Energy exchange in coupled interactions between a shock wave and metallic plates

by

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ABSTRACT

An experimental investigation was carried out to consider the partitioning of energy deposited by a planar shock impinging on thin metallic plates mounted at the end wall of a large scale shock tube. A split-view time-dependent digital image correlation technique was employed to capture the three-dimensional motion of the external surface of the specimens. Steel plate and aluminum plates were tested in the present work. The experimental results were analyzed by using the control volume approach with moving boundaries and the presence of discontinuities. The energy deposition by the flow on the specimens were examined and discussed in two different frames of reference to demonstrate that energy is not Galilean invariant. It was found that most of the deposited energy is spent on plastic deformation of the plates followed by the kinetic energy, which reached significant values only in the initial stages of the shock-material interaction.

1. Introduction

Dynamic loading of elastic structures by shock or blast wave impact has been investigated for many years. The impingement of these waves on a structure is associated with a number of complex phenomena which include reflection of the wave off the surface of the structure followed by a substantial transient aerodynamic load, which can cause significant plastic deformation of the structure that may subsequently cause damage or lead to phase transitions. In addition, the interaction between the shock/blast wave and the structure can induce reverberation within an enclosure and cause substantial overpressure through multiple reflections of the wave.

The mechanical response and characterization of materials under transient dynamic loading caused by shock or blast wave impingement is not well understood. Understanding the response of different materials to shock wave loading is critical in the design of structures. When materials are subjected to extreme thermomechanical conditions and shocked above a threshold strength known as Rankine–Hugoniot elastic limit, plastic deformation relaxes the compressive stress along the direction of shock wave propagation and therefore reduces its strength while at the same time a plastic wave appears behind travelling at lower velocities. However, in many materials (including metals), strength has been observed to increase during and after the shock impact due to strain and strain-rate effects such as dislocation build-up and hardening.

The assessment of the effects of blasts on structural systems was studied systematically after World War I. An early published work was Hopkinson's [1] study of pressure from high explosives or bullet impacts. One of the earliest analyses related to structures subjected to air-blast loading in the literature was performed by Taylor [2], in which he developed a solution for a one-dimensional wave impinging on a free-standing plate to compute the momentum transmitted to the plate by the shock pulse. The prediction and measurement of the structural response of ship panels to free-field air-blast explosions were first investigated by Houlston et al. (1985) [3]. In this research, square plates and stiffened panels subjected to air-blast and underwater shock loads were investigated. Later, assuming a form of displacement function and a modified Friedlander equation reflected blast overpressure loading that exponentially decays with time was estimated.

Gupta et al. [4] conducted a single-degree-of-freedom elastodynamic analysis of the response of a rectangular plate subjected to an explosive blast. A literature review of works on the response of thin plates to impulse loading prior to 1989 was presented in Nurick and Martin [5,6]. These authors developed a theoretical platform based on non-dimensional quantities involving the impulse of the load,

material properties and plate geometry and deflections to provide a unified behavior of plate response to blast loading.

Beshara [7] investigated the prediction of dynamic effects of unconfined explosions needed for the structural analysis of blast-loaded above-ground structures and introduced a unified approach to model the associated phenomena related to loading for mathematical and practical applications. Türkmen and Mecitoğlu [8,9] carried out a study on the dynamic behavior of laminated composite plates subjected to blast loading. In this study, theoretical analysis and experiment results of the strain time histories were obtained.

Xue and Hutchinson [10] studied the quasi-static dynamic responses of clamped circular sandwich plates subject to uniformly distributed impulsive loads. Numerical analysis on the plate neglecting the effects of fluid-structure interaction was carried out and a new constitutive relation for sandwich plates was introduced as well [11]. In similar research, the dynamic response of a clamped sandwich beam/plate was assessed. The fluid-structure interaction effect was evaluated utilizing Taylor's approximation by including a segregated fluid-structure interaction phase in research by Deshpande, Fleck, and Qui et al. [12,13].

Impact events are very complex to treat analytically and understand physically even in very simplified cases which involve one dimensional setups. This is primarily due to the variety of materials, and geometries of the plates and variations in the impulse loading. Strain and strain-rate effects are present in metals, polymers, composites and bio-materials with different results depending on the material's dynamic strength [14,15]. In particular, Whiteman and Millet [14] have demonstrated experimentally how an initially one-dimensional deformation due to a corresponding one-dimensional compression leads to the development of a strongly three dimensional stress field causing an increase in shear strength of austenitic steel due to hardening. Manganin stress gauges were used in this investigation which were tested in different encapsulating materials in [15] and proved to be free of shock-dispersion effects.

In the present work, thermal effects are considered very small since the value of the parameters used are well below the Rankine-Hugoniot limit and therefore the focus is on extreme mechanical environments. We will consider the mutual energy exchange between a shock wave of moderate strength and an idealized structure in the form of a thin plate. Such information is very critical in characterizing materials that can dissipate or absorb the energy in the shock waves causing them to propagate slower and therefore become less destructive. It is also known, that the mechanical properties of materials, particularly metals, under dynamic loading improved substantially. Such behavior is classified as strain-rate effects. The mechanisms for the high energy absorption capability at high strain rates are not fully understood. In the context of our experimental conditions a mechanical dynamic load is applied to a plate by an impinging shock wave with a rate of the order of 10^{13} Pa/s. This results in a complex aeroelastic interaction between the shock wave and the plate. The shock wave starts to load the structure while at the same time starts to reflect.

This problem is of particular importance in the design of building structures and vehicles where maximum survivability is sought. In the past, this interaction has been considered as "one-way" only, i.e. the impacting shock wave creates a load which deforms the structure. However, this deformation affects the shock wave reflection and therefore the interaction is a two-way process. The mutual interaction between a blast or shock wave and an elastic structure has been studied experimentally by Gong and Andreopoulos [16] and computationally by Gong and Andreopoulos [17] and for the case of shock impacted composite plates by Andreopoulos [18].

The objective of the present work is to develop an understanding of the mutual energy exchange between the impinging shock and thin plates. Particularly, we will start this work by revisiting the question how much energy is being transferred to a deformable body by the shock. Subsequently we will investigate how this energy is redistributed or partitioned into various other forms and its pathways of dissipation within the structure. This will be accomplished by introducing appropriate control volumes with moving boundaries and corresponding reference frames. We will explore the fact that energy is *not* a relativistic invariant; instead it is frame dependent, by considering two cases, one with energy transfer in a coordinate system moving with the shock and second with energy exchanges for a stationary observer. In that respect, the present work is different from the work of Wang & Shukla [19] who used Rankine–

Hugoniot relations to solve for the flow particle velocities, density and pressure and found that less than 5% of the shock energy is used to deform the structure.

The present work was conducted in a laboratory environment involving a large scale shock tube facility as opposed to carrying out full-scale field experiments with actual geometries and charges that are quite involved and costly, both in terms of preparation and measurements. The major advantage of a laboratory simulation as opposed to field measurements is its controllable environment which can provide a platform for systematic investigation of complex phenomena with well-established initial and boundary conditions. This is critically important in basic research where a better understanding of the underlying physics is sought or high fidelity experimental data are needed to test and verify advanced computational methods with specific predictive capabilities. An additional advantage of laboratory testing is the ability to vary boundary conditions.

