Effects of temperature and hydrogen-like impurity on the coherence time of RbCl parabolic quantum dot qubit

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Abstract By using a variational method of Pekar type, the Fermi Golden Rule and the quantum statistics theory (VMPTFGRQST), we investigate the effects of the hydrogen-like impurity and temperature on the coherence time of a parabolic quantum dot (PQD) qubit with a hydrogen-like impurity at the center. We then derive the ground and the first excited states' (GFES) eigenenergies and the eigenfunctions in a PQD. A single qubit can be realized in this two-level quantum system. The phonon spontaneous emission causes the decoherence of the qubit. The numerical results show that the coherence time is a decreasing function of the temperature, the strength of the Coulombic impurity potential (CIP) and the polaron radius (PR).

Keywords RbCl parabolic quantum dot · Qubit · Temperature · Hydrogen-like impurity

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1 Introduction

Quantum information processing devices outperform their classical counterparts by preserving and exploiting the correlated phases of their constituent quantum oscillators, which are usually two-state systems called as “qubits”. [1] An increasing number of theoretical proposals [2-4] have shown that such devices allow secure long-distance communication and improved computational power. Solid-state implementations of these devices are favored due to both scalability and ease of integration with existing hardware, although previous experiments [5-7] have shown limited coherence times for solid-state qubits. The development of quantum error-correcting codes and fault-tolerant quantum computation (QC) [8] showed that largescale quantum algorithms are still theoretically possible in the presence of decoherence. However, the coherence time must be dauntingly long: approximately 10^5 times the duration of a single quantum gate, and probably longer depending on the quantum computer architecture. [1,9] The question of whether a scalable implementation can surpass this coherence time threshold is not only important for the technological future of QC, but also for fundamental understanding of the border between microscopic quantum behavior and macroscopic classical behavior.[9-11] Recently, Sun[12] et al. obtained effects of magnetic field on the coherence time of a parabolic quantum dot qubit. Gale [13] et al. measured the coherence time in high energy Proton-Nucleus collisions. Hayashi et al. studied coherent manipulation of electronic states in a double quantum dot [14]. Effects of temperature and hydrogen-like impurity on the coherence time of a parabolic QD qubit, however, have not previously been investigated yet.

In this article, we investigate the effects of the temperature, the strength of CIP, PR of the PQD

on the coherence time by using VMPTFGRQST. The temperature effects on the coherence time in parabolic QD under hydrogen-like impurity field are important subject in quantum information and quantum computing processing.

2 Theoretical Model and Calculations

Consider the electron confined in the parabolic potential QD. The electrons are confined much stronger in one direction (taken as the Z-growth direction) than in the other two directions. Therefore, we will focus on the condition of the electron and LO phonon only on the X-Y plane [15,16]. The electron under consideration is moving in the RbCl crystal PQD. The Hamiltonian of an electron-phonon interaction system with a hydrogen-like impurity at the center takes the form of [17]:

$$H = -\frac{\hbar^2 \nabla^2}{2m} + \sum_{\mathbf{q}} \hbar \omega_{LO} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \frac{1}{2} m \omega_0^2 \rho^2 + \sum_{\mathbf{q}} [V_{\mathbf{q}} a_{\mathbf{q}} \exp(i\mathbf{q} \cdot \mathbf{r}) + h.c.] - \frac{\beta}{r}, \quad (1)$$

where $a_{\mathbf{q}}^{\dagger} (a_{\mathbf{q}})$ denotes the creation (annihilation) operator of the bulk LO phonons with wave vector $\mathbf{q}$. $\mathbf{p} = (\mathbf{p}_{\rho}, p_z)$ and $\mathbf{r} = (\boldsymbol{\rho}, z)$ are respectively the momentum and position vectors of the electron. $-\beta/r$ denotes the CIP between the electron and the hydrogen-like impurity. The strength of the CIP β is $\beta = e^2/\epsilon_0$. $V_{\mathbf{q}}$ and α in Eq.(1) are

$$V_{\mathbf{q}} = i \left(\frac{\hbar \omega_{LO}}{q} \right) \left(\frac{\hbar}{2m\omega_{LO}} \right)^{\frac{1}{4}} \left(\frac{4\pi\alpha}{v} \right)^{\frac{1}{2}}, \alpha = \left(\frac{e^2}{2\hbar\omega_{LO}} \right) \left(\frac{2m\omega_{LO}}{\hbar} \right)^{\frac{1}{2}} \left(\frac{1}{\epsilon_{\infty}} - \frac{1}{\epsilon_0} \right). \quad (2)$$

