On the Feasibility of Crack Propagation Tracking and Full Field Strain Imaging via a Strain Compatibility Functional and the Direct Strain Imaging Method

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Abstract

The simultaneous tracking of the trajectory of a moving crack along with the corresponding full field strain fields associated with this motion due to quasi-static or dynamic impact loading, is a problem of high interest from both the perspectives of the engineering practice and engineering science alike. However, full field optical methodologies haven't yet been able to address the issues arising in experiments with moving boundaries such as dynamically evolving cracks. Such a problem requires solving simultaneously the problem of identifying the dynamically evolving geometry of the cracks as well as the problem of introducing these cracks into the full-field representation of a relevant full field method. In the present work we are introducing a methodology that can solve this problem. Specifically, we describe an optical method that can be used to detect dynamically evolving discontinuities on

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the surface of a body even if they are invisible to the eye, by using a strain compatibility functional that makes no continuity assumptions about the underlying medium. The relevant data from this process are then introduced in a full field representation that leads to an acceptable estimation of the discontinuous strain field. The proposed method is based on the recently introduced Direct Strain Imaging method that can be used to visualize and quantify the full fields of the strain tensor components. Synthetic numerical experiments were performed to generate synthetic crack propagation data along with the associated strain fields in order to assess the feasibility of the proposed method. The approach employed for this purpose was that of applying the extended finite element method to solve the problem of a propagating crack in an elastic medium with inclusions and holes under quasi-static tension. Finally, we present the results of detecting the synthetically produced crack trajectory as well as the relevant strain fields.

Keywords:

Full Field measurement, Strain, Direct Strain Imaging, Crack Detection, Propagation, Mesh Free

1. Introduction

Being able to track the trajectory of one or more dynamically propagating cracks in a dynamically loaded body is of great interest both from a structural health monitoring perspective but also from a perspective of engineering mechanics mainly because of its relevance to the validation of crack initiation, propagation and branching models [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In a historical setting, crack propagation can be monitored by using a number of techniques. Here we present some examples along with the associated major disadvantages.

One example is the utilization of crack detection gages but it requires simple geometries and is applicable only when the shape of trajectory of the crack is known a priori so that the crack will brake the electric conduction of the wires associated with each gage.

Another example is the method of dynamic Photoelasticity [19, 20] that requires the existence of a transparent birefringent material or an adhesive layer on top of non-birefringent material of interest and the assumption that the adhesive photoelastic layer is perfectly bonded with the substrate material of interest as the dynamic crack propagates.

The method of caustics [21, 22, 19] is another approach that tracks the approximate position of the crack tip because of the light caustic formed from the lights reflected (or diffracted) when a light beam impinges on the deformed surface of the specimen in the neighborhood of the crack tip. In addition, to the crack tip location this method can also be used to extract the dynamic stress intensity factors and therefore reconstruct the local form of the stress and strain fields, but it requires reflective and diffracting deformable media and therefore it is not general enough.

Another method that can be used is that of visual monitoring by image capturing. This method can be used on most materials and is based on the fact that cracks may be visible (through their contrast with the material surface color) and hence tracked as they propagate. Although this may be the case for many experiments, there is no guarantee that visual aspects of the

crack reflect the actual discontinuity state of the body. The same issues arises when similar assumptions are made in automated digital imaging systems [23, 24, 25].

Finally, simple visual inspection of captured images or digital imaging methods [23, 24, 25, 26] can also be used to identify the approximate location of the crack tip, but no other field quantity, while at the same time this method requires simple geometries and an a priori knowledge of where the crack trajectory. Certain automated digital imaging methods based on photogrammetry and image processing [26] but their applicability is constrained by the need of a priori knowledge of the material failure behavior.

The advent of full field optical methods has motivated recently some investigators to attempt crack trajectory identification and a limited amount of full field approximations. For example the Digital Image Correlation (DIC) methods has been used for simultaneous approximate crack trajectory detection and approximate full displacement field evaluation [27, 28]. The difficulties of the moving boundaries generated by the dynamically propagating crack(s) do not allow the evaluation of the strain fields throughout the domain of the specimen.

An important limitation of all of the optical methods described above, is that they are mostly applicable and suitable for planar specimens that can be described as rectangular disk geometries. However, the evolution of multiaxial testing capabilities such as those developed at the Naval Research Laboratory (NRL) by our group, generate out of plane deformations. Specifically, in contrast to the conventional uniaxial loading machines, a system that is capable of applying six degrees of freedom kinematic specification at the

Figure 1: The NRL 66.3 is a six degree-of-freedom robotic testing system capable of multi-axial testing.

movable edge of any specimen is the so-called NRL66.3 hexapod robotic loader [29] as shown in Fig. 1. This system has the ability to apply three translations and three rotations of the movable grip relative to a global frame of reference in the center of the specimen [30, 31, 32, 33]. Two of the translations and one of the rotations generate in-plane displacement and strain states, but the remaining two rotations and one translation generate out of plane displacement and strain states. Therefore, when a crack trajectory manifests under multi-axial conditions, the method used to achieve its tracking and identification must be able to operate for such out-of plane conditions.