Most laboratory-scale research has focused on the deformation of plates subjected to blast loads. Approaches using a shock tube platform appear to have certain distinct advantages over approaches using detonations. A major advantage is that the shock front in the shock tube is planar, thus yielding uniform pressure behind it which loads the structure. This simplifies the geometry and boundary conditions and therefore it allows for a better evaluation of modeling issues associated with numerical simulations of the phenomena involved. In contrast, shock wave fronts generated by detonations are three dimensional in space and lead to complex pressure loadings which can mask the underlying basic physics of the impact. Once the physics of the impact is better understood in simple geometries then the next level of complication can be introduced.

The complicated interactions under investigation need to be set up by using a simple, yet relevant geometry of the metallic material structure and simplified flow fields which have the potential to reveal basic features of the associated phenomena. Figure 1 shows a schematic of the present setup. A planar shock impinges a flat metallic plate mounted at the end wall flange of a shock tube. The pressure behind the incident shock wave, p , remains constant across the flow field before the shock reflection. As the shock wave contacts the surface of the plate, the process of reflection starts, the plate is loaded and deformation begins which leads to a vibration pattern.

Figure 1: Schematic of shock impact on flat plate.

2. The process of reflection and the onset of deformation

In the analysis described below, we will follow the classical shock tube nomenclature as shown in fig. 2, where region 4 is the driver section between the diaphragm and its end wall, region 2 is the region behind the moving incident shock, region 1 is the upstream region between the end wall downstream and the incident wave while the region behind the reflected shock is designated as region 5.

Figure 2: Shock tube schematic; (a) regions after rupture of diaphragm; (b) region after end wall reflection.

Figure 3: Schematic of the process of wave reflection

It is very useful to consider the interaction of the plate and the moving shock by first considering the process of reflection over the surface of the plate. The incident shock wave which is traveling with velocity W_S and although traditionally it is treated as a discontinuity, here we assume that it has a thickness δ . The impact of the incident shock wave on the flat plate is associated with the generation of a transmitted wave with propagation velocity W_{PW} within the plate of thickness t_p (see fig. 3) and the appearance of a reflection which starts to propagate within the gas and is moving in the opposite direction with velocity W_R . The time scale, τ , of the shock reflection can be approximated by $\tau \approx \delta / W_S$ and a typical estimate of the shock thickness according to Thompson (1972) [20] is obtain by $\delta = \lambda_p \sqrt{\gamma}$ where λ_p is the mean free path, γ is the specific heat ratio for air and p and ρ are the pressure behind the and upstream of the incident shock respectively. An estimate of τ can be obtained as $\tau \approx \lambda_p \sqrt{\gamma} / W_S$, where η , ρ and c are the viscosity, density and speed of sound of air upstream of the shock. For the shock strengths investigated here, the time-scale of reflection appears to be 1.6τ . The transmitted wave within the plate has a time scale τ_p with $\tau_p = t_p / W_{PW}$ where E and ρ are the Young's bulk modulus of

elasticity and density of the plate material. It is very instructive to compare with . If , the deformation will start much earlier than the completion of the wave reflection process and will affect that process dramatically by interfering with the shock. In the case of, the reflection will be much faster than the onset of the plate movement and the reflected shock won't be affected. In the present case, steel and aluminum plates were used and after approximating the wave speeds within plates to be about 5km/s for both, the time scale of the transmitted wave is estimated to be . Thus, the time scale of the shock reflection is about 4×10^3 faster than the time scale of the transmitted shock inside the plate. In that respect, the plate is not moving and the reflection is practically taken place with the plate behaving like a rigid body. This implies that the flow velocity will acquire the wall velocity so that and is now the stagnation pressure behind the reflected shock which loads the plate. The plate, in this case, will start moving much later, in time scales related to vibrational motions which are of the order of . If the deflection of the plate is assuming axisymmetric deformation and its velocity , then it is reasonable to assume that the local fluid speed will acquire the plate velocity so that .

The short time scale of the shock reflection affects also the loading rate for the plate which is of the order of Pa/s. As was mentioned earlier, the mechanisms of high energy absorption rates of such plates under high loading or strain rates over a range of time scales are not fully understood and it is believed that focusing first on the vibrational time-scales may be more feasible to characterize the material and structures.

3. Experimental facilities and setup

The experiments were carried out in the large-scale Shock Tube Research Facility of the Department of Mechanical Engineering at CCNY. The facility is shown schematically in figure 4 and is described in detail by Briassulis, et al. (1998) [21]. It has been used extensively for research on shock wave interactions with turbulence and vortices (Andreopoulos et al., 2000; Briassulis et al., 2001, Agui et al., 2005) [22–24] and shock interactions with monolithic and composite plates Gong & Andreopoulos (2008) [16]. The specimen was mounted at the end flange of the working section.

The rupture of the diaphragm between driver and driven sections releases a shock wave which travels downstream at pressure to the end of the working section where the test plates are mounted. By rearranging the various parts of the driven section the duration of the pressure load can be varied as well as the type of loading, shock or blast wave. In the present work, the shock tube was configured for uniform pressure loading behind a reflected shock wave. The pressure in the shock tube was measured by Kulite and optical pressure transducers installed in the working section. Aluminum or Mylar diaphragms were used in the present experiments.

Figure 4: Schematic of the CCNY High Resolution Shock Tube Facility, with the unused features shown in grey.

The test specimens used in this work were thin circular plates 380 mm (15") in diameter. Mild steel and 6060 Aluminum plates were used as specimens. In this study the thickness of the plates was 3.2 mm (1/8"). Table 1 shows the parameters and properties of the materials tested.

Table 1: Parameters and properties of materials tested.

Material	Free radius, a [mm]	Thickness t_p , [mm]	Density ρ_p , [kg/m ³]	Elastic Modulus E [GPa]	Poisson's Ratio	Flexural Rigidity [Pa·m ³]	Bending Wave Speed U_p [m/s]	Time Scale [ms]
Aluminum	152	3.2	2,700	69	0.33	207	4.91	31.05
Mild Steel	152	3.2	7,900	200	0.30	586	4.83	31.52

4. Digital Image Correlation Techniques

An experimental technique which was initiated in Jahnke and Andreopoulos (2011) [25] has been developed that is based on digital image correlation (DIC) to capture the 3-dimensional deformation of the plate under high-rate loading. It consists of a camera acquiring two split-views of the motion by two different mirror pairs located at different positions that are inclined and tilted appropriately to capture the time-dependent motion of the whole plate specimen under blast/shock loading. The optical set up includes two large size planar primary mirrors A & B viewing the specimen from two opposite locations tilted to about 30 degrees to the specimen as shown schematically in figure 5a. Two secondary mirrors A&B are positioned about the center line to reflect the image from the primary mirrors into the camera. An optical system consisting of a combination of light reflecting (mirrors) and refracting elements (lenses) is called Catadioptric. The present optical setup consists of a single high-frame-rate camera, Vision Research v710 Phantom high speed camera with a native resolution of 1280×800 pixels, which can accommodate two simultaneous stereo images of the deforming structure on its CMOS chip and therefore it is a real Time-Resolved Catadioptric Stereo Digital Image Correlation (TR-SDIC) system. Figure 5b shows one photograph of the TR-SDIC setup. Details of the technique including its development and validation are described in [26].

