Void ratio based representative volume element for modelling the high strain rate behaviour of granular materials

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ABSTRACT

The ability to model impact and penetration behaviour of granular materials – such as sand – largely depends on the understanding of the stress-strain and volume change characteristics at high strain rates. Split Hopkinson Pressure Bar has been used extensively in the past to evaluate strain rate dependent constitutive response of a wide range of advanced and natural materials. However, the deformation behavior of geo-materials – including granular media – needs careful interpretation in order to provide data for calibration and validation of numerical models. In this paper, a new procedure for the determination of the smallest Representative Volume Element (RVE) of granular media is proposed. The procedure relies upon the simulations of consolidated granular assembly using Discrete Element Method (DEM). Results indicate the role of void ratio as an important state variable, which influences the dynamic mechanical performance of granular materials considerably. The existence of a lower limit for the dimensions of specimens used in the high-strain rate

30 experiments on granular materials as a function of their consolidation state is demonstrated.
 31 The adoption of the RVE as the smallest specimen size allows the time for achieving
 32 dynamic equilibrium that is essential for reliable characterization of rate dependent behaviour
 33 of granular or particulate materials. Using the concept of RVE, experimental results are
 34 generated, demonstrating the success of our proposed methodology. The identified RVE size
 35 will favourably impact the next generation of predictive modelling tools for impact and
 36 penetration problems in granular materials.

37

38 Keywords: Computational mechanics, Dynamics, Sand, High strain rate.

39

40 LIST OF NOTATIONS

e	void ratio
n	porosity
V_v	volume of voids
V_s	volume of solids
V_{tot}	total volume available
d_0	distance between the boundary and the centre of each particle
R	radius of the particle
V_{sph}	volume of each particle
V_{sph_cup}	difference between the volume of the sphere and of a spherical cup of height d_0
d	dimension of the fictitious volume
D	diameter of the real container
E	Young's modulus
ν	Poisson ratio

L	length of the sample in the loading direction
m	mass of the sand poured in the container
e_{mean}	mean value of the void ratio
$e_{variance}$	standard deviation of the void ratio
ρ_{sp}	bulk density of the silica sand
n_p	number of particles in the assembly
$\varepsilon_i(t)$	history of the incident strain signal
$\varepsilon_r(t)$	history of the reflected strain signal
$\varepsilon_t(t)$	history of the transmitted strain signal
A_0 and A_s	cross sectional area of bars and specimen
E_0 and C_0	Young's modulus and elastic wave speed of the bars

41 **1. INTRODUCTION**

42 A representative length scale is one of the first considerations when characterising the
43 behaviour of solid materials subjected to mechanical loading. When considered
44 macroscopically, some materials appear homogeneous. However, at a smaller length scale, it
45 is often possible to identify diffused or discrete discontinuities, associated with the material's
46 heterogeneous micro or nano structure corresponding to voids, cracks, grains or crystals,
47 molecules, dislocations, atoms. In fact, every macroscopic property of an intrinsically
48 heterogeneous and discontinuous medium is the consequence of its microscopic structure
49 (Andreotti et al. 2013). Modelling an apparently homogeneous material without taking into
50 account its structure at the smaller scale would yield inaccurate results, unless its behaviour is
51 comprehensively characterised experimentally at the larger scale.

52 Granular materials are highly heterogeneous assemblies whose response to mechanical
53 loading is strongly affected by the distribution of grain sizes and shapes, as well as the

consolidation state, moisture content, surface roughness, inter-particles interactions and several other parameters (Georgiannou et al. 1990, Leroueil & Vaughan 1990, Omidvar et al. 2012). At certain larger scales, even highly heterogeneous and discontinuous assemblies – such as powders, sands and gravels – can exhibit homogeneous behaviour, which can be modelled using equivalent continuum mechanics approaches. This poses a requirement upon the smallest Representative Volume Element (RVE) of materials and loading conditions under consideration.

Several definitions of RVE are available in literature. For example, (Freudenthal 1950) defines the RVE as the volume representing the entire system and its heterogeneity, including its regions of disorder. In (Hashin 1983), it is defined as the volume used to determine the corresponding effective properties of a given material to be adopted in a homogenised macroscopic model. Therefore, the dimensions of the representative volume element should be large enough to contain statistically representative information about its microstructure while describing the behaviour of the macroscopic body. When the sample is smaller than the RVE, the measurements of any physical property start to vary thus resulting in uncertainty of the obtained experimental results.