In order to overcome as many as possible of the issues and limitations arising from all of the above mentioned methods, we present here a new approach based on the Direct Strain Imaging (DSI) method [34] that can be used to simultaneously detect medium discontinuities when the medium has a distribution of strain states and provides the full field distributions of the strain tensor components during a dynamically propagating crack experiment. The method is based on the mesh-less derivation of the strain tensor field that makes no compatibility assumptions (implicit or explicit) and hence is valid to the extent of the representation adequacy of the shape functions used, for any body regardless of its continuity state. The DSI method is different from other full-field strain measurement methods developed by various investigators [35, 36, 37, 38], and methods developed by our group [39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49], in that it does not require the knowledge of the emplacement vector components fields to construct the strain fields via numerical differentiation, but instead it directly approximates the local strain tensor hence avoiding the errors emanating by the differentiation of an already noisy signal (that of the displacements fields).

The paper begins with an introduction of the full field DSI method and continues with the algebraic considerations that are needed in order to formulate the strain compatibility functional that is based on strain measurements. The deviation of the evaluation of the strain compatibility functional from zero implies violation of compatibility and therefore existence of discontinuity. The paper continues with a section describing crack detection based on quantitative observations that show the departure of appropriate probabilistic metrics from the case of a body without discontinuities. A framework for

establishing the certainty of detection is presented and exercised next. Due to the lack of actual experimental data under the conditions of DSI, the proposed method is applied on synthetic data of a curved trajectory of a propagating crack that has been generated also synthetically using the extended finite element (XFEM) method to solve the elastic problem of a propagating crack in a rectangular medium containing holes and inclusions under a tensile load. After the discussion of the performance of the method, the paper ends with concluding remarks and future plans.

2. The DSI Method

The DSI method is a combination of experimental measurement technique and an analytical tensor approximation technique that can be used to evaluate the strain tensor component field distributions on the surface of a deforming structure. The experimental part consists of a setup involving one or more cameras that observe a properly marked specimen that is subjected to some strain generating loading. The analytical part consists of the calculation of the strain tensor field components through a mesh-less method that is based on the approximation of pairs of directional values of the strain tensor over a not necessarily regular grid. A brief description of the method follows in the next subsections, while a more detailed one can be found in [34]

2.1. Outline of the DSI Method

The typical experimental procedure for measuring the full field of deformation quantities according to DSI can be outlined as follows:

1. A specimen is marked with an appropriate visible pattern that consists

of a distribution (random or regular) of dots distinguishable from the background.

2. If the experiment is to take into consideration out of plane motion, two or more cameras are used so that the deformation is stereoscopically reconstructed. The projective characteristics of the cameras are identified through an appropriate calibration procedure [50, 42].
3. The specimen is placed in the mechanical testing machine and one image per camera is captured in the undeformed configuration prior to the initiation of the loading sequence.
4. While the experiment is taking place, successive images of the deforming specimen are captured.
5. The images are processed and for each frame, the coordinates of appropriate points (nodes) are calculated. These nodes may be for example geometric centroids of dots (for the case of grid methods), appropriate boundaries of geometric entities, correlated sub regions, etc.
6. A set of nodes in the vicinity of each point $\mathbf{w} = \{x_w, y_w\}^T$ in the full field representation are considered and their in-between distances l_{ij} and l'_{ij} are calculated in the undeformed (Fig. 2a) and deformed (Fig. 2b) configurations respectively.
7. For each of these node pairs the engineering strain is calculated by $e_{ij} = (l'_{ij} - l_{ij}) / l_{ij}, i = 1 \dots n - 1, j = 1 \dots n, i < j$.
8. The strain tensor components at the point $\mathbf{w}$ are calculated using the DSI approximation scheme as described in the section that follows.

The digital imaging setup introduces noise by various sources is the most dominant source of error and it plagues all full field measurement methods.

(a) Undeformed

(b) Deformed

Figure 2: Deformation configurations and distances of selected node pairs.

Because of the very good noise mitigating attributes of the DSI method [34], the degrading effects of this noise is greatly reduced in comparison to other methods, even when compared to MRG [39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49].

2.2. Direct Strain Approximation Analysis

The engineering strain between pairs of adjacent centroids of dots (nodes) along the directions defined by them, is the digitally determined data input for the DSI analysis. A domain Ω in the undeformed configuration is shown in Fig. 2a, populated with a number of such nodes. The same domain in the deformed configuration is shown in 2b. For a set of n nodes in the vicinity of

an interest point described by a position vector $\mathbf{w} = \{x_w, y_w\}^T$, it is possible to form a number of engineering strain quantities $e_{ij} = (l'_{ij} - l_{ij}) / l_{ij}$, $i = 1 \dots n - 1, j = 2 \dots n, i > j$ [51] in terms of the distances l_{ij}, l'_{ij} in the initial and deformed configurations respectively. Each of these quantities can be considered as the mean of the strain component in the direction of the line connecting the two nodes over the respective line segment. Given these engineering strains and the coordinates of the nodes, DSI can be used to calculate the strain tensor at any point $\mathbf{w}$ in the domain. It is important to note that similarly with traditional meshless methods [52, 53, 54] the vicinity of a point of interest $\mathbf{w} = \{x_w, y_w\}^T$ is bound by an appropriate Domain Of Support (DOS), and in this work we will consider the DOS to be circular.

A detailed description of the solution to this problem can be found in [34]. In the following paragraphs, only the essential formulas for the DSI evaluation will be presented.