Figure 5a: TR-SDIC setup in the shock tube with two pair of mirrors

Figure 5b: Mirror setup for high-speed video capture of displacement for TRC-SDIC. The lens of the camera can be seen coming out of the tube section at the right. The two large circular mirrors are the primary mirrors. The secondary mirrors are partially visible between the two primary mirrors.

An in-house MATLAB-based software solution was developed to reconstruct the 3D motion of the external surface of the specimen by combining the two cross-correlation maps of the speckle displacements from two different split-views. Shape modes were employed to fit the experimental data with Bessel functions. Further extensive validation tests of our optical techniques were undertaken out which included comparisons with static and dynamic data obtained from embedded sensors in specimens time-synched to the high-speed video images. The TRC-SDIC system is designed to capture a global view of the specimen and therefore the displacement information is available on every point on the external surface of the deformed plate. The CMOS chip of the Phantom V7.1 camera contains 1280×800 pixels and therefore each of the two split views roughly contains 640×800 pixels. Since the diameter of the specimen which occupies most of the chip area is $D=304$ mm, the resolution appears to be between 2 and 2.6 px/mm. In reality, the capture image size is about 400 mm and therefore the effective resolution is between 1.45 and 2 px/mm. The size of the speckles on the specimen surface is between 5 and 8 pixels. This resolution is adequate to evaluate displacement but inadequate to resolve directly their gradients and therefore obtain strain measurements. Instead, the displacement data were fitted with a least-error curve which provided a continuous variation between discrete displacement points and allowed a more accurate differentiation of the displacement data to obtain the strains and . Finding the best data-fitting curve is very challenging since the strain results are very sensitive to the local curvatures and inflexion points. It was found in our previous work with shock waves impinging on composite or monolithic material plates (see [27] and [16]) that immediately after the impact the plate starts to deform and engage in a vibrational motion first in 1st mode and subsequently in the 2nd mode. It was then not surprising that the best fit to the experimental displacement data was provided by the mode shapes of the vibration.

5. Results

5.1 Plate displacement

Figure 6 shows a sequence of 6 frames describing the evolution of the Z-displacement contours on the external surface of the steel plate at various times which are multiples of the frame-acquisition rate $1/7500$ s = 0.133 ms. A video of continuous frames is also available through this submission. At $t=0$ ms, the beginning of the interaction, no displacement is detected as required. The pattern of deformation is evident from the following 3-D plots. It appears that the displacement is not uniform across the surface. It is, however, reasonably axisymmetric during the time of deformation within the experimental uncertainty. The area of the plate near the rim of the flange exhibits the least displacement throughout all frames because of the constraints imposed by the boundaries. The plot at $t=0.4$ ms shows an interesting behavior. The region of the plate around $r=85$ mm experiences the largest displacement of about 3 mm at this

moment while the middle of the plate around the center lags by about 1 mm. Subsequently at $t=0.67$ ms the plate's displacement increases rapidly to 9 mm at the center. The maximum displacement occurs at the center at $t=1.33$ ms and by this time the plate exhibits signs of permanent deformation and plasticity. After this time, the middle part of plate experiences vibrations where the displacement increases and decreases with a frequency which appears to be close to the 2nd mode. Our observations indicate that this vibrating part of the plate is defined approximately by a circle with $r=60$ to 75 mm. A sequence of contours of the in-plane radial displacement is shown in figure 7. Detectable radial displacements between 0 and 1.25 mm appear first at $t=0.67$ ms. At later times the radial displacement increases to a maximum of 1.5 mm in the locations of the plate defined by $100 \text{ mm} > r > 30 \text{ mm}$. It is very interesting to observe that the pattern of the iso-contours is circular and axisymmetric, with zero radial displacement around the center of the plate and close to the supporting flange as expected.

5.2 Strain

TRC-SDIC method described above provides the out of plane displacement (z) in each mesh element over the plate based on an axially symmetric grid of 3,000 elements as shown in figure 8. In order to obtain strain values over the testing plate, Lagrangian strain definition is employed in this study [28]

Figure 6: Contours of out-of-plane displacement of steel plate at various times after shock impingement.

and

Noting that ϵ_{xx} are nonlinear strains, ϵ_{yy} are linear strains and ϵ_{zz} are rotations of an element about the i -axis.

Figure 7: Radial displacement U_r contours of steel plate at various times after shock impingement.

Figure 8: Generated mesh on testing plate

According to Kirchhoff's plate theory, tractions on surfaces parallel to the reference plane are very small compared to in-plane stresses and in-plane displacements are linearly proportional to the z . Based on this assumption, the in-plane displacements (u , v) and out of plane displacement (w) at any random point of the plate in the x , y , z directions can be estimated by

$$(2)$$

where $u^o(x,y)$, $v^o(x,y)$ and $w^o(x,y)$ are the reference plane displacements, (U, V, W) , is the general displacement at any point, and θ_x, θ_y are the rotation vector components of the reference plane.

The relations in Cartesian coordinates described above can be transformed to polar coordinates.

$$(3)$$

Axisymmetric condition of the test plate yields the following relation between displacement and strain.

$$(4)$$

The evolution of the strain field on the external surface of the plate is shown in figure 9 where contours of the radial strain are plotted. The first band of tensile (positive) strain appears at about $r=90$ mm just around the inflection point of the Z-displacement at $t=0.4$ ms and then it propagates towards the center. At the same time compressive (negative) strain starts to develop in the outer part of the plate and close to the support. In general, throughout the rest of the shock loading, the strain appears to be compressive in the outer part of the plate and tensile in the vicinity of the center while in the region between, it changes sign to adjust to these highly stressed areas. Maximum tensile and minimum compressive strains of about 6000 and -16,000 respectively are observed at $t=1.67$ ms. After this time, the radial strain field exhibits very small fluctuations.

Contours of the tangential strain at various times are shown in figure 10. The strain field is tensile throughout all locations of the plate and remains always positive during the deformation. The area first strained appears to be close to the rim of the flange as shown at $t=0.4$ ms. Subsequently, tensile strain starts to develop in the rest of the plate and reaches maximum value of about 5000 near its center at $t=1.67$ ms. Minor asymmetries appear to develop which are of the order of the measurement uncertainties due to limits of the optical setup including variations in mirror positioning.

In the case of the aluminum plate specimen, the out-of-plane displacement after the shock impingement is much higher than that of the steel plate. This is shown in figure 11 where a sequence of Z-displacement contours is displayed as measured at different times. At the beginning, the observed displacement appears to be relatively uniform throughout the plate except close to the flange as is shown at $t=0.4$ ms. This is a feature which has been also observed in the case of the steel plate. Maximum displacement of 22 mm is observed at $t=0.66$ ms, which is earlier than the corresponding time in the case of steel plate. Thus, the aluminum plate not only deforms more than the steel plate but this deformation peaks at earlier times too.

