A Description of the Initial Fire Sprinkler Spray

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Abstract

Accurate representation of the fire sprinkler spray enables quantitative engineering analysis of fire suppression performance. Increasingly, fire sprinkler systems are analyzed with computational fluid dynamics (CFD) fire models where the sprinkler spray is simulated with Lagrangian particles dispersed throughout the fire induced flow. However, there is limited guidance for representing the complex, spatio-stochastic characteristics of the initial sprinkler sprays in terms of these Lagrangian particles. The present work establishes a descriptive analytical framework for the initial sprinkler spray that is rigorously grounded in statistical theory, related to local spray properties, and capable of translating high-fidelity measurements into CFD inputs. This framework describes the initial sprinkler spray as a unified probability distribution function, varying over an initialization surface, and statistically representing measurements of near field local spray properties (volume flux, drop size distribution, and drop size-velocity correlation). Lagrangian particles accurately representing the sprinkler spray may be initialized by a stochastic sampling of this probability distribution function. This novel representation enables high-fidelity initialization of the sprinkler spray in CFD fire models, improving their utility in quantitative engineering analysis.

Keywords: fire sprinkler; spray modeling; atomization; suppression; spray theory; drop distribution function; Lagrangian particle; fire model; computational fluid dynamics; CFD

Nomenclature

$\vec{a}$	acceleration
d	drop diameter
D	rate of change of drop diameter
d_{v50}	median drop size
f^*	drop distribution function
f_N	drop number probability density function
f_V	volume flux probability density function
$f_{V,\Omega}$	angular volume flux probability density function
$f_{V,d}$	drop size volume flux probability density function
F_V	volume flux cumulative distribution function
$F_{V,\Omega}$	angular volume flux cumulative distribution function
$F_{V,d}$	drop size volume flux cumulative distribution function
G_V	inverse volume flux cumulative distribution function
N	drop number
N_p	Lagrangian particle number
$\dot{N}$	drop flow rate
$\dot{N}_p$	Lagrangian particle injection rate
$\hat{n}$	normal vector to a surface
p	probability space spanning 0 to 1
Q	rate of change of drop number
r	radius
r_0	initialization radius
r_{bu}	breakup radius
r_m	measurement radius
t	time
$\vec{u}$	velocity
u_0	initialization velocity
u_{bu}	breakup velocity
u_m	measured velocity

Nomenclature (cont.)

$\dot{V}$	volume flow
$\dot{V}''$	volume flux
W_p	particle weighting factor
$\vec{x}$	position

Greek Symbols

Γ	drop size distribution width
θ	elevation angle
ψ	azimuthal angle
Ω	solid angle

1. Introduction

Fire sprinklers provide a simple and cost effective spray dispersion method to arrest fire growth [1]. A typical pendent sprinkler, as seen in Fig. 1, consists of a thermal activation element and flow controlling elements; specifically, the orifice, frame arms, boss, and deflector. Upon activation, a water jet (created by the orifice) strikes the frame arms, boss, and deflector, transforming the jet into a complex, cascading series of sheets, jets, and drops [2]. These sheets and jets move outward away from the sprinkler, interact with the surrounding air, and, because of aerodynamic instabilities, eventually break into drops. These drops form a dilute spray flying outward; dispersing through combustion products, plume, flame, and reactants; and interacting with surfaces.

The jet impingement atomization method produces a highly non-uniform spray, consistent with measurements by Sheppard [3, 4]. Recent work by Ren [5] introduced a method characterizing measurements of the near-field fire sprinkler spray in support of numerical simulation of sprinkler spray dispersion. These measurements, as well as detailed measurements by Zhou [6], reveal large elevation and azimuthal variations in near-field volume flux, drop size, and velocity corresponding to geometric features of the fire sprinkler head. An example typifying these spatial variations is seen in Fig. 2, a measurement of the near-field

20 volume flux for a typical, pendent-type sprinkler head, measured in the University of Maryland’s spray lab with the Spatially-resolved Spray Scanning System (4S) [7]. Any description of the fire sprinkler spray used for high fidelity fire protection analysis must capture these spatial variations.

