PHENOMENOLOGY OF RAPID PROJECTILE PENETRATION INTO GRANULAR SOILS

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Abstract

Eighty subscale penetration tests were performed to investigate penetration phenomenology in granular media, with impact velocities in the range of 70-300 m/s. The parameter space consisted of (1) four natural and synthetic granular materials, including Ottawa sand, crushed fused quartz, aragonite, and crushed coral, (2) loose and dense packings, (3) dry and wet targets, and (4) four different projectiles, including spheres and long rods with conical, hemispherical and blunt nose shapes. Two techniques were employed to obtain penetration time histories, including photonic Doppler velocimetry for high velocity impact tests, and high speed imaging for low velocity impact tests. Penetration time histories were differentiated to obtain velocity and acceleration time histories. Analysis of the time histories revealed that the role of nose shape, packing density, and saturation is material dependent. Silica sands and calcareous sands showed contrasting behavior across the parameter space considered. These observations point to the significance of particle crushing in rapid penetration into granular media.

Keywords: Photonic Doppler velocimetry; High-speed imaging; Sand; Granular media; Impact; Projectile

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1. Introduction

There has been a recent surge of interest in penetration of granular materials. Impact and penetration into granular media is important for many engineering applications, including subsurface investigations, planetary impact, and offshore foundations and anchors, among others (Omidvar et al. 2014). Many aspects of penetration mechanics have been addressed in recent literature. A general picture is emerging in which there appears to be several distinct regimes in the global records of penetration into granular media. These regimes appear with different fidelity in different materials, and they appear to depend on the penetration velocity. The existence of different penetration regimes may point to changes in energy transfer mechanisms at different stages of penetration. This paper is an effort to further investigate these regimes.

The study of rapid penetration into granular media is closely related to their high strain rate response (Omidvar et al. 2012). Thus study of penetration effects provides both qualitative and quantitative insights into the mechanical properties of granular materials at high stresses and high strain rates. For example it has been shown that during penetration of an intruder in granular media, networks of stressed particles known as force chains transfer the intruder load to the target media at particle contacts (*e.g.*, Clark et al. 2014). As the magnitude of load carried by the particles increases, displacement of particles results in buckling of these force chains. Load is therefore redistributed along other force chains as new contacts form. Formation and buckling of force chains appears dependent on the penetration velocity due to the time scale associated with these phenomena. Investigating these hypotheses requires observations of penetration dynamics across a wide range of penetration velocities.

In this paper, results of experiments performed with projectiles penetrating various granular media are reported. Two methods were employed to measure time history of projectile penetration; *photonic Doppler velocimetry* (PDV) for spherical projectiles impacting at 300 m/s and *high-speed imaging* (HSI) for rod projectiles penetrating at 50-100 m/sec. Three natural sands and one synthetic material are tested, and the penetration histories are recorded. First, the experimental setups used to obtain penetration time histories in various materials and using different projectiles are briefly described. Next, observations of penetration dynamics along with general patterns observed from the tests in various materials are reported. Finally, the observed response is related to the phenomenology of rapid penetration into granular media.

2. Experiments

2.1. Granular media

Four different granular media were tested in this study (Fig. 1). The materials were selected to cover a range of parameters representative of natural and synthetic granular media, including specific gravity, packing density, particle shape, mineralogy, and internal flaws. Several standard characterization tests were performed on the tested media, including minimum and maximum density measurements according to ASTM D4254 and ASTM D4253, specific gravity according to ASTM 854, and particle size distribution according to ASTM 6913. In addition to the aforementioned characterization, typical particle shapes were also characterized using a dynamic imaging particle size analyzer (Sympatec QicPic). Typical 2D projections of particle shapes corresponding to the median diameter, by weight, known as d_{50} were obtained (Fig 2). Particles were characterized using sphericity, ψ , and aspect ratio, A_R . Sphericity is defined as the ratio of the surface area of a sphere with the same volume as the given particle to the surface area of the particle, and aspect ratio is defined as the ratio of the largest diameter to the smallest diameter orthogonal to it. Distribution of sphericity with particle size is shown in Fig 2 for the four tested materials, which are as follows:

- **Ottawa sand**, which is a siliceous sand with a particle specific gravity of 2.65. The size distribution of the particles is shown in Fig 1. Ottawa sand has a narrow size distribution, classified as poorly graded sand (SP) in the *Unified Soil Classification System* (USCS) (ASTM D2487). Ottawa sand is comprised mainly of round particles (Fig. 2), having a d_{50} of 0.65 mm, median sphericity of 0.90. Minimum and maximum densities of the Ottawa sand were 1.56 g/cm³ and 1.83 g/cm³, respectively.
- **Crushed fused quartz**, a product of grinding fused silica glass, is also classified as SP according to USCS. Crushed fused quartz consists of 99% SiO₂, and has a specific gravity of 2.20 (Guzman et al 2014). Unlike Ottawa sand, it is amorphous; presumably the grains are much stronger, since there are no sub-grains that can break apart. The material is comprised of angular particles, having d_{50} of 0.21 mm, with a median sphericity of 0.83, but with a broad distribution of particles with smaller sphericity down to approximately 0.45. Minimum and maximum densities of the tested crushed fused quartz were 0.99 g/cm³ and 1.31 g/cm³, respectively.
- **Aragonite sand** is an oolitic calcium carbonate based granular material with round particle shapes, as shown in Fig 2. Aragonite results from precipitation of calcite; the particles have a

specific gravity of 2.74, which is higher than the other materials tested. Aragonite was chosen because it resembles the sphericity of Ottawa sand, but is more susceptible to crushing. The tested Argonite has a d_{50} of 0.38, with an average sphericity of 0.88 and aspect ratio of 0.74. It is classified as SP according to USCS. Minimum and maximum densities of the aragonite sand were 1.55 g/cm³ and 1.73 g/cm³, respectively.

