Investigation of the dynamic stress-strain response of compressible polymeric foam using a non-parametric analysis

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Abstract

Dynamic stress-strain response of rigid closed-cell polymeric foams exposed to direct impact loading is investigated in this work by subjecting high toughness polyurethane foam specimens to direct impact with different projectile velocities and quantifying their deformation response with high speed stereo-photography together with 3D digital image correlation. The measured transient displacement field developed in the specimens during high strain rate loading is used to calculate the transient axial acceleration field throughout the specimen. A simple mathematical formulation based on conservation of mass is also proposed to determine the local change of density in the specimen during deformation. By obtaining the full-field acceleration and density distributions, the inertia stresses at each point in the specimen are determined through a non-parametric analysis and superimposed on the stress magnitudes measured at specimen ends, to obtain the full-field stress distribution. The process outlined above overcomes a major challenge in high strain rate experiments with low impedance polymeric foam specimens, i.e. the delayed equilibrium conditions can be quantified.

Keywords: Polymeric foam; Direct impact; Digital image correlation; Non-parametric analysis, Parameter identification

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1. Introduction

Determination of a material's constitutive behavior at intermediate to high strain rate ranges has been a subject of interest for quite a long time [1, 2]. Historically, the importance of obtaining equilibrium state has required special attention in the experimental setup, since configurations where equilibrium can be readily identified make it easier to study the dynamic stress-strain response of the materials. The issue is particularly significant in the case of engineering materials with low strength and low mechanical impedance [3], where conditions of stress/strain equilibrium oftentimes require a time duration that exceeds the time required for the material to undergo failure. In such cases, the determination of stress-strain response of the material based on the load and displacement measurement at specimen ends can introduce a significant degree of inaccuracy in the obtained constitutive response, with continued deterioration in accuracy at higher strain rates [4-6].

Polymeric foams, as a class of materials widely used in applications which require light weight structural design with superior energy dissipation characteristics, are among the materials possessing a low wave propagation speed [7-10]. Their application in areas such as automotive industries, ships and packaging requires precise knowledge of the deformation response of the material at different loading rates. Dynamic deformation and failure behavior of these materials have been studied both experimentally and numerically in recent years [11-16]. In general, most polymeric foams possess remarkable strain rate sensitivity at strain rates above 500 s^{-1} [14]. However, owing to non-homogeneity in the stress and deformation states, particularly during the early stages of dynamic loading, quantifying the dynamic deformation of foams has been a major challenge. One of the most widely used methods in studying the dynamic deformation behavior of soft materials is the split Hopkinson pressure bar technique [17]. Different approaches have been practiced in recent years to increase the accuracy of the measurements in this technique, particularly in testing of low impedance materials. These

approaches include pulse shaping techniques [17], use of polymeric bars [18] and long projectiles [19]. A recent study by Liu et al. [16] indicates that in addition to the solutions proposed above, full-field measurements must be incorporated in order to accurately measure the deformation response and reveal the active failure mechanisms in foam specimens subjected to high strain rate loading conditions.

The advent of full-field measurement techniques such as digital image correlation (DIC), in conjunction with different experimental techniques has facilitated the study of deformation of materials in a wide range of time and length scales [16, 20-24]. More importantly, the recent work by Pierron and his group [21, 25, 26] using virtual fields and inverse methods has identified a unique way of analyzing the dynamic deformation of materials by using D'Alembert's principle and incorporating "inertia forces" into the analysis. Though the inverse methods have already been demonstrated to be effective in calculating the stress-stress response of materials at high strain rate loading conditions, the effect of compressibility has not been addressed in previous studies.

The present work focuses on the study of rigid closed-cell polymeric foams subjected to direct impact loading by accounting for material compressibility and the effect of inertia. A shock tube apparatus is used to apply dynamic loading on the foam specimens, while high speed stereovision imaging together with 3D digital image correlation is used to study the full-field deformation of the material under high strain rate loading conditions. A simple mathematical model has also been proposed to account for the material compressibility and the local variation of density during deformation of the specimens. Also, based on the full-field displacement distribution captured by DIC and considering the instantaneous change of material density, a non-parametric analysis is developed to incorporate "inertia effects" into the analysis, following references [5, 6]. Using the proposed methodology, the full-field stress-strain response of the material has been determined during high strain rate loading and

the global constitutive behavior of the foam quantified. The original contribution here is the inclusion of the material compressibility into the analysis. In addition, attempts have been made to generalize the methodology to study the dynamic deformation of any low-impedance compressible solid. To the authors' knowledge, this is the first time a thorough analysis of the dynamic deformation of compressible polymeric foams has been performed taking into account the concurrent effects of material compressibility and inertia loading to capture the full-field stress-strain response of the material.

2. Experimental Procedure

2.1. Material and Specimen Geometry

The material used in this study is a rigid closed-cell polyurethane foam of 560 kg/m³ (35 pcf) nominal density obtained from Sandia National Laboratories [27]. The initial density of the foam specimens was measured in-house and confirmed to be consistent with density values reported in the literature [27]; the measured value was 95% of literature data. A cylindrical foam specimen with a high contrast speckle pattern is shown in Figure 1a. The specimen is 25.4 mm in diameter and 25.4 mm in height. Each specimen was extracted from the as-received billets using a waterjet system, resulting in a relatively smooth lateral surface finish with +/- 0.1 mm dimensional variability. The high contrast speckle pattern shown in Figure 1a consists of a thin white sub-layer with randomly-distributed black speckles; the black speckles were applied using an airbrush. A histogram of the speckle pattern is shown in Figure 1b.

Fig. 1- (a) Typical speckled specimen with its corresponding grey scale histogram shown in (b).