Physical and geometric properties such as density, particle size and distribution, and void ratio or porosity are often used to determine the RVE (Graham & Yang 2003, Kanit et al. 2003, Nielsen 2005). In other cases, mechanical parameters such as elastic modulus, shear modulus, stress fluctuations are taken into account (Evesque & Adjemian 2002, Ren & Zheng 2002, Stroeven et al. 2004). However, no information is available about the relationship between RVE of granular materials and sample size used in experiments at elevated strain rates such that the response to rapidly applied loading can be successfully characterised ensuring dynamic equilibrium. For the cases where dynamic equilibrium remains debatable

78 (Scheidler et al. 2011), the experimental data generated is used as validation of macroscopic
79 material models.

80 The choice of a particular type of sand in its dry state (Ottawa sand in this study) defines
81 size distribution, grain shape and moisture content. Therefore, only the effect of the initial
82 consolidation state, which affects considerably the dynamic mechanical response of granular
83 materials (Georgiannou et al. 1990, Leroueil & Vaughan 1990), needs to be defined.

84 In fact, the very same granular medium, if subjected to external loading without
85 controlling its initial consolidation state, could provide very different results, similarly to
86 what would happen when testing different materials. Therefore, the initial void ratio (e) is a
87 key control state parameter for obtaining the RVE adopted in high strain rate compression
88 tests using the split Hopkinson pressure bar.

89 The initial void ratio is defined as the ratio of the volume of voids V_v to the volume of
90 solids V_s (Craig 2004).

$$e = \frac{V_v}{V_s} \quad (1)$$

91 Another equivalent parameter largely used in soil mechanics is the porosity (n), defined
92 as the ratio between the volume of voids and the total volume of the soil V_{tot}

$$n = \frac{V_v}{V_{tot}} \quad (2)$$

93 The two definitions are linked by the following

$$n = \frac{e}{1 + e} \quad (3)$$
$$e = \frac{n}{1 - n}$$

94 Most of the experimental work on the mechanical characterisation of granular material
95 does not take into account the definition of the RVE.

96 The definition of the dimensions of the sample to be used in SHPB tests is often based on
97 conventions or obtained by trial and error procedures. (Kabir et al. 2010) present an example

of preliminary definition of the sample dimensions using high-speed cameras to define the smallest volume characterised by uniform deformation. However, no check of the correct representation of the initial consolidation state of the sample is conducted.

Moreover, according to the existing literature, it is common practice to adopt granular samples with a very short thickness during SHPB experiments (Lu et al. 2009, Omidvar et al. 2012, Zhang & Zhao 2013) to obtain higher strain rates and enhance the dynamic equilibrium within the sample, which is otherwise difficult to obtain because of the low wave propagation speed in granular media (Brown et al. 2007, Charlie et al. 1990). For instance, (Felice 1985, Felice et al. 1991) use specimens with a slenderness L/D lower than 0.2 to carry out Hopkinson pressure bar experiments on saturated sand. Also (Lu et al. 2009, Meunier 1974, Veyera & Ross) adopt samples with very low L/D values.

The work presented in this paper proves the existence of a lower limit on the dimensions of the specimens to be used in high rate experiments. This limit is represented by the RVE, herein based on the concept of void ratio, for the considered granular material. As a result, this work proves that it is not possible to use arbitrarily small samples. In fact, samples smaller than the RVE do not statistically represent the desired consolidation state of the medium.

In addition, the adoption of samples whose dimensions are equal to the pre-determined RVE ensures an improved experimental repeatability of the results when compared against existing data (Kabir et al. 2010).

Section two of this paper presents the procedure used to determine the void ratio and its variation within the samples. Section three includes the description of the method adopted to compute the RVE. It also reports the outcomes of the numerical simulations and the experimental results obtained after adopting thus determined dimensions. Finally, section four summarizes all the key outcomes from this investigation.

2. NUMERICAL COMPUTATION OF THE VOID RATIO WITHIN THE SAMPLE

The generation of three-dimensional assemblies of grains, representing the sand samples under examination, is the first step required for the determination of the RVE.