- **Crushed coral** is a calcareous granular material consisting mainly of calcium carbonate or calcite, with marine origin. The material was chosen due to its angularity which resembles fused quartz, and susceptibility to crushing, as the particles have significantly lower strength compared to silica based natural sand. Typical particle shapes, shown in Fig 2, reveal that the material is somewhat less angular than fused quartz. The tested material has a d_{50} of 0.36, with an average sphericity of 0.84 and aspect ratio of 0.74. The crushed coral used in this study is classified as poorly-graded sand (SP) according to USCS, and has a specific gravity of 2.66. Minimum and maximum densities of the crushed coral were 1.44 g/cm³ and 1.73 g/cm³, respectively.

Samples were prepared in loose and dense packings by means of pluviation (raining). Consistent packing can be produced throughout the target by allowing granular media to be pluviated from a controlled height, to pass through a set of sieves, and to sediment in the container, according to guidelines suggested by Raad and Tumay (1987). Reducing the pluviation rate results in increase in density of the target media, whereas loose packing can be achieved by increasing the pluviation rate. In this paper, loose and dense packings were achieved by pluviating at a rate of 130 and 6 g/cm²/min. Consistency of the resulting packing density was verified in trial tests by placing metal cans within the target during pluviation and calculating density of retrieved samples. Samples with a coefficient of variation of approximately 2–5% of the calculated densities, were achieved.

Wet samples were prepared by first preparing dry targets following the pluviation method described above, then adding water from the bottom of the samples, at a rate of 30 cc/min, in order to ensure that pores were replaced by water to the extent possible. Visual inspection of the targets revealed that the samples were nearly saturated, although few pores remained unsaturated. Typical degrees of saturation exceeded 0.96 for the tested materials; samples were assumed to be in a ‘wet’ state.

2.2. Penetrators

The penetrator designs are shown in Fig 3. Rods with three different nose shapes were considered: hemispherical, conical, and blunt, along with a sphere. The rods were precision machined from 6061 aluminum, to a diameter of 10 mm, and a length-to-diameter ratio of 10. The blunt, hemispherical, and conical rods weighed 16.64, 16.03, and 16.10 g, respectively. In order to increase trajectory stability and decrease mass, a counter bore was drilled in the backs, to a depth of 65 mm. The low impact velocities employed for rods did not result in high enough stresses to cause wear and deformation of the rods, which was confirmed by post mortem visual inspection; thus the same projectiles were used throughout the program. Motion of the projectiles following full embedment was tracked by fitting a slender rod to the end of the projectile, which extended to a length of 100 mm beyond it. In order to provide reference points for tracking movement, the projectile and the extending follower were marked with stripes along their length using nonmetallic black spray paint.

High velocity tests were conducted with spheres made of stainless steel, having a diameter of 14 mm and a mass of 11.17 g. In the high velocity tests, wear was evident in post mortem inspection, and a new sphere was used in each test.

2.3. Projectile launchers

Two different electro-pneumatic accelerators were employed, one for the lower velocity tests, and the other for higher velocities. The lower velocity tests in the range of 50-100 m/s were performed using a single stage electro-pneumatic accelerator, described by Cave et al. (2014). The higher velocity tests (300 m/s) were performed using a single stage double burst diaphragm electro-pneumatic accelerator. This launcher consisted of two chambers, separated by a burst diaphragm designed to burst at a prescribed pressure. The setup therefore allowed achieving desired pressures in shorter times, allowing for accelerating the projectiles to higher impact velocities. Further details of the second launcher are available in Peden et al. (2014).

2.4. Measurements

Two optical diagnostic techniques were employed to measure time history of projectile penetration into dry and wet granular media. The first method was Photonic Doppler Velocimetry (PDV), which was used for spherical projectiles impacting at 300 m/s. The second method was high-speed imaging (HSI) which was used for rod projectiles.

PDV is a heterodyne technique, in which Doppler shifted coherent light reflected from a moving object is collected and used to measure velocity of the moving object with high temporal resolution. In penetration tests, coherent laser light was projected onto the rear of the projectile, and the reflected light was continuously collected. The interference of the two signals produced an interference pattern resulting in signals with the sum and difference of the emitted and reflected light frequencies. The signal produced at the difference, known as the beat signal, was used to obtain the velocity of the moving projectile. First, the beat frequency, f_b , is measured in the tests, and is related to the Doppler shifted frequency, f_D , and the original laser source wave frequency, f_0 , as follows

$$f_b = |f_D - f_0| \quad (1)$$

The time varying beat frequency can be related to the instantaneous velocity of the projectile, $v(t)$, using the following relationship:

$$f_b(t) = 2 \left[\frac{v(t)}{c} \right] f_0 = 2 \left[\frac{v(t)}{\lambda_0} \right]$$

$$v(t) = \frac{1}{2} \lambda_0 f_b(t) \quad (2)$$

where c is the light wave speed, and λ_0 is the wavelength of the laser light source. Eq. (2) can therefore be used to calculate the velocity of the projectile during penetration. Details of the PDV setup used to obtain velocity time history of penetration into granular media reported in this study can be found in Peden et al. (2014).