2.2. Impact Loading

A shock tube apparatus was utilized to apply controlled dynamic loading on the specimens. The shock tube used in this work is shown in Figure 2a. The apparatus has an overall length of 7.2 m, consisting of a 1.8 m driven section and a 5.4 m driver section. The inner diameter of the entire tube, except the last 0.9 m section near to the muzzle, is 75 mm. The inner diameter of the final 0.9 m end section of the tube is 50 mm, resulting in increasing shock velocity over the final stage.

To perform the experiment, the speckled cylindrical specimen shown in Figure 1a is placed at the muzzle of the shock tube and affixed to a custom fabricated strong-back. As shown in Figure 2b, three piezotronic load-cells were placed behind the specimen and mounted on a specially fabricated fixture. Use of three load-cells in this work assures the accuracy of the force measurements and compensates for any possible misalignment of the loading which might occur during high strain rate experiments. The specimen is held on the loading fixture

by the use of lithium grease, which also acts as the lubricant. Further details on the shock tube used in this work can be found elsewhere [4, 24, 28, 29].

Fig. 2- (a) The shock tube at its entirety, with the loading fixture shown in (b). The schematic of the shock tube muzzle and the pressure transducer are depicted in (c).

As shown in Figure 2b, direct impact loading is applied using a projectile placed inside the tube at the beginning of the reducing section. The projectile was made of high strength 7068 aluminum alloy with a diameter 2 mm smaller than the inside diameter of the reducing section. The projectile was fabricated as a cylinder hollowed from one end, to reduce its mass, with an overall length of 72 mm, and a 0.07 kg mass. Prior to performing the experiment, the high pressure driver section and the low pressure driven section are separated by a diaphragm, a 0.18 mm thick stretched polyester film, under the commercial name Mylar. During testing, the driver section is pressurized using helium gas and as the pressure difference between the driver and driven sections reaches a critical value, the polyester film

bursts, forming a shock wave that pushes the projectile towards the specimen side. The pressure and the velocity of the shock and the associated strain rate can be manipulated by varying the number and the thickness of diaphragms separating the driven and driver sections. In this work, two different strain rates were successfully applied on the specimen by using one and two sheets of diaphragms. The pressure profile developed inside the shock tube was measured using a piezotronic pressure sensor placed inside the tube near to the muzzle (See Figure 2c). Maximum internal pressures of 250 kPa and 350 kPa were accordingly measured, by using one and two Mylar sheets, respectively.

Upon the formation of the shock inside the shock tube, the projectile is shot directly at the specimen, imposing a large strain rate magnitude on the deforming material. Stereo-imaging of the specimen was performed using two Photron SAX-2 high speed cameras. Triggering of the cameras was facilitated using the pressure transducer located inside and 180 mm behind the muzzle outlet of the tube (See Figure 2c). The pressure transducer was connected to the oscilloscope so that the signal from the transducer could be used to trigger the cameras, as the internal pressure increased inside the tube. Repeatability of the results obtained from dynamic tests in this work was assured by testing two specimens for each strain rate conditions.

2.3. Imaging Setup and Digital Image Correlation

The Photron SAX-2 cameras used in this work are equipped with 100 mm lenses. Images were acquired at a rate of 10^5 fps at 384×264 pix² full-field resolutions resulting in inter-frame intervals of 10 μ s. At 10^5 fps, the camera system used in this work allows the smallest exposure time of 1/996923 s; thus, an exposure time of 1 μ s was used here. The load-cell data acquisition rate was synchronized with the imaging rate through a high speed data acquisition system and an oscilloscope.

Images acquired during the specimen deformation period were used as the input to the commercial DIC software Vic-3D[®] for quantitative 3D digital image correlation analysis. In this software, subset and step sizes of 39 pix and 9 pix were selected, respectively; and the spatial resolution was calculated as 9.16 pix/mm. Image correlation in Vic-3D was performed using ‘Normalized Squared Differences’ matching criterion, with optimized 8-tap interpolation, and first order (affine) shape functions. In this software, strains were derived from the full-field displacement using Gaussian filtering with a filter size of 13. Further details of the stereovision camera system parameters are listed in Table 1. The experimental setup including the shock tube and the stereovision camera arrangement system used in this work is shown in Figure 3.

Fig. 3- The experimental setup including the shock tube and the stereovision camera system used in this work.

Table 1- System parameters obtained from calibration of the stereo camera arrangements
used in this work

Parameter	Camera 1	Camera 2
Center - x (pix)	148.7	167.5
Center - y (pix)	24.2	43.9
Focal length - x (pix)	5799.5	5708.3
Focal length - y (pix)	5796.1	5705.3
Skew (°)	-0.89	-0.48

The diagram illustrates the stereo camera system. Camera 1 is on the left with origin O_1 and coordinate axes X_1 , Y_1 , and Z_1 . Camera 2 is on the right with origin O_2 and coordinate axes X_2 , Y_2 , and Z_2 . A transformation matrix T is shown between the two origins. The rotation angles for Camera 1 are α (around Y_1), β (around X_1), and γ (around Z_1).

Alpha (°)	20.6
Beta (°)	-0.8
Gamma (°)	-0.6
Tx (mm)	6.8
Ty (mm)	223.7
Tz (mm)	41.9

2.4. Measurements Performance

Performance of the measurements in the present work was investigated through a set of bias and resolution calculations. For this purpose, a set of 7 stationary images that were acquired before the onset of the impact experiments are analyzed. Displacement, strain, and full-field acceleration were computed for each image, following the procedure described later in Sec. 3.2. Average values and standard deviation of the full-field data were calculated and used for characterizing the measurement performance.

Figure 4 shows the standard deviation (resolution) and the mean (bias) obtained for different variables. The presence of non-zero mean values in displacement (Figure 4a) indicates a

systematic bias in the form of rigid body motion, the source of which is not known at this point. The systematic bias in displacement is transferred to the calculated strain and acceleration fields, as show in Figure 4b and 4c. In addition, the larger magnitudes of standard deviation in all cases confirm the presence of noise in the measurements. It has been documented that the value of this noise can be significantly reduced by applying appropriate smoothing schemes [30]. However, no data smoothing was performed in the present work, and the results shown hereafter are those obtained directly from image correlation with no smoothing applied.