According to [34], the DSI strains are calculated by,

$$\varepsilon_{kl}^h(\mathbf{w}) = \Phi_{kl}(\mathbf{w}) \mathbf{e}_{\mathbf{w}} \text{ for } k, l \in \{x, y\}, \quad (1)$$

where,

$$\mathbf{e}_{\mathbf{w}} = \{e_{21}, e_{31}, e_{32}, e_{41}, e_{42}, \dots, e_{ij}, \dots, e_{n(n-1)}\}^T, \quad (2)$$

is a vector that collects all the measured engineering strains in the vicinity of $\mathbf{w}$ for all pairs of points i, j with position vectors $\mathbf{x}_i = \{x_i, y_i\}^T$ and $\mathbf{x}_j = \{x_j, y_j\}^T$. Additionally,

$$\Phi_{\alpha}(\mathbf{w}) = \mathbf{p}^T(\mathbf{w}) \mathbf{M}_{\alpha}(\mathbf{w}), \quad (3)$$

are shape functions of the strain components for the point $\mathbf{w}$ for $\alpha = xx, yy, xy$. $\mathbf{p}(\mathbf{x})$ is a vector of basis functions that can be chosen to consist of m monomials

of the lowest orders to ensure minimum completeness. A polynomial basis of order $M < m$ has the general form,

$$\mathbf{p}(\mathbf{x}) = \mathbf{p}(x, y) = \{1, x, y, xy, x^2, y^2, \dots, x^M, y^M\}^T. \quad (4)$$

Such a polynomial basis can be constructed by concatenation of the complete order terms using the Pascal triangle of monomials [52, 54]. The terms $\mathbf{M}_\alpha$ in Eq. (3) are identified by a proper partitioning of the matrix,

$$\mathbf{M}(\mathbf{w}) = \begin{bmatrix} \mathbf{M}_{xx}(\mathbf{w}) \\ \mathbf{M}_{yy}(\mathbf{w}) \\ \mathbf{M}_{xy}(\mathbf{w}) \end{bmatrix}. \quad (5)$$

that can be calculated by solving,

$$\mathbf{Q}(\mathbf{w}) \mathbf{M}(\mathbf{w}) = \mathbf{B}(\mathbf{w}) \quad (6)$$

$\mathbf{B}$ is a matrix that has the form,

$$\left. \begin{aligned} \mathbf{B}(\mathbf{w}) &= [\mathbf{b}_{21}, \mathbf{b}_{31}, \mathbf{b}_{32}, \mathbf{b}_{41}, \mathbf{b}_{42}, \dots, \mathbf{b}_{n(n-1)}] \\ \mathbf{b}_{ij}(\mathbf{w}) &= W\left(\mathbf{w} - \frac{\mathbf{x}_i + \mathbf{x}_j}{2}\right) \mathbf{q}(\mathbf{x}_i, \mathbf{x}_j) \end{aligned} \right\}, \quad (7)$$

and $\mathbf{Q}(\mathbf{w})$ is defined by,

$$\mathbf{Q}(\mathbf{w}) = \sum_{i=1, j=2, i < j}^{n-1, n} W\left(\mathbf{w} - \frac{\mathbf{x}_i + \mathbf{x}_j}{2}\right) \mathbf{q}(\mathbf{x}_i, \mathbf{x}_j) \mathbf{q}^T(\mathbf{x}_i, \mathbf{x}_j), \quad (8)$$

where $W(\mathbf{w} - (\mathbf{x}_i + \mathbf{x}_j)/2) = W_{ij} \geq 0$ is a weight function that decreases with distance as introduced by [52, 54]. A common weight function is the cubic spline weight function given by,

$$W(\mathbf{d}) \equiv W(\hat{d}) = \begin{cases} \frac{2}{3} - 4\hat{d}^2 + 4\hat{d}^3 & , \hat{d} \leq \frac{1}{2} \\ \frac{4}{3} - 4\hat{d} + 4\hat{d}^2 - \frac{4}{3}\hat{d}^3 & , \frac{1}{2} < \hat{d} \leq 1 \\ 0 & , \hat{d} > 1 \end{cases} \quad (9)$$

Figure 3: Coordinate definitions for a pair of points A, B in the vicinity of $\mathbf{w}$ relative to the local coordinate system at point A.

with $\hat{d} = \frac{|\mathbf{d}|}{d_w}$, the normalized distance, and d_w the smoothing length that is usually equal to the extend of the domain of support.

The terms $\mathbf{q}(\mathbf{x}_i, \mathbf{x}_j)$ in Eqn. (8) are given by,

$$\left. \begin{aligned} \mathbf{q}_{xx}^T(\mathbf{x}_i, \mathbf{x}_j) &= \mathbf{r}^T(\mathbf{x}_i, \mathbf{x}_j) \cos^2 \theta \\ \mathbf{q}_{yy}^T(\mathbf{x}_i, \mathbf{x}_j) &= \mathbf{r}^T(\mathbf{x}_i, \mathbf{x}_j) \sin^2 \theta \\ \mathbf{q}_{xy}^T(\mathbf{x}_i, \mathbf{x}_j) &= \mathbf{r}^T(\mathbf{x}_i, \mathbf{x}_j) \sin 2\theta \end{aligned} \right\}. \quad (10)$$

with $\mathbf{x}_i, \mathbf{x}_j$ the vector representations of each pair of points i, j and θ the angle relative to the global system of reference of the line segment that connects these points. For two points A and B these items are shown in Fig. 3.