The PDV setup described above was employed in measurement of tests performed at 300 m/s. The reflected laser light was tracked during penetration, from impact down to approximately 25-50 m/s, after which the signal was usually lost due to the collapse of the cavity in the sand behind the projectile. In some tests, signals were lost, and retrieved at later stages of penetration. The method became increasingly difficult as the penetrator approached terminal penetration depth.

The HSI technique was employed in order to obtain better data at relatively greater depths and lower velocities. In these tests rod penetrators were used. Tracking the marks on the penetrator and its follower provided the penetration time history. A single differentiation then yielded the velocity time history.

The high-speed camera used to acquire images of the penetrator was an NAC HX5 unit. A Nikkor 55 mm, 1.2 f/stop (aperture) lens was mounted on the camera, along with a close up lens with a magnification of +2, to increase the spatial resolution of the images in physical units. The lens was adjusted to an aperture opening corresponding to f/5.6. The camera was operated at an imaging frequency of 50 kHz in the tests reported herein, which resulted in a field of view of 384 by 304 pixels (vertical x horizontal). An exposure time of 5 μ s was used to prevent image blurring. The setup is shown in Fig 4. In order to deliver adequate lighting to the penetrator for high-speed imaging, a set of two 500W and one 250W tungsten halogen lights were used to front light the projectile. The setup described above resulted in a spatial resolution of 54 μ m and a temporal resolution of 20 μ s. Snapshots of the penetrator traversing through the field of view of the high-speed camera are shown in Fig 5 for penetration of blunt nose shaped rod into dry fused quartz.

In the HSI tests the field of view of the camera was constricted to a region extending approximately 20.8mm by 16.4mm above the surface of the target container. The ratio was selected so as to ensure that a marker would always be visible along the field of view and that the projectile trajectory would be captured, even if there was a deviation of the projectile flight from the vertical trajectory following impact. Following each test, the acquired images were analyzed by counting the inter frame integer pixel displacement of the projectile, over the entire penetration event, or for the duration for which the projectile and the attached rod were visible in the field of view. The recorded integer displacements were then assembled into penetration time histories, by means of conversion of displacements from image coordinates to physical coordinates.

The visual record of each penetration event was analyzed by three independent observers to obtain three displacement time histories. Tests were accepted when two of the displacement time histories matched one another (± 2 mm terminal penetration depth), otherwise the test was repeated. Tests were not averaged in order to maintain the high frequency component of the record, thus one of the matching tests was selected for further analysis.

A feature of the above procedure was that the position of the penetrator with respect to the surface of the target was not precisely known. Penetration time histories therefore needed to be analyzed to ensure the point of impact was correctly identified for plotting and comparison purposes. Velocity time histories included pre-impact and post impact data. Since velocity decay following impact resulted in a change in the slope of the velocity time history, the two distinct

sections of the velocity time history were examined to determine the point of impact. First, a straight line was fit to approximately 0.2-0.3 ms of the region that constituted the post impact portions of smoothed velocity time histories. Second, the pre impact straight line was then extended so that the two lines intersected. Next, a line was constructed to bisect the angle formed by the two lines. The intersection of this line and the velocity time history was determined as the point of impact, as shown in Fig 6a for several representative tests. The error in the impact time determined through the above-described procedure was less than 0.04 ms.

Acceleration time histories were calculated by means of single differentiation of the velocity time histories. Since integer pixel locations were recorded in the penetration time histories, a step-type artifact occurs in the position-time data, which causes high frequency noise in the velocity-time data. The velocity time histories were therefore smoothed using a sliding local polynomial fit. This was performed by means of employing a Savitsky-Golay smoothing algorithm. A fourth order polynomial was fitted to 10-20 points on both ends of each point of the velocity time histories, that is, a 21-point or a 41-point sliding polynomial fit was obtained, depending on the level of noise observed in the time histories. Following the above procedure, smoothed velocity time histories were obtained, from which single differentiation yielded acceleration time histories. A similar smoothing algorithm was then applied to the acceleration time histories, to reduce the noise produced by differentiation.

The procedure was highly repeatable; for example repeatability and scatter of velocity-time record is investigated in Fig 6b for 3 representative tests, with standard error shown at ten point intervals. The figure shows that the tests had a maximum standard error of 0.015. Therefore, only representative test results are shown for each test in subsequent figures, to preserve clarity of presentation.