Fig. 4- mean and standard deviation of (a) axial displacement, (b) axial strain and (c) axial acceleration.

3. Data Analysis

3.1. Modeling of Material Compressibility

The polymeric foam specimens examined in this work are compressible, and hence their density varies when subjected to deformation [31]. To take the material compressibility into account, a simple one-dimensional model was proposed in this work, which facilitates the

calculation of material's instantaneous density, as a function of its initial density and the applied strain components.

Let us consider a compressible cylindrical specimen of initial volume Ω_0 and initial density ρ_0 , exposed to axial compression from one side. The specimen is then deformed into a cylinder of volume Ω , and density ρ at time t , as schematically shown in Figure 5. The temporal rate of change of the specimen volume, $\dot{\Omega}$, can be written as:

$$\frac{\dot{\Omega}}{\Omega} = 2 \frac{\dot{r}}{r} + \frac{\dot{h}}{h} \quad (1)$$

Where $\dot{r}$ and $\dot{h}$ are the temporal rate of change of radius and height of the cylindrical specimen, respectively. Assuming logarithmic strain metrics for both the radial and axial strain components, and the apparent Poisson's ratio as ν , the strain rate equations can be written;

$$\dot{\epsilon}_r = \frac{\dot{r}}{r} \quad \dot{\epsilon}_z = \frac{\dot{h}}{h} \quad (2a)$$

$$\dot{\epsilon}_r = -\nu \dot{\epsilon}_z \quad (2b)$$

Thus, Eq. (1) can be written in the following form;

$$\frac{\dot{\Omega}}{\Omega} = (1 - 2\nu) \dot{\epsilon}_z \quad (3)$$

Eq. (3) can be further simplified by defining the instantaneous axial strain rate as [32]:

$$\dot{\epsilon}_z = -\frac{V}{h} \quad (4)$$

where V is the magnitude of the velocity of the specimen at the impacted surface and h denotes the instantaneous specimen height, as shown schematically in Figure 5. Using Eqs.

(3) and (4), the rate of change of the specimen volume can be rewritten in terms of the Poisson's ratio and the impacted end velocity, as:

$$\frac{\dot{\Omega}}{\Omega} = (2\nu - 1) \frac{V}{h} \quad (5)$$

Now, by considering the principle of conservation of mass for the compressible cylindrical specimen shown in Figure 4, the rate of change in density can be written as:

$$\dot{\rho} = -\left(\frac{\dot{\Omega}}{\Omega}\right)\rho \quad (6)$$

Assuming that the density, ρ , varies only along the z -direction and applying a simple mathematical chain rule, Eq. (6) can be rewritten as:

$$\frac{d\rho}{dh} \cdot \frac{dh}{dt} = -\left(\frac{\dot{\Omega}}{\Omega}\right)\rho \quad (7)$$

where by defining $\frac{dh}{dt} = -V$ and upon rearrangement, the final ordinary differential equation can be obtained as:

$$\frac{d\rho}{\rho} = (2\nu - 1) \frac{dh}{h} \quad (8)$$

Eq. (8) can be integrated to yield:

$$\rho = \rho_0 \left(\frac{h}{H}\right)^{2\nu-1} \quad (9)$$

Eq. (9) can also be written in terms of the axial strain, $\varepsilon_z = \ln\left(\frac{h}{H}\right)$, as:

$$\rho = \rho_0 [\exp(\varepsilon_z)]^{2\nu-1} \quad (10)$$

227 The significance of Eqs. (9) and (10) is that the variables h , ε_z and ν can be experimentally
228 measured using digital image correlation at any given point along the specimen axis. The
229 Poisson's ratio, ν , can be measured for a cylindrical specimen as the ratio between the radial
230 and axial strain components. In this work, to measure the local evolution of ν , a cylindrical
231 coordinate system was first assigned to the initial geometry of the specimen. Then, by
232 tracking the evolution of the radial displacement component during the entire time of
233 deformation, the radial strain component was determined. Note that by implementation of a
234 cylindrical coordinate system, the radial strain component can be measured directly from the
235 growth in the radius of the specimen, at any given time and any location on the surface. This
236 'growth in radius' will be independent of the amount of out-of-plane motion. Indeed, this is
237 valid assuming that the radial strain values remain constant through the thickness the
238 specimen, and that a circular section in the material remains circle during the deformation.
239 On the other hand, the out-of-plane displacement in a Cartesian coordinate system contains
240 both deformation and rigid body motion components, which cannot be readily separated
241 when surface measurement is conducted. Therefore, if the radial expansion was to be
242 measured from the out-of-plane displacement in a Cartesian system, significant error would
243 have been introduced into the analysis. The change of coordinate system from Cartesian to
244 cylindrical in this work was performed using the built-in 'coordinate' tool in Vic-3D².
245 Accordingly, the Poisson's ratio was calculated at different locations along the axis of the
246 specimen. Finally, it should be noticed that the mathematical expression shown in Eq. (10)
247 predicts perfect incompressibility when $\nu = 0.5$, i.e. the condition existing in the classical
248 incompressible plasticity.

² More details on the software features can be found in the technical notes available in <http://www.correlatedsolutions.com>.

Fig. 5- The schematic view of a compressible cylindrical specimen subjected to axial loading.

3.2. Dynamic Stress-Strain Response Using a Non-Parametric Method

In order to include the effects of inertia force and stress in the measurements in this work, a non-parametric analysis was performed in which the inertia stress within the specimen was reconstructed using the full-field deformation response.