Contours of the in-plane displacement in the radial direction are shown in figure 7. The overall characteristics of this displacement are qualitatively similar to those observed in the experiments with the steel plate. However, there are many quantitative differences. Since this aluminum plate deforms more than the steel plate, it is expected that the in-plane displacements be larger. Values of U_r about 6mm are measured at $t=0.8$ ms while the maximum value of 8 mm is observed at later time $t=3.3$ ms (not shown in fig. 7). Similarly, the strains developed on the external surface are also higher as shown in the next two figures 9 and 10 where the contour sequences are plotted for the radial and tangential strains respectively. The maximum tensile radial strain of 7000 is also observed around the center of the plate while the minimum (negative) value of compressive strain is -50,000 observed close to the rim. It is also

interesting to observe that this maximum strained field is developed at $t=0.67$ ms and remains about the same afterwards, most probably due to permanent deformation that has taken place.

Figure 9: Radial strain contours of steel plate at various times after shock impingement.

Figure 10: Tangential strain contours of steel plate at various times after shock impingement.

An examination of the available videos of deformations in the case of the aluminum plate specimen indicates that the out-of-plane displacement after the shock impingement is much higher than that of the steel plate. At the beginning, the observed displacement appears to be relatively uniform throughout the plate except close to the flange. This is a feature which has been also observed in the case of the steel plate. Maximum displacement of 22 mm is observed at $t=0.66$ ms, which is earlier than the corresponding time in the case of steel plate. Thus, the aluminum plate not only deforms more than the steel plate but this deformation peaks at earlier times too.

The overall characteristics of the in-line displacement are qualitatively similar to those observed in the experiments with the steel plate. However, there are many quantitative differences. Since this aluminum plate deforms more than the steel plate, it is expected that the in-plane displacements be larger. Values of U_r about 6mm are measured at $t=0.8$ ms while the maximum value of 8 mm is observed at later time $t=3.3$ ms. Similarly, the strains developed on the external surface are also higher as shown in the corresponding videos where the contour sequences are plotted. The maximum tensile radial strain of 7000 is also observed around the center of the plate while the minimum (negative) value of compressive strain is -50,000 observed close to the rim. It is also interesting to observe that this maximum strained field is developed at $t=0.67$ ms and remains about the same afterwards, most probably due to permanent deformation that has taken place.

6. Shock tube flow with moving wall

In order to provide some additional insights into the physics of the interaction of the shock with the metallic plates, the Euler equations for 1-D flow expressing the conservation of mass, momentum and energy have been solved by using Roe's numerical approach [29]. The hyperbolic equations for this Riemann problem are

$$(5)$$

where ρ is the fluid density, u is the particle (flow) velocity, p is the pressure and E is the total energy (internal and kinetic) per unit mass. By assuming the ideal gas model and the corresponding equation of state, the above system of equations is closed and a numerical solution can be obtained. Considering the column vector of unknowns, the Euler system of equations can be written in a conservative form as . In addition to increasing the physical understanding of the complex interaction between the shock and the plate, the present computations will help extrapolate the pressure profiles measured slightly upstream of the end-wall of the shock tube to the end-wall where the specimen is mounted. This approach was necessary, because the pressure at the location of the plates could not be measured directly without compromising the structural integrity of the plates.

In the present work, we extended Roe's numerical scheme, which was written for a rigid or a fully open end-wall of the shock tube, to include the mutual interaction between the impinging shock and the deforming metallic plate. This was achieved by introducing an end-wall motion very close in magnitude to what has been measured experimentally. In particular, we modified Roe's scheme by introducing a 1-D velocity of the end wall boundary which is assumed to be the fluid's velocity. The algorithm detects the arrival of the shock at the specimen and then applies a waveform for the measured area-averaged deformation velocity

$$(6)$$

In Roe's scheme, the cell center rather the cell edge is aligned with the boundary i.e. end wall and the time-advancing formula for the wall velocity at grid point J is

$$(7)$$

where n is the time step count, F denotes the flux at or locations and α is a known coefficient. In this relation we substitute . The present experimental data indicated that the time-dependent deformation of the plate is axisymmetric and therefore the previous relation (7), which expresses 1-D equivalent of the actual 2-D deformation velocity is

$$= (8)$$

where R is the radius of the plate i.e. shock tube radius. The moving wall boundary condition was simulated by a fixed grid, oscillating fluid slug boundary condition imposed on the mass flux at the ghost points downstream of the computational points. This condition required that

$$\text{and}$$

Symmetry downstream boundary conditions in the plane of the fixed boundary were imposed on the density and energy variables, between points $J-1$ and J and their corresponding ghost points $J+1$ and $J+2$. Sufficiently removed from the swept volume, the imposed numerical boundary conditions yield results equivalent to an oscillating wall (see Gu et al., [30]).

This simplified 1-D model approximates reasonably well the two-way flow-structure interaction between the shock-flow and the deforming plates: the shock deforms the plate and the plate distorts the flow by introducing pressure waves. Values of the computed are shown in fig. 11 for the experiments with the aluminum and steel plates respectively. The origin of time, $t=0$ corresponds with the onset of reflection of the shock wave off the end plate. In the case of steel plate there is small lag in its motion due to inertia. The peak of the deformation velocity takes place about 0.5 ms after the shock reflection for the steel plate and about 0.4 ms for the aluminum plate. This deformation velocity is approximated by fitting these data to a waveform which was introduced into the 1-D model. This waveform was found to be for $0.12 \text{ ms} < t < 0.7 \text{ ms}$ and for $0.7 \text{ ms} < t$ and it is plotted in fig. 11. The first semi cycle defined between 0.12 and 1 ms represents the initial deformation of the plate which occurs at the 1st vibrational mode, while the motion after $t=1 \text{ ms}$ represents a damped oscillation of the 2nd mode. It appears that this data-fitted waveform represents rather well the processes involved in the present interactions while its differences from the measured pressured have no detectable effects in the computed results.

The predictive capabilities of this numerical solution are demonstrated in fig. 12a where the measured time-dependent pressure at a location $z=-0.306 \text{ m}$ upstream of the end wall is compared to the predicted one for the overall long-time duration of the shock tube flow under the present configuration

Figure 11: Area-averaged velocity of plate displacement at different times. Load of steel plate MPa; load of aluminum plate MPa.

and arrangement of components. This includes multiple shock reflections off the end-wall (the specimen at the end of the working section), and the contact surface or expansion wave reflections off the driver section end-wall.

The measured and computed pressure signals clearly show the passage of the incident shock which takes place at $t = -0.7$ ms and the passage of the reflected shock wave at about 1 ms. The computed signal at the end wall, where the specimen is placed, indicates that the shock impingement and reflection occur simultaneously.

The computations compare fairly well with the measured time-dependent pressure in this long-time duration of the shock tube processes at nearly every moment of the signal except towards the tail of the expansion waves where differences are evident at times > 40 ms. The reason for this disagreement is that the expansion waves as they are coming towards the end wall of the driven section after their reflection off the driver's wall pass through the ruptured deformed diaphragm and are distorted substantially. Additionally, partial reflections occur off the parts of the failed diaphragm that extend into the flow. This distortion is not simulated in the present computations.