3. Test results

3.1. Results for dry materials

A total of 80 tests were performed as part of this study. The variables probed, along with the range of each variable as well as details of the test matrix are shown in Table 1. Penetration-time histories and corresponding velocity-time histories for penetration into various loose and densely packed dry granular media using hemispherical-nose rods are shown in Fig 7. Penetration-time histories are shown from the time of impact to that of the terminal penetration depth in the case of

densely packed media. For loosely packed targets the follower was not visible in the field of view once the penetrator had slowed to velocities of 15-35 m/s, depending on the target material.

It can be seen from the penetration vs time plots that results are somewhat material dependent. For loose packing, the two silica-based materials are very similar, as are the two carbonaceous materials. The calcareous material is more difficult to penetrate. For densely packed materials, each target medium has a distinct penetration curve. Silica sand is still the easiest material to penetrate. Calcareous aragonite sand is the most difficult. The fused quartz and crushed coral are intermediate. Considering the different range of specific gravities of the various materials tested, these penetration curves suggest that density is not the sole factor determining penetration dynamics, and particle shape and strength characteristics may also play a role.

There are consistent trends in the velocity-time data. There appears to be two penetration regimes in the loosely packed media and three in the case of densely packed media. For both packing conditions, there is an initial steep drop in velocity associated with impact and embedment of the projectile, followed by a second stage in which the deceleration (dv/dt) is nearly constant, and, in the dense material, a final regime towards the terminal penetration depth, where the deceleration increases just before the projectile comes to rest. Evidence of the existence of different penetration regimes can be seen from tests reported by Thompson (1975) as well. Existence of these contrasting regimes may point to underlying change in penetration resistance mechanisms.

Velocity data was differentiated to obtain the acceleration-time histories (Fig. 7). Several features can be identified: (1) there is an initial peak deceleration in all traces, associated with impact and embedment. Embedment time for all traces are similar, with a duration on the order of D/v_0 , the time required for the first diameter penetration, where D is the projectile diameter and v_0 is the impact velocity. The peak deceleration in this transient depends strongly on the material. Its value is proportional to density. (2) Following embedment deceleration drops sharply, and depends only slightly on material. It is greatest for the densely packed materials. (3) After about 1–1.5 ms, the deceleration reaches a plateau at a relatively low value where it is nearly constant. (4) There are quasi-periodic oscillations in the acceleration records. Amplitudes of these oscillations are higher in loose materials. These oscillations are not artifacts of the pixilation of displacement, and there is evidence in the literature that they correspond to formation and buckling of force chains during penetration (*e.g.*, Clark et al. 2014).

3.2. Role of nose shape

The effect of nose shape was investigated by penetrating dense Ottawa sand and Aragonite targets using long rods with three different nose shapes, namely, hemispherical, conical and blunt. The comparisons are shown in Figs 8 and 9. It can be seen that the nose shape has a much more significant effect in Ottawa sand compared to aragonite sand. Nose shape has a noticeable effect in the case of dense Ottawa sand, where the conical nose has the largest penetration depth, followed by hemispherical and blunt nose shapes.

Contrary to Ottawa sand, nose shape has a less significant effect in the case of penetration into dense aragonite sand. It can be seen in Fig 9 that blunt and hemispherical nose shapes produce nearly identical penetration depths, while the conical nose shaped projectile only marginally increases penetration depth. Since both materials have rounded particles, and the main difference between the two is the higher susceptibility of aragonite sand to crushing compared to Ottawa sand, it can be postulated that the difference in the nose shape dependence of the two materials may be related to particle crushing during penetration.

There is evidence in the literature that when projectiles penetrate granular media at high enough penetration velocities to induce particle damage, crushed particles form a dense region immediately ahead of the projectile (*e.g.*, Tanaka 2011; Kotov et al. 2013). This region of compacted crushed materials forms a conical shape, and may behave as a ‘false cone’, effectively reshaping the projectile nose during penetration. Post-mortem inspection of the penetration front in aragonite samples confirmed the formation of a false cone in high velocity sphere penetration tests, as evidenced in Fig 10. Preliminary analysis of the cone revealed that it exhibited a bimodal size distribution, with materials either relatively intact, or comminuted to micron-size particles. It is hypothesized that the formation of a false cone reduces the nose shape effect. In the case of Ottawa sand, nose shape affects momentum transfer, as particles have to gain momentum to move out of the trajectory of the penetrator. However, in the case of aragonite sand, the false nose formed by the crushed particles gives penetrators with different nose shapes an effectively conical nose shape during penetration.

3.3. Effect of water saturation

The effect of saturation was studied by performing penetration tests using the hemispherical projectile in wet samples. At the strain rates experienced during penetration all the target materials are expected to behave undrained in the vicinity of penetration (Omidvar et al. 2014). Comparison of penetration tests at impact velocities in the range of 80 ± 5 m/s is shown in Fig 11 for loosely packed and densely packed Ottawa sand, aragonite and crushed coral targets. Despite

having similar relative densities (relative to minimum and maximum densities), the material possess different void ratios (defined as the ratio of volume of voids to volume of solids), as listed in Table 1. It can be seen from the results that penetration into dry and wet targets produce contrasting response.