Consider a cylindrical specimen, as schematically shown in Figure 6, subjected to direct impact loading on its right end, exerted by a projectile. The load-cell is located at the opposite side of the specimen. Let Ω be the current domain of interest. Each point inside the domain is represented by a position vector $\underline{z} = z_1 \underline{e}_1 + z_2 \underline{e}_2 + z_3 \underline{e}_3$ in the domain. The force applied at any given position along the specimen axis $\underline{z}$ and at time t can be expressed as:

$$\underline{F}(z, t) = F_z(z_1, t)\underline{e}_1 \quad (11)$$

Considering the free body diagram of the sliced section in Figure 6 and applying Newton's law of motion in axial direction, the following equation can be written:

$$F_z(z_1 + dz, t) - F_z(z_1, t) = \rho(z_1, t)A(z_1, t)dz \frac{\partial^2 u_z}{\partial t^2}(z_1, t) \quad (12)$$

where $\rho(z_1, t)$ and $A(z_1, t)$ are specimen density and cross sectional area at position z_1 and time t , respectively. $u_z(z_1, t)$ represents the displacement component along the axial direction. Note that the F_z values in Eq. (12) are algebraic quantities. Eq. (12) can be rewritten in differential form as:

$$\frac{\partial F_z}{\partial z}(z_1, t) = \rho(z_1, t)A(z_1, t) \frac{\partial^2 u_z}{\partial t^2}(z_1, t) \quad (13)$$

which upon integration yields:

$$F_z(z_1, t) = F_z(0, t) + \int_{\xi=0}^{\xi=z_1} \rho(\xi, t)A(\xi, t) \frac{\partial^2 u_z}{\partial t^2}(\xi, t) d\xi \quad (14)$$

Eq. (14) was previously proposed by Othman et al. [5] and Aloui et al. [6]. This equation indicates that the force at any position $z = z_1$ can be reconstructed from the force at position $z = 0$, i.e. at the location of the load-cells, along with the acceleration and density at position $z = z_1$. The density component at any location can be obtained using the equations described in section 3.1. The acceleration component at any given position along the z -direction can also be calculated from the displacement field obtained from DIC. In this work, a central difference scheme was utilized to numerically calculate the full-field acceleration distribution

within the deforming specimen. Accordingly, the acceleration component

$a_z(z_1, t) = \frac{\partial^2 u_z}{\partial t^2}(z_1, t)$ was calculated as:

$$a_z(z_1, t) = \frac{1}{\Delta t^2} [u_z(z_1, t - \Delta t) - 2u_z(z_1, t) + u_z(z_1, t + \Delta t)] \quad (15)$$

where Δt denotes the time interval between two consecutive images, i.e. $\Delta t = 1/(\text{frame rate})$.

Similar to Eq. (14), the stress component in z -direction can be recovered as:

$$\sigma_z(z_1, t) = \frac{F_z(0, t)}{A(z_1, t)} + \int_{\xi=0}^{\xi=z_1} \rho(\xi, t) \frac{\partial^2 u_z}{\partial t^2}(\xi, t) d\xi \quad (16)$$

The stress component $\sigma_z(z_1, t)$ can be reconstructed from the stress magnitude measured at the load-cell side plus the inertia stress shown in integral term in Eq. (16).

Note that true stress value at any given position across the domain of interest can be calculated only if the current cross sectional area $A(z_1, t)$ at the same position is identified. As described in the previous section, the change of the cross section of the cylindrical specimen can be measured directly from DIC. In this case also, a cylindrical coordinate system can be assigned on the initial configuration to facilitate measuring of the current dimensions of the cylindrical specimen at any given position in space and time.

By including compressibility of the foam into the analysis, the final closed form equation that can be used to reconstruct the axial stress at any given position across the domain of interest can be written as:

$$\sigma_z(z_1, t) = \frac{F_z(0, t)}{A(z_1, t)} + \int_{\xi=0}^{\xi=z_1} \rho_0 [\exp(\varepsilon_z(\xi, t))^{2\nu(\xi, t)-1}] \frac{\partial^2 u_z}{\partial t^2}(\xi, t) d\xi \quad (17)$$

Assuming that the deformation of the specimens is only axial, to calculate the stress using the full-field data extracted from DIC and Eq. (17), the cylindrical geometry of the specimen is sliced into a finite number of thin sections of thickness s along the specimen length. Then, density and acceleration values within each section are calculated by averaging these parameters over the volume of each thin sliced section. Now, the integral term on the right hand side of Eq. (16) can be approximated as:

$$\int_{\xi=0}^{\xi=z_1} \rho(\xi, t) \frac{\partial^2 u_z}{\partial t^2}(\xi, t) d\xi \approx \sum_{i=1}^n \rho^{(i)} a^{(i)} s^{(i)} \quad (18)$$

Where $\rho^{(i)}$ and $a^{(i)}$ are the average density and axial acceleration over the i th slice., respectively. $s^{(i)}$ represents the slice thickness, and n denotes the total number of sliced sections up to the location of z_1 (See Figure 7). In this work, the entire region of interest was sectioned into 24 thin slices of about 1 mm thick, each containing 36 data points. Thus, the averaging has been conducted on a population of 36 data points in each slice.

It should be emphasized that the compressibility model proposed in this work, i.e. Eq. (10), assumes homogeneous deformation condition. However, Eq. (10) can be used in the case of heterogeneous deformation condition, assuming that each individual slice undergoes a rather homogeneous deformation. This is studied in detail in Sec. 4.

Fig. 6- Schematic of a cylindrical specimen subjected to direct impact. The dynamic load is applied on the specimen from the projectile side.

Fig. 7- Schematic geometry of the specimen sliced into a finite number of thinner segments, used to calculate the inertia force and stress applied at each point along the specimen axis.

4. Results and Discussion

Figure 8 shows the force measured at the load-cell side of the specimen for two independent experiments performed using one Mylar sheet. Consistent results shown here confirm the repeatability of the experimental results in this work.