The terms $\mathbf{r}^T(\mathbf{x}_i, \mathbf{x}_j)$ are vector integrals and can be derived as described in [34]. For example for a 6 term basis vector $\mathbf{p}(\mathbf{x}) = \mathbf{p}(x, y) =$

$\{1, x, y, xy, x^2, y^2\}^T$, the integral basis vector will be,

$$\begin{aligned} \mathbf{r}^T(\mathbf{x}_A, \mathbf{x}_B) &= \int_0^1 \mathbf{p}^T(\mathbf{s}(\mathbf{x}_A, \mathbf{x}_B; \lambda)) d\lambda = \\ &\left\{ 1, \frac{1}{2}(x_A + x_B), \frac{1}{2}(y_A + y_B), \right. \\ &\frac{1}{6}(x_A(2y_A + y_B) + x_B(y_A + 2y_B)), \\ &\left. \frac{1}{3}(x_A^2 + x_A x_B + x_B^2), \frac{1}{3}(y_A^2 + y_A y_B + y_B^2) \right\}. \end{aligned} \quad (11)$$

with $\mathbf{s}(\mathbf{x}_A, \mathbf{x}_B; \lambda)$, $\lambda \in [0, 1]$, the parametric representation of the line segment AB. In any case, the choice of an appropriate polynomial basis will yield an integral vector basis $\mathbf{r}^T(\mathbf{x}_A, \mathbf{x}_B)$.

3. The Surface Strain Compatibility Functional

For a simply connected continuous medium under deformation, the compatibility conditions for infinitesimal strain on the surface should hold and are given by:

$$\frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} - 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} = 0 \quad (12)$$

For the points of the surface where the previous equation doesn't hold, the continuity or simple connectivity assumptions are no longer valid and hence the body can be considered as having developed discontinuities, that are a clear manifestation of the onset of failure. Quantitatively this corresponds to the departure of the left hand side quantity of Eqn. (12) from 0. Based on this observation, we can define as a metric representation of the loss of continuity of the body to be the Surface Strain Compatibility Functional (SSCF),

$$s_v = \frac{\partial^2 \epsilon_{xx}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial x^2} - 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \quad (13)$$

The calculation of the SSCF from full field measurements requires the calculation of the second derivative of strain components. Introducing Eqns. (1) into the definitions of the second derivatives we obtain,

$$\left. \begin{aligned} \frac{\partial^2 \varepsilon_{xx}^h(\mathbf{x})}{\partial y^2} &= \frac{\partial^2 \Phi_{xx}(\mathbf{x})}{\partial y^2} \mathbf{e}_w \\ \frac{\partial^2 \varepsilon_{yy}^h(\mathbf{x})}{\partial x^2} &= \frac{\partial^2 \Phi_{yy}(\mathbf{x})}{\partial x^2} \mathbf{e}_w \\ \frac{\partial^2 \varepsilon_{xy}^h(\mathbf{x})}{\partial x \partial y} &= \frac{\partial^2 \Phi_{xy}(\mathbf{x})}{\partial x \partial y} \mathbf{e}_w \end{aligned} \right|_{\mathbf{x} \rightarrow \mathbf{w}}, \quad (14)$$

where $\mathbf{x} = \{x, y\}^T$. The second derivatives of the shape functions are derived from Eqn. (3),

$$\left. \begin{aligned} \frac{\partial^2 \Phi_{xx}(\mathbf{x})}{\partial y^2} &= \frac{\partial^2 \mathbf{p}^T(\mathbf{x})}{\partial y^2} \mathbf{M}_{xx}(\mathbf{w}) + 2 \frac{\partial \mathbf{p}^T(\mathbf{x})}{\partial y} \frac{\partial \mathbf{M}_{xx}(\mathbf{x})}{\partial y} + \\ &\quad + \mathbf{p}^T(\mathbf{x}) \frac{\partial^2 \mathbf{M}_{xx}(\mathbf{x})}{\partial y^2} \\ \frac{\partial^2 \Phi_{yy}(\mathbf{x})}{\partial x^2} &= \frac{\partial^2 \mathbf{p}^T(\mathbf{x})}{\partial x^2} \mathbf{M}_{yy}(\mathbf{w}) + 2 \frac{\partial \mathbf{p}^T(\mathbf{x})}{\partial x} \frac{\partial \mathbf{M}_{yy}(\mathbf{x})}{\partial x} + \\ &\quad + \mathbf{p}^T(\mathbf{x}) \frac{\partial^2 \mathbf{M}_{yy}(\mathbf{x})}{\partial x^2} \\ \frac{\partial^2 \Phi_{xy}(\mathbf{x})}{\partial x \partial y} &= \frac{\partial^2 \mathbf{p}^T(\mathbf{x})}{\partial x \partial y} \mathbf{M}_{xy}(\mathbf{w}) + \frac{\partial \mathbf{p}^T(\mathbf{x})}{\partial x} \frac{\partial \mathbf{M}_{xy}(\mathbf{x})}{\partial y} + \\ &\quad + \frac{\partial \mathbf{p}^T(\mathbf{x})}{\partial y} \frac{\partial \mathbf{M}_{xy}(\mathbf{x})}{\partial x} + \mathbf{p}^T(\mathbf{x}) \frac{\partial^2 \mathbf{M}_{xy}(\mathbf{x})}{\partial x \partial y} \end{aligned} \right|_{\mathbf{x} \rightarrow \mathbf{w}}. \quad (15)$$