For all loosely packed materials, saturation clearly decreases penetration, although the effect on final penetration depth cannot be quantified because for the dry condition, there was total embedment of the projectile and its follower in the target. For densely packed materials the terminal penetration depth decreased by approximately 67% and 5% for dense Ottawa sand and crushed coral targets, respectively due to saturation. Accordingly, there is little effect in the calcareous sand, while significant reduction in penetration depth can be seen in Ottawa sand. For dense materials, the main effect of saturation was manifested in the second velocity regime, where deceleration is significantly higher in the wet sample. The result is that the velocity-time history and velocity-penetration curves of the loosely-packed wet target resembles that of the dense dry target. In particular, in the case of Ottawa sand, saturation appears to suppress the change in the slope of the velocity-time history, which occurs at approximately $0.55 V_0$ in the dry target (Figure 11). The result is that there is a significant decrease in penetration depth in the wet target, owing almost entirely to the aforementioned contrasting response below approximately $0.55 V_0$. In contrast, saturation has nearly no effect on penetration into dense crushed coral. This may be because penetration into dense Ottawa sand is accommodated by movement of particles away from the trajectory of the projectile, whereas penetration into dense crushed coral is facilitated mainly by crushing of particles. The effect of saturation is therefore to restrict particle motion in dense Ottawa sand, while in crushed coral the pore fluid does not significantly affect particle crushing.

It is noteworthy that while the large difference in penetration depth due to saturation in Ottawa sand and crushed coral are due to intrinsic behavior, the actual values are to a small degree affected by the variation in impact velocity between the tests, which is up to 6 m/s in all tests except in loose aragonite, where the difference is more significant, at approximately 20 m/s.

3.4. Penetration of Spheres at High Velocity

Penetration time histories for spheres at impact velocities in the range of 150-300 m/s were obtained by means of the PDV described in Section 2.4. PDV measurements provide higher temporal resolution for small changes in velocity.

Results of penetration tests into dry Ottawa sand and crushed fused quartz are shown in Fig 12. Several features can be identified from these plots. First, it can be seen that at high velocities, packing density does not significantly affect penetration into fused quartz, whereas it has a marked effect on penetration into Ottawa sand. Moreover, it can be seen that a transition regime exists in the velocity time histories of Ottawa sand at 60-80 m/s penetration velocity. This transition regime is more evident in dense Ottawa sand compared to loose Ottawa sand. However, the transition is not discernable in the case of crushed fused quartz. The difference between the two materials can also be seen in the deceleration plots of Fig 12c, where regions with different slopes can be identified in the Ottawa sand curves, whereas both loose and dense crushed fused quartz show similar, relatively monotonic deceleration throughout the measurement.

The effect of saturation was investigated for Ottawa sand. Penetration and velocity time histories are shown in Fig 13. It can be seen that in the wet tests the PDV signal was lost earlier in the penetration compared to dry tests, as a result of ejecta obscuring the path of the laser light source. Comparison of the dry and wet time histories reveals that contrary to the lower velocity tests, resistance to penetration at high velocities is lower in wet samples compared to dry samples. The effect of saturation is more significant in the case of loose sand. For example, at approximately $0.5V_0$, penetration in loose wet sand is approximately 77% greater in the case of wet sand compared to dry sand, whereas in dense sand the increase in depth due to saturation is approximately 35%.

The data portray the somewhat surprising result that the effect of saturation varies greatly with velocity. For dense sand especially, in the range of 300 to about 60 m/s, saturation decreases penetration resistance, but the opposite is true at lower velocities. It is believed that projectile penetration gives rise to a number of related complex effects including (1) changes of porosity ahead of the projectile (positive or negative depending on initial packing); (2) increase in pore water pressure, may lead to loss of stiffness and strength which can reach a limit state known as liquefaction, depending on porosity and provided the degree of saturation is sufficiently high; and (3) particle fracture or comminution which contribute to changes in porosity and may also affect the rate of dissipation of pore pressures due to plugging of pore space. Although the data is not yet conclusive, one possible explanation of this effect is that dynamic shear dilatancy is resisted by pore water. Since shear strength comes to dominate penetration resistance as velocity falls, a wet material appears to be harder to penetrate than a dry one. There could be at least two meso scale processes involved: the viscosity of the water suppresses relative movement of particles, and pore creation is difficult because water resists dilation. Another hypothesis is that at high

velocity water may suppress stress gradients within grains, thereby reducing fracture, but this effect is less important at low speeds where fracture does not occur even in dry material. Eventually, as penetration becomes quasi static, the influence of pore water likely diminishes.

4. Discussion

4.1. Phenomenology

Phenomenological models provide a framework for approaching penetration mechanics, in which penetration is a function of assumed forces with non-dimensional fitting coefficients. Newton's third law of motion implies that the force exerted by the penetrator on the target equals the force exerted by the target on the penetrator. Force is obtained from Newton's second law of motion. The resistance to penetration offered by the target, F , is expanded into a polynomial with various orders of dependence on velocity. The general form of the equation can be written as:

$$m_p \frac{dv}{dt} = F = m_p g - \alpha v^2 - \beta v - \gamma \quad (3)$$

where m_p is the projectile mass, v is instantaneous projectile velocity at a given time t , g is gravitational acceleration, γ is a parameter (with units of force) dominated by the bearing strength of the soil; β controls the contribution of viscous resistance of soil (with units of mass over time); and α represents the contribution of inertial stresses (with units of mass over length). The terms in Eq 3 were individually investigated for their ability to describe the data. In initial fits it was found that the viscous component had a negligible role in improving fits to the data, and only the inertial and frictional terms were considered in the analyses. First a velocity range was identified that was well fit by the quadratic velocity term. Next, a residual force was calculated as the difference between dynamic resistance and the inertial fit to the entire time history of the data. The residual force was then used to evaluate the role of frictional resistance. In order to do so, quasi static penetration tests were performed, and the residual force was compared to the quasi static penetration resistance.