The correct numerical reproduction of experimental conditions implies the appropriate modelling of the microstructure of granular materials (in terms of grain size distribution, grain shape, and physical properties), confinement and boundary conditions.

An in-house software (DEST – Discrete Elements Simulation Tools) is used to model the motion and interaction of 3D particles during gravitationally assisted deposition and consolidation. This approach allows for numerous simulations to evaluate the final packing densities as a function of the granular media and deposition process. The detailed description of the adopted numerical method is included in the next section.

Once the numerical packing is generated, the variation of void ratio within the sample and its mean value have to be estimated. Moreover, it has to be verified whether the numerical values match the experimental ones, thus correctly reproducing the initial consolidation state of the packing.

The definition of the RVE should take into account the variations of the packing density within the sample. In fact, the walls of the container induce an ordered distribution of particles in their vicinity while the particles far from the walls are randomly arranged.

As a result, the particles immediately close to the walls, typically within a distance of 2-4 equivalent diameters, should not be considered statistically representative of the consolidation state of the medium (Blumenfeld et al. 2005, Landry et al. 2003, Man et al. 2005).

Figure 1: Two-dimensional schematic representation of the three dimensional effect of the wall and free surfaces on the RVE

For non-spherical particles, instead, the variation of local density due to the presence of the walls is less significant. In fact, non-spherical particles tend to generate a more random packing (Cumberland & Crawford 1987). Furthermore, when only gravity is applied, the presence of the uneven free surface on the top of the samples (Figure 1) produces significant oscillations of the void ratio.

The numerical approach used to compute the void ratio and its variation within the sample is based on equation (4), which is obtained by rewriting (1) as a function of the volume of particles V_s enclosed in a container of volume V_{tot} :

$$e = \frac{V_v}{V_s} = \frac{V_{tot} - V_s}{V_s} = \frac{V_{tot}}{V_s} - 1 \quad (4)$$

V_s and V_{tot} are numerically computed considering smaller fictitious volumes (which locally define V_{tot} in (4)) enclosed in the real container and computing only the volume of the particles and their fractions contained in them (V_s in (4)). This approach takes into account the effect of walls and free surface on the consolidation state. The fictitious volumes can be defined in two ways:

1. By gradually increasing their height (Figure 2) starting from the bottom of the real container. This approach assesses the variation of the consolidation state across the height of the sample.

Figure 2: Schematic representation of the variation of the height of the fictitious volume

2. By simultaneously increasing all three spatial dimensions starting from the centre of the sample (Figure 3).

Figure 3: Schematic representation of the variation of all the dimensions of the fictitious volume

Both cases present six different configurations to be taken into account when computing the particle fractions contained in the virtual volumes. These are shown in Figure 4, where the fraction of particles comprised in the volume is coloured white.

The ratio between the distance d_o , from the centre of the particle to the boundary, and the radius R of the particle delineates all possible locations (Figure 4a). The distance d_o is positive if the centre of the particle is inside the volume of interest.

Table 1 summarizes the results of calculations of the volume of particles taken into account in all configurations.

Figure 4: Possible intersection between boundaries and spheres

Ref. Figure	Condition	Description	Definition of particle volume
Figure 4a	$\frac{d_0}{R} > 1$	Spheres totally contained in the volume	$V = V_{sph}$
Figure 4b	$0 < \frac{d_0}{R} < 1$	The sphere is inside with more than half of its volume	$V = V_{sph} - V_{sph-cup} = \frac{4}{3}\pi R^3 - \pi d_0^2 \left(R - \frac{d_0}{3}\right)$
Figure 4c	$\frac{d_0}{R} = 0$	The centre of the sphere is on the boundary	$V = \frac{V_{sph}}{2}$
Figure 4d	$-1 < \frac{d_0}{R} < 0$	Less than a half of the sphere is inside the volume	$V = V_{sph-cup} = -\pi(R - d_0)^2 \left(R - \frac{d_0}{3}\right)$
Figure 4e	$\frac{d_0}{R} < -1$	Spheres totally outside the volume	$V = 0$
Figure 4f		The spheres intersects two corner of the box at the same time	Procedure reported in the Appendix A of (Faure et al. 2009)

Table 1: Volume evaluation of the spheres contained in the fictitious volume

The void ratio e is computed for each considered fictitious volume (for different d/D , Figure 3), hence defining its variation within the sample.