The derivatives of the basis $\mathbf{p}^T$ are easily calculated by differentiation of its vector representation (Eqn. (4)), while the derivatives of $\mathbf{M}_\alpha$, $\alpha = xx, yy, xy$ are calculated by solving the equations resulting from the differentiation of Eqn. (6), i.e. by solving for the derivatives of $\mathbf{M}$ each of the equations,

$$\left. \begin{aligned} \mathbf{Q}(\mathbf{x}) \frac{\partial \mathbf{M}(\mathbf{x})}{\partial x} &= \frac{\partial \mathbf{B}(\mathbf{x})}{\partial x} - \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial x} \mathbf{M}(\mathbf{x}) \\ \mathbf{Q}(\mathbf{x}) \frac{\partial \mathbf{M}(\mathbf{x})}{\partial y} &= \frac{\partial \mathbf{B}(\mathbf{x})}{\partial y} - \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial y} \mathbf{M}(\mathbf{x}) \end{aligned} \right\} \quad (16)$$

and

$$\left. \begin{aligned} \mathbf{Q}(\mathbf{x}) \frac{\partial^2 \mathbf{M}(\mathbf{x})}{\partial x^2} &= \frac{\partial^2 \mathbf{B}(\mathbf{x})}{\partial x^2} - \frac{\partial^2 \mathbf{Q}(\mathbf{x})}{\partial x^2} \mathbf{M}(\mathbf{x}) - 2 \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial x} \frac{\partial \mathbf{M}(\mathbf{x})}{\partial x} \\ \mathbf{Q}(\mathbf{x}) \frac{\partial^2 \mathbf{M}(\mathbf{x})}{\partial y^2} &= \frac{\partial^2 \mathbf{B}(\mathbf{x})}{\partial y^2} - \frac{\partial^2 \mathbf{Q}(\mathbf{x})}{\partial y^2} \mathbf{M}(\mathbf{x}) - 2 \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial y} \frac{\partial \mathbf{M}(\mathbf{x})}{\partial y} \\ \mathbf{Q}(\mathbf{x}) \frac{\partial^2 \mathbf{M}(\mathbf{x})}{\partial x \partial y} &= \frac{\partial^2 \mathbf{B}(\mathbf{x})}{\partial x \partial y} - \frac{\partial^2 \mathbf{Q}(\mathbf{x})}{\partial x \partial y} \mathbf{M}(\mathbf{x}) - \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial x} \frac{\partial \mathbf{M}(\mathbf{x})}{\partial y} - \\ &\quad - \frac{\partial \mathbf{Q}(\mathbf{x})}{\partial y} \frac{\partial \mathbf{M}(\mathbf{x})}{\partial x} \end{aligned} \right\} \quad (17)$$

and then partitioning similarly to Eqn. (5) in order to obtain the separate $\alpha = xx, yy, xy$ components.

Since $\mathbf{q}(\mathbf{x}_i, \mathbf{x}_j)$ can be considered independent of $\mathbf{w}$ (i.e. the set of nodes in the vicinity of $\mathbf{w}$ is invariant for an infinitesimal change of $\mathbf{w}$), $\mathbf{Q}$ and $\mathbf{B}$ can be considered to vary only with W . Hence their derivatives will depend on the derivatives of the weight function only. For $\mathbf{d} = \{x_d, y_d\}^T = \mathbf{x} - \frac{\mathbf{x}_i + \mathbf{x}_j}{2}$ it follows,

$$\frac{\partial W(\mathbf{d})}{\partial x} = \begin{cases} \frac{4(3\hat{d} - 2)x_d}{d_w^2} & , \hat{d} \leq \frac{1}{2} \\ -\frac{4(\hat{d} - 1)^2 x_d}{\hat{d} d_w^2} & , \frac{1}{2} < \hat{d} \leq 1 \\ 0 & , \hat{d} > 1 \end{cases} \quad (18)$$

$$\frac{\partial^2 W(\mathbf{d})}{\partial x^2} = \begin{cases} \frac{4(-2\hat{d} d_w^2 + 6x_d^2 + 3y_d^2)}{\hat{d} d_w^4} & , \hat{d} \leq \frac{1}{2} \\ -\frac{4(2\hat{d}^2(x_d^2 - \hat{d} d_w^2) + (\hat{d}^2 + 1)y_d^2)}{\hat{d}^3 d_w^4} & , \frac{1}{2} < \hat{d} \leq 1 \\ 0 & , \hat{d} > 1 \end{cases} \quad (19)$$

$$\frac{\partial^2 W(\mathbf{d})}{\partial x \partial y} = \begin{cases} \frac{12x_d y_d}{\hat{d} d_w^4}, & \hat{d} \leq \frac{1}{2} \\ -\frac{4(\hat{d}^2 - 1)x_d y_d}{\hat{d}^3 d_w^4}, & \frac{1}{2} < \hat{d} \leq 1 \\ 0, & \hat{d} > 1 \end{cases} \quad (20)$$

The derivatives of W with respect to y are given by expressions similar to these in Eqns. ((18) - (20)), but with x_d and y_d swapped. It should be noted that Eqns. (18) - (20) are not appropriate for modelling highly irregular geometries appearing within the domain of support. In such cases more suitable techniques can be used, the detailed description of which can be found in [54] and can be easily adopted here as well. It is important to emphasize that this approach needs to be employed only when the irregular geometry is known and is necessary to be modeled and not when the proposed method is used to detect unknown cracks and discontinuities.

