Characterizing Questions and Their Focus When Pre-Service Teachers Implement
Dynamic Geometry Tasks

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Abstract

When technology is used in classrooms new interactions among students, the teacher, and technology are enabled. The purpose of this study was to examine the ways pre-service mathematics teachers implemented technology-based tasks with individual advanced middle-school students. Pre-service teachers posed questions that focused students on features of technology and geometry in different classifiable ways. In particular, there were instances when teachers focused only on mathematics or technology. There were also instances when the teacher suggested students use the technology for the purpose of noticing mathematics and other times when the teacher would pose a mathematics question or statement with the assumption that students would use technology in response. Analysis of six pre-service teachers' is provided along with a classification system.

Keywords: Dynamic geometry, technology, pre-service teachers, questioning

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Introduction

Dick (2011) states, “the value of technology to the teacher lies not so much to the answers it provides, but rather to the questions it affords” (p. 2). For over two decades, the National Council of Teachers of Mathematics (NCTM, 1991) has emphasized the importance of teachers selecting and implementing worthwhile mathematical tasks and asking quality questions that engage all students. Researchers have shown that worthwhile tasks and good questions are critical components of effective mathematics classrooms (e.g., Stein, Smith, Henningsen & Silver, 2000). Technology has also been identified as a critical component of mathematics classrooms (e.g., NCTM, 2000) with dynamic geometry programs considered an important aspect of geometry instruction (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

Some studies have examined how teachers design tasks that utilize technology (e.g., Laborde, 2002) or implement technology-based tasks with students (Hollebrands & Heid, 2005; Lee, 2005). Research has also focused on the implementation of non-technology based tasks in mathematics classrooms (e.g., Stein, Grover, Henningsen, 1996). However, few studies have considered how pre-service teachers pose questions to students while implementing geometry tasks that employ technology. When teachers are using technology, they need knowledge of mathematics, and technology and the ways technology represents mathematics to pose questions that will focus students on important ideas, relationships, and invariances related to the mathematical goal they want students to learn. The effectiveness of instruction and use of technology depends on the ways tasks are implemented with students. Thus, it is important to know more about how teachers implement technology-based tasks with students. Knowledge about ways teachers can support students learning when using technology can inform the design of mathematical tasks for students and implementation

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guides for teachers. In the current study, we examined questions and statements posed by pre-service teachers and their interactions with students during the implementation of geometric tasks that incorporated technology. In this paper we address the following research question: How do pre-service teachers focus students' attention on mathematics and technology when engaging with dynamic geometry software tasks?

Literature Review

Questioning

Questioning has long been identified as an important practice of teaching (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956; Hargie, 1978) and an important factor in students' mathematical learning (e.g., Martino & Maher, 1999; Sloan & Pate, 1966). Many beginning teachers are familiar with different taxonomies (e.g., Bloom's Taxonomy) and frameworks for analyzing and creating mathematical questions and tasks (e.g., Mathematical Task Framework), but they lack experience working with students. As teachers gain experience with students, they are better able to anticipate how a student will respond to a posed question and are able to formulate an appropriate follow-up question or response.

Teachers' Uses of Technology

Technology adds another layer of complexity to posing tasks and questions. Early work by Farrell (1996), Doerr and Zangor (2000), and Goos, Galbraith, Renshaw, and Geiger (2003) identified various ways technology tools, particularly graphing calculators, were used and the roles they played in the classroom. Such roles were characterized as technology as a partner, and teachers serving as an explainer, facilitator and mediator. In particular, Doerr and Zangor (2000) found that when teachers assumed the role of facilitator they asked questions that shifted students' attention from a focus on computational results to considering how to interpret and explain results returned by the calculator. However, other studies have suggested that teachers' questions can narrowly focus students and result in students

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following a specified set of directions and procedures, rather than considering conceptual aspects of the task teachers have created (e.g., Heid, Blume, Zbiek, & Edwards, 1998; Wood, 1998). More recent research utilizing Rabardel's instrumental approach (Verillon & Rabardel, 1995) has focused on teachers' instrumental orchestrations (Drijvers, et al., 2010; Trouche, 2004). Leung (2011) proposed three epistemic modes to support students' acquisition of new mathematical knowledge abstracted from interactions with dynamic geometry environments (DGEs) and a corresponding techno-pedagogic task design model. These modes take into consideration the instrumentalization process specific to DGEs.

While all of these studies have added significantly to an understanding of teachers' practices and interactions with students and technological tools, we do not know much about how novice and pre-service teachers learn to interact with students while using a technology tool to solve a task. Bowers and Doerr (2001) and Lee (2005) investigated pre-service teachers' interactions with students while using a technology tool. Both found that pre-service teachers were able to use representations available within the technology environments to focus students' attention on important ideas in a task or to pose additional questions for them to consider. In addition, Lee (2005) also found that the three pre-service teachers in her study each made decisions in their interactions with students to step into a role of explainer (Farrell, 1996). However, in both of these prior studies, the pre-service teachers did not design the tasks that were being implemented. Bussi & Mariotti (2008) analyzed how teachers use artifacts or tools (e.g., abacus, Cabri) as a semiotic mediator of mathematical signs. A recurrent sequence of actions performed by teachers was identified: revisit the task, focus on specific aspects of the use of the tool, request synthesis, and then synthesize. The researchers indicate that this "didactical cycle" was carefully orchestrated by the teacher in response to her analysis of students' uses of the tools and mathematical meanings they were developing. Unlike the studies conducted by Bowers and Doerr (2001) and Lee (2005),

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teachers in this study had the opportunity to formulate questions, tasks, and new activities in response to their interpretations of students' geometric and technological work.

Teachers' Uses of Dynamic Geometry Software

The use of dynamic geometry programs presents unique opportunities and challenges to students and teachers. Although many different dynamic geometry programs are available, they all share basic features: primitive objects (e.g., points, lines, circles), basic construction tools (e.g., parallel, perpendicular), transformations, measuring and dragging capabilities. Dragging allows one to interact directly with geometric objects and modify them in ways that maintain the properties that were used in their construction (Goldenberg & Cuoco, 1998). Because these programs use properties to respond to the actions of students, students have opportunities to observe and abstract geometrical relationships and theorems from those experiences (Laborde, 1993). In addition, because technology provides fairly accurate measures and representations, students may not feel the need to verify the truth of conjectures, but may seek to know why a particular invariance always occurs (Arzarello, Olivero, Paola, & Robutti, 2002; Christou, Mousoulides, Pittalis, and Pitta-Pantazi; Jones, 2000; Leung, 2011).

Teachers need knowledge not only of technical aspects of a tool, but the cognitive and pedagogical issues and opportunities that may arise. In a three-year study of teachers learning to design tasks to incorporate dynamic geometry technology, Laborde (2002) found that several teachers initially designed tasks for which the technology served as a visual aid or used the tool to do similar non-technology based activities in a more efficient manner. The use of technology in these cases could be characterized as an amplifier (Pea, 1985). In essence, the tasks themselves remain unchanged, but the technology enables one to do them more efficiently or accurately. Over time, Laborde (2002) found that teachers began to develop tasks that could only exist in the technology-based environment. This use of

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technology might be characterized as a reorganizer (Pea, 1985). The technology changes the way one approaches and thinks about mathematics. It is clear that teachers need to think carefully about interactions among mathematics, technology, and student thinking.

In their study of two high school mathematics teachers using a dynamic geometry program to teach properties of quadrilaterals Sinclair & Yurita (2008) examined the shifts that occurred in the teachers' discourse when technology was used. In particular, four different routines related to the identification of quadrilaterals were identified when technology was used. These shifts moved from static discourse that relied on perception to dynamic discourse patterns that focused on transformations, measures, and change. The tasks that were used by teachers in this study seemed to support this transition from static to dynamic discourse and teachers appeared comfortable using the technology. More research is needed to know how to support teachers' design or selection of technology-based tasks and their implementation with students.