Figure 5 shows an example of the typical e vs d/D curves, depicting the effects of free-surfaces and boundaries upon the void ratio. These charts are obtained by varying the dimension (d) of the fictitious bounding volumes (within which the void ratio is computed) with respect to the dimension (D) of the real container. Three different zones can be identified:

ZONE ① in this portion of the figure, the dimensions of the fictitious samples are comparable to the dimensions of the grains. Small volumes in the centre of the assembly contain only few voids and, consequently, the void ratio is very low.

ZONE ② in the central part of the plot the void ratio is not affected by the presence of walls and free surface. This portion of the chart is used to calculate the average void ratio and its standard deviation, adopted in the following analysis (Section 3.1).

ZONE ③ the dimensions of the fictitious volume are comparable with those of the real container; the void ratio is very high because both the effects of free surface and walls are present.

Figure 5: Qualitative distribution of the void ratio within granular material, varying the dimension (d) of the imaginary volume with respect to the real container (D)

Applying this numerical method for every packing, the mean value of the void ratio is computed and it is, then, used for the estimation of the RVE.

3. METHOD FOR THE ESTIMATION OF THE RVE AND RESULTS

The response of granular materials at high rates of strain (HR) by means of Split Hopkinson Pressure Bar experiments (ASM 2000) has been extensively studied in the past decades (Bragov et al. 1996, Felice 1985, Hagerty et al. 1993, Kabir et al. 2010, Lu et al. 2009, Omidvar et al. 2012, Pierce 1989).

The main aim of this work is to demonstrate the existence, for the first time in literature, of a lower limit for the dimensions of the sample to be used in SHPB tests on granular materials. This limit is represented by the smallest RVE with respect to the void ratio of the granular material under consideration. Additionally, this work demonstrates that the adoption of specimen size comparable to the determined RVE improves the repeatability of the experimental results with respect to what is presented in literature.

The numerical method for the calculation of the RVE presented in this paper has to be considered as a general approach applicable to any kind of granular material. However, for the sake of simplicity, it has been adopted to define the RVE of a specific type of sand, namely the dry Ottawa sand in a dense initial consolidation state (corresponding to a void ratio of 0.56).

In order to define the minimum statistically representative sample, at prescribed consolidation states, numerical simulations are conducted before carrying out the experiments.

3.1. Numerical determination of the RVE

The preparation of the sample is crucial for any experimentation on granular materials since each test has to be representative of a fixed and reproducible consolidation state.

223 Experimentally specific consolidation procedures have been adopted to produce samples
224 in initial dense consolidation state.

225 The sand is dropped from a controlled height into a container and subsequently, it is
226 compacted by tapping and compressing it (with a dropping weight of 240 g). A detailed
227 description of the experimental preparation procedure is presented in Section 3.2.

228 The numerical simulations have to be representative of the experimentally achieved initial
229 consolidation state.

230 The Ottawa sand is composed of quasi-spherical grains (Figure 6). For this reason, the grains
231 are modelled as spherical particles. Table 2 reports a discretisation of the experimental
232 particle size distribution obtained by sieving. Table 3 lists the physical properties of the sand
233 used in the simulations.

234 All packing simulations are conducted using frictionless contact. This assumption is based on
235 previous numerical studies (Vandewalle, N. et al. 2007, Zavaliangos 2003) which compared
236 the results obtained by either using inter-particle friction or frictionless contact. It has been
237 demonstrated that both approaches converge to the same solution when evaluating the
238 consolidation state of very dense granular assemblies.

239 Simulations carried out in presence of friction require the application of pressure loading and
240 initial perturbative boundary conditions. On the contrary, the adoption of frictionless contact
241 allows for the obtainment of physically equilibrated densely packed granular systems
242 exclusively by applying gravity (Olsson & Larsson 2012, Skrinjar & Larsson 2004).

243 The assemblies considered in this study are consolidated under gravity and, therefore, no
244 significant deformations are expected other than relatively small compliance along interacting
245 surfaces of individual grains. Consequently, the particles are modelled as rigid spheres that
246 can overlap by the amount predictable by Hertzian contact law.