Fig. 8- Load-cell response for two independent experiments performed using one Mylar sheet.

The velocity of the projectile can be varied by changing the thickness and/or the number of diaphragms separating the driver and driven sections of the shock tube. Projectile velocity in this work was calculated by differentiating the projectile displacement with respect to time. Projectile displacement was measured by tracking the motion of a single pixel, located at the tip of the projectile, in time. Thus, the projectile velocity measurements in this work are of pixel accuracy level. Displacement of the tip of the projectile with respect to time is shown in Figure 9. The projectile velocity was shown to remain constant well after incidence with the

specimen. In addition, experimental data indicate that the average projectile velocity increased by 35% when the number of Mylar diaphragms was increased from one to two.

Fig. 9- Variation of the projectile displacement with time. Increasing the number of plastic diaphragms from one to two has resulted in a 35% increase in the velocity of the projectile.

Two different strain rates were applied on the specimens in this work, associated with one and two Mylar sheets used to separate the driver and the driven sections of the shock tube. To calculate the average strain rate applied to the specimen, the axial strain magnitudes over the entire domain of interest were averaged and plotted as a function of deformation time, as shown in Figure 10. The slopes of the best linear fits were then considered as the average strain rate in each case. The average strain rates obtained through this procedure are 1437 s^{-1} and 2458 s^{-1} corresponding to one and two Mylar sheets, respectively. The error bars indicate standard deviation of the full-field strain data, which can be considered as a measure of the inhomogeneity of the strain field at any given time during deformation. Considering the strain

history curves shown in Figure 10, the level of inhomogeneity of axial strain is observed to be higher in higher strain rate condition. Furthermore, the strain inhomogeneity is found to be higher at earlier stages of deformation, i.e. the time during which equilibrium has not yet achieved. The heterogeneous distribution of strain in earlier stages of deformation yields in the strain rates to be highly heterogeneous, as well. In materials with strong rate sensitivity, such as the one examined in this work, the heterogeneous strain rate field might result in a spatial variation of stress-strain response over the gage area.

Figure 11 shows the variation of average axial and radial strains per slice, at different positions along the specimen axis and at various times. Strain heterogeneity along the specimen axis is clearly observed in both sets of curves. It is also seen that at earlier deformation times, both axial and radial strains take significantly larger values at the impacted side of the specimen ($\bar{Z} = 1$); while, at longer durations, this behavior is reversed and the load-cell side ($\bar{Z} = 0$) undergoes larger strains. In addition, the radial strain heterogeneity seems higher compared with the axial strain component. This might be due to the frictional state and boundary conditions at specimen ends which partially restrain the lateral expansion of the specimen within areas closer to specimen ends.

Fig. 10- Evolution of the average axial strain applied on the specimen as a functions of time, for two different number of Mylar sheets.

Using the obtained radial and axial strain distributions, evolution of the Poisson's ratio in the region of interest was also determined. Contours showing the evolution of Poisson's ratio for the specimen deformed under the higher strain rate conditions have been depicted in Figure 12. The evolution of ν at three different locations along the specimen axis has also been plotted in Figure 13, for both strain rate conditions. The calculated Poisson's ratios indicate maximum values at $t = 20 \mu s$ for both strain rate conditions. This could be associated with the boundary condition and strain heterogeneity at early stage of loading. During early deformation stage, the specimen is relatively free to deform laterally. As the loading progress the friction and boundary conditions at the specimen ends could restrain the radial deformation to some extent. Therefore, the rate of change of lateral strain with respect to that of axial strain significantly increases during the first $20 \mu s$ after the impact, thus resulting in

larger Poisson's ratios at $t = 20 \mu\text{s}$. In addition, larger Poisson's ratio values were obtained for the specimen deformed under lower strain rate conditions, indicating that the foam specimens examined in this work experience a larger compressibility at higher strain rate (See Eq. (10)). It should be emphasized here that no conclusive information can be extracted on the rate sensitivity of the Poisson's ratio of the specimens in this work. The evolution of Poisson's ratio for specimens deformed under different strain rates was considered only to facilitate the application of the proposed compressibility model.

Fig. 11- Variation of average (a) axial and (b) radial strains per slice, at different locations and times. $\bar{Z} = 0$ indicates the load-cell side.

Fig. 12- full-field distribution of Poisson's ratio, ν , for the specimen subjected to direct impact with two Mylar sheets.

Fig. 13- Local Poisson's ratio, ν , of the specimens extracted from different positions along the axis of foam specimens deformed at (a) 1437 s⁻¹ and (b) 2458 s⁻¹ strain rates.

The local change of density of the specimens at different strain rate conditions can be determined once the evolution of the Poisson ratio, ν , is obtained. Figure 14 illustrates the typical form obtained for the local change of density of the specimens at three different

locations along the specimen axis. For the case of lower strain rate condition, the foam specimen shows less compressibility with a negligible ($\sim 2\%$) change of density (see Figure 14a). However, the density of the specimen changes more noticeably under higher strain rate conditions ($> 5\%$), as ν deviates from a perfect incompressible condition, i.e. $\nu = 0.5$. Considering these results, it is important to include the material change of density in the calculation of inertia forces and stresses, to increase the accuracy of the predictions. Note that the change of density calculated for materials considered in this work are relatively small. However, the proposed model can be used for any compressible solid, regardless of the degree of compressibility

Fig. 14- variation of local density as a function of deformation time, extracted from different positions along the axis of foam specimens deformed at (a) 1437 s^{-1} and (b) 2458 s^{-1} strain rates.