Theories are available to enable researchers to consider more carefully teachers' interactions among mathematics content, student thinking, and the use of technology.

Framework

Didactic Triangle and Tetrahedron

Designing and implementing tasks with technology places pre-service teachers in a didactic situation — a situation in which they have an opportunity to develop their understandings about teaching students mathematics with technology. Many researchers (e.g., Brousseau, 1997; Freudenthal, 1991; Steinbring, 2005) have considered mathematics learning and teaching by considering the interactions that occur among the student, teacher, and content. Typically, these interactions have been represented using a didactic triangle (see Figure 1).

Figure 1. A representation of the didactic triangle

To make explicit how one considers the role of technology among interactions with students, a teacher, and mathematics, the didactic triangle was extended by Tall (1986), and more recently by Olive et al. (2010) and Ruthven (2012). The expansion of the didactic triangle to take into consideration technology was the focus of a special issue of *ZDM: The International Journal on Mathematics Education* in 2012. Olive et al.'s view is that "the introduction of technology into the didactic situation could have a transforming effect on the didactical situation that is better represented by a didactic tetrahedron (see Figure 2), the four vertices indicating interactions among Teacher, Student and Mathematical Knowledge, mediated by Technology." (p. 168).

Figure 2. Didactic tetrahedron from Olive et al. (2010)

Tools fundamentally shape, influence, and mediate learning and their use can both assist and constrain how one thinks about particular ideas (Miera, 1995; Vygotsky, 1978). The use of tools can also impose additional cognitive demands on both teachers and students.

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Teachers need to know how to use the tools, be mindful of when they are appropriate to use, know to what extent they are accurate and reliable representations of abstract constructs, and be aware of limitations that may be experienced by students when one uses a particular tool. In addition, teachers must be able to pose questions that focus students on mathematics and take advantage of technological affordances. When a teacher designs a task that utilizes technology, he or she likely considers the relationship between mathematical ideas in the task, and the extent to which technology represents those mathematical ideas in a “faithful” way (Zbiek, Heid, Blume, & Dick, 2007). Also within the teacher-technology-mathematics face of the tetrahedron (see Figure 2), teachers may consider the types of actions on objects that are enabled by technology. Teachers may then use these analyses to inform the design of a task. During this design phase, a teacher is likely anticipating ways in which a student will engage with mathematics using the technology tool (i.e., the student-mathematics-technology face of the tetrahedron). This anticipation may assist a teacher in developing specific questions or directions for students. Several researchers have described the importance of carefully designed questions for technology tasks. Specifically, questions should focus students on particular mathematical relationships and be aligned with the features available and actions allowed in a technology tool (Dreyfus, 1991; Leung, 2011; Sinclair, 2003).

While the design of a task is a critical component of the learning process, its implementation by teachers is also important. During implementation of a technology-enabled task, the two faces of the tetrahedron that involve student-teacher interaction with mathematics or technology come to the fore (student-mathematics-teacher, and student-technology-teacher). How teachers introduce the task and technology, pose questions, and offer assistance to students as they are working can impact what is learned. In particular, how teachers and technology tools focus students' attention on particular aspects of mathematics

can influence what students notice and learn from those interactions (Lobato & Ellis, 2002; Lobato, Ellis, & Munoz, 2003).

Focusing Effect

Lobato & Ellis (2002) define the focusing effect of technology as “the regular direction of users’ attention toward certain subject matter properties over others” (p. 299). For example, in a dynamic geometry environment, because so much changes when an object in the environment is dragged, a student might be more likely to notice what does not change. These invariances may be a reflection of the underlying geometric relationships or an artifact of the way in which the technology was designed. For example, a student may create a segment, $\overline{AB}$, and place a point C arbitrarily on it. By dragging point A or point B , the student may note, in most dynamic geometry environments, that the ratio $AC:CB$ remains fixed (Cuoco & Goldenberg, 1997). On the other hand, a student might construct the midpoint, C , of $\overline{AB}$ and notice that the ratio $AC:CB$ remains fixed as 1:1. In the second example, the invariance is related to a geometric definition the teacher might want students to observe and notice. The first example is related to how the programmer designed the technology tool and may not be something the teacher wants students to focus upon. It is important for teachers to be aware of these differences when they are using technology with students.

Lobato, Ellis, and Munoz (2003) expand their notion of focusing effect to consider the focusing phenomena, which relates to how aspects of a classroom environment (e.g., teachers, curricula, tasks, tools) “direct attention toward certain mathematical properties over others” (p. 1). The focusing phenomena influence mathematical abstractions students make. When teachers are working with students on technology-based tasks, there may be features of the task, uses of the tool, questions posed by the teacher, or language that is used by students and teacher that regularly direct students’ attention toward particular mathematical features.

Epistemic Modes

Leung (2011) suggests the consideration of three epistemic modes to guide the design of a task that takes advantage of the specific technological affordance in DGE: Establishing Practices Mode, Critical Discernment Mode, and Situated Discourse Mode. In Establishing Practices Mode, students are engaged in activities that support instrumental genesis. The task may provide directions that guide students in learning how to construct particular objects, find specific measurements, or drag in certain ways. Students are becoming familiar with the tool and developing routines. With the Critical Discernment Mode, students are moving from a focus on the tool and particular ways to use it, to thinking about the mathematics. Students may be encouraged to use their empirical experiences with DGE to generate conjectures and explanations. Constructing generalizations and proofs using deduction is encouraged in tasks within the Situated Discourses Mode. Each of these modes focuses differently on mathematics and technology.

Initial Categories of Focusing Questions

With the epistemic modes and didactic tetrahedron in mind (see Figure 2), we used the didactic tetrahedron to develop four initial categories that we anticipated could be used to describe the focus of the questions, prompts, and statements exchanged between pre-service teachers (PSTs) and students while working on a task: 1) *focus on technology*, 2) *focus on technology to notice mathematics*, 3) *focus on mathematics with the use of technology*, and 4) *focus on mathematics*. The direction of the conversation (student to teacher or teacher to student) was also considered.

The four initial categories of questions/statements, their definitions, and diagrams to illustrate their relationships in the didactic tetrahedron (third column) are included in Figure 3. With a 3-D drawing of the didactic tetrahedron, it is not possible to see how some of the types of interactions are represented. However, if the tetrahedron is opened into a net, it is

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easier to examine the interactions among different vertices. A prompt to answer a mathematical question with the assumption technology will be used can be considered as an interaction among the teacher-mathematics-technology-student vertices of the tetrahedron (as shown in Figure 3). Similarly, a teacher might prompt students to do something with the technology with an assumption that they would notice a mathematical relationship (*focus on technology to notice mathematics*). By looking at the open tetrahedron, this can be viewed as an interaction among the teacher-technology-mathematics-student vertices of the tetrahedron. We also depict the four categories in two dimensions to illustrate interactions between a student and teacher with respect to mathematics and technology (see fourth column in Figure 3). Examples of questions in each category can further illustrate the way we anticipated viewing the focus of interactions between the preservice teachers, students, mathematics, and technology.

A *focus on technology* includes questions or statements related to the technology itself. For example, a set of directions like “click on the arrow tool and then move your cursor to a point, click the mouse and you can drag the point around the screen” are in this category. Or, a student might ask how to do something with technology. This would be an example of an interaction within the student-technology-teacher face of the tetrahedron. Other questions may prompt students to focus on a technological action that is related to a geometric idea (*focus on technology to notice mathematics*). For example, the question “what happens to the incenter if you drag the vertex of the triangle?” focuses attention to the dragging feature that the technology affords to notice a geometric relationship between the incenter and the triangle.