The inertial component of resistance to penetration can be expressed as a quadratic drag force by employing momentum transfer and Newton's third law. Accordingly, an object having a cross sectional area A advancing at a velocity v in an incompressible fluid over a time period dt , encounters an increment of mass, $dm_g = \rho_g A v dt$. This material is either moved aside or pushed forward, but in either event acquires a velocity proportional to v . Therefore, a momentum proportional to $dp = v dm_g = \rho_g A v^2 dt$ is imparted to the medium. According to Saslow and Lu

(2008), Newton derived this relationship, and noted that the above momentum transfer is doubled by virtue of perfect reflection for a flat projectile. In practice, a new empirical parameter, C_D , referred to as the drag coefficient is introduced to the equation, which is often assumed to be independent of penetration velocity. The common convention in fluid dynamics is to adopt $1/2C_D$ in the momentum equation. The force F in Eq 3 is therefore obtained as $F=-dp/dt$, resulting in the following relationship:

$$F = -\frac{1}{2}C_D\rho_g v^2 A \quad (4)$$

In order to evaluate the efficacy of Eq 4 in describing penetration dynamics, the acceleration time histories shown in Fig 7 were normalized by the dynamic stress by dividing ma/A by $\rho_g v^2$. The resulting non-dimensional term was fitted with a constant, as shown in Fig 14 (dashed lines). Comparing the normalized term and Eq. 4 reveals that the average constant value corresponds to $1/2C_D$. For both loose and dense targets a reasonable fit using a constant drag coefficient can be found for velocities down to approximately 20–30 m/s. The values of the drag coefficient obtained by the above described method are listed in Table 2. Drag coefficients were in the range of 0.92-1.27 for loosely packed media and 1.86-2.26 for densely packed media.

A significant finding was that for a given relative density, the drag coefficient was only moderately influenced by the material (Table 2). However, the drag coefficient does depend upon bulk density. For dense materials, the resistance to penetration apparently involves more than simple momentum transfer to the target grains in the path of the projectile. Comparison of drag coefficients obtained from loose and dense targets is also shown in Table 2. The data from the high impact velocity tests using spheres into Ottawa sand are consistent with the data from hemispherical rod tests at similar penetration velocities. Based on the data from high velocity and low velocity tests, two transition regimes can be identified in the velocity time histories (Fig. 15). The first occurs at penetration velocities in the range of 60-80 m/s, and the second transition occurs at approximately 15-20 m/s. The observation persists for other materials in dense states, where data is available for all HSI measurements but for the high-speed measurements laser reflection from the projectile was interrupted, which prevented capturing data in PDV measurements. These two transition regimes may point to change in the mechanism of resistance to penetration, as discussed in the next sections.

4.2. High velocity transition

It appears that there is a transition regime in penetration resistance, occurring at approximately 60-80 m/s. There is a distinct change in the slope of the velocity time history over these velocities, which could be due to a change in the energy transfer mechanism. The transition zones are more pronounced in the case of densely packed targets, and are more evident in Ottawa sand compared to fused quartz. Previous studies have found that these penetration velocities also correspond to the velocity range above which particle damage is significant, and below which there is a marked reduction in the damage to particles during penetration. For example, Bless et al. (2009) and Cooper and Breau (2010) have reported a trail of comminuted sand along the trajectory of the projectile at velocities greater than those associated with this transition regime, and diminishing trails of comminuted sand at lower velocities (Fig. 16). Preliminary inspection of the crushed material It is believed that the observed transition in penetration resistance is associated with the threshold of significant particle damage.

4.3. Low velocity transition

The sharp increase observed in drag at lower velocities is analogous to behavior of objects moving through fluids. At high Reynolds numbers (high velocities), resistance to penetration in fluids is also proportional to velocity squared, and drag coefficient is constant. At low velocities, viscosity dominates and resistance is proportional to velocity to the first power, and drag coefficient increases rapidly. The transition between the inertial and viscous regimes is determined by the Reynolds number, that is, the ratio of inertial to viscous forces. It was found that resistance passes from a v^2 dependence to one dominated by a v^0 dependence. In these circumstances, the transition between behavior regimes is controlled by the so-called solid Reynolds number (Bernard and Hanagud 1975; Yankelevsky and Gluck 1980), which can be written as follows

$$\text{Re}_s = \frac{\rho v^2}{\tau} \quad (5)$$

where τ is the quasi static resistance to penetration. In order to investigate this transition, quasi static penetration tests were performed into dry targets under dense packing. The tests were performed by penetrating a long rod with a hemispherical nose and a diameter matching that of the projectiles into the targets. A hydraulic actuator was used to penetrate the rod into the targets at a penetration rate of 0.05 mm/s. Penetration resistance was measured by means of a load cell attached to the end of the long rod. Results of the measurements are shown in Fig (17) for penetration into dense aragonite sand. The residual dynamic force calculated as the difference

between the measured dynamic resistance to penetration (calculated from the acceleration record) and the quadratic fit to the same data using Eq. 4. is also shown in the figure.