Increasingly, computational fluid dynamics (CFD) fire models, like the Fire Dynamics Simulator (FDS) or FireFOAM, are trusted to inform fire protection
25 decisions [8, 9]. Both FDS and FireFOAM use an Eulerian-Lagrangian (EL) approach to describe multiphase flows. In the EL approach, the continuous phase (i.e. the gas phase, including the combustion products, plume, flame, and reactants) is represented as an evolving Eulerian field, while the dispersed
30 phase (i.e. the sprinkler spray) is modeled using Lagrangian particle tracking [10]. In these models, Lagrangian particles, representing the sprinkler spray, are injected into the modeled domain. Particle motion is determined by solving the Lagrangian equations of motion and interaction with the Eulerian continuous phase is handled through a variety of sub-models (e.g. heat transfer, drop
35 evaporation, turbulent dispersion, etc.).

There is limited guidance as to how the complex, spatio-stochastic characteristics of the initial fire sprinkler sprays can be represented in terms of these Lagrangian particles. Modeling is further complicated by the limited ability of current fire models to predict fire sprinkler atomization [6]. As a result, the
40 initial fire sprinkler spray must be specified from measurements. Work by Ren [5] successfully identified the importance of measured local spray properties, but did not fully connect these filtered measurements to a robust statistical description of the flow or provide guidance to their implementation in high fidelity Lagrangian particle spray dispersion simulations.

45 The present work seeks to provide an initial spray description grounded in statistical theory, tied to near-field measurements, and suitable for translating these measurements into a Lagrangian spray modeling framework. First, the drop distribution function is introduced, providing a rigorous statistical description for any dilute spray. This drop distribution function is then applied
50 and adapted, whereby the initial sprinkler spray is described by a multivariate

probability density function varying across an initialization surface. This probability density function, and the corresponding cumulative distribution function are related to locally measured spray properties. The net result of the current study is an initial fire sprinkler spray description scheme, for use in measurement and modeling, presented with requisite theoretical basis, implementation framework, and CFD application guidance.

2. Theory

A dilute spray is stochastic and locally unsteady. Despite this behavior, at sufficiently large spatial and temporal scales, a spray may be statistically described by a probability distribution function, f^* , called the drop distribution function [11]. This distribution function is defined such that the number of drops, N , in a phase space is assumed to be quasi-steady with time and is given as

$$\int_{[\vec{x}, \vec{u}, d]} f^*(\vec{x}, \vec{u}, d) d\vec{x} d\vec{u} dd = N, \quad (1)$$

where the phase space spans the volume space, $\vec{x}$, the velocity space, $\vec{u}$, and the drop diameter space, d . For convenience, the phase space over which the drop distribution function exists may vary, and is denoted by the variables in parenthesis. As a result, the dimensions of f^* may vary depending on the defined phase space.

The drop distribution function description of a spray has the advantage of being conceptually consistent with the way dilute sprays are measured, where the statistical properties of the spray are determined by measuring the spray in a limited volume of space over a sufficiently long period of time.

The evolution equation for the drop distribution function, called the spray equation, is given as

$$\frac{\partial f^*}{\partial t} = -\frac{\partial}{\partial d} (Df^*) - \vec{\nabla}_x \cdot (\vec{u}f^*) - \vec{\nabla}_u \cdot (\vec{a}f^*) + Q, \quad (2)$$

75 where

$$\vec{u} \equiv \frac{d\vec{x}}{dt}, \quad (3)$$

$$\vec{a} \equiv \frac{d\vec{u}}{dt}, \quad (4)$$

$$D \equiv \frac{dd}{dt}, \quad (5)$$

and Q accounts for the formation of new drops in a non-colliding spray.

The direct solution of the spray equation, Eq. 2, is non-trivial and computationally expensive with traditional finite-volume or finite difference techniques [12, 13]. Instead, an alternative solution based on Lagrangian particle methods
80 is commonly used to approximate the solution to Eq. 2.