A teachers' question may also *focus on mathematics through the use of technology* when students are prompted to answer a geometric question with the assumption that technology would be used. For example, with the intention of focusing a student's attention to the

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relationship between the incenter and the circle by dragging the incenter, a teacher may ask the student “what do you notice about the incenter? Is it always inside the circle?” The questions are focused on the mathematical idea of invariance of the incenter, but the use of “always inside” may imply the use of technology to explore different configurations without saying what to specifically do with the technology.

At times, a teacher might pose a mathematical question to a student (*focus on mathematics*). In this case, we are considering the teacher-mathematics-student face of the tetrahedron. For example a teacher might ask, “How could you prove the base angles of an isosceles triangle are congruent?” Or a teacher might ask a student for a particular geometric definition, “What is the definition of perpendicular lines?”

In some ways these four categories build on and further elaborate the epistemic modes described by Leung (2011). While his modes are useful in designing tasks, our categories provide additional guidance to teachers about what they may do when enacting a DGE task with students. The first category (*focus on technology*) is related to Leung’s Establishing Practices Mode. The second and third categories (*focus on technology to notice mathematics*; *focus on mathematics with the use of technology*) could be considered within the Critical Discernment Mode. The fourth category (*focus on mathematics*) is similar to the Situated Discourses Mode. In the remainder of this article, we will describe the study we conducted that informed the initial development of these categories and provide examples from our data to illustrate these categories and several subcategories that emerged from our analysis.

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Category	Definition	Tetrahedron Face	Coding Diagrams
Focus on technology	The teacher makes a statement or poses a question to the student that is only about the technology.
Focus on technology to notice mathematics	The teacher makes a statement or poses a question related to a technological action that is related to a mathematical idea.
Focus on mathematics with use of technology	The teacher makes a statement or asks the student a mathematical question with the assumption that the student will do something with the technology to respond.
Focus on mathematics	The teacher makes a statement or poses a question to the student that is only about the mathematics. The question could be answered without using technology (or could use the technology sketch in a static manner).

Figure 3. Categories, definitions, and diagrams of focusing questions in technology-enabled mathematics tasks

Context and Methods

This study was conducted in the context of a mathematics education methods course emphasizing the use of technology in teaching and learning mathematics, in which twenty pre-service teachers (PSTs) were enrolled at a large public university. The course served PSTs preparing to teach in middle school (typically ages 11-13) and high school (typically ages 14-18). Prior to this study, PSTs received approximately five weeks of instruction on the use of The Geometer's Sketchpad (GSP) using materials from Hollebrands & Lee (2012). They had experience using GSP as learners of mathematics to engage in geometric explorations and reflected on those experiences from the perspective of a teacher. They were introduced to the mathematics task framework (Stein, Grover, Henningsen, 1996) and had created and critiqued GSP sketches and questions.

Partnering with a nearby middle school, PSTs were paired and assigned to design, share, and implement a thirty-minute, exploratory task using GSP with a single advanced middle school student taking a geometry course (typically taught at the high school level in the U.S.). The task consisted of three components: (1) a pre-constructed GSP file, (2) a handout posing the task for the student, including questions related to the task and/or simple technology directions for using GSP, and (3) a separate document for the pair of pre-service teachers to refer to while implementing the task. This third document described what the PSTs' anticipated students would do, included a list of questions to guide students in the discovery process, and also described how the task could be adjusted for a student at a lower level or extended for a higher-level student. The instructor of the course did not provide feedback or assistance to PSTs as they developed these documents.

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Each of the ten pairs of PSTs worked together to design a task. The pairs were then separated and matched with different PSTs in the class to pilot their task and receive feedback about how it could be modified. After the feedback session, the original pairs met to make modifications. During the next class meeting, PSTs implemented their tasks individually with one middle school student. Afterwards, the PSTs gathered to reflect on the experience.

Three pairs, or six PSTs, were selected to serve as cases for this study because they represented different subgroups in the mathematics education program (See Figure 4 – all names are pseudonyms). One pair (Jessica and Matthew) were pursuing a single degree to teach middle or high school mathematics, another pair (Delta and Jonah) were pursuing two different degrees (Mathematics Education and Mathematics or Statistics) and the third pair (Audrey and Victoria) were adult students choosing to return to the university to earn a degree and a license to teach high school mathematics. Delta took more statistics courses than the other five PSTs and Jonah took more courses in mathematics. Audrey and Victoria were both parents of children and had more experience interacting with young adolescents. It is important to note that our focus on three pairs constitutes a small but purposeful sample and may not be representative of the general population of pre-service mathematics teachers. For these three pairs, the piloting, teaching, and reflective sessions were videotaped.

Pseudonym	Year	Major
<i>Pair 1</i>		
Jessica	Senior	Mathematics Education – Middle Grades
Matthew	Junior	Mathematics Education – High School
<i>Pair 2</i>		
Delta	Junior	Double Degree: Mathematics Education – High School and Statistics
Jonah	Senior	Double Degree: Mathematics Education – High School and Mathematics
<i>Pair 3</i>		
Audrey	Senior (returning adult)	Mathematics Education – High School

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Victoria	Returning adult	Licensure Only Mathematics Education - High School
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Figure 4. Description of six participants

Because we were interested in understanding the types of questions that PSTs posed, and the ways in which these questions focused students' attention, it was appropriate to use qualitative methodology. Primarily, analysis was conducted using the qualitative data analysis tool, Transana. Codes were developed related to how teachers' focused students' attention. Sample codes included: student noticing relationship, teacher refocusing student, teacher learning math, focus on math, focus on technology, do technology action to notice math, consider math while using technology. These codes informed the development of the four categories (Figure 3) that reflected a focus. The researchers coded a portion of a single transcript linked to the video recording and discussed differences in coding until a consensus was reached. Examples and definitions of codes were refined during this process of analyzing one PST's work. Then, to analyze the remaining five PSTs' work, each researcher analyzed one or two additional video recordings and met with another researcher who had analyzed the same video recording(s) to discuss commonalities and differences in the ways pairs of PSTs implemented the same task. Codes from each of the six PSTs' implementations were agreed upon through consensus of at least two researchers. Codes from all six implementations were analyzed by each of the four categories to identify trends and themes in questions and statements made by PSTs and students. This cross-case analysis resulted in creating better descriptors of the foci, with subcategories emerging that indicated the apparent purposes of the questions posed within each category.

Description of the Tasks

The size and scope of the tasks designed by the three pairs varied. Jessica and Matthew designed and implemented a task to teach one specific theorem. In contrast, Audrey and Victoria

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constructed a sketch for students to explore four different triangle centers. Delta and Jonah created a sketch for students to examine a single triangle center, the circumcenter. Screen shots of the tasks that included pre-constructed sketches are included in Figure 5. What follows is a brief description of each of the tasks.

Pair	Screenshot of pre-constructed sketch.
Jessica and Matthew
Delta and Jonah
Audrey and Victoria

Figure 5. Screenshots of pre-constructed sketches

Jessica and Matthew created a task designed to explore the idea that the shortest distance between a point and a line is the perpendicular distance. Their pre-constructed sketch included a

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drawing of a basketball court with labeled positions of a “Teammate” (always on the half-court line) and “You” (always on the baseline). A “Coach” and “Ref” were also placed on opposite sidelines at the half-court line (See Figure 5). Students were asked to attend to measurements included in the sketch (e.g., segment lengths and angle measures) that changed as a result of dragging. The task only asked students to determine the “shortest pass that you were able to make to your teammate” and “the angle measure created between the points ref, teammate, and you.” Thus, the mathematical goal of the task was inferred to be for students to observe that the shortest pass created a right angle with, and was therefore perpendicular to the half-court line. There was one anticipated answer and the task did not allow for much exploration. While PSTs were given approximately thirty minutes to implement their task with students, both Jessica and Matthew finished their tasks early.