Inspection of the dynamic and quasi static penetration resistance reveals that (1) during the initial portion of penetration corresponding to impact and partial embedment, dynamic resistance is up to two orders of magnitude larger than quasi static penetration resistance. (2) As dynamic penetration depth increases, dynamic resistance reduces to values of the same order as that of the quasi static resistance. At approximately 5D penetration, the two values match. (3) Beyond this penetration depth, the residual dynamic resistance matches to a reasonable extent to the quasi static resistance, as shown in the inset. The match between the two values can be further improved by shifting the quasi static penetration resistance to the left of the penetration axis by approximately 3D, as shown in Fig 17, and the inset of the figure. This shift is simply a convenient means to account for the greater volume of soil involved in frictional resistance to dynamic penetration relative to static penetration as suggested by Collins et al. (2011). This match suggests that the term γ in Eq (3) can be derived from quasi static penetration resistance. Moreover, the depth dependent, nonlinear increase in the quasi static penetration resistance suggests that γ is depth dependent, and the $\gamma(z)$ can be reasonably derived from standard penetrometer tests in soils.

The term γ in Eq. 3 is material dependent, as shown in Fig 18, and there is a significant difference between the different materials. For example, at 10D penetration depth, quasi static resistance of fused quartz and aragonite differ by as much as 500%. For the materials tested, Aragonite has the largest quasi static resistance, followed by crushed coral, Ottawa sand, and fused quartz. This strong material dependence points to the importance of frictional response in the penetration dynamics, particularly at lower velocities.

5. Conclusions

Data for penetration resistance of four different granular media including Ottawa sand, crushed coral, aragonite, and crushed fused quartz has been assembled. The parameter space included two packing densities of loosely packed and densely packed targets, four different projectiles including spheres and long rods with conical, hemispherical and blunt nose shapes, and two pore fluid conditions of dry and wet media. Impact velocities in the range of 65 to 300 m/s were considered. Several observations were reported from the penetration tests, as follows:

Effect of packing density: Increasing the packing density generally reduced penetration depth, and increased deceleration of the projectile. Moreover, there appeared to be two transition regimes in penetration into densely packed targets, where a discernable change in deceleration could be identified in dense soils.

Effect of saturation: The role of saturation was found to be material dependent. In silica sand, saturation generally reduced penetration depth in tests performed at impact velocities below 100 m/s. The reduction was greater in densely packed targets compared to loosely packed targets. In calcareous targets, saturation reduced penetration depth in loose targets but had negligible effect on penetration into dense targets.

Effect of projectile nose shape: The role of nose shape was also material dependent. In Ottawa sand, conical projectiles penetrated deeper than hemispherical nose rods, followed by blunt nose projectiles. However, in calcareous sand, the penetrator nose shape had negligible effect on penetration. The role of particle crushing and the apparent ‘false cone’ was suggested as the justification for this contrasting behavior in silica and calcareous sands.

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Table 1: Matrix of penetration tests performed

Soil Type	Relative Packing	Projectile	Saturation	Test ID	Impact Velocity (m/s)	Target Dry Density (g/cm ³)	Void Ratio*	# of repeats
Ottawa Sand	Loosely Packed	Sphere	Dry	OS-LP-S-D	297.4	1.587	0.67	1
			Wet	OS-LP-S-W	303.0	1.600	0.66	1
		Hemispherical Rod	Dry	OS-LP-HR-D	83.9	1.670	0.59	4
			Wet	OS-LP-HR-W	77.3	1.660	0.60	4
		Blunt Rod	Dry	OS-LP-BR-D	74.1	1.640	0.62	2
		Conical Rod	Dry	OS-LP-CR-D	76.8	1.680	0.58	1
	Densely Packed	Sphere	Dry	OS-DP-S-D	299.2	1.817	0.46	1
			Wet	OS-DP-S-W	302.5	1.819	0.46	3
		Hemispherical Rod	Dry	OS-DP-HR-D	79.7	1.830	0.45	4
			Wet	OS-DP-HR-W	73.9	1.830	0.45	1
		Conical Rod	Dry	OS-DP-CR-D	72.8	1.832	0.45	1
		Blunt Rod	Dry	OS-DP-BR-D	78.3	1.820	0.46	1
Fused Quartz	Loosely Packed	Sphere	Dry	FQ-LP-S-D	303.0	1.187	1.14	2
		Hemispherical Rod	Dry	FQ-LP-HR-D	83.7	1.030		3
	Densely Packed	Sphere	Dry	FQ-DP-S-D	302.5	1.298	0.69	1
		Hemispherical Rod	Dry	FQ-DP-HR-D	71.2	1.170	0.88	4
Crushed Coral	Loosely Packed	Hemispherical Rod	Dry	CC-LP-HR-D	77.1	1.460	0.82	1
			Wet	CC-LP-HR-W	70.1	1.460	0.82	1
	Densely Packed	Rod	Dry	CC-DP-HR-D	76.2	1.640	0.62	3
			Wet	CC-DP-HR-W	80.1	1.630	0.63	1
Aragonite	Loosely Packed	Hemispherical Rod	Dry	CA-LP-HR-D	82.0	1.600	0.71	1
			Wet	CA-LP-HR-W	60.2	1.580	0.73	2
		Blunt Rod	Dry	CA-LP-BR-D	67.0	1.610	0.70	1
		Conical Rod	Dry	CA-LP-CR-D	72.9	1.580	0.73	1
	Densely Packed	Hemispherical Rod	Dry	CA-DP-HR-D	76.2	1.850	0.48	3
			Wet	CA-DP-HR-W	79.3	1.730	0.58	1
		Blunt Rod	Dry	CA-DP-BR-D	78.0	1.810	0.51	1
		Conical Rod	Dry	CA-DP-CR-D	77.2	1.790	0.53	2