Figure 6: Grain shapes for Ottawa sand.

Radius	Percentage
0.355 mm	7 %
0.25 mm	76 %
0.175 mm	17 %

Table 2: Radii distribution of the simulated sand particles

Material	Quartz
Density	$2.65 \text{ E-3 } \frac{\text{g}}{\text{mm}^3}$
E	$76 \text{ E9 } \frac{\text{g}}{\text{mm s}^2}$
ν	0.31

Table 3: Material properties of the Ottawa sand

The sand assemblies used in the simulations are packed by means of a combination of geometric deposition methods and mechanical approaches (application of gravity). Representative 3D geometrical packings of particles, based on the experimentally measured size distribution reported in Table 2, are generated using the “planar” packing algorithm detailed in (De Cola et al. 2015). This geometric deposition method is based on the tangency between neighbouring particles and it can effectively consider poly-dispersity and different particle shape. This algorithm produces several parallel layers of particles, thus reproducing a prescribed mass flow through a funnel. The application of gravity, using the DEM, reproduces the pouring procedure required to have physically meaningful and stable systems (Figure 8).

Figure 7: Flowchart of the iterative algorithm for the calculation of the RVE

The RVE is calculated by means of the iterative procedure summarised in Figure 7.

The starting point is the definition of the diameter of the container used in the experiments (point A in Figure 7). In this study, a stainless steel cylindrical container of diameter equal to 20 mm and 5 mm thick walls is used. The diameter of the samples is equal to the diameter of the Split Hopkinson bar (20 mm) and it respects the requirements for countering the effects of inertia and friction (ASM 2000). The thickness of the container is designed to resist applied axial and resulting lateral stresses avoiding plastic deformation of the container and its lateral expansion.

Once the diameter is fixed, only the thickness of the sample along the direction of loading (L) needs to be defined. This depends, in turn, upon the amount of sand (m) to be poured in the

container. From the numerical point of view, this is equivalent of fixing the number of particles to be included in the simulations.

The first iteration of the process starts choosing an exploratory number of particles n_p equal to 5000. A number of n three-dimensional DEM simulations with same number n_p of particles are run, in analogy with repeated samples preparation in the experiments (point B in Figure 7). In our test case, 3 simulations per configurations – characterised by the same radii distribution but different arrangements – are carried out (Figure 8).

Figure 8: Three different initial and final assemblies produced with the combination of geometrical deposition method and mechanical method based on the application of gravity

The simulations are interrupted when the total kinetic energy of the system decreases to around 10^{-3} J thus reproducing static equilibrium conditions (Figure 9).

Figure 9: Example of the Kinetic Energy of the system during the deposition.

At this stage, it is possible to measure the resulting length L of the sample in the loading direction and the corresponding slenderness (L/D).

For the case of 5000 particles, the average length of the sample L after compaction is approximately 2.3 mm ($L/D = 0.12$), as reported in Table 4.

Afterwards, the results of the simulations are analysed to evaluate the mean value of the void ratio and its distribution within the samples (point C in Figure 7). For this purpose, the second procedure defined in Section 2 – based on the progressive increase of the dimensions of a virtual volume of interest (d , with diameter equal to length) – is used.

The average values of the void ratio are extracted taking into account the portion of plot included between $d/D = 0.4$ and $d/D = 0.7$, characterised by a maximum variation of the void ratio of less than 0.1. Therefore, the void ratio resulting from each simulation is defined as the average value calculated in ZONE $\mathcal{Q}$ as identified earlier in Figure 5.

Table 4 reports the results obtained from the simulations, in terms of void ratio and final height of the sample, after gravity is applied.

Sample	e_{mean}	$e_{variance}$	Thickness L of the sample [mm]
5000_1	0.50	0.07	2.35
5000_2	0.57	0.06	2.24
5000_3	0.59	0.04	2.36

Table 4: Numerical results for the 5000 spheres simulations

Figure 10 summarizes the results obtained from the simulations of assemblies containing 5000 particles.

Figure 10: Void ratio for three different numerical samples with 5000 particles, varying the dimension of the imaginary box (d) with respect to the real one (D)

The charts produced by plotting the distribution of the void ratio within the sample (as in Figure 10) are used to conduct a convergence study to assess if the considered samples are statistically representative of the consolidation state of the material under examination (point D in Figure 7).