Delta and Jonah designed a two-part task for students to explore the circumcenter of a triangle. For the initial task, students were provided with a sketch that contained a “generic” triangle with vertices that could be dragged (Figure 5). The PSTs challenged students to find the center of a circle that would circumscribe a given triangle. Based on the questions that followed, it was clear they wanted the student to construct the circumscribed circle, although it was not explicitly stated in the question. Then, questions were posed to encourage the student to consider relationships between the circle and triangle. In the second part of the task, students were provided with a sketch that contained a circle. They were challenged to construct an inscribed right triangle. Questions were posed to focus students on the relationship between the hypotenuse of the right triangle and the diameter of the circle. They also asked the student to construct the midpoint of the hypotenuse and consider how it was related to the circle. Two sets of technological directions were created in case the student needed assistance. One set provided

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information about which tools to select and click (e.g., use the point tool and click to create three different points). The other set of directions was more general. The task allowed opportunities for students to make conjectures and test them using the technology. The task included questions that required students to extend what they learned in the technological exploration to consider how to find the circumcenter of a triangle drawn on paper.

Delta's and Jonah's individual sessions with their students began in very different ways. Delta handed her student the paper that contained detailed step-by-step directions for the task after the student indicated a lack of familiarity with Geometer's Sketchpad. In contrast, Jonah gave his student the task sheet without the detailed directions after the student indicated that she was familiar with some geometric objects.

The goal for Audrey and Victoria was for students to use a pre-constructed sketch to explore properties and relationships of four different triangle centers: the circumcenter, incenter, orthocenter, and centroid. Five sketches were created in the GSP file; one for each of the four centers, and a fifth sketch that included a "generic triangle" with all four of the centers constructed. On each sketch, measurements were displayed for particular objects (Figure 5). In addition to the pre-constructed sketch, they also designed a worksheet for their students, which included a table that listed each of the four centers. Students were to write what they noticed about how the center relates to the triangle and whether the center always remained inside the triangle. Questions focused students on considering similarities and differences among the four centers.

Audrey and Victoria wanted each student to develop their own definitions for the four centers and notice similarities and differences among them. However, the way in which students arrived at those definitions, and the similarities and differences noted, could vary.

Refinement and Illustration of Focusing Questions

All three of the PSTs' tasks seemed to go beyond simply “amplifying” what they might also do on paper (Laborde, 2002). The tasks required students to construct or drag and observe measurements or other invariants; however, how these interactions were encouraged during task implementation varied. While the four broad categories arose from the literature and initial codes, subcategories were developed from the data (see Figure 5) to capture variations within the four broad categories and reflected the *purpose* for the statement/question. These categories and purposes may enable researchers to better describe the ways in which teachers focus students on mathematics and/or technology while enacting technology-based tasks, and the purpose behind such a focus. We view these categories and purposes (subcategories) as the main contribution of this research. Examples and descriptions of the purposes are provided in the following sections.

Focus on Technology - The teacher makes a statement or poses a question to the student that is only about the technology.	
Purpose	Definition
Orienting students	Focus on providing an overview of elements and structure of a pre-constructed sketch
Specifying software interactions	Focus on actions that are specific to the software being used
Creating or interacting with mathematical objects and measures	Focus on actions for constructing geometric objects and measures
Addressing general technology issues	Focus on ways to complete general technology actions such as clicking in the white space or selecting a menu
Focus on Technology to Notice Mathematics - The teacher makes a statement or poses a question related to a technological action that is related to a mathematical idea.	
Purpose	Definition
Focus on specific actions	Focus on specific actions to be performed, actions that were being performed, or actions that had been performed within the dynamic geometry environment that could provide opportunities to notice or consider mathematical ideas and relationships.
Focus on specific objects or	Focus on a specific object or measure

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measures	without specifying the condition or the particular action to perform.
Focus on meeting conditions	Focus is on meeting a specific condition without indicating the particular action to perform.
Focus on Mathematics with the Use of Technology - The teacher makes a statement or asks the student a mathematical question with the assumption that the student will do something with the technology to respond.	
Purpose	Definition
Focus on connecting mathematics with technological representations	Focus on using mathematics to make sense of pre-created representations or to create a representation with specific mathematical properties
Focus on variance or invariance	Focus on the use of mathematics to describe variances and/or invariances in the GSP sketches.
Focus on Mathematics - The teacher makes a statement or poses a question to the student that is only about the mathematics. The question could be answered without using technology (or could use the sketch in a static manner).	
Purpose	Definition
Making a conjecture	Focus on making a prediction or conjecture in response to a question posed by a student or teacher
Recalling a definition	Focus on recalling a particular definition
Stating a generalization	Focus on describing an invariance using the language of mathematics
Constructing an argument	Focus on providing a justification for a claim

Figure 6. Categories and purposes of questions and statements

The four categories, which emphasize different aspects of mathematics and technology, were useful in better understanding how the PSTs were implementing their tasks with students. What follows are examples from the work of the six PSTs that illustrate the different purposes for questions and statements made within each category for the main focus.

Focus on Technology

PSTs' questions that focused on technology made up a small proportion of questions and statements. How and when these types of questions were posed varied among the PSTs. Statements and questions that were categorized as *focus on technology* were further classified

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into four different purposes: orienting students, specifying software interactions, interacting with geometric objects and measures, and addressing general technology issues.

Orienting students

Questions or statements identified as *orienting students* were those used for the purpose of orienting a student to the pre-constructed sketch or to GSP. These types of statements were typically posed at the beginning of implementation or when a student started working on a different sketch. When introducing their tasks, Victoria, Jessica and Matthew guided their students through an overview of the elements in the sketch. Victoria's orientation was typical of this interaction and seemed to serve the purpose of giving a broad overview of the elements in the sketches.

Victoria: If you click on these tabs here you can see, ... The first page is the circumcenter. [student clicks on a different tab at bottom of file.] Now you're on the incenter. Each triangle, you're going to be able to drag the vertices, you can click on the vertex of the triangle and drag it around and see how the triangle changes. And over here on the left [within the sketch], you can look at different measurements we've chosen to give you some ideas about properties about the centers.

Specifying software interactions

Sometimes PSTs focused on technology for the purpose of *specifying software interactions*. PSTs posed questions or statements about unique ways of interacting with the software. Such actions were used frequently and were critical for students' success. Within the dynamic geometry program used by the PSTs in this study (The Geometer's Sketchpad), these ways of interacting included actions such as choosing a primitive tool (point, circle, segment) from the toolbox when needed for creating the desired geometric object, choosing the arrow tool to be

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able to select or drag objects in a sketch, clicking in the white space of a sketch to deselect all objects, selecting several objects to make certain menu options available, and dragging an endpoint of a segment or ray to increase its length and change its placement in the sketch. These types of interactions are critical for accomplishing simple tasks within the software.

Several of the PSTs often had to direct their students to click in certain ways to select or deselect objects. Jonah's direction to his student was quite typical "but you don't want to select this line. Click there [pointing to white space], and now click on this line." There were also instances of directions and reminders for using the various tools, particularly a focus on selecting the arrow tool to stop using a tool to create geometric objects and to be able to interact with the sketch by selecting and dragging objects (e.g., Audrey: "This command constructs segments [the segment tool]...you press here [the arrow tool] and it will go away"). While almost all of these types of technology focused questions or statements were initiated by a PST, a few were initiated by students, such as the student working with Jonah trying to figure out how to get the measure angle option to appear in the measure menu "Wouldn't it be like measure [clicking on measure menu), hold on, I'd have to select the angle, right?"