4. Probabilistic Crack Detection and Propagation for Full Field Measurements

Assuming the existence a distribution of nodes $\mathbf{w}_i$ on the surface of a body, we can evaluate of the SSCF as derived from Eqns. (13) through (20). This will yield values that can be used to estimate the continuity state of the surface. If the SSCF is close to 0 it can be assumed that the surface medium remains continuous while if it differs from 0 it can be assumed that it has developed discontinuities or cracks. The values obtained are considered an estimation because of the errors introduced by: (a) the discretization error of the mesh-free scheme applied and (b) the digital imaging measurement noise of the locations of the nodes used in the DSI method.

In order to address the uncertainties emanating from error sources, a probabilistic framework was developed in [55]. The framework assumes a Probability Density Function (PDF) f_N that describes the position vector $\mathbf{w}_i$ of each of the nodes in the field ([56, 47]. The contribution of the errors of the various sources is propagated through the all computational steps involved in the execution of the DSI analysis to the SSCF s_v . Hence the variability of s_v over a surface can be represented by another PDF f_v .

A number of images of the nodes on a surface at rest can be used to determine the f_{cv} PDF for a certain noise level for the case of a surface that doesn't have any discontinuities. This PDF can then be used to compute the levels of uncertainty that indicate whether the SSCF value of a point in another domain of unknown continuity state originates from f_{cv} or not. If the inference indicates that it originates from f_{cv} , then the location of the point can be assumed to be continuous while if the inference indicates that it doesn't originate from f_{cv} , the location can be considered discontinuous.

For a two - tailed symmetric PDF, the certainty $c(\mathbf{w}_i)$ that $s_v(\mathbf{w}_i)$ does not belong to a distribution represented by $f_{cv}(x)$ can be defined by [57],

$$c(\mathbf{w}_i) = 2 \left(1 - \int_{-\infty}^{|s_v(\mathbf{w}_i)|} f_{cv}(x) dx \right), \quad (21)$$

where $\mathbf{w}_i = \{x_{wi}, y_{wi}\}^T$ is the vector representing the position of point of interest i and $|s_v(\mathbf{w}_i)|$ is the absolute value of SSCF at this point. In other words $c(\mathbf{w}_i)$ is the probability that the vicinity of point $\mathbf{w}_i$ contains discontinuities. Therefore, calculation and plotting of c for all the points of interest $\mathbf{w}_i$ will generate a full field representation of the discontinuity certainty.

In [55], the framework was developed solely on an image processing context, assuming regularly distributed nodes. The requirement to introduce the crack as a boundary in the full field representation of the domain necessitates the need to alter this framework in order to obtain a semantically meaningful crack representation that can be used to calculate the strain field.

4.1. Moving Crack Trajectory Detection

In this new framework some of the steps presented in [55] have been modified and new ones have been introduced to accommodate a multivariate adaptive regression that can be used to compute a multi-line representation of a propagating crack at various time steps. The specific method is based on spline representation and is called Multivariate Adaptive Regression Splines (MARS^{®1}) [58, 59, 60]. The MARS[®] representation has the form of a product spline basis functions expansion. Both the number of the basis functions and the relevant parameters are determined by an adaptive method [58].

The modified approach that can be used for transient crack detection and results in semantically meaningful crack representation has as follows:

1. For a given continuous domain, the SSCF is repeatedly calculated so that the uncertainty propagation of the various sources of noise result in histogram representation. An appropriate PDF is regressed over the histogram. The calculation need to be performed on an actual experiment where the presence of noise is inherent. If this is not possible, a synthetic simulation can be used instead. In both cases a PDF f_{cv} that describes the noise characteristics of the SSCF will be calculated.

¹MARS[®] is a trademark of Jeril, Inc.

2. For the domain of interest the SSCF $s_v(\mathbf{w}_i)$ is calculated over a number of points $\mathbf{w}_i$ so that an adequately dense full field representation becomes available.
3. The certainty value $c(\mathbf{w}_i)$ given by Eqn. (21) is calculated for each node of the field defined in the previous step.
4. If the noise of the previous step is high, steps 2 - 3 are repeated under the same loading conditions and an average frame is calculated.
5. For a predetermined certainty level, a threshold operation is applied on the previous nodal values, such that all the points with lower certainty are removed.
6. The MARS[®] algorithm is applied on the resulting points and a poly-line (or a product spline basis functions expansion) that represents the crack is identified.