In case the graphs obtained for the n samples show repeatability and the plateau area is wide and horizontal (ZONE 2 of Figure 5), then the dimension L averaged from the n simulations determines the thickness of the specimen corresponding to the RVE. On the contrary, if the

charts show significant scatter or the plateau region does not exist, the number of particles n_p (and consequently the specimen length L) has to be increased.

It is evident, observing Figure 10, that the curves do not show any plateau – as they exhibit a positive slope – and that the results obtained for the three samples are visibly different. The relevant scatter in the measurements demonstrates that the considered dimensions do not correspond to the RVE.

Therefore, a second series of simulations containing 10000 particles s was conducted.

Table 5 and Figure 11 summarize the results of the simulations.

Sample	e_{mean}	$e_{variance}$	Thickness L of the sample [mm]
10000_1	0.56	0.08	5.2
10000_2	0.51	0.07	5.0
10000_3	0.57	0.09	4.9

Table 5: Numerical results for the 10000 spheres simulations

Also in this case, the variation of void ratio among the three samples shows that the current volume cannot be considered as the RVE of the material.

Figure 11: Void ratio for three different samples with 10000 particles, varying the dimension of the imaginary box (d) with respect to the real one (D)

The dimensions of the samples are then further increased using 15000 particles in two different arrangements.

Table 6 and Figure 12 report the outcomes of both simulations.

Sample	e_{mean}	$e_{variance}$	Thickness L of the sample [mm]
15000_1	0.56	0.05	6.34
15000_2	0.55	0.04	6.24

Table 6: Numerical results for the 15000 spheres simulations

Figure 12: Void ratio for two different samples with 15000 particles, varying the dimension of the imaginary box (d) with respect to the real one (D)

The thickness of the samples containing 15000 particles, after compaction, is about 6.3 mm.

The void ratio is nearly constant in a broader region ($0.2 < d/D < 0.8$) with a value approximately equal to 0.56.

There is no significant scatter between the data obtained in these two simulations. Therefore, the considered virtual sample, corresponding to a length of the sample in the loading direction L equal to 6.3 mm, is considered as the RVE.

The thickness L of the obtained sample can be expressed in terms of average particle diameter ($\bar{d} \approx 0.5 \text{ mm}$). In this case, the length of the representative sample is approximately

12 times the mean diameter $\bar{d}$. X-ray computed tomography experiments (Razavi et al. 2006) confirm our estimation of the RVE dimensions, thus proving the correctness of the numerical results obtained.

For completeness, a sample containing 20000 particles is considered. The application of gravity yields a sample of final height equal to 8.2 mm and void ratio equal to 0.56.

In Figure 13, the curve representing the void ratio of the sample containing 20000 particles is superimposed to the curves shown in Figure 12. It is evident that the three curves are in perfect agreement. Therefore, the adoption of a volume of sand bigger than the RVE does not modify significantly the results in terms of void ratio.

Sample	e_{mean}	$e_{variance}$	Thickness L of the sample [mm]
20000	0.56	0.04	8.2

Table 7: Numerical results for the 20000 spheres simulation

Figure 13: Comparison of the void ratio for the 15000 particles samples and the 20000 particles sample, varying the dimension of the imaginary box (d) with respect to the real one (D)

In conclusion, the RVE of the sand under investigation is considered to be a cylindrical sample of diameter and length equal to 20 mm and 6.3 mm respectively ($L/D \cong 0.3$). This is obtained for the specific compaction state and diameter of the container considered in this

study. However, the method presented herein is intended to be a universally applicable numerical approach, which can be used for the estimation of RVE of any specific granular media.

3.2. Experiments

A series of laboratory Hopkinson bar compression experiments are conducted to assess the behaviour of Ottawa dry sand in its dense state ($e=0.56$) at high strain rate.

The Split Hopkinson Pressure Bar (SHPB) apparatus, used for the experiments, is composed of a projectile and instrumented input and output bars made of Ti64, with length of 2.8 m and diameter of 20 mm (Figure 14). Both bars are supported using low-friction polymeric circular bearings in order to avoid any lateral movements and buckling. The projectile accelerated using low-pressure compressed air, impacts the input bar. The impact produces a stress pulse that loads the specimen under consideration, inserted between the input and output bar.