The phenomena of interest, however, are associated with the impact of the shock on the end wall where the specimen is mounted and they last no more than 3 ms after the passage of the shock. Measurements and observations have indicated that the plates are deformed permanently within this time scale and no additional plastic deformation is observed after this time. Figure 12b shows the same data as fig 12a in an expanded time scale after the shock reflection. The time of the onset of plastic deformation of the plate is also marked by red arrows in this figure together with the time of full plastic deformation at about 3 ms. These data were deduced from the strain information as explained earlier. It is evident that the loading of the plate is under constant pressure. The effects of the end wall vibrational motion are also present as small perturbations on the constant level pressure load. The computed pressure at this location $z = -0.306$ m immediately after the passage of the reflected shock contains the effects of the damped vibrating plate motion which agree reasonably well with those measured experimentally. In fact, the measured and predicted pressure waves on top of the overall unsteady motion agree remarkably well, not only in their amplitude but also in their phase during the first three cycles of their oscillations. The difference between the computations and measurements is only in the small random fluctuations observed

Figure 12a: Comparison of computations and measurements of pressure at the location of pressure transducer (ptc) at $z = -0.306$ m. Steel plate with load MPa.

in the measured signal which are due to turbulent motions in the flow behind the shock that are not part of the present computations.

Based on the good agreement between the measured and computed time-dependent pressure signals and wave characteristics one can conclude that the computed quantities at the end wall which could not be measured directly, represent a reasonably close approximation to the actual ones. In addition, the computed pressure at the end wall, for instance, shown in fig 12b, exhibits the same level of pressure and wave perturbations as the measured and computed signals at the measurement location $z=-0.306$. This suggests that there are no substantial quantitative differences between the end wall quantities and those at $z=-0.307$ m.

Figure 12b: Comparison of computations and measurements of pressure at the location of pressure transducer (ptc) at $z=-0.306$ m in an expanded time scale. Steel plate with load MPa.

7. Momentum and energy exchange between the shock wave and the plate

Figure 13: Schematic of the three domains and corresponding control volumes after the shock impact.

In order to investigate the various processes associated with these interaction phenomena between the impinging shock wave and the plates, it is useful, first, to recognize the fact that the shock wave and the flow behind it carry momentum and energy which could strongly affect the interaction. In that respect, it is obvious that the initial conditions of the shock wave and fluid which control the beginning of the interaction and the exit conditions which indicate the changes in momentum and energy need to be determined accurately. Thus focusing first on the shock and fluid seems to be appropriate and logical. The three different domains involved are considered first by designating and assigning the corresponding control volumes (CV) all with time-dependent (moving) boundaries. The first control volume, CV I, encompasses the fluid behind the reflected shock and the inner surface of the plate where the shock has impinged, as shown in fig. 13. The second one, CV II, includes the moving volume of the plate and the third one, CV III, involves the fluid on the external side of the plate and it is extended into the ambient environment. We will consider these three control volumes with moving boundaries and the shock wave as a discontinuity by using the transport equation known as generalized Leibniz equation (see Panton, [31]):

$$(9)$$

where ϕ stands for any quantity scalar, vector or tensor function and $\mathbf{v}_k$ is the velocity of moving boundary k of surface S . Usually the term $\frac{d}{dt} \int_V \phi dV$ can be expressed further in terms of a transport equation. In the present case will assume that ρ obtains the value of 1, or it is equal to density of air, ρ ; momentum, $\rho \mathbf{u}$; and total energy where e is the internal energy and $\frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u}$ is the kinetic per unit mass respectively.

Applying equation 5 for ϕ we can obtain the rate of volume change in CV I as

$$(10)$$

It should be noted that the velocity of the reflected shock, W_R , is the same everywhere on the surface of the shock wave. It is also apparent that this volume always increases because the left-running reflected shock front moves with velocity W_R in the same direction as the boundary of the CV while the deformed volume by the right boundary of CV I vibrates always about a mean position. Thus, within the time from 0 to t i.e. from impact till time t after, the change in volume will be

$$(11)$$

where $L_R(t)$ is the distance from the origin traveled by the shock, A is the cross sectional area of the shock tube, $\delta(t)$ is the displacement of the plate at various locations on its inner surface and n_i is the local normal vector to the surface of the plate. The two terms per unit area A on the right hand side of equation (11) are plotted in fig. 14 for the steel and aluminum plates respectively. These terms basically express how the distance $L_R(t)$ and the area-averaged displacement of the plate vary with time. It appears that the shock distance $L_R(t)$ varies linearly with time because its velocity W_R is constant while $\delta(t)$ reaches a plateau after some time. The reflected shock speed in the case of steel plate is slightly higher, since the loading pressure is marginally higher than that in the aluminum case.

Figure 14: Flow volume behind shock per unit area A , area-averaged plate displacement and deformation volume of plate per unit area A , $L_R(t)$ measured at different frames. Load of steel plate: MPa; load of aluminum plate: MPa.

The onset of detectable deformation in the case of steel is delayed somehow due to its inertia while the aluminum plate starts deforming immediately after the arrival of the shock. It should be also mentioned that the loading pressure was about the same in both cases of steel and aluminum plates. The aluminum plate deforms substantially more than the steel plate does. Both plates reach a plateau after sometime. It is interesting to observe that the maximum occurs almost at the same time in both. It should be also mentioned that the area-averaged velocity of surface displacement reaches its maximum much earlier than the maximum displacement as shown in fig. 14 for both cases. The major feature, however, here is that the shock is located far away when the maximum of occurs. In the case of aluminum, for instance, the maximum area-averaged displacement of 20.7mm occurs at time=0.67 ms, when the shock is at $L_R(t)=0.25$ m. This suggests that the shock is very unlikely to be affected by the deforming plate in its wake.

8. Moving and stationary frames of reference

In the following we need to recognize two distinct cases in further applying equation (5). One is the case of a frame of reference associated with the moving shock wave and a second associated with a stationary observer.

In the case of the i.e. density, then equation 5 expresses the rate of change of mass which in the present case of moving shocks and boundaries is not zero.

(12)

The first non-zero integral in the right-hand side of equation (8) represents the mass flux across the shock wave surface while the surface in the interface between the deforming plate and the CVI/CVII is denoted as SII . We have assumed here that the fluid behind the moving boundary SII always acquires the velocity of the plate locally so that . Thus the 2nd integral above is zero. If the plate were solid, the velocity will be . Under these assumptions the rate of change of mass within CVI is entirely due to the shock motion

(13)

In the present notation and and therefore the mass within CVI always increases after the shock reflection. It should be also mentioned here, that the previous mass influx is for an observer riding with

the shock wave. It is also the rate of mass change within the CVI due to the shock travelling, even for a stationary observer.

In the case of momentum, equation (9) according to Panton becomes

$$(14)$$

For an observer moving with the shock velocity W_R , we apply this equation for to obtain

$$(15)$$

Since there is no net mass flux through S_{II} , the momentum flux integral disappears i.e. and therefore we have

$$(16)$$

Evidently the oscillatory motion of the end-wall plate has an effect throughout CVI. The inner surface of the plated is subjected to load determined by which represents the transmitted momentum/force to the plate by the flow, while atmospheric pressure acts on the external surface of the plate and therefore the deforming force is .