The full-field acceleration distribution was calculated based on the procedure described in earlier sections. Full-field axial displacement and acceleration maps at each time step are

shown in Figure 15 for the specimen subjected to higher strain rate conditions. Figure 15a shows a rather circumferentially-homogeneous distribution of the axial displacement. Relatively uniform distribution of acceleration in hoop direction is also evident in Figure 15b. However, both displacement and acceleration contour maps indicate strong axial gradients. This is shown in the highly nonhomogeneous distribution of acceleration along the axis of the specimen, particularly in the earlier stages of deformation, i.e. $0 < t < 20 \mu\text{s}$ (See Figure 15b). The axial acceleration then becomes smaller in magnitude and more homogeneous at longer durations, as shown in Figure 16, where the axial acceleration component was averaged over the entire domain of interest and plotted at each deformation time step. Error bars in Figure 16 indicate standard deviation of the full-field acceleration data. For both strain rate conditions, a sudden change in the magnitude of the average acceleration takes place after $10 \mu\text{s}$ of loading. Though, the limitation of the temporal resolution ($10 \mu\text{s}$) of the experiment makes it difficult to track the trend of the acceleration, it clear that the acceleration decreases as a function of time as shown in Figure 16.

The large value of acceleration obtained in early stages of deformation results in a significant increase of inertia force and inertia stress (see Eqs. (14) to (17)) at earlier deformation times, as also reported previously in [5]. Note that the accuracy of the acceleration field highly depends on the temporal resolution of the measurements, i.e. the frame rate. A low frame rate can filter out the calculated acceleration, resulting in the inertia stresses to be smeared out; whereas a high framing rate may amplify noise in the acceleration curves. Due to limitation of the temporal resolution of our camera, parametric studies on the framing rate and acceleration noise level have not been conducted. The accuracy of the acceleration fields determined in this work may have deteriorated due to relatively low framing rate of the available camera system. However, the concept of the study of concurrent influences of

442 inertia and compressibility remain valid as long as the methodology detailed in this work is
 443 followed.

444

445 Fig. 15- full-field distribution of (a) axial displacement (mm) and (b) axial acceleration (m/s^2)
 446 for the specimen subjected to higher strain rate conditions.

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Fig. 16- Variation of the axial acceleration averaged over the entire domain of interest with respect to time.

Having obtained all the required parameters, the values of force and stress can be reconstructed at any given position along the specimen axis, following the procedure detailed in Sec.3.2. The force applied on the load-cell side of the specimen can be directly measured from the load-cells inserted behind the specimen. However, the force applied on the other side, i.e. the projectile side, is required to be reconstructed using the load measured on the load-cell side along with the acceleration and density fields obtained earlier. The forces acting on two sides of the foam specimens are plotted in Figure 17. In both strain rate conditions, the load measured on the load-cell side exhibits a very slight rate of change during the first 10 μs of the deformation. This can be attributed to the low wave propagation speed in the specimen. Note that due to the low impedance of the foam specimens examined

here, there would be an elapsed time between the instant at which the projectile hits the specimen and the time at which the compressive wave reaches the other end of the specimen, i.e. the location of the load-cells. Therefore, the load-cells are not capable of recording the load exerted on the specimen during the early stages of deformation. During this early time period, the force applied on the projectile side of the specimen is significantly larger, resulting in significant deformation within the specimen, particularly within the regions close to the projectile (see Figure 11). This non-equilibrium state gradually diminishes after $t = 20$ μs , resulting in the load magnitudes on the load-cell and projectile sides of the specimen to become more equilibrated.

Fig. 17- Forces applied on the load-cell and projectile sides of the specimen for specimens deformed under (a) 1437 s^{-1} and (b) 2458 s^{-1} strain rate.

The non-equilibrium state of deformation that occurs during the early stages of impact can be studied through the stress-strain response of the specimen. Full-field axial strain and stress contour maps for the specimen subjected to 2458 s^{-1} strain rate are plotted in Figure 18. In this case also, the strain maps were obtained by slicing the specimen geometry into 24 thinner

sections and averaging the strain values over each and every sliced section. The full-field strain data utilized for this purpose were those directly measured from DIC. Using the previously obtained full-field acceleration and applying Eq. (18), the stress distribution was also obtained along the axis of the specimen. The stress-strain responses of the specimens are very similar to those described previously for the load history. The full-field contour maps shown in Figure 18 exhibit a highly non-homogeneous distribution of strain and stress during the first 20 μs of the deformation. For $t > 20\mu\text{s}$, the level of inhomogeneity in the deformation is substantially reduced and the specimen approaches equilibrium in the deformation state.

Fig. 18- full-field distribution of (a) axial strain and (b) axial stress (MPa) components for the specimen subjected to 2458 s^{-1} strain rate. Note that the strains and stresses are compressive in nature, but the values reported in here are the absolute values.

The significance of using the methodology outlined above is that the stress and strain histories within the material at any given position along its axis can be calculated and studied, even when “quasi-static equilibrium” conditions typically employed are not present. For instance, the evolution of strains and stresses extracted from different locations along the

specimen axis are plotted in Figures 19 and 20, respectively. Similar to the load curves, remarkable differences between the local curves for both local strain and local stress responses shown in Figures 19 and 20 are evident when $t < 20\mu\text{s}$. Considering the local stress curves shown in Figure 20, one can notice that the large variation in local stress and local strain lasts for about $25\mu\text{s}$, the time required for the wave to propagate through the specimen. All local stress curves are then expected to collapse to the same curve, as shown in Figures 20a and 20b, until the failure of the specimen. However, there exists a systematic offset between the curves even after $t = 25\mu\text{s}$, and this can be attributed to the spurious acceleration behavior shown and discussed earlier in Figure 16. Note that the low temporal resolution of the measurements in this work results in interframe times of the order of the wave propagation time. This means that the acceleration, at least in the early stages of deformation, might have been underestimated. The low accuracy of the acceleration measurements can generate a systematic error on the stress-strain curves, as well. One way to address this issue would be to simulate the measurements using, for instance, the procedure proposed in [33, 34]. This is beyond the scope of the present work, but must be accounted for in highly accurate analyses of the low impedance materials.