The stress pulse travels through the input bar reaching the sample. At the interface between sample and input bar, due to the mechanical impedance mismatch, part of the signal is transmitted and part reflected. The transmitted wave propagates through the specimen into the output bar. The material response is obtained by means of stress wave analysis using the signals measured by strain gauges.

Figure 14: Split Hopkinson Pressure Bar setup

The strain rate, strain, and stress histories in the specimen are calculated according to the one-dimensional wave analysis (Kolsky 1949), as follows:

$$\dot{\varepsilon}(t) = \frac{C_0}{L_s} [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)] \quad (5)$$

$$\varepsilon = \int_0^T \dot{\varepsilon}(t) dt = \frac{C_0}{L_s} \int_0^T [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)] dt \quad (6)$$

$$\sigma(t) = \frac{A_0}{2A_s} E_0 [\varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t(t)] \quad (7)$$

Where L_s is the length of the sample, $\varepsilon_i(t)$, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ represent the incident, reflected and transmitted strain histories respectively. A_0 and A_s are the cross sectional area of bars and specimen; E_0 and C_0 are Young's modulus and elastic wave speed of the material of the bars. Cardboard shims (436 g/m²) of 1 mm thickness were used to reduce the high-frequency content in the signal, wave dispersion and Pochhammer–Chree oscillations.

The peculiarity of this setup is the significant length of the projectile (2.8 m). Since the length of the projectile influences the duration of the incident stress pulse (Kolsky 1949), the use of this setup allows for longer loading durations and, in turn, higher strains. The velocity of the projectile is regulated by changing the pressure in the chamber. In these experiments, the projectile moves, before impact, at a speed of around 10 m/s.

The first tests are conducted using cylindrical sand specimens characterized by a diameter equal to 20 mm and length equal to 10 mm, hence using a volume bigger than the calculated RVE.

The porosity and void ratio definitions, used for the experiments, are:

$$n = 100 * \left(1 - \frac{V_s}{V_{tot}}\right) = 100 * \left(1 - \frac{\frac{m}{\rho_{sp}}}{V_{tot}}\right) \quad (8)$$

$$e = \frac{n}{1 - n} \quad (9)$$

Where V_s is the volume of solid particles, ρ_{sp} is the bulk density for the silica sand (equal to 2.65 g/cm³) and $V_{tot} = \frac{\pi D^2}{4} L$ is the available volume. Figure 15 illustrates the sample in its

398 assembly. The two titanium anvils confine the sand within the cylinder and protect the input
 399 and output bars from indentation.
 400 A specimen filling procedure is established to obtain experimentally a fixed and reproducible
 401 void ratio. The amount m of sand required to obtain a given compaction is determined by
 402 means of a precision scale. The sand is then poured in the container using a funnel.
 403 The container is thereafter tapped five times on four diametrical points. The sand has to be
 404 further compressed using a dropping weight of a fixed mass (240 g) until it reaches the final
 405 required height L (measured by means of Vernier-callipers). The weight is dropped 10 times
 406 starting from a height of 10 cm.

Figure 15: Schematic of sand specimen confined in steel tube

409 A SHPB test is valid only if dynamic equilibrium conditions, within the specimens, are
 410 achieved during the experiment (ASM 2000). In fact, the mechanical response obtained is
 411 representative of the whole specimen only when this condition is fulfilled.

During SHPB tests the stress pulse is produced by the impact of a striker on the input bar. A perfect dynamic equilibrium would require several reflections of this stress wave along the specimen.

Granular materials are characterized by a low wave propagation velocity (Lu et al. 2009, Martin et al. 2009). Consequently, the time required to achieve dynamic equilibrium is high.

The dimensions of the sample need to be reduced as much as possible to allow multiple stress wave reflections within the specimen during the test.

The time history of the forces at the interfaces between sample and bars (Figure 16) is obtained from the measurement data associated with the strain gauges positioned along the input and output bars. The comparison of these forces provides a measure of the dynamic equilibrium within the sample.

Figure 16: Forces at the sample-bars interfaces

Figure 17 shows such comparison for an experiment conducted on a sample of length equal to 10 mm.