* ratio of volume of voids to volume of solids

OS: Ottawa sand, FQ: fused quartz, CA: calcareous aragonite, CC: crushed coral, LP: loosely packed, DP: densely packed, HR: hemispherical rod, CR: conical rod, BR: blunt rod, S: spherical projectile, D: dry, W: wet

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Table 2: Drag coefficients calculated from low and high velocity tests

Target Parameters	Impact Velocity (m/s)	Packing Density (g/cm ³)	Void Ratio	Drag Coefficient from partial fit (1/2 C _D)
OS-LP-HR-D	83.9	1.670	0.59	0.92
OS-DP-HR-D	79.7	1.830	0.45	1.64
AR-LP-HR-D	82.0	1.600	0.71	1.21
AR-DP-HR-D	76.2	1.850	0.48	2.26
CC-LP-HR-D	77.1	1.460	0.82	1.27
CC-DP-HR-D	76.2	1.640	0.62	2.12
FQ-LP-HR-D	83.7	1.030	1.14	1.22
FQ-DP-HR-D	71.2	1.170	0.88	2.89

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Graphical abstract

Figure 1

Fig 1: Particle size distribution of four granular media tested

Figure 2

Fig 2: (a) Density distribution of particle sphericity, and (b) Typical two dimensional projections of particle shapes for granular media tested

Figure 3

Fig 3: Projectiles used in the penetration tests

Figure 4

Fig 4: High speed imaging setup used to track projectiles during penetration into granular media

Figure 5

Fig 5: Snapshots of penetrator passing through the field of view of the camera during HSI of a penetration event $t = -0.8, 0, 0.4, 3.2$, and 4.0 ms, respectively ($t = 0$ ms is point of impact)

Figure 6

Fig 6: (a) Procedure used to identify point of impact, and (b) Repeatability of tests performed, shown for penetration into loose and dense dry Ottawa sand using hemispherical rods

Figure 7

Fig 11: Penetration, velocity, and acceleration time histories for penetration of hemispherical nose rods into loosely packed (a, c, e) and densely packed (b, d, e) granular media (dashed lines represent raw data)

Figure 8

Fig 8: Effect of nose shape on penetration into dense Ottawa sand: (a) Penetration time history, (b) velocity vs. penetration, and (c) velocity vs. time (dashed lines represent raw data).

Figure 9

Fig 9: Effect of nose shape on penetration into dense Aragonite sand: (a) Penetration time history, (b) velocity vs. penetration, and (c) velocity vs. time (dashed lines represent raw data).

Figure 10

Fig 10: Image of false cone formed ahead of the projectile: (a) Spherical projectile and retrieved cone, (b) scanning electron microscope image of the false cone

Fig 11: Effect of saturation on normalized velocity-time and penetration-time history of penetration into (a, b) loose and dense Ottawa sand, (c, d) loose and dense aragonite sand, and (e, f) loose and dense crushed coral

Figure 12

Fig 12: Penetration and velocity-time histories for high-speed tests; data collected using PDV (dashed lines represent raw data).

Figure 13

Fig 13: Comparison of wet and dry penetration time histories at high velocities; (a) Penetration time histories, (b) Velocity time histories, and (c) Velocity vs. penetration

Figure 14

Fig 14: Time history of projected frontal nose pressure (ma/A) normalized by dynamic pressure (ρv^2) for penetration of hemispherical nose rods into targets (a) Ottawa sand, (b) Aragonite sand, (c) Crushed coral, (d) Fused quartz. Normalization is by dynamic pressure, ρv^2 . Dashed lines are constant fits to the data, representing the drag coefficient.

Figure 15

Fig. 15 Schematic representation of velocity regimes during projectile penetration in granular media.

Figure 16

Fig 16: Trail of comminuted sand along the penetration trajectory from high velocity penetration tests (after Omidvar et al. 2014)

Figure 17

Fig 17: Comparison of dynamic and quasi static penetration resistance for dense aragonite sand (dashed line shows quasi static penetration resistance shifted on the horizontal axis by 3D)

Figure 18

Fig 18: Quasi static penetration resistance of the four materials tested (penetration rate: 0.05mm/s)