We now consider the accumulated effects of momentum and force by integrating (16) over time where the left hand side becomes - where. The time denotes the time immediately after the shock reflection which corresponds to the beginning of the reflected shock wave travel with speed . This change of momentum is the classical impulse relation which in this particular situation takes the form

$$(17)$$

The two pressure terms cancel each other and therefore we have for the momentum change in the control volume being given by

$$- (18)$$

For , the two limits of the integral coincide and the result is

It is always interesting to consider the case of a stationary observer by applying 5 with to obtain

$$(19)$$

Identical procedures involving the rate of change of mass and momentum can also be applied to derive the equations for the incoming flow of the incident shock wave before its impact on the surface of the plate. The time variable is now negative and it is denoted as t since the flow behind the incident shock takes place before the impact. The time denotes the time the incident shock travelling with speed touches the plate at the end wall. Thus, the change of momentum leads to

$$(20)$$

$$\text{or} (21)$$

It is very tempting to form the change of momentum between the incoming flow behind the incident shock and the outgoing flow with the reflected shock after the impact of the shock on the plate. However, the two expressions (18) and (21) are relatively to the moving shocks which propagate with different speeds and . The two reference velocities do not cancel each other and therefore the momentum difference here is not Galilean invariant. This difference makes the interpretation of the results very ambiguous at least if not erroneous. Determining the flow quantities across the shock discontinuity at the wall is better addressed in a coordinate system with a stationary observer. Nevertheless, it is acceptable to compare kinematic quantities based on the same coordinate system but it is not sensible or acceptable to compare results obtained in two different coordinate systems, one based on and the other on .

For a stationary observer equation (18) becomes

$$(22)$$

which is zero at the initial stages after the reflection, when while before the interaction the corresponding momentum difference is In this case both equations are referenced to the same coordinate system and therefore a comparison is possible so that

during the time Δt , or when $\Delta t \rightarrow 0$.

9. The energy exchange

The total energy is not a relativistic invariant and depends on the reference coordinate system which in this particular case is associated with the shock motion. In this case of the total energy per unit mass in a coordinate system associated with shock motion or total enthalpy h where γ is the ratio of the specific heat and equation (9) becomes

In this notation $\mathbf{q}$ is the heat flux vector. Although the thermodynamic state of the processes involved changes during and after shock reflection, its effect on the thermodynamics of the plate is expected to be small. The temperature of the gas T_5 , for instance, increases after reflection, but because the time scales are extremely short, the transmitted heat flux to the plate has practically no effect in raising its temperature. In that respect, the heat flux integral. Then the above equation becomes

(26)

The first term on the right hand side of the above equation describes the flux of total enthalpy and the second term denotes the rate of work done by the moving shock. Finally, the rate of energy/work deposited on the plate by the flow in the coordinate system moving with the shock is

while the rate of work done by the plate is

(27)

The incoming energy flux in to the CVI is

(28)

In the above notation the subscripts p and H denote the plate and total enthalpy respectively.

If h is the total enthalpy in the volume V then after integrating in time (26) to determine the accumulated effects over time we can obtain

(29)

Accordingly, the accumulated work done by the plate is

(30)

while the energy accumulated in the flow is

(31)

We can define the ratio of the accumulated work done by the plate to the accumulated energy

(32)

In the case of stationary observer and equation (9) becomes

(33)

The rate of energy/work deposited on the plate by the flow in this case is

(34)

while the rate of work done by the plate is

(35)

while the flow energy flux is

(36)

Similarly, we can define the following ratio expressing the relative size of the accumulated rate of work done by the plate relatively to the incoming enthalpy:

(37)

The accumulated effects over time are characterized in a similar way through the following definitions

(38)

which indicates the accumulate work done by the plate and

(39)

Then the ratio is defined as

(40)

Figure 15: Rate of work by deforming plate, $\dot{W}_p$, in coordinate system moving with the shock; total enthalpy flux for an observer moving with the shock front and rate of work done by the shock wave $\dot{W}_s$; total enthalpy flux for a stationary observer and rate of work done by the shock wave $\dot{W}_s$. Case of steel and aluminum plates with load 1 MPa.

Figure 15 shows the results of the rate of work done by the vibrating plate, $\dot{W}_p$, as a function of time after the shock reflection in the case of the coordinate system moving with shock front. It reaches a maximum of 25 MW at about 0.5 ms, which corresponds to the maximum plate velocity shown in fig. 11. It is interesting to observe that during the first half of the first cycle of the plates vibration, which ends at about 1 ms, the energy rate is substantially higher than the amplitude of the following cycles which decay continuously due to damping effects.

The rate of work by the plate $\dot{W}_p$ in a stationary observer changes sign for the first time at about 1 ms and subsequently alternates around zero with a frequency close to its 2nd vibrational mode. For steel reaches a value of 0.65 MW and tends to zero at large times. During the negative periods of $\dot{W}_p$, the plate velocity v_p is also negative, and therefore energy flux changes direction and energy is returned to the fluid from that stored into the plate. A comparison of the values with those of $\dot{W}_s$ indicates that the former are at least 38 times smaller than the latter.

Values of $\dot{W}_p$ and $\dot{W}_s$ are also plotted in fig. 15. They increased very fast immediately after the shock impact and then after some oscillatory behavior attributed to the plate vibrational motion, settle to constant values during the constant pressure loading. These data also demonstrate the effect of the reference coordinate system used to evaluate energy rates. For an observer moving with shock wave, the incoming enthalpy flux and pressure work rate are substantially higher than in the case of a stationary observer. Fig. 15 also exhibits the behavior of the rate of work done by the pressure forces on the plate, $\dot{W}_p$, and $\dot{W}_s$, for the case of aluminum plate. It appears that this rate of work $\dot{W}_p$, has a maximum value of about 1.67

MW which is more than 2 times higher and occurs much earlier than in the case of steel plate. Also interesting to observe are the oscillations of $\dot{W}$ after the initial cycle, which are smaller and are damped much faster than in the case of steel. In reference to moving with the shock coordinates, $\dot{W}_{sh}$ for the case of aluminum is slightly higher than in that of steel because of higher plate velocity.

Figure 16: Work done by deforming plate, $\dot{W}$ for an observer moving with the shock front; total enthalpy and work done by the shock wave for an observer moving with the shock front $\dot{W}_{sh}$; total enthalpy impulse for a stationary observer I_p and impulse of work done by the shock wave I_H . Case of steel and aluminum plates with load 10 MPa.

The accumulated effects over time of the data shown in fig. 15 are plotted in fig 16. They are the results of temporal integration between t_0 and t times of the corresponding quantities. $\dot{W}_{sh}$ seems to increase linearly with time since $\dot{W}$ remains constant after the shock impact. The data for $\dot{W}$ indicate the amount of energy/work done by the plate in a stationary coordinate system which is always positive. In the case of steel plate, it reaches its maximum value of 400 J at about of 1.25 ms and then it starts to oscillate about an average of 310 J. The incoming energy by the flow expressed in two different reference frames, $\dot{W}_{sh}$ and $\dot{W}$, have values of 50 kJ and 13.5 kJ respectively at the time of maximum $\dot{W}$, $t=1.25$ ms. These quantities increase their values with time almost linearly in the instances of interest when the pressure and the flow velocities are constant.