Creating or interacting with geometric objects and measures

The purpose of questions or statements within this subcategory related to constructing objects or measures. This includes understanding parent-child relationships and differences between drawing and constructing. These actions are at the crux of how a dynamic geometry program enables users to engage with geometric objects and properties. In a pre-constructed sketch, a teacher could opt to have all objects and specific measures of interest completed for a student. Audrey and Victoria's sketch is an example in which the students did not have to construct objects or measures. Thus, there were no interactions in their episodes that fell into this

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purpose. Given the nature of Jonah and Delta's task, which started with a nearly blank sketch, there were many interactions in their teaching episodes that involved creating objects and measures. A major difference was that most interactions in Delta's teaching session were initiated by the teacher with statements such as "If you click this point, then another point, then this point, then construct angle bisector." In contrast, in Jonah's teaching session, most were initiated by his student who would ask how to complete certain constructions with questions such as "Is there a way to like construct midpoints for that?" and "How do you draw the circle?" A few interactions occurred with Jessica and Matthew's task implementation because there was a need to measure angles (e.g., Matthew: "click, click on ref and then click on teammate click it again see how it highlights pink and then ... go up to measure and then go down to angle.").

General technological issues

As one might imagine, when using a technology tool, it is often necessary to give general technology directions to assist students in navigating a tool or doing a common, non-mathematical operation. For example, when his student did something in the sketch that appeared to have an undesirable result, Jonah said, "if you hit Ctrl Z you can undo it." Other statements or questions of this type may include how to open a file or save work, or how to access some menu items by using a "right-click" mouse action. The actions are not mathematical, though they can be important in facilitating progress on a task.

Focus on Technology to Notice Mathematics

Not surprisingly, there were often interactions between the teacher and student in which statements or questions would focus on technological actions with an opportunity to notice or further consider mathematical ideas. In examining the multitude of instances within this focus, several different purposes of questions and statements were identified. We found evidence to

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suggest three main purposes in questions and statements within this category: 1) a focus on specific actions with technology, 2) a focus on specific objects or measures while interacting with technology, and 3) a focus on using technology to meet certain conditions. How each of these purposes may provide opportunities to notice mathematics is described below.

Focus on specific actions

The purpose of interactions among the teachers and students was often to focus on specific actions to be performed, actions that were being performed, or actions that had been performed within the dynamic geometry environment that could provide opportunities to notice or consider mathematical ideas and relationships. At times the questions and statements specified a particular action, but were open about what might be noticed (e.g., Jonah: And now you can move the vertices around and see how that changes things). However, more often, questions and statements occurred that not only had a specified action, but pointed directly to specific objects or measures that should be considered. These included questions like “Now try moving the vertices around and see how the circle changes” (Jonah), and “if you click off of B and click on to C you can move that left and right [student doing actions], do you see how that's changing this angle?” (Matthew) and statements from students such as “Whenever you drag a point, the side that... so... you've got the point and then you've got the legs going off of the point, and from that point the hypotenuse is never changing” (Student working with Victoria). The questions and statements point directly to what actions are performed and the object or measure as the focus of attention in considering changes.

At times there were particular actions discussed, but the focus on measurements or objects was left vague to invite students to notice an invariance on their own or to make a conjecture. Thus, a student might have noticed aspects of objects or measurements that were more or less

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aligned with a teacher's intention. An example of this occurred when Victoria had her student interacting with a sketch in which the centroid of a triangle was already constructed along with several measurements. She asked the student to drag a vertex and focused her attention on measurements on the left side of the screen (Figure 7), though not to specific measures.

Victoria: [Student has cursor near or on vertex F in Figure 7] You can move [vertex], and see what you notice over there [pointing to measures on left side of screen in Figure 7].

Student: So this measurement does not change, so... (inaudible, talking to herself). Okay, so when you move this vertex towards the base, then both of these angles will be getting smaller. Yeah. So they get smaller. Then as you move away they get larger. Alright, so ED is right here. As you move the vertex closer to the base, all these areas will get smaller.

Victoria: Mmm okay. As the big triangle gets... right, the areas are getting smaller.

Student: Yeah, all of the areas are getting smaller. And as it gets larger....

Figure 7. The sketch Victoria and her student discussed as they focused on specific actions. Though Matthew's and Jonah's questions were more specific, they also provided opportunities for students to consider variant and invariant relationships among geometric objects.

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Focus on specific objects or measures

As seen in the focus on specific actions section, there is often a dual focus on actions and the objects and measures that may change as a result of those actions. However, there were times when what was discussed did not specify any actions to be performed, but would have a focus on a specific object or measure. In this way it seemed that the actions were a secondary, unspoken, focus. This was sometimes due to conversations happening while a student or teacher was performing an action. Thus, it is possible the actions were already an understood focus.

Consider the following question posed by Audrey and Victoria on their worksheet and a student's resultant focus.

Student: [Reading question on worksheet] How are the centroid and the incenter similar and how are they different? (dragging - clicks on different tabs) It seems like the centroid is in the very center of the triangle and the incenter...(drags a vertex)

Audrey: okay

Student: the centroid is always in the very center of the triangle and the incenter moves with the radius of the circle. They are always inside the triangle...

The question focuses on two objects, but does not give any hint at what actions the student should perform to explore the similarity and differences between these two objects. Thus, the student chooses the actions and their focus becomes the relationship between each of these objects and its relation to the triangle from which it is constructed.

Focus on meeting conditions

The focus of attention in many exchanges between a teacher and student was often not about specific actions with technology, but the doing of actions necessary to meet a specific condition. It was this desired condition that became the focus of discussion, rather than the action to meet

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the condition. In this way, the purpose of using technology was to assist students in considering what may be true if a condition occurs and could potentially lead to conjectures or generalizations. A typical question of this type was posed by Delta and Jonah on the handout given to students: “What do you notice about the angles of the triangle when one of the legs intersects the center of the circle?” While this question may also look similar to focus on mathematics with the use of technology, the main goal of the question was to prompt a particular action. The student needed to use various technology actions to meet the condition of legs intersecting the center of the circle, and was then directed to notice something about specific measures, in this case, the measures of angles of the triangle.

A question about meeting a condition may rely on a student or teacher knowing about actions that can be performed (or were performed) and being able to focus on a measure or object in a sketch without being explicitly told to do so. This seemed to happen most often when students were already familiar with a sketch because they had been working on the task for a while, or if the sketch was limited in the ways a student could interact with it and what was visible on the screen. Jessica and Matthew both had their student working with a sketch in which they were told the point labeled “teammate” could be moved along the segment representing the half-court line. Then they were asked on a worksheet “What is the shortest pass that you were able to make to your teammate?” Thus, an action of dragging the teammate is implied but the purpose of the question is on meeting a condition. One student working with Jessica, after dragging, noted that the shortest pass was 7.1. A single measure labeled Distance of Pass was visible on the screen, thus the student likely did not have to be directed to focus on it.

Even though a teacher often initiated the focus on a specific condition, this was not always the case. At times a student decided to consider a specific condition. Audrey and Victoria’s

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students illustrate this in the following episodes that follow from questions posed on a handout: “Write down what you notice about the four centers in relationship to the triangle and to each other when you drag the vertices.” The question gave a specific action (drag vertices) and directed the student to attend to the relationship among five objects (4 centers and the triangle). Victoria had initiated the request to do a specific action with an open opportunity for noticing.

Victoria: This is all four centers on one triangle. So you can move the vertices around and see what happens.

Student: [dragging a vertex] That's a right triangle.

Victoria: Yeah.

Student: So when you have a right triangle, the orthocenter will be on the 90 degree point and circumcenter will be on the hypotenuse.

After being asked the same question by Audrey, we see again that a student initiates a specific condition to help answer the question. While dragging, the student chooses to focus on meeting a specific condition, the case of an isosceles triangle.