Fig. 19- Local strain evolutions extracted from different positions along the specimen axis for the foam specimens deformed at (a) 1437 s⁻¹ and (b) 2458 s⁻¹ strain rates.

Next, using the strain-time and stress-time curves obtained and shown earlier in Figures 19 and 20, the local stress-strain curves were determined. The local stress-strain curves for both strain rate conditions have been illustrated in Figure 21, where good agreement between each set of curves is indicated. Finally, the local stress-strain curves obtained at different positions along the specimen axis were averaged and the resultant average curves were compared with the stress-strain response conventionally determined using the specimen-end force measurement. Note that in the conventional methodology, the strain was obtained by averaging the axial strain magnitudes, obtained directly from DIC, over the entire field of view. Consequently, only one strain value at each point of time is obtained in this method. Figure 22 compares the stress-strain curves using two methods, i.e. non-parametric analysis and the conventional method. It is clearly observed that by using the conventional methodology substantial error can be imposed on the results, making this method ineffective

for the case of low impedance polymeric foams. In particular, the highest discrepancy between the curves is evident at earlier stages of deformation, as discussed in detail earlier.

Finally, it is worth noting that the analysis performed in this work allows the measurement of the elastic modulus of viscoelastic materials at high strain rate conditions. Accordingly, the elastic modulus of the examined foam was estimated as 352 MPa and 411 MPa at 1437 s^{-1} and 2458 s^{-1} strain rates, respectively. Indeed, due to the low temporal resolution of the measurements in the present work, these values are estimates only. Clearly, the elastic properties of the examined material could have been estimated more accurately, in case higher resolution measurements were conducted. However, the authors believe that using the methodology described in this work, while taking advantage of ultra high speed photography, measurement of the elasto-viscoplastic properties of low-impedance materials can be facilitated.

Fig. 20- Evolution of local stress at different positions along the specimen axis for the foam specimens deformed at (a) 1437 s^{-1} and (b) 2458 s^{-1} strain rates.

Fig. 21- Local stress-strain curves obtained at different positions along the specimen axis for the foam specimens deformed at (a) 1437 s^{-1} and (b) 2458 s^{-1} strain rates.

Fig. 22- Global stress-strain curves obtained using two different methods for foam specimens deformed at (a) 1437 s^{-1} and (b) 2458 s^{-1} strain rates.

5. Conclusions

The dynamic stress-strain response of closed-cell rigid polyurethane foam was investigated using high speed photography with 3D digital image correlation. Owing to the low wave propagation speed of the examined foam specimens, a highly non-homogeneous state of deformation was observed during impact loading, a condition that requires the inclusion of inertia stresses in the analysis. The change in density, inertia forces and stresses were first calculated using the full-field strain and acceleration distributions. By superimposing the locally developed inertia stresses to the stress magnitudes measured at specimen end, the full-field distribution of axial stress developed within the material was calculated. Considering the results obtained in this work, the following remarks should be noted:

- It has been well established in the literature, and was clearly demonstrated in the present work, that in the case of low impedance materials, conventional boundary measurements can introduce substantial error into the analysis.
- The method described in the present work took advantage of full-field 3D digital image correlation measurements throughout the transient loading process to include the effects of compressibility and inertia forces and to overcome the limitations associated with conventional boundary measurements. The main limitation in the present work was the temporal resolution of the DIC measurements. This issue can be resolved by using ultra high speed cameras commercially available today.
- Attempts were made to include the concurrent influences of material compressibility and inertia effect in the analyses. Although the material examined here shows minimal change of density, the analysis can be applied to any low-impedance compressible solid, regardless of the degree of compressibility.

- The authors believe that following the procedure detailed in this work, it would be possible to carry out high strain rate experiments on low impedance and/or soft materials and extract accurate stress-strain relations, a concept that continues to be of great interest in the experimental mechanics' community.
- Clearly, the effort in this work was to demonstrate the applicability of the utilized method of analysis. The authors believe that using the methodology described in the present work, along with more powerful full-field measurement techniques and more advanced dynamic experiment apparatuses, the rate dependent elastic properties of materials, regardless of their high or low impedance characteristics, can be obtained. This would be basically achieved following the improvement of ultra-high speed imaging techniques in the near future.

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References

- [1] Field JE, Walley SM, Proud WG, Goldrein HT, Siviour CR. Review of experimental techniques for high rate deformation and shock studies. *Int J Impact Eng* 2004; 30: 725-775.
- [2] Hopkinson B. A method of measuring the pressure produced in the detonation of high explosives or by the impact of bullets. *Phil Trans R Soc Lond* 1914; 213: 437-456.
- [3] Song B, Chen W. Dynamic stress equilibrium in split Hopkinson pressure bar tests on soft materials. *Exp Mech* 2004; 44: 300-312.
- [4] Mallon S, Kidane A, Lu WY. Full-field deformation observation of polymer foam subjected to shock loading. In: Song B, Casem D, Kimberley J (Editors), *Dynamic behavior of materials – Volume 1, Proceedings of the 2014 Annual Conference on Experimental and Applied Mechanics*, Springer, 2015, pp 83-89. DOI 10.1007/978-3-319-06995-1_14
- [5] Othman R, Aloui S, Poitou A. Identification of non-homogeneous stress fields in dynamic experiments with a non-parametric method. *Polym Test* 2010; 29: 616-623.
- [6] Aloui S, Othman R, Poitou A, Guegan P, El-Borgi S. Non-parametric identification of the non-homogeneous stress in high strain-rate uni-axial experiments. *Mech Res Commun* 2008; 35: 392-397.
- [7] Zhang X, Zhang H. Optimal design of functionally graded foam material under impact loading. *Int J Mech Sci* 2013; 68: 199-211.
- [8] Avalle M, Belingardi G, Montanini R. Characterization of polymeric structural foams under compressive impact loading by means of energy-absorption diagram. *Int J Impact Eng* 2001; 25: 445-472.