In the case of the aluminum specimen the work done by the plate is about 740 J in the stationary coordinate system, a value which is about twice that of the case of steel plate. The work, $\dot{W}$, exhibits less fluctuations about a mean value, at different times in the case of aluminum than in the case of steel which is a direct outcome of the nature of the behavior shown in fig. 15.

Figure 17: Ratio of work done by plate to total incoming flow enthalpy and work by shock front, and respectively for an observer moving with reflected shock or stationary. Case of steel and aluminum plates with load MPa.

The ratios and can also be formed by appropriately dividing the data shown in the previous figure 16 and therefore illustrate the ratio of the work being done by the plate as a fraction to the available by the flow energy. These results are shown in figure 17, where these ratios are plotted as a function of time. The maximum value of is 0.5 which is reached immediately after the shock impact and remains constant because the rate of work, shown in fig. 16, is constant for an observer moving with the reflected shock. For a stationary observer the maximum values of are 0.0325 and 0.067 for the case of steel and aluminum respectively. Values of monotonically decay afterwards. The aluminum plate due to its large deformation receives more work per unit time (8.2%) and energy (6.7%) from the shock tube flow in the stationary reference system. The values of the ratios, , are also different here and there is no reason to expect that these will be Galilean invariant and therefore the same. As was mentioned earlier in the case of momentum, these differences make the interpretation of the results very ambiguous when the two reference systems are mixed. Thus, it is acceptable to compare kinematic quantities based always on the same coordinate system but it is not acceptable to compare results obtained in two different coordinate systems, one based on a moving with the shock coordinate system and the other on a stationary observer. To illustrate the difference, we have form the ratio of the work of the plate in a stationary coordinate system to that of the incoming enthalpy associated with the moving shock, , as which reflects the way energy and energy rate is defined by Wang and Shukla. This ratio is also plotted in figure 17 and reaches maximum values 0.01 and 0.02 in the case of steel and aluminum respectively.

10. Plate energy partitioning

If we consider the dynamic equilibrium of forces and momentum of the plate in control volume CV II, the differential equation for thin plates is

$$(41)$$

where u is the displacement, F is the external force per unit volume and σ is the local stress. If the above relation is multiplied by the velocity of displacement, $\dot{u}$, we can obtain the equation of the rate of mechanical energy which can be integrated over time to yield the energy equilibrium

(42)

The surface integral is expanded as $\int_V \sigma_{ij} \epsilon_{ij} dV$. In the present application under Kirchhoff's theory we have $\sigma_{11} = \sigma_{22} = \sigma_{33} = 0$ and the surviving stresses are σ_{12} and σ_{21} and the main displacement is in Z i.e. w direction. The last part is zero because $\sigma_{12} = \sigma_{21} = 0$ and the other two vanish because the displacement w is orthogonal to the stresses. Thus, the rate of work done by the surface forces is zero. However, if we consider a possible pull out of the plate from its position in the flange, then the surface friction should be accounted for.

As was mentioned earlier, the external forcing is only the pressure load acting on both sides of the surface and therefore their work done is $\dot{W}_p$. This energy is distributed into kinetic energy, elastic and plastic strain energy, the acoustically emitted energy, AE, that is transmitted into the domain CVIII and the residual energy which contains the unaccounted energy, frictional energy losses and uncertainties in the estimates of energy.

(43)

In the formulation above the acoustic energy has been absorbed in the residual energy RE because it is relatively small in magnitude.

The strain energy U has been computed by considering both elastic and plastic deformations separately and assuming that the one-dimensional stress-strain relation can be expanded into the other two directions by using Prandtl's plasticity theory of equivalent stresses. As was mentioned earlier the strain ϵ was computed from the out-of-plane displacement W . Accordingly, the strain includes both the linear and plastic components

(44)

The elastic-plastic behavior of metallic plates under blast loading has been investigated in the past [1]-[4] both theoretically and experimentally. In these studies, high impact loadings caused large plastic deformation that may control elastic behavior and led to a failure by tearing or shearing. Jones [32,33] developed a rigid, perfectly plastic model to predict the inelastic deformation on the metallic plates, he stated that this approach give good agreement in most cases when the ratio of the initial kinetic energy to maximum possible strain energy is larger than about 10. Jones [32,33] also shows that elastic effect can be ignored when the impulse duration is adequately short with respect to the fundamental natural period of elastic vibration. However, Symonds and Frye [34] show that when the impulse duration is long enough and the plate is stiff, the elastic-plastic solution is more accurate than rigid plastic model. In this study, since the impulse duration is moderate, linear elastic-perfectly plastic model is used for both aluminum and steel plates. This simplification allows for a straight forward decomposition of the strain into elastic and plastic parts without additional substantial changes in data processing time or loss of accuracy.

Von Mises yielding criterion is employed to separate the elastic region and plastic region of the plate during the impingement of the shock wave. It is well known that inertia and strain rate effects cannot be neglected under dynamic loading condition. In order to take strain rate effect on the metallic plate into account, Cowper-Symonds constitutive equation (43) is employed and dynamic yield stress (σ_y) is obtained for mild steel plate and aluminum plate. Cowper Symonds coefficients for several materials are obtained from previous studies [35,36] are listed in Table 2.

(45)

Table 2: Cowper-Symonds coefficients [35,36]

Material	D(s ⁻¹)	q
Mild Steel	40.4	5

TRC-SDIC method provides displacement on each mesh element as shown in figure 8. Total strain values are calculated by using classical plate theory as described earlier. An algorithm is developed to separate elastic region and plastic region of each mesh element on the testing plate at each time step. The flow chart of the algorithm as shown in Figure 18 is described as follows. As shown below, the total strain is transformed into the principal strain by (44) for the sake of simplicity.

(46)

The initial deformation assumed to be elastic to compute the pseudo principal stresses by (44) in order to employ von Mises yielding criterion.

Based on von Mises yielding criterion, each mesh element on the testing plate is evaluated at each time step. There are two different cases considered in the algorithm. If the von Mises stress is higher than the dynamic yielding stress of the material in a mesh element, the mesh element is registered as in elastic-plastic region then, its principal stress components are validated individually. While only one of the principal stress components is higher than dynamic yield stress, the principal stress which is higher than dynamic yield stress is equated to the dynamic yielding stress by means of linear elastic-perfectly plastic model assumption. Therefore, its elastic strain can be calculated by Hooke's law and calculated elastic strain is subtracted from the total measured strain in the mesh element and the principal stress component that is lower than yield stress is used to compute elastic strain by Hooke's law. While two principal stress components are higher than von Mises stress, these stresses are equated to the dynamic yielding stress, and elastic strain of the mesh element is computed by Hooke's law, and the computed elastic strain is subtracted from total measured strain in the mesh element to obtain plastic strain component in the mesh element. In the last case, when the von Mises stress is less than dynamic yield stress, the elastic strain in the mesh element is equal to the total measured strain.

Figure 18: Block diagram of algorithm for decomposition of the measured strain into plastic and elastic strain

The elastic strain energy is calculated as

(47)

after elastic strain in each mesh element is determined and integration of strain energy over the plate with respect to the plate volume and time step yields total elastic strain energy

In the unrecoverable deformation stage, plastic strain energy is computed by

(48)

after plastic strain in each mesh element is obtained. The integration of strain energy over the plate with respect to the plate volume and time step gives total plastic strain energy. Total strain energy of the plate is the sum of elastic strain energy and plastic strain energy,

A linear distribution of the strain is also assumed across the thickness of the plate.