Student: it seems like if you create an isosceles triangle they all lie in one line...

Audrey: right

Student: if you make an isosceles triangle they are on one line (dragging). Looks like they are always lined in the same order... orthocenter, incenter, centroid, and circumcenter

As illustrated in the examples, the questions and statements that were posed to consider a specific condition often led to goal-directed activity with the technology and often resulted in a noticing or conjecture.

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Focus on Mathematics with the Use of Technology

Throughout each task, there were also times when a mathematical question was posed without a specific direction for how to use the sketch, although the use of technology was expected. These types of questions took two different forms. Sometimes they served as questions prompting students to make connections between mathematical properties and constructed representations. Other questions required students to notice or make conjectures based on variances/invariances in the sketch.

Focus on connecting mathematics with technological representations

The purpose of these types of questions and statements was to involve students in trying to make sense of pre-created representations or trying to create a representation with specific mathematical properties. In the first case, Audrey prompts her student to explain the mathematics behind a construction and the student responds:

Audrey: how were those triangles created?

Student: they were created by bisecting the three angles and by finding the midpoint of the three sides... and well this is kind of like the other triangle where you created perpendicular lines...

Jonah's student was asked to construct a circle that circumscribed a triangle in GSP. She responded, "Okay. So can I just get the circle, and go...?" Both students were trying to make sense of the mathematical properties involved in constructions. Teachers prompted students to make connections either by asking how something was created, like Audrey did, or by asking students to create something on their own. For example, "how are we going to make a circle that circumscribes this triangle?"

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A more open-ended approach to this type of focus came from Jonah. He reminded the student of the type of relationships or constructions that might be useful to consider with technology, but did not provide specific directions or connections. Two examples include:

“So use what you know of midpoints, medians, angle bisectors, perpendicular bisectors, um, altitudes...anything like that. And see how they fit together and see what you can do from there okay?”

“Um, so if the medians don't work, try something different.”

Focus on variance or invariance

There were many instances of teachers directing students to look for or notice variances and/or invariances in the GSP sketches. Sometimes they were given a very specific variance/invariance to investigate, for example, “Does the orthocenter always stay in the triangle?” or “What do you notice about the angles of the triangle when one of the legs intersects the center of the circle?”

Other times, the questions were more open: “Do you notice anything about that center in relation to the circle?” This type of question or statement often resulted in the student simultaneously making a conjecture while dragging something in the sketch. Often, their conjecture would change while engaging with the sketch, which would imply that the students were responding to this type of prompt with Doing a Technological Action and Noticing Something Mathematical.

For example, Jonah's student was asked about the relationship between the circumcenter and one of the triangle angles. She proceeded to drag the vertex of the triangle and describe what was happening to the circumcenter as she dragged:

Student: Yeah it is. But then it's up above 90 degrees the whole time. Yep I see that. Oh it's

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inside of the triangle when the, what should this angle be called? Like, hold on. But that doesn't. But. When the circumcenter is outside of the triangle, it's under 90 degrees, but when it's inside, it's above 90 degrees.... No it's not, it's under 90 degrees. Hold on, there it is. But then when it is outside, hold on, wait, there's the triangle, that's is that the circumcenter? Oh, oh, when it's outside. Hold on. This is the circumcenter right?

Jonah: Okay. So, it's inside, so its' ab...under...

Student: Yeah. So, b [the angle] is under 90 degrees when it's [the circumcenter] outside.

Hold on. Whoa, there, what happened?

Some questions were even more open-ended. "What do you notice about those measurements?" or "What do you notice about these two triangles?"

In one case, a student made an incorrect generalization based on a static sketch so the teacher prompted the student to use dragging to check their conjecture. Audrey wanted her student to determine whether the altitudes of a triangle (which form the orthocenter) also bisected the angles of the triangle. The student had created a triangle that appeared to be equilateral and generalized that the altitudes also bisected the angles. To make sure the student did not infer this, Audrey wanted the student to create other types of triangles by dragging. In response, the student dragged a vertex of the triangle and determined that the altitudes did not necessarily bisect the angles.

Audrey: They are perpendicular to the sides and you said they go through the vertices, right?

(Student: yes) Do they necessarily bisect, you said they bisect... you remember what we said about bisecting the angles? Means it creates two congruent angles, right? Do they necessarily bisect? Try to drag the triangle...

Student: no, not always... okay!

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In general, these questions were not specific about how to use technology, but suggested that a mathematical relationship existed and could be explored using the technology. Sometimes the suggested use of technology was inferred from previous statements and actions, other times it was more specifically stated within a question.

Focus on Mathematics

Statements and questions from teachers and students that were categorized as focusing on mathematics were typically for four different purposes: making a conjecture, recalling a definition, stating a generalization, or constructing an argument. There were also instances when the teacher would pose a similar or related question or problem that was also categorized as focusing on mathematics.

Making a conjecture

There were times when a teacher would pose a question and the student would make a prediction or conjecture about the answer to that question. For example, after examining the location of the circumcenter in relation to the triangle Delta asked, "What about if I constructed [pause] If I constructed angle bisectors, are they in(side) or out(side) (the triangle)? The student immediately stated, "they'd be in." However it was not clear whether he was convinced of his answer because he spent several minutes considering the question using the technology after he provided his response. In this case, the question was posed by the teacher and the student initially made a prediction about the answer using what he knew about mathematics.

Other examples from Jonah and Jessica can further illustrate the purpose of making a conjecture. Jonah's student was working on a sketch that contained a triangle and trying to figure out how to locate the center of a circle that would circumscribe the triangle. After trying medians and noticing that they would not work the student posed a question: "Student: Um, I could use...

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angle bisectors. Do you think that would work? Angle bisectors?" She went ahead and constructed angle bisectors using the technology, suggesting that she believed they might work and was using the technology to determine whether it would work. Jessica explicitly asked her student to make a conjecture about the shortest pass between a player and his teammate. In response to this request the student responded, "Wouldn't the shortest one be straight ahead from you?"

Conjectures were identified in the form of questions posed by the teacher or student and responses from the student about which there may have been some doubt expressed.

Recalling a definition

Sometimes PSTs posed questions and statements for the purpose of assisting students in recalling a definition. Victoria typically asked questions of her student to determine whether they recalled the name of a particular geometric object. For example, when her student was working on the "Circumcenter" sketch of the triangle centers task she asked a series of questions:

Victoria: Circumcenters where those three lines meet, right? So, where they intersect, and these lines here, you notice those are always 90 degrees.

Student: Yeah.

Victoria: Do you know what you call that when this is 90 degrees with that? Starts with a "p." I don't know if you've had that before.

Student: Uh uh.

Victoria: It's perpendicular.

Student: Oh, right.

Victoria: Yeah. Those are perpendiculars. So you notice that. Yeah, that's good, you notice the angles there

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Similar to Victoria, Audrey asked questions of her student to determine whether she was familiar with the term altitude.

Audrey: you remember what we call it, maybe you are not familiar with the terms how we call it... but know what it is, the line. It goes through a vertex and perpendicular to the opposite side. You remember what we call it?

Student: Oh. No.

Audrey: We call it an altitude

Student: Oh yeah

Audrey: You remember now?

Student: yes, I do!

For both Audrey and Victoria, teaching students the correct language and terminology was an important goal related to their task and when these conversations took place they typically focused only on mathematics. For Jessica, Matthew, Delta, and Jonah there were also instances of teacher prompts to determine the knowledge students had about a particular term or word. The occurrence of these questions may be because the PSTs had limited knowledge about what these students had already studied and learned during their geometry class.