In the Lagrangian particle method, the flow is seeded with N_p Lagrangian particles chosen represent the drop distribution function. Each computational particle may represent many drops, having an associated position vector, $\vec{x}$, velocity vector, $\vec{u}$, diameter d , and particle weighting factor, W , corresponding
85 to the number of drop represented. These particles are, in effect, a statistical sampling of the drop distribution function. These N_p particles may then evolve individually according to Eqs. 3-5, indirectly satisfying the spray equation, Eq. 2.

Lagrangian particle evolution methods are well developed and well understood
90 [12, 13]. The challenge for fire sprinkler sprays is describing the strong spatial variations measured in the initial spray and initializing Lagrangian particles to accurately represent these measurements.

3. Initial Spray Description

Because the state of the art in sprinkler sprays does not yet allow for predic-
95 tion of atomization, initial sprays are generally described through a set of measured spray properties: volume flux, drop size, and velocity. Further, because

strong spatial variations exist in typical sprinkler sprays, these spray properties must be measured locally. These local spray properties can be related to the probability density function, f , to completely describe the spray.

100 The probability density function is directly related drop distribution function, Eq 1 by

$$f_N(\vec{x}, \vec{u}, d) = f^*(\vec{x}, \vec{u}, d) / N, \quad (6)$$

where f_N describes the number fraction of drops in the multivariate phase space. The drop number probability density function, f_N , has the property

$$\int_{[\vec{x}, \vec{u}, d]} f_N(\vec{x}, \vec{u}, d) d\vec{x} d\vec{u} dd = 1. \quad (7)$$

The drop number probability density function, f_N , can be used to stochastically inject Lagrangian particles to represent measurements.

3.1. Relationship to Measurement Topology

To facilitate conversion between measurements and modeling, the probability density function should reflect measurement topology and be directly related to measured properties.

110 To minimize the influence of the gas phase on the spray, the spray is measured as close to the sprinkler head as possible. However, the spray can only be measured after complete atomization. Atomization occurs at a radius from the sprinkler head of between 0.1 and 0.4 m for most sprinklers [14]. Further, experimental observations have shown that the spray, when very near the sprinkler head, tends to move radially outwards from the sprinkler head [5]. As a result, measurements are generally made along a spherical surface with its origin on the sprinkler head.

The probability density function, f_N , given in Eq. 7, may be redefined in spherical coordinates as

$$\int_{[\vec{r}, \vec{u}, d]} f_N(\vec{r}, \theta, \phi, \vec{u}, d) r^2 dr d\Omega d\vec{u} dd = 1, \quad (8)$$

120 where $\vec{r}$ is the radial coordinate with an origin on the sprinkler head, θ is elevation angle, and ϕ is azimuthal angle, and where $d\Omega$ is the differential solid angle defined as

$$d\Omega = \sin(\theta) d\theta d\phi. \quad (9)$$

For sprinkler sprays it is generally not the number of drops per unit volume but the volume flux at surfaces that governs sprinkler performance [1]. Keeping
125 with this flux perspective, the drop distribution can be described in terms of a drop volume flux through a given surface (instead of in terms of a drop number in a volume) given by

$$\int_{[r,\theta,\phi,\vec{u},d]} f_V(r, \theta, \phi, \vec{u}, d) r(\theta, \phi)^2 dr d\Omega d\vec{u} dd = 1, \quad (10)$$

where

$$f_V(r, \theta, \phi, \vec{u}, d) = f_N(r, \theta, \phi, \vec{u}, d) (\vec{u} \cdot \hat{n}) (\pi/6) d^3 / \dot{V}, \quad (11)$$

$\hat{n}$ is the vector normal to the surface, and

$$\dot{V} = \dot{N}(\pi d^3/6). \quad (12)$$

130 This function describes the density of water volume flux passing through a given angular position on the surface $r(\theta, \phi)$. While Eq. 10 is valid in general, the surface flux based formulation is especially well suited for describing the near-field spray characteristics on the surface where measurements are made, $r_m(\theta, \phi)$, and correspondingly where Lagrangian particles may be initialized
135 with known characteristics.