610 [9] Ouellet S, Cronin DS, Moulton J, Petel EO. High rate characterization of polymeric
611 closed-cell foams: Challenges related to size effects. In: Chalivendra V, Song B, Casem D
612 (Editors), Dynamic behavior of materials – Volume 1, Proceedings of the 2012 Annual
613 Conference on Experimental and Applied Mechanics, Springer, 2013, pp 21-28. DOI
614 10.1007/978-1-4614-4238-7_4

615 [10] Petel OE, Ouellet S, Frost DL, Higgins AJ. Shock Hugoniot measurements in foam. J
616 Phys: Conf Ser 2014; 500: 1-6.

617 [11] Viot P, Beani F, Lataillade JL. Polymeric foam behavior under dynamic compressive
618 loading. J Mater Sci 2005; 40: 5829-5837.

619 [12] Flores-Johnson EA, Li QM. Low velocity impact on polymeric foams. J Cell Plast 2011;
620 47: 45-63.

621 [13] Linul E, Marsavina L, Voiconi T, Sadowski T. Study of factors influencing the
622 mechanical properties of polyurethane foams under dynamic compression. J Phys: Conf Ser
623 2013; 451: 1-6.

624 [14] Ouellet S, Cronin D, Worswick M. Compressive response of polymeric foams under
625 quasi-static, medium and high strain rate conditions. Polym Test 2006; 25: 731-743.

626 [15] Whisler D, Kim H. Experimental and simulated high strain dynamic loading of
627 polyurethane foam. Polym Test 2015; 41: 219-230.

628 [16] Liu J, Saletti D, Pattofatto S, Zhao H. Impact testing of polymer foam using Hopkinson
629 bars and digital image analysis. Polym Test 2014; 36: 101-109.

630 [17] Song B, Chen W. Dynamic stress equilibrium in split Hopkinson pressure bar tests on
631 soft materials. Exp Mech 2004; 44: 300-312

632 [18] Oullet S, Cronin D, Worswick M. Compressive response of polymeric foams under
633 quasi-static, medium and high strain rate conditions. *Polym Test* 2006; 25: 731-743.

634 [19] Chen R, Huang S, Xia K, Lu F. A modified Hopkinson bar system for testing ultra-soft
635 materials under intermediate strain rates. In: *DYMAT-International Conference on the*
636 *Mechanical and Physical Behaviour of Materials under Dynamic Loading*, vol. 1, pp. 389-
637 394. EDP Sciences, 2009. DOI: 10.1051/dymat/2009055

638 [20] M.A Sutton, J.J Ortu, H.W Schrier. Image correlation for shape, motion and
639 deformation measurements. Springer. NY. 2009

640 [21] Pierron F, Zhu H, Siviour C. Beyond Hopkinson's bar. *Phil Trans R Soc A* 2014; 372:
641 20130195.

642 [22] Gilat A, Schmidt TE, Walker AL. Full field measurement in compression and tensile
643 split Hopkinson bar experiments. *Exp Mech* 2009; 49: 291-302.

644 [23] Koohbor B, Mallon S, Kidane A, Sutton MA. A DIC-based study of in-plane mechanical
645 response and fracture of orthotropic carbon fiber reinforced composites. *Compos Part B-Eng*
646 2014; 66: 388-399.

647 [24] Mallon S, Koohbor B, Kidane A, Sutton MA. Fracture behavior of prestressed
648 composites subjected to shock loading: A DIC-based study. *Exp Mech* 2015; 55: 211-225.

649 [25] Pierron F, Forquin P. Ultra-high-speed full-field deformation measurements on concrete
650 spalling specimens and stiffness identification with the virtual fields method. *Strain* 2012; 48:
651 388-405.

652 [26] Moulart R, Pierron F, Hallett SR, Wisnom MR. Full-field strain measurement and
653 identification of composites moduli at high strain rate with the virtual fields methods. *Exp*
654 *Mech* 2011; 51: 509-536.

655 [27] Lu WY. Mechanical characterization of rigid polyurethane foams. Sandia Report 2014:
656 SADN2014-20708.

657 [28] Mallon S, Koohbor B, Kidane A. Fracture of pre-stressed woven glass fiber composite
658 exposed to shock loading. In: Song B, Casem D, Kimberley J (Editors), *Dynamic behavior of*
659 *materials – Volume 1, Proceedings of the 2014 Annual Conference on Experimental and*
660 *Applied Mechanics*, Springer, 2015, pp 213-219. DOI 10.1007/978-3-319-06995-1_32

661 [29] Koohbor B, Mallon S, Kidane A, Lu WY. The deformation and failure response of
662 closed-cell PMDI foams subjected to dynamic impact loading. *Polym Test* 2015; 44: 112-
663 124.

664 [30] Pierron F, Sutton MA, Tiwari V. Ultra high speed DIC and virtual fields method analysis
665 of a three point bending impact test on an aluminum bar. *Exp Mech* 2011; 51: 537-563.

666 [31] L.J Gibson, M.F Ashby. *Cellular solids: Structure and properties*. 2nd ed. Oxford. 1997

667 [32] Samanta SK. Dynamic deformation of aluminum and copper at elevated temperatures. *J*
668 *Mech Phys Solids* 1971; 19: 117.

669 [33] Lava P, Cooreman S, Coppieters S, De Strycker M, Debruyne D. Assessment of
670 measuring errors in DIC using deformation fields generated by plastic FEA. *Opt Laser Eng*
671 2009; 47: 747-753.

672 [34] Rossi M, Lava P, Pierron F, Debruyne D, Sasso M. Effect of DIC resolution, noise and
673 interpolation error on identification results with the VFM. *Strain* 2015; 51: 206-222.