4.2. Full field measurement crack propagation

Since new information regarding the existence of the crack has become available, it is trivial to introduce it in the DSI analysis. For every time-step, where the crack has a new geometry, new shape functions are calculated, considering an irregularity treatment approach [42, 54]

The introduction of the crack will provide with more accurate results in the vicinity of the crack. Indeed although the DSI formulation retains its validity even if the discontinuities are not explicitly modelled in the creation of shape functions, it is the shape functions form that define the consistency of the results with the actual deformation. Hence it is expected that simple linear or low order polynomial shape functions will not be able to accurately capture the deformation characteristics around discontinuities as accurately

as an explicit definition of the geometry would do. Therefore, the introduction of the crack geometry as calculated by the final step of the previous section into DSI will provide with more accurate strain values around the crack.

It is possible to use other methods to calculate the new strains including the Mesh-Less Random Grid method [39, 40, 41, 42, 43, 44, 45, 47], but also element based or XFEM methods. The investigation of the applicability of other methods is the purpose of another study.

5. Synthetic Modeling of Crack Propagation

Synthetic experiments were designed in order to verify the performance of the proposed method in the absence of real experimental data. These numerical experiments were conducted for of a flat specimen as shown in the drawing of Fig. 4. The specimen has a width of 40 *mm* a height of 80 *mm* an initial crack of a length of 5 *mm*, two circular openings of a diameter 14 *mm* each and two hard inclusions of diameter 10 *mm* each. The specific geometry was chosen such that the trajectory of the propagating crack is not trivial. The elastic properties of the materials used are shown in table 1. The specimen is considered to be loaded in tension under plane stress conditions in the vertical direction with a distributed load of 16×10^6 *N/m* applied in the top edge, while the bottom edge is considered with roller boundary conditions.

The steps of the analysis are the following:

1. The defined geometry is analyzed using an XFEM code such as the initial crack is propagated through a number of steps.

(a) Overview drawing of the elastic plate. The hatched areas represent inclusions. The dimension numbers are in *mm*.

(b) Detail at the crack region, where the location of the initial crack relative to the mesh is shown along with representative points tracked by the DSI method.

Figure 4: Geometry of the specimen used in the XFEM simulation.

Table 1: Material properties used in the XFEM simulations

Region	E (N/m^2)	ν
Bulk material	70×10^9	0.33
Inclusion	200×10^9	0.30

2. The locations of the nodes for each time step are calculated based on the XFEM analysis.
3. In order to simulate imaging sensor noise, the nodal locations are perturbed by random vectors sampled from an appropriate distribution [47]. It has been found that a good approximation of the noise distribution is that of Gaussian distribution [43, 47].
4. The nodal locations are used as the points necessary to perform the DSI analysis (section 2) together with the calculation of the SSCF (section 3).
5. The SSCF distribution is used to detect the crack at each time step using the procedure described in section 4.
6. For each time step the crack geometry is used in a new DSI analysis and more accurate strains are calculated based on the assumption of the existence of a crack in a new location.

5.1. XFEM Analysis for Crack Propagation Data Generation

The crack propagation analysis was performed using the XFEM method [61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74]. In XFEM special functions are introduced to the finite element approximation using a partition of unity approach. In isotropic linear elasticity, a discontinuous function and

Figure 5: Vertical strain component contours at various stages of crack propagation as calculated by the XFEM simulation. The lengths next to the captions represent crack propagation levels. The x and y axis units are in m .

two-dimensional asymptotic crack-tip displacement fields are used to model the local deformation in the vicinity of the crack. Using this approach the crack propagation can be simulated without the need for remeshing.

The plate was meshed with a regular grid of square elements of a width of $0.5mm$ each. The initial crack was slightly offset from the center line of the plate so that the results are not biased in the application of the DSI method. A detail of the mesh near the crack is presented in Fig. 4b, that depicts the local mesh in the vicinity of the crack. The XFEM analysis was performed using the MXFEM scripts [75]. MXFEM has the ability to model homogeneous cracks [76, 66], as well as voids [69] and interfaces [77]. The XFEM analysis was performed for 80 iterations with a growth rate of $3.3mm$ per iteration.

Some results from the XFEM analysis are presented in Fig. 5. In these figures the vertical strain component, which is parallel to the loading is presented. The openings and the inclusions have caused the propagation to

take a non straight course as intended. It is noteworthy to mention that the strain fields are plotted over the initial (undeformed) configuration.

5.2. Transient Crack Detection

Since the region of interest is only at the vicinity of the crack, the subsequent analyses focus at an area with a height of $0.035m$ centered at the initial crack.

The nodal displacements of the XFEM analysis were perturbed by random vectors sampled from a Gaussian distribution. The distribution parameters were chosen such as noise manifests the equivalent of an imaging noise of $1/128$ pixels for a sensor of 4000 pixels wide, a performance very typical of the capabilities of modern digital cameras.

Performing the analysis described in section 4 results in a piecewise spline representation of the crack for each time step. The shape function chosen for the analysis was linear and the extend of the domain of support was 0.015 m . It should be noted that in the current realization, the identification of the crack geometry is performed independently for each time step. A possible re-factoring of the process would involve the identification of the crack geometry using data from multiple time steps at a time.

A comparison of the identified crack geometries and the synthetically generated by the XFEM analysis are presented in Fig. 6. The crack geometry produced by the XFEM solution is represented with a thick dashed line and the detected ones with a thin solid line crossed by $\times$ marks. It can be seen that overall the method works very well in identifying the location and geometry of the crack for every time step.