An estimate of the radiative acoustic energy AE was obtained by measuring the pressure fluctuations in CVII at a distance of about 0.4 m from the plate. It appears that this is the energy transmitted by the interaction into the external side of the plate outside of the shock tube. Figure 19 shows a typical pressure signal obtained in the case of the aluminum plate. Two large excursions with amplitudes 15 and -25 kPa are observed in the beginning of the signal which are highly correlated with the onset of the plate vibration which sets into motion the nearby air and therefore generates the pressure fluctuations at the location of measurements. The pressure signal in fig. 19 decays rather quickly and within 20 ms its amplitude is down to less than 1 kPa. An estimate of pressure fluctuations away from the plate is provided by where ρ is the air density, v is particle velocity assume to be that of the deforming plate and c is the speed of sound assume to be 340m/s. For the maximum v measured over the plate the pressure fluctuation amplitude is Δp and therefore the radiated sound energy is E_{AE} if A is assumed. Thus, over the plate area we can have an average of E_{AE} . Effectively, these pressure fluctuations change the local pressure acting on the external surface of the plate which counteracts the deforming pressure by the fluid. In that respect, positive pressure fluctuations decrease the work done by the plate and therefore this AE should be added to right hand side of equation (42).

Figure 19: External pressure measurement in CVIII.

Fig. 20a and 20b show the balance of the terms of equation (42). The work done by the external forces increases quickly with time and reaches a maximum value of about 360 J and 750 J in the case of steel and aluminum respectively. This rise time is about 0.8 ms and 0.6 ms respectively. During this time a considerable amount of kinetic energy is generated which peaks very early and then disappears by the end of the rise time. The work done by the external forces appears to level off after the rise time in both cases which is a reflection of the constant load applied. On the same figures the computed work done by the external forces is plotted for direct comparison with the work directly obtained from the experimental data of the plate displacement measurements. In the computed data, which were obtained by using Roe's approach with a modified end-wall velocity to account for the fluid-structure interaction, the area-averaged plate velocity data were used while the experimental data are based on the actual measurements of the 3-dimensional plate displacement as they were measured by DIC under a constant pressure load measured at $x=-0.306\text{m}$ upstream of the plate. The computed work done by the forces seems to agree with the experimentally obtained work at the maximum value immediately after the end of the rise time and on the phase of some of the fluctuations. In general, it is smaller by about 8%, on average, in the case of the steel plate vs. the aluminum plate. In the case of aluminum, the agreement is much better up to about 3 ms and then the measured values increased slightly.

The amount of elastic strain energy estimated is on average 50 J and 100 J for the cases of steel and aluminum plates respectively. One characteristic, evident from both signals of the elastic strain energy, is the appearance of a periodicity which is associated with the 2nd vibrational mode. The estimated inelastic dissipative energy (plastic strain energy) spent on damaging the plates accounts for the largest part of the initially available plate energy.

Finally, the residual energy represents all types of energy losses other than those mentioned above. These include frictional forces in the flange holding the specimen, acoustic radiative energy, thermal energy due to dissipative processes within the plate. In addition, the residual energy contains all the errors due to the uncertainties involved in the measurements. The averaged values of the residual energy are about 10% of the maximum value of the deformation work done by the plates.

Figure 20: Accumulated energy balance in the case of aluminum plate subjected to shock impact. (a) steel plate; (b) aluminum plate.

Figure 21: Ratios of accumulated energy of various forms to work by pressure in the case of aluminum and steel plates under shock impact.

Of interest is the partitioning of the deformation energy into kinetic energy, elastic strain energy, inelastic dissipative (plastic strain) energy and residual energy. The corresponding ratios are defined as R_{KE} , R_{ES} , R_{ID} , and R_R respectively so that $R_{KE} + R_{ES} + R_{ID} + R_R = 1$ at all times. The results are shown in fig. 21. It appears that most of the available energy is converted into inelastic dissipative energy. On average about the same percentage of the available energy, 75% is spent on plastic deformation in both cases. It is also very interesting to observe that the KE accounts for about 55% of the available deformation energy at the beginning of the interaction of the shock with the steel plate. In contrast, KE accounts for 40%, at most, of the available energy in the case of aluminum. The motion of the plates diminishes quickly when plastic deformation starts to dominate the energy processes and the corresponding energy allocations just after 1 ms time. The elastic strain energy accounts for about 20% to 30% of the available energy in the beginning of the interaction and then drops to about 10% and 15% for the steel and aluminum respectively.

11. Conclusions

In the present work, we considered the interaction of a shock wave and clamped circular thin metallic plates. The mechanical response and characterization of such materials under high transient dynamic loading caused by shock or blast wave impingement is not well understood. One of the least investigated topics in these interactions is the pathways of energy starting with the compressible flow energy and ending with the dissipative energy inside the material plates and its acoustic emission to the environment.

An analysis of the time scales of the incident shock wave, its reflection and its transmitted wave inside the structure and that of the structure itself, indicated very short time scales that affect the loading rate for the plate which in the present experiments reached values on the order of Pa/s

The experiments involved steel and aluminum material plates mounted at the end-wall of a large scale shock tube. Both specimens were loaded by approximately the same shock strength. A split-view time-dependent digital image correlation technique was employed to capture the three-dimensional motion of the external surface of the specimens. The optical set up consists of a high frame-rate camera acquiring two split-views of the motion by two different mirror pairs located at different positions that are inclined and tilted appropriately to capture the time-dependent motion of the whole plate specimen under blast/shock loading. Strains were computed from displacements by using Kirchhoff's theory and the corresponding strain energy included both linear and non-linear strain contributions. It was also assumed that the material behavior was linear elastic and perfectly plastic.

The experimental results were analyzed by using the control volume approach with moving boundaries and discontinuities. The energy deposition by the flow on the specimens were examined and discussed using two different frames of reference to demonstrate that energy is not Galilean invariant. In a frame of reference moving with shock approximately 50% of the shock flow energy is transferred to the deforming plate. For a stationary observer, the deposited energy is 3.25% for steel and 6.75% for steel and aluminum respectively. This deforming energy is subsequently partitioned into kinetic energy (KE), elastic and inelastic energy, and residual energy including acoustic emission which for aluminum it was estimated from measurements of fluctuating pressure to be about 6.9 J. KE is significant at the beginning of the interaction and it is then subdued by the onset of plasticity. In absolute terms, the aluminum plate absorbs more energy than the steel one. Interestingly enough, the fraction of KE in terms of the initially supplied deforming work is higher in the case of steel, close to 55%, than in the case of aluminum, close to 40%. The reason for this behavior is that during this time, the fraction of inelastic (plastic dissipation) is higher in the case of aluminum which apparently damps the motion of the plate. Thus, the aluminum plate appears to absorb relatively more energy during the early stages of the interaction while both materials exhibit the same relative absorption of energy at later stages of the interaction. Since in the beginning of the interaction the time scales are shorter than in the later stages, this difference in the energy absorption represents a time scale dependence across the time scale range investigated in the present work.

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