown as solid lines in Figs. 11. (There was no difference between the OSU/Abaqus results (shown) and the Toyota/LS-Dyna ones. These results were found to be insensitive to physical and simulation perturbations including pillar geometric distortions, off-axis loading, and altered spot weld integrity.

The standard FE results were compared with two advanced representations, a QPE-4 model with one parameter set by the ultrasonically-measured Young's modulus, $E_0 = E_{US} = 196 \text{ GPa}$ ²⁴, and the currently-proposed **Universal Law**²⁵, with no adjustable or fit parameters and with no experimental input. Figures 11 show the results of those comparisons. The two advanced models provide equal fidelity; both reduce the error by a factor of ~ 2 compared with the standard FE result²⁶.

In summary, the **Universal Law** provides a simple and powerful tool for engineers to improve the accuracy of small-strain / transitional simulations, such as for springback or automotive structural crush, without a corresponding time and effort penalty for experiments.

²⁴ The other QPE parameters, Eqs. 6 and 7, were found from testing of a coupon cut from a tested pillar to be: $A_1 = 85.76 \text{ GPa}$, $A_2 = 595.3$, and $\epsilon_c = 0.0011$.

²⁵ In order to implement the **Universal Law** for SPC440, an unloading stress is required. Since this is unknown for the particular condition of the pillar before crush testing, an average value between the yield stress and tensile strength was used to establish the relationship.

²⁶ It can be noted that even the advanced models do not perfectly reproduce the experiments, particularly in the initial "take-up" or "lash elimination" stage of loading. That is, small variations in length at various points of the end section cause uneven contact and soft response until the entire end is in contact. This is quite common and nearly unavoidable, particularly in view of the very small strains involved and the sheet-formed and welded nature of the structure.

(a)

(b)

Figures 11: Toyota compression test data, original QPE FE results and **Universal Law** FE results
(a) flat design (b) jogged design.

DISCUSSION

The conclusions presented above raise many questions, a few of which are addressed briefly here.

Micro-Mechanism(s)

Two alloys tested here having notably fine microstructures showed more nearly linear behavior than for the 24 more typical alloys. A detailed look at the degradation of the stress-strain slope for these alloys showed that the deviations from the **Universal Law** occurred primarily at larger strains following unloading, more than 0.002. This suggests that both of the plausible mechanisms identified in the *Micromechanisms Governing the Transition Region* sub-section of this paper operate for most alloys and that Mechanism 2 (dislocation pile-up and relaxation) was suppressed by the very fine microstructure. This makes sense if the distance between obstacles

is insufficient to create substantial “pile-ups” (or generalized polarized structures having a similar role).

Continuum FE simulations based on an RVE representation of a DP600 microstructure²⁷ have been presented (Govik et al., 2014). Figure 12 replots predictions from Govik (Govik Figs. 11a and 11b) along with results from the current work. The continuum mechanism shows a smaller decrease of modulus than that observed experimentally. The divergence is consistent with a concurrent mechanism unaccounted for by the continuum framework, presumably a dislocation pile-up mechanism. This tentative conclusion is in agreement with the suggestion made in the Introduction section of this paper.

Figure 12: Comparison of E_T reduction for the current experimental data and from RVE-based FE simulations (after Figs. 11 of Govik et al., 2014) for DP 600.

Is E_0 equal to Young's Modulus for all 26 Alloys?

²⁷ It should be noted that the Govik simulations make use of assumed constitutive relations for constituent martensite and ferrite that are each fit with modulus degradation using an inverse method applied to the macroscopic behavior. That is, the predictions are not independent of the measured macroscopic behavior.

It was concluded that stress-strain nonlinearity begins at the transition point for all materials tested, and that at least for the 12 core steels the initial slope was equal to Young's modulus. In order to test the second conclusion more consistently with the first conclusion, a "2-P" form was created, identical to 1-P except that E_0 is an adjustable parameter representing the optimized initial slope.

The 1-P fit is compared with all the other forms tested here for the 26 alloys, as follows:

$E=E_{\text{Handbook}}$	72 MPa (0.849)	41 GPa (0)
$E=E_{\text{Chord}}$	25 MPa (0.980)	19 GPa (0)
<i>Test-to-test scatter</i> ²⁸ :	<i>10 MPa (0.992)</i>	<i>11 GPa (0.646)</i>
1-P (Eq. 4):	5 MPa (0.999)	8 GPa (0.767)
QPE-2 (Eqs 6, 7):	4 MPa (0.999)	7 GPa (0.805)
2-P (Eq. 4):	3 MPa (1.000)	6 GPa (0.843)
QPE-3 (Eqs 6, 7):	2 MPa (1.000)	6 GPa (0.838)
QPE-4 (Eqs 6, 7):	2 MPa (1.000)	6 GPa (0.827)

The conclusion from the 12 core steels is confirmed for all alloys. The data does not justify a postulate of an initial slope different from the physically-measured Young's modulus. That is, the nonlinear mechanisms, while vanishingly small at the transition point, become progressively more active from the start of reverse deformation.

CONCLUSIONS

Hundreds of carefully-controlled, high-precision, load-unload-load tensile tests were carried out for 26 diverse alloys using a Class A extensometer. The transitional mechanical responses were analyzed linearly and nonlinearly and compared with traditionally-measured Young's modulus and chord modulus representations. The following conclusions were reached:

- 1) Within the limits of material and experimental variability, even using the most sensitive techniques available, there is no significant evidence of a finite linear region. The data is consistent with nonlinearity throughout the transition region approaching a line only at zero change of strain.²⁹
- 2) Within the limits of material and experimental variability, even using the most sensitive techniques available, a simple 1-parameter nonlinear equation represents the entire transition region for metals adequately. Multi-parametric fits do not significantly improve the reproduction of the experimental data.

²⁸ The quoted test-to-test scatter is from the 12 core steels only: the only ones for which multiple tests were performed. The results for the remaining alloys seem consistent with these numbers, but have not been measured directly.

²⁹ While tempting to refer to this conclusion as demanding a "yield stress of zero," the nature of the deformation in the transitional region is not limited to elastic or plastic, or even a combination of elastic and plastic, as outlined by Sun and Wagoner, (2011). It is only possible to conclude that no finite proportional limit is justified by the experimental data.

- 3) The transitional stress-strain behavior for all alloys obeys a “Universal Law” that depends only on two independently-measured and commonly-known material values: strength and Young’s modulus. There are *no undetermined or adjustable parameters* required.
- 4) The Universal Law does not take into account the composition, the microstructure, the hardening/deformation mechanisms, or the crystal structure of the alloy. Therefore, no testing of individual alloys beyond obtaining those two properties is required to use it effectively.
- 5) The Universal Law was shown to predict the collapse curve for an applied automotive structure twice as accurately as a standard linear representation, and equally accurately as a complex multi-parameter representation that requires special testing and fitting to obtain.
- 6) The results suggest the operation of two concurrent micro-mechanisms producing nonlinear and reversible transitional behavior: continuum inhomogeneity and dislocation pile-up mechanisms. Of the two, the pile-up mechanism appears to dominate.

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