Stating a generalization

Often after working with technology, a student would use the language of mathematics and their knowledge of mathematics to describe the invariance they observed in terms of a generalization. There were also instances when a teacher encouraged students to create a generalization. For example, after Delta's student had examined relationships between a right triangle and the circumscribed circle, she engaged in the following discussion in an attempt to guide her student in making a generalization, albeit not the one she had in mind:

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Delta: The hypotenuse of the right triangle. What is always constant about that?

Student: Oh, it's perpendicular to the angle bisector of the 90 degree angle. And it forms two congruent line segments. Two radii.

Delta: That's true. It's not what I'm looking for, but it's true. But think about it. You said it all along. The hypotenuse of the right triangle lies where on your circle.

In other cases, the student would make a statement about a relationship that would "always" be true. For example, when Matthew's student was determining the shortest pass for a basketball player to make to his teammate he was able to abstract from that context that, "Student: So like the shortest path is always going to be 90 degrees."

Constructing an argument

The last type of statement or question that had a *focus on mathematics* were those in which the student was focused on providing a mathematical justification or warrant for a claim that they made. There were instances when a PST pushed a student to provide an explanation, and also instances of when a PST provided the explanation to a student.

For example, a student working on the task about determining the shortest distance between a point and a line made specific reference to the theorem that she was using. She stated, "I used the linear pair conjecture most of the time because that just states that two angles have to equal to 180 degrees on a straight line." There were other instances when the connection between the claim and justification provided by a student was less clear. For example a student posed and answered the following question, "Is the incenter always inside the triangle? Yes, because the circle is inside the triangle." Another student responded to a question Delta posed about when the circumcenter would be inside the triangle. Her student stated, "Wouldn't it be when it's

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equilateral? Cause, like when all the sides are the same, the circle's going to be on the inside, cause it can touch all of the sides, all of the points, all of the midpoints?"

In each case the student attempted to provide reasons for their statements. This might have been in part because the teacher encouraged this. Jonah posed a question to encourage his student to explain why, "Okay, now why would both of these angle measures be 90 degrees?" In another instance, Delta stated, "I'm just asking yes or no...If you are yes or no, you just have to provide a reason, explanation. " In both of these examples, the PST prompted the student to provide an explanation.

Discussion

The goal of this study was to examine questions posed and statements made by pre-service teachers and students while engaged with a dynamic geometry task. From the literature and our data, we described four broad foci a question or statement could have when learning mathematics while using technology. These included, 1) *focus on technology*, 2) *focus on technology to notice mathematics*, 3) *focus on mathematics with the use of technology*, and 4) *focus on mathematics*. These categories are similar to the epistemic modes described by Leung (2011). However, they go a step further by considering within each focus a purpose for which a student or pre-service teacher might pose a question or make a statement. For example, if a pre-service teacher wants to focus students on technology, there are particular questions we observe him/her using to achieve that focus. And while questions that suggest a single focus are easier to generate, posing questions and statements that address relationships between technology and mathematics is more difficult to do. Pre-service teachers tended to ask broad questions such as, "What do you notice?" when really what they wanted students to notice is a particular relationship or invariance. It is possible that mathematics educators can help support pre-service

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teachers in posing questions that focus students on the invariant relationship they are interested in. This research study provides additional information about how pre-service teachers might pose questions that address the relationship between actions and behaviors in a technological environment and abstract mathematical properties. If pre-service teachers are more aware of how their statements and questions are focusing on mathematics and/or technology, perhaps they can be more purposeful in how they are posed and sequenced when working with students. This may be an interesting study to conduct in the future.

In our study we found that while PSTs did ask questions that considered mathematics and technology, they did not push students to explain why a relationship they observed might be true. Rather, our PSTs simply asked students to record what they observed. This might be related to the mathematical knowledge of the PST and their inability to apply it to the students' thinking that was emerging from a technological-based task. It may be that PSTs were having difficulty considering student thinking as expressed in an environment different from the environment the PST may have learned that mathematics. Perhaps, the PSTs assumed they were to take on the role of an observer rather than a teacher. Sometimes PSTs would ask students to go back to the task, but not for the purpose of "making signs emerge" while using the technology (Bussi & Mariotti, 2008, p. 42) or for abstracting new mathematical knowledge (Leung, 2011). This is similar to the findings of Even and Tirosh (1995) as well as Son and Crespo (2009) who reported teachers rarely went beyond the surface aspects of a student's solution when analyzing and responding to students' work.

We also noticed that in all but one task implementation, PSTs began with questions focused only on technology. We might consider this as an important first step if we believe it is important for students to be familiar with the environment and experience instrumental genesis. However,

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it may be that PSTs are themselves still in the process of learning how to use these tools and the questions they pose reflect their own limited understanding of the artifact and embody their own discomfort with it as a tool. Perhaps PSTs' own instrumental genesis is reflected in the ways they interact with students learning mathematics with technology. Maybe the different purposes for posing statements and questions with technology we have identified are linked with particular stages of teachers' acceptance of technology or comfort with using it as a learning and teaching tool. Future research that examines relationships between teachers' mathematics and technological knowledge and their interactions with students may shed more light on some of these issues.

Although the focus of our analysis was on questions that Mason (2000) described as suggesting a particular focus, we found that there were several instances of questions that were posed to seek or test information. Questions of the latter two types (seeking or testing information) tended to occur within the student-teacher-mathematics face of the tetrahedron (e.g., Recalling a definition) whereas questions that suggested a particular focus were not bound to a particular face of the tetrahedron. These were identified as prompts from the student or teacher to use the technology to notice something mathematical or to consider a mathematical question with the assumption that students would use the technology. For these types of questions, opening up the tetrahedron to examine interactions among the faces as depicted in a net was helpful. Future researchers may find it interesting to examine how the use of particular types of questions influences how students learn particular mathematical ideas.

While Olive et al. (2010) chose to modify the tetrahedron to consider task, student, teacher, and technology as the vertices and mathematical knowledge and practices as the space that is created within the vertex, we believe it is still important to include mathematics as a vertex so as

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to carefully analyze interactions between students and teachers around mathematical ideas. We agree with Ruthven (2012) who states, “the operational core of any didactic lies in the forms of activity and discourse through which curricula content is animated so as to become appropriable by students” (p. 628). The animation of the curricula content in this study was embodied in the particular tasks PSTs created and influenced greatly by the way these tasks were implemented.

While the focus on mathematics and technology was a deliberate choice made by the researchers, it may also be viewed as a limitation in terms of what we were able to observe and describe. Other limitations include the limited opportunities our pre-service teachers were provided to work with students and short amount of time they were provided to become familiar with the technology tools they were using and the mathematics content they were teaching. A more extended study, or even one that is longitudinal in nature, may provide a broader picture of the ways teachers develop their abilities to develop and implement technology-based tasks with students.

References

- Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. *Zentralblatt für Didaktik der Mathematik*, 34(3), 66-72.
<http://dx.doi.org/10.1007/BF02655708>
- Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). Taxonomy of educational objectives - Handbook 1: Cognitive domain. *New York: David McKay*.
- Bowers, J., & Doerr, H. M. (2001). An analysis of prospective teachers' dual roles in understanding the mathematics of change: Eliciting growth with technology. *Journal of*

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

- Mathematics Teacher Education*, 4(2), 115-137.
<http://dx.doi.org/10.1023/A:1011488100551>
- Brousseau, G. (1997). Theory of didactical situations in mathematics (Edited and translated by N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield). *Dordrecht, NL: Kluwer*.
- Bussi, M. B., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. *Handbook of international research in mathematics education (2nd ed., pp. 746-783)*. New York: Routledge.
- Christou, C., Mousoulides, N., Pittalis, M., & Pitta-Pantazi, D. (2004). Proofs through exploration in dynamic geometry environments. *International Journal of Science and Mathematics Education*, 2(3), 339-352. <http://dx.doi.org/10.1007/s10763-004-6785-1>
- Cuoco, A. & Goldenberg, P. (1997). Dynamic geometry as a bridge from Euclidean geometry to analysis. In J. R. King and D. Schattschneider (Eds.) *Geometry turned on!: Dynamic software in learning, teaching, and research* (pp. 33-46). Washington, D.C.: The Mathematical Association of America.
- Dick, T. (2011). Introduction. In T. Dick & K. Hollebrands (Eds.), *Focus in high school mathematics: Technology to support reasoning and sense making* (pp. xi-xvii). Reston, VA: NCTM. <http://dx.doi.org/10.1515/9781400840007.1>
- Doerr, H. M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics*, 41(2), 143-163.
<http://dx.doi.org/10.1023/A:1003905929557>
- Dreyfus, T. (1991). *Advanced mathematical thinking processes* (pp. 25-41). Springer Netherlands.