Following the VMPT [18,19], the system's trial GFES' wavefunction is given by [20,21]

$$|\varphi_0(\lambda_0)\rangle = |0\rangle |\xi(z)\rangle |0_{ph}\rangle = \frac{\lambda_0}{\sqrt{\pi}} \exp\left[-\frac{\lambda_0^2 \rho^2}{2}\right] |\xi(z)\rangle |0_{ph}\rangle, \quad (3)$$

$$|\varphi_1(\lambda_1)\rangle = |1\rangle |\xi(z)\rangle |0_{ph}\rangle = \frac{\lambda_1^2}{\sqrt{\pi}} \rho \exp\left(-\frac{\lambda_1^2 \rho^2}{2}\right) \exp(\pm i\phi) |\xi(z)\rangle |0_{ph}\rangle, \quad (4)$$

where λ_0 and λ_1 are the variational parameters, $|\xi(z)\rangle$ is the electron's wavefunction in the z- growth direction, since the electrons are much more strongly confined in z- growth direction than in other two directions and considered to be confined in an infinitesimally narrow layer, so does $\langle \xi(z) | \xi(z) \rangle = \delta(z)$. The electron's GFES energies in a PQD can be derived in the following form:

$$E_0(\lambda_0) = \frac{\hbar^2 \lambda_0^2}{2m} + \frac{\hbar^2}{2\lambda_0^2 m l_0^4} + \frac{m\omega_c^2}{8\lambda_0^2} - \frac{1}{2} (2\pi)^{\frac{1}{2}} \alpha \hbar \omega_{LO} r_0 \lambda_0 - \sqrt{\pi} \beta \lambda_0, \quad (5)$$

and

$$E_1(\lambda_1) = \frac{\hbar^2 \lambda_1^2}{2m} + \frac{\hbar^2}{m \lambda_1^2 l_0^4} + \frac{m \omega_c^2}{4 \lambda_1^2} - \frac{11}{32} \alpha \hbar \omega_{LO} (2\pi)^{1/2} r_0 \lambda_1 - \frac{\sqrt{\pi}}{2} \beta \lambda_1, \quad (6)$$

where $l_0 = (\hbar/m\omega_0)^{1/2}$, $r_0 = (\hbar/2m\omega_{LO})^{1/2}$, l_0 , r_0 is the effective confinement length (ECL) and the PR, respectively. A single qubit can be realized in this two-level quantum system. The electron's superposition state reads

$$|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad (7)$$

Under the dipole approximation, based on the Fermi Golden Rule [22], the spontaneous emission rate can be calculated using

$$\tau^{-1} = \frac{e^2 (\Delta E)^3}{\varepsilon_0 \hbar^4 C^3} |\langle 0 | \rho | 1 \rangle|^2 = \frac{e^2 (\Delta E)^3}{\varepsilon_0 \hbar^4 C^3} \frac{16 \lambda_0^2 \lambda_1^4}{(\lambda_0^2 + \lambda_1^2)^4}, \quad (8)$$

where $\Delta E = E_1 - E_0$ denotes the energy separation between the GFES and C is the speed of light in vacuum. ε_0 is the material (vacuum) dielectric constant and τ is the coherence time.

3 Temperature effec

At finite temperature, the properties of the polaron are statistical averages of various states. According to the quantum statistics theory, the statistical average number of the bulk LO phonons is in the following form

$$\bar{N} = \left[\exp\left(\frac{\hbar \omega_{LO}}{K_B T}\right) - 1 \right]^{-1}, \quad (9)$$

where K_B presents the Boltzmann constant and T is the temperature of the system. The optical phonon's mean number in superposition state around the electron is expressed as

$$N = \frac{1}{4} (2\pi)^{1/2} \alpha r_0 \lambda_0 + \frac{11}{64} (2\pi)^{1/2} \alpha r_0 \lambda_1. \quad (10)$$

Through self-consistent calculation based on Eq. (9) and Eq. (10), we can obtain the relationship between variational parameters λ_0 and λ_1 and the temperature T . From Eq. (8) we can see that the coherence time in a PQD depends on the variational parameters λ_0 and λ_1 , and then is related to the temperature T .