However, a minor issue is the noticeable slight offset at the left side of the

crack and another minor issue is a small deviation at identifying the exact tip of the crack. It is very plausible that the main reason for the appearance of these two minor issues is the presence of the two main sources of error; the discretization error and the simulated imaging sensor noise level. It is our experience that with a denser mesh and lower noise levels, these small differences will become even smaller. A sensitivity analysis of the method on these parameters will be the subject of a future study.

5.3. Full field calculation of propagating crack

After the geometry of the crack had been identified, it was introduced in a new DSI analysis that was performed as described in section 4.2. The domain of support and the shape function used in this case were the same ones used in the analysis of section 5.2.

The results of all three analyses (XFEM, DSI based on XFEM data, DSI with explicit crack geometry) are presented in Fig. 7 for 4 characteristic time steps.

From these figures it is evident that the strain field recovered by DSI (Figs. 7b, 7e, 7h and 7k) and by DSI with the assumed crack (Figs. 7c, 7f, 7i and 7l) are very close to the synthetic experiment ones (Figs. 7a, 7d, 7g and 7j) for most of the area of interest. It is clear that when the DSI is unaware of the crack presence, the shape functions do not adequately capture the strain field at the vicinity of the crack.

On the other hand, when the DSI analysis is performed using the identified crack geometry, the strains are recovered for most of the regions around the crack. Indeed, these regions no longer show the saturated strains that appear in the middle column of Fig. 7 but are very close to the ones of the synthetic

experiment corresponding to the first column of the same figure. However, some errors still appear to exist exactly on the tip of the crack and they are attributed to two reasons: (a) The crack detection on the tip is not ideal as demonstrated in Fig. 6 and (b) There has been no attempt to find an optimal domain of support extent to count for the high gradient of deformation at the region near the crack.

In addition, the absolute difference between the full field values of the DSI results and those of the DSI with the identified crack taken into consideration relative to the synthetic data produced by the XFEM analysis are presented in Fig. 8. In these figures, the observations that are made in Fig. 7 are more pronounced. The error for the bulk of the domain is close to 0 for both cases, but it is high in the vicinity of the crack trajectory in the simple DSI calculation, as expected. When DSI is applied while incorporating the identified crack, only a small error in the neighborhood of the crack tip appears.

In Fig. 9 we are presenting a comparison of the contour plot distributions for all three strain components (ε_{xx} , ε_{yy} , ε_{xy}) for a propagation length of 26.4 *mm* as generated from the XFEM generated synthetic data (left column) and the DSI full field results when the identified crack has taken into consideration. It can be seen that the fields of all strain components are identified in a quite satisfactory manner despite the noise injected into the synthetic data.

6. Conclusions

In this study a new method was presented that can be used to determine the geometric characteristics of a propagating crack using full field measurement data. The method is based on the property of the Direct Strain Imaging method that no assumptions are made for the continuity state of the material and hence a proper strain compatibility functional can be used to identify the areas of the surface of a specimen that contain crack and discontinuities. The method is material and loading independent, to the extent the strain compatibility functional takes a value actuated by the strains induced by an external load. The method then uses a multivariate adaptive regression splines scheme to calculate an analytic expression of the geometry of the propagating crack. Subsequently, the identified crack was used again in conjunction with the DSI method to evaluate a more accurate representation of the strain field over a synthetically generated deformed configuration. An error analysis was performed that showed good agreement between the synthetically known field and the method's results.

In the future it is planned to validate the proposed method with actual experimental data by using both uniaxial and multiaxial loading machines for testing specimens made by both isotropic and anisotropic materials. We also plan to implement an adaptive scheme that finds the optimal domain of support for the DSI method under certain assumptions for noise, in order to increase the accuracy of the method in regions with high deformation gradient.

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(a) $9.9mm$

(b) $13.2mm$

(c) $16.5mm$

(d) $19.8mm$

(e) $23.1mm$

(f) $26.4mm$

Figure 6: Comparison between the vertical synthetic propagating crack profile (bold dashed line) and the full-field data identified crack profile (thin line with $\times$) for various crack propagation lengths.

Figure 7: Comparison between the vertical strain component contours at various crack propagation lengths of the XFEM simulation (left column) the DSI analysis results of the synthetic experiment (middle column) and the DSI assuming the existence of the identified crack (right column) at an equivalent noise level of 1/128 pixels for an image sensor width of 4000 pixels.

(a) DSI at 9.9 *mm* (b) DSI with identified crack 9.9 *mm*

(c) DSI at 9.9 *mm* (d) DSI with identified crack at 13.2 *mm*

(e) DSI at 13.2 *mm* (f) DSI with identified crack at 13.2 *mm*

(g) DSI at 19.8 *mm* (h) DSI with identified crack at 19.8 *mm*

Figure 8: Absolute error distribution plots of the DSI and DSI with identified crack results of Fig. 7

Figure 9: Comparison between the contour plots of the three strain component fields for a crack propagation length of 26.4 mm generated via XFEM simulation (left column) and the corresponding fields generated via the DSI method, assuming the existence of the identified crack (right column) with an equivalent noise level of 1/128 pixels for an image sensor width of 4000 pixels.