3.2. Relationship to Measured Properties

The volume flux based probability density function, f_V , is chosen to represent the initial spray because it conveniently reflects measurement methods. What remains is to relate the measured properties to the probability density
140 function.

As previously mentioned in Sec. 3.1, a spherical coordinate system was chosen to describe the spray because observation of the spray reveals that all drops in the initial spray tend to move radially away from the sprinkler head [5]. Because the velocity vector of the initial spray is assumed to be directed
145 approximately radially out of the sprinkler head, it may be seen that

$$\vec{u} \cdot \hat{n} \approx |\vec{u}|, \quad (13)$$

where $\hat{n}$ is aligned with $\vec{r}$ at any given angular location. Additionally, drops of the same size tend to move at the same velocity, as seen in Fig. 3, which shows the drop size-velocity correlation for drops measured in the same pendent sprinkler as Fig. 2 at $\theta = 115^\circ$ and $\phi = 245^\circ$. As a result, drop velocity can be
150 described as a characteristic velocity, $u_0(\theta, \psi, d)$.

If all drops are assumed to emerge from the surface $r_0(\theta, \psi)$ with an initial velocity, $u_0(\theta, \psi, d)$, the phase space of the volume flux probability density function, f_V , from Eq. 10, may be further restricted to vary only with angular location, (θ, ϕ) , and drop size, d , as

$$\int_{[\theta, \phi, d]} f_V(\theta, \phi, d) r_0(\theta, \phi)^2 d\Omega dd = 1. \quad (14)$$

155 This probability density function may be decomposed into the conditional probabilities

$$f_V(\theta, \phi, d) = f_{V,\Omega}(\theta, \phi) f_{V,d}(d|\theta, \phi), \quad (15)$$

where $f_{V,\Omega}(\theta, \phi)$ is the probability of finding volume flux from *any* drops emerging from the angular coordinates (θ, ϕ) , and $f_{V,d}(d|\theta, \phi)$ is the probability of finding volume flux from drops of size d given an angular location (θ, ϕ) .

To match the literature descriptions of drop size distributions and for convenience in initializing Lagrangian particles, the spray may also be described by a cumulative distribution function. The volume flux cumulative distribution function corresponding to the volume flux probability density function, f_V , is

defined as

$$F_V(\theta, \phi, d) = \int_0^d \int_0^\phi \int_0^\theta f_V(\theta, \phi, d) r_0(\theta, \phi)^2 d\Omega dd, \quad (16)$$

160 and may similarly be decomposed into conditional probabilities

$$F_V(\theta, \phi, d) = F_{V,\Omega}(\theta, \phi) F_{V,d}(d|\theta, \phi), \quad (17)$$

where $F_{V,\Omega}$ is the cumulative volume flow angular distribution, corresponding to the probability distribution function $f_{V,\Omega}$ and $F_{V,d}$ is the cumulative volume flow drop size distribution, corresponding to the probability distribution function $f_{V,d}$. The volume flow cumulative distribution function, $F_V(\theta, \phi, d)$, is
 165 constructed such that every phase space coordinate, (θ, ϕ, d) , corresponds to a unique value of F_V between 0 and 1, which enables its later sampling to generate Lagrangian particles, as described in Sec. 4.

Both $F_{V,\Omega}$ and $F_{V,d}$ may be related to the measured volume flux and drop size, respectively, and the multiplied to provide a complete description of the
 170 flow, as per Eq. 17.

The cumulative volume flow angular distribution, $F_{V,\Omega}$, is related to the measured local volume flux normal to the initialization surface, $\dot{V}''(\theta, \phi)$, by Eq. 16 and

$$f_{V,\Omega}(\theta, \phi) = \frac{\dot{V}''(\theta, \phi)}{\dot{V}}. \quad (18)$$

The local volume flux of the initial spray may be obtained with spatially resolved
 175 measurements such as those measured with the University of Maryland's 4S, as seen in Fig. 2 and described by Jordan, et. al. [7].