Figure 17: Force equilibrium and strain rate in dry, dense Ottawa sand, $L = 10$ mm

The two curves – representing the input (grey line) and output (black line) interfacial forces – are not parallel and do not overlap during the compaction phase of the specimen. This denotes a lack of dynamic equilibrium in the sample.

As already said, the dynamic equilibrium conditions improve reducing the longitudinal dimensions of the specimen (i.e. reducing the length over diameter ratio L/D for a defined cross section). However, the minimum dimensions of the sample are dictated by the lower limit defined by the RVE, as explained previously in this paper.

Figure 18 shows the time history of the forces at the interfaces between bars and sample for a specimen having dimensions equal to the RVE.

Figure 18: Force equilibrium and strain rate in dry, dense Ottawa sand, $L = 6$ mm.

The dynamic equilibrium conditions are visibly improved with respect to the previous case.

Furthermore, the strain rate (dotted line in Figure 18) is approximately constant for a large portion of the stress-strain curve.

Figure 19 shows two stress-strain curves obtained using samples having dimensions corresponding to the RVE ($L = 6.3$ mm equivalent to a slenderness of the sample $L/D \cong 0.3$).

The results obtained from a series of five tests, in a same initial consolidation state, are all included between the two curves depicted in Figure 19.

This demonstrates that the choice of specimens having dimensions corresponding to the RVE produces also a significant improvement in the repeatability of the results, thus capturing better the mechanical behaviour of sand subjected to high strain rates.

Figure 19: Upper and lower limit of the experimental variation (over 5 tests) of the stress-strain curves of Ottawa sand at high strain rate (1400 s^{-1}) and void ratio $e=0.56$ for samples of length $L = 6 \text{ mm}$.

It must be stressed that it is not possible to further improve the dynamic equilibrium in the sample reducing its dimensions below the RVE limit as this would affect the initial representative consolidation state. However, it is worthwhile mentioning that pulse-shaping techniques could additionally enhance the equilibrium conditions.

Figure 20: Upper and lower limit of the experimental variation (over 5 tests) of the stress-strain curves for samples smaller than the RVE with $L/D = 0.1$

Finally, for the sake of completeness and to demonstrably confirm the correct determination of the RVE, an additional series of 5 tests were carried out on specimens of thickness equal to 2 mm ($L/D = 0.1$). All the corresponding stress-strain curves are comprised in between the two curves depicted in Figure 20. The adoption of samples smaller than the RVE affects the consistency of the void ratio within the specimen as demonstrated by the increased scatter in the obtained results (Figure 20).

4. CONCLUSIONS

Achieving the dynamic equilibrium within the sample, at constant strain rates, is of utmost importance in experimental characterisation of the mechanical performance of any material when relying upon the Split Hopkinson bar apparatus. The most common approach used in literature to improve the dynamic equilibrium conditions is the reduction of the sample dimensions. However, such approach may have missed the important aspect of considering a representative volume element size.

A rigorous characterisation of the high strain rate behaviour of granular materials requires careful consideration of the initial consolidation state of the particulate or granular assembly in order to achieve consistent results that represent the stress-strain behaviour of its continuum. Choosing an arbitrarily small sample would invalidate the correct representation of the initial compaction state. Therefore, it is necessary to evaluate the volume element representing the consolidation state of the tested granular material. This paper demonstrates the effectiveness of using Discrete Element Method for the estimation of the RVE.

The RVE is the smallest volume of sand that ensures the attainment of a compaction state consistent with the material under investigation in its continuum. A new numerical procedure for the definition of the RVE based on the estimation of the void ratio is presented.

The RVE proposed in this paper is based on the phenomenological observation on the importance of initial geometric arrangement of granular materials captured using the void ratio parameter. This is an important descriptor of the initial consolidation state of assemblies that affects significantly the high strain rate behaviour. Furthermore, authors are currently investigating the role of the void ratio on particle crushing during confined compression and overall dynamic behaviour.

The representative volume element of the sand under consideration is determined using an iterative numerical procedure based on DEM simulations.

The results obtained from the Split Hopkinson Pressure Bar experiments and reported in this paper demonstrate that the achievement of the dynamic equilibrium, in samples of granular materials, strongly depends upon the RVE dimensions.

Finally, the use of samples with dimensions comparable to the RVE enabled better repeatability of measurements when compared against results available in open literature.

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