4 Numerical results and discussion

The experimental parameters used in the calculation are $\hbar \omega_{LO} = 21.45$ meV,

$m = 0.432m_0$, $\alpha = 3.81$. The numerical results on the coherence time τ of the two-level quantum system in the RbCl PQDs versus the temperature T , the strength of the CIP β and the PR r_0 are presented in Figs.1-2.

Figure 1 shows the coherence time τ versus the temperature T and the strength of the CIP β for RbCl crystal with ECL $l_0 = 10.0\text{nm}$ and the PR $r_0 = 1.0\text{nm}$. It turns out that the coherence time is a decreasing function of the temperature. The influences of the temperature in the RbCl QPD are stronger in the ground-state than that in the first-excited state with increasing temperature. For this reason, the energy space between the GFES increases with increasing temperature and then the coherence time decreases, which is in good agreement with the results of Makhlin et al. [23], Wang et al. [24] and Xiao et al. [25], whereas this result is the opposite of asymmetric QD qubit [26]. From another point of view, with the enhancement of the temperature, the velocity of the electrons and the phonons is increased, and then there are more electrons to interact with phonons. The contribution from the electrons interacting with more phonons, which destructs the superposition state, is larger than that from the increased electron velocity that makes more electrons to be located on the superposition state. Hence, the coherence time decreases with increasing temperature. Figure 1 also indicates that the coherence time will increase with decreasing strength of the CIP. The reason is that the increase in the first-excited state is smaller than that in the ground-state with increasing CIP. So the increasing of the energy space between the GFES induces the reduction of the coherence time.

In Fig. 2, we plot the coherence time τ varying with the temperature T and PR r_0 for RbCl crystal with the CIP $\beta = 4.0\text{meV} \cdot \text{nm}$ and the ECL $l_0 = 10.0\text{nm}$. We find that the coherence time increases with decreasing temperature and the polaron radius. Because the influence of the polaron radius in the PQD is stronger in the ground state than that in the first-excited state with increasing polaron radius. Therefore, the energy space between the GFES increases and the coherence time decreases. This result is in agreement with the result of Xiao et al. [25], which is for the case without the hydrogen-like impurity. The magnitude of the coherence time obtained in the present paper is 0-100 microseconds, which is in agreement with Refs. [27] and [28], which were obtained respectively in experiments and by the effective mass envelope function theory.

This two-level quantum system is very frail due to the interactions between the quantum memories and their surroundings. The coherence of qubit is crucial to the investigations of quantum computation, where the coherence time is one of the physical quantities representing the properties of coherence. In addition, the electron's probability density and oscillation period also are the physical quantities representing the properties of coherence. The variations of the coherence time and oscillating period with the energy space between the GFES are the same as in [24,26]. Because with increasing electron's probability density and oscillation period, the life time of the qubit is prolonged, and then the coherence time is longer too. Moreover, one can adjust in experiment the size, the effective confinement length, the polaron radius, the temperature and the

materials (different electron-phonon coupling) of the QD to tune the coherence time, the electron probability density and the oscillation period. These three quantities can also be adjusted by external fields, such as electric field, magnetic field and the impurities (Coulomb field).

Due to the existence of the hydrogen-like impurity, the coherence time is decreased. To compensate the effects of the hydrogen-like impurity on the coherence time, one may diminish or shield the hydrogen-like impurity and reduce the PR. Here we can find that we can adjust the coherence time by changing the temperature, the strength of the CIP and the PR.

5 Conclusion

We investigate the effects of the hydrogen-like impurity and temperature on the coherence time of a PQD qubit by using the VMPTFGRQST. The calculated results illuminate that the coherence time will decrease with increasing temperature. The coherence time is decreasing function of the strength of the CIP and the PR. We find that by changing the temperature, the strength of the CIP and the PR one can adjust the coherence time. Our calculated results may be useful for designing the solid-state QC.

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Fig.1 The coherence time τ changes with the temperature T and strength of the CIP β .

Fig.2 The coherence time τ changes with the temperature T and PR r_0 .