The cumulative drop size distribution, $F_{V,d}$, has been well explored in the literature and, for a typical fire sprinkler, can be well represented by a combination of log-normal and Rosin-Rammler distributions [15]. The cumulative
 180 volume fraction may be fit with two spatially varying parameters, median drop size, d_{v50} , and drop size distribution width, Γ . This distribution may be written

as

$$F_{V,d}(d|\theta, \psi) = \begin{cases} \frac{1}{\sqrt{2\pi}} \int_0^d \frac{\Gamma}{1.15d'} e^{\left(\frac{-\ln\left(\frac{d'}{d_{v50}}\right)^2}{2\left(\frac{1.15}{\Gamma}\right)^2}\right)} dd' & d < d_{v50}, \\ 1 - e^{-0.693\left(\frac{d}{d_{v50}}\right)^\Gamma} & d \geq d_{v50} \end{cases} \quad (19)$$

For typical sprinklers, d_{v50} varies between 0.5 and 3 mm and Γ is between 2 and 3. Figure 4 shows $F_{V,d}$ and $f_{v,d}$ for the spray measured in the same pendent sprinkler as Fig. 2 at $\theta = 115^\circ$ and $\phi = 245^\circ$. At this location, $d_{v50} = 0.57\text{mm}$ and $\Gamma = 2.2$.

The impact of spatial variations in drop size can be seen in Fig. 5, which shows the same spatially resolved volume flux from Fig. 2 decomposed into volume flux distributions from small drops ($d \leq 0.6\text{mm}$) and large drops ($d > 0.6\text{mm}$). The small and large drop spheres are calculated by multiplying the spatially resolved volume flux, $\dot{V}^n(\theta, \phi)$, by $F_{V,d}(0.6|\theta, \phi)$ and $1 - F_{V,d}(0.6|\theta, \phi)$, respectively. It becomes clear from this filtering that for this particular spray, the larger drops come primarily from the sprinkler tines, while smaller drops are produced uniformly across tines and slots.

The initial spray may thus be described by the volume flow cumulative volume fraction, $F_V(\theta, \phi, d)$, where F_V is given, as in Eq. 15, by the product of $F_{V,\Omega}$ and $F_{V,d}$, given by Eqs. 18 and 19, respectively, where all drops at a given angular location are assumed to originate from an initial radius $r_0(\theta, \phi)$, with a velocity magnitude, $u_0(\theta, \psi, d)$, and a velocity angle aligned with the radial direction at a given angular location. $F_V(\theta, \phi, d)$ provides a detailed description of the spray grounded in spray theory and measurements, and capable of use for initializing Lagrangian particles for the numerical simulation of spray evolution.

4. Initializing Lagrangian Particles

The stochastic nature of the spray makes it impossible to precisely simulate every drop in the sprinkler spray. Instead, Lagrangian particles reflecting the statistical properties of the spray are chosen. To capture measured properties the volume flow cumulative distribution function, F_V , is sampled.

Every phase space coordinate, (θ, ϕ, d) , corresponds to a unique value of the volume flow cumulative distribution function, F_V between 0 and 1. Because F_V is strictly increasing and continuous, an inverse distribution function or quantile function, G_V , may be defined as

$$G_V(p) = (F_V)^{-1}(\theta, \psi, d), \quad (20)$$

where p spans 0 to 1 and each value of p corresponds to a unique value of θ , ψ , and d . No analytic solution is available for the volume flow cumulative distribution function, F_V , and so numerical methods must be used to calculate its inverse, G_V .

To initialize N_p Lagrangian particles, N_p values of p are selected randomly between 0 and 1. Each p is used with the numerical solution to Eq. 20 to determine the particle's angular location, θ and ψ , and drop size, d . This numerical particle is then initialized at a radius $r_0(\theta, \psi)$ from the sprinkler head with a velocity $u_0(\theta, \psi, d)$. Each phase space location will be sampled with a frequency proportional to its volume flow, and thus each Lagrangian particle chosen in this manner must carry an equal volume. To conserve volume at each time step, a particle weighting factor, W_p , corresponding to the number of physical drops contained in each numerical drop is given by

$$W_p = \frac{\dot{V}/\dot{N}}{\pi d^3/6}, \quad (21)$$

where drops are assumed to be spherical.