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

- Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213-234. <http://dx.doi.org/10.1007/s10649-010-9254-5>
- Even, R., & Tirosh, D. (1995). Subject-matter knowledge and knowledge about students as sources of teacher presentations of the subject-matter. *Educational studies in mathematics*, 29(1), 1-20. <http://dx.doi.org/10.1007/BF01273897>
- Farrell, A. M. (1996). Roles and behaviors in technology-integrated precalculus classrooms. *The Journal of Mathematical Behavior*, 15(1), 35-53. [http://dx.doi.org/10.1016/S0732-3123\(96\)90038-3](http://dx.doi.org/10.1016/S0732-3123(96)90038-3)
- Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer.
- Goldenberg, E. P & Cuoco, A. (1998). What is dynamic geometry? In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 351- 367). Mahwah, NJ: Lawrence Erlbaum Associates
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior*, 22(1), 73-89. [http://dx.doi.org/10.1016/S0732-3123\(03\)00005-1](http://dx.doi.org/10.1016/S0732-3123(03)00005-1)
- Hargie, O. D. (1978). The importance of teacher questions in the classroom. *Educational Research*, 20(2), 99-102. <http://dx.doi.org/10.1080/0013188780200203>
- Heid, M. K., Blume, G. W., Zbiek, R. M., & Edwards, B. S. (1998). Factors that influence teachers learning to do interviews to understand students' mathematical understandings. *Educational Studies in Mathematics*, 37(3), 223-249. <http://dx.doi.org/10.1023/A:1003657820047>

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

- Hollebrands, K., & Heid, M. K., (2005) Patterns of secondary mathematics students' representational acts and task engagement in a small-group technology-intensive context. In M. Wilson & G. Lloyd (Eds.) *Proceedings of the Twenty-Seventh Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.
- Hollebrands, K. F., & Lee, H. S. (2012). *Preparing to teach mathematics with technology: An integrated approach to geometry*. Dubuque, IA: Kendall Hunt Publishers.
- Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics*, 44(1-3), 55-85.
<http://dx.doi.org/10.1023/A:1012789201736>
- Laborde, C. (1993). The computer as part of the learning environment: The case of geometry. In C. Keitel & K. Ruthven (Eds.), *Learning from computers: Mathematics education and technology* (pp. 48-67). Berlin, Germany: Springer Verlag. http://dx.doi.org/10.1007/978-3-642-78542-9_3
- Laborde, C. (2002). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6, 283-317.
<http://dx.doi.org/10.1023/A:1013309728825>
- Lee, H.S. (2005). Facilitating students' problem solving in a technological context: Prospective teachers' learning trajectory. *Journal of Mathematics Teacher Education*, 8(3), 223-254.
<http://dx.doi.org/10.1007/s10857-005-2618-6>
- Leung, A. (2011). An epistemic model of task design in dynamic geometry environment. *ZDM*, 43(3), 325-336. <http://dx.doi.org/10.1007/s11858-011-0329-2>

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

- Lobato, J. & Ellis, A. (2002). The focusing effect of technology: Implications for teacher education. *Journal for Technology and Teacher Education*, 10(2), 297-314.
- Lobato, J. Ellis, A., & Munoz, R. (2003). How focusing phenomena in the instructional environment support individual students' generalizations. *Mathematical Thinking and Learning*, 5(1), 1-36. http://dx.doi.org/10.1207/S15327833MTL0501_01
- Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *Journal of Mathematical Behavior*, 18(1), 53-78. [http://dx.doi.org/10.1016/S0732-3123\(99\)00017-6](http://dx.doi.org/10.1016/S0732-3123(99)00017-6)
- Mason, J. (2000). Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology*, 31(1), 97-111. <http://dx.doi.org/10.1080/002073900287426>
- Miera, L. (1995). The microevolution of mathematical representations in children's activities. *Cognition and Instruction*, 13, 269-313. http://dx.doi.org/10.1207/s1532690xci1302_5
- National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. NCTM: Reston, VA.
- National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.
- Olive, J., Makar, K., Hoyos, V., Kor, L., Kosheleva, O., & Sträßer, R., (2010). Mathematical knowledge and practices resulting from access to digital technologies. In C. Hoyles & J.B. Langrange (Eds.), *Mathematics education and technology — rethinking the terrain* (pp. 133-177). New York: Springer.

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

Ozgun-Koca, S.A., Meagher, M., & Edwards, M.T. (2009). Preservice teachers emerging TPACK in a technology-rich methods class. *The Mathematics Educator*, 19(2), 10-20.

Ruthven, K. (2012). The didactical tetrahedron as a heuristic for analysing the incorporation of digital technologies into classroom practice in support of investigative approaches to teaching mathematics. *ZDM Mathematics Education*, 44, 627-640.

<http://dx.doi.org/10.1007/s11858-011-0376-8>

Sinclair, M. (2003). Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials. *Educational Studies in Mathematics*, 52(3), 289-317. <http://dx.doi.org/10.1023/A:1024305603330>

Sinclair, N., & Yurita, V. (2008). To be or to become: How dynamic geometry changes discourse. *Research in Mathematics Education*, 10(2), 135-150.

<http://dx.doi.org/10.1080/14794800802233670>

Sloan, F. A., & Pate, R. T. (1966). Teacher-pupil interaction in two approaches to mathematics. *The Elementary School Journal*, 67(3), 161-167. <http://dx.doi.org/10.1086/460353>

Son, J. W., & Crespo, S. (2009). Prospective teachers' reasoning and response to a student's non-traditional strategy when dividing fractions. *Journal of Mathematics Teacher Education*, 12(4), 235-261. <http://dx.doi.org/10.1007/s10857-009-9112-5>

Stein, M.K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488.

<http://dx.doi.org/10.3102/00028312033002455>

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

- Stein, M. K., Smith, M., Henningsen, M., & Silver, E. (2000). Implementing standards-based mathematics instruction: A casebook for professional development. Teachers College Press
- Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: An epistemological perspective. New York: Springer.
- Tall, D. (1986). *Using the computer as an environment for building and testing mathematical concepts: A tribute to Richard Skemp*. Retrieved from:
<http://www.warwick.ac.uk/staff/David.Tall/themes/computers.html>
- Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9(3), 281-307. <http://dx.doi.org/10.1007/s10758-004-3468-5>
- Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrument activity. *European Journal of Psychology in Education*, 9(3), 77-101. <http://dx.doi.org/10.1007/BF03172796>
- Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.
- Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. Bartolini-Bussi & A. Sierpiska (Eds.), *Language and communication in the mathematics classroom*. pp. 167-178. Reston, VA: National Council of Teachers of Mathematics.
- Zbiek, R.M., Heid, M.K., Blume, G., & Dick, T. (2007). Research on technology in mathematics education: The perspective of constructs. In F. Lester (Ed.), *Second handbook of research*

TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY TASKS

on mathematics teaching and learning. (pp. 1169-1208). Charlotte, NC: Information Age Publishing.

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