In this way, Lagrangian particles are preferentially injected at the locations in phase space corresponding to the fraction of the total spray volume flow they carry. This method may be used with any particle injection rate, $\dot{N}_p$. Limited computational resources make it impractical to simulate the motion of every drop in the sprinkler spray, but improved accuracy is seen with increasing particle injection rate. The increase in accuracy results because selecting particles amounts to a statistical sampling of the specified mean properties of the spray, represented by F_V . The standard error of a sampled mean from the true mean

is proportional to $1/\sqrt{N}$ for normal distributions. This $1/\sqrt{N}$ scale is thus a
 235 reasonable approximation for error introduced by the representation of a spray
 over a particular scale of interest by limited Lagrangian particles.

Initial spray particles from a pendent type sprinkler, using the measured
 values of volume flux, drop size, and drop velocity highlighted in Figs. 2 - 4
 according to Eqs. 15 - 19, are displayed on the measurement surface in Fig.
 240 6. This group of 1000 Lagrangian particles passing through the measurement
 surface represents a statistical sample of the millions of drops generated by
 the spray every second, capturing the measured sprays complex phase space
 dependence with great fidelity. Fig. 6 shows regions of high particle density
 corresponding to the regions of high volume flux in Fig. 2. It is also clear that
 245 many more small drops (white) are present than large drops (blue) according to
 the drop size distribution presented in Fig. 4. The rays displayed in Fig. 6 also
 reflect the volume weighted average local velocity based on measurement, but
 do not reflect the local detailed drop size - velocity correlation presented in Fig.
 3. While this information is available from measurement, it is not convenient
 250 to include these correlations for each local measurement.

An alternative effective breakup surface, also derived from measurements,
 is included in Fig. 7 (a). This surface accounts for the local drop size velocity
 correlation with the added benefit of providing a facility to include the transfer
 of drop momentum to air (i.e. entrainment within the measurement surface).
 255 The breakup surface is constructed from local drop size-velocity correlations
 recognizing from atomization theory [16] that drops have identical injection
 velocities at breakup. The effective local breakup radius (i.e. effective break-
 up surface) and initial velocities shown in Fig. 7 are determined from best-fits
 of these local drop size-velocity correlations. Figure 7 (a) reveals a complex
 260 breakup surface consistent with qualitative images of the initial sprinkler spray.
 Figure 7 (b) shows the same statistically sampled drops displayed in Fig. 6 but
 superimposed on the breakup surface. The rays show consistent drop velocities
 while variations in radial drop location are apparent. It is clear from Fig. 7 (b)
 that larger drops are generated closer to the sprinkler where the sheets formed

265 by the deflector are expected to be thicker; whereas, smaller drops are typically
located radially outward where sheets are thinner.

5. Conclusions

The drop distribution function provides the mathematical foundation necessary to completely describe the complex spatio-stochastic characteristics of
270 the sprinkler spray first explored by Ren [5]. This description of the spray is
very general and provides unification between Lagrangian and Eulerian spray
dispersion perspectives in a stochastic framework. Recent advances in spray
measurement techniques applied to fire sprinklers can now produce detailed
local volume fluxes, local drop size distributions, and local drop size-velocity
275 correlations. A probability density function description of these spray properties
on an initial surface, as presented in Eqs. 16-19 and directly connected
to the drop distribution function, provides a mathematical grounding for their
robust integration and interpretation in a broader physical context. The presented
framework for the description of the initial spray provides useful insight
280 when applied in the analysis of high fidelity spray measurements and is readily
applied to stochastically generate an initial spray in modern detail rich CFD
based fire models.

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Figure 1: Partial cross section of a pendent sprinkler illustrating components; (a) orifice, (b) boss, (c) deflector, (d) frame arm (ref), (e) tines, and (f) slots [7].

Figure 2: Highly non-uniform volume flux for a typical pendent sprinkler head, measured at a radius of 0.4 m from the sprinkler head [7].

Figure 3: Local drop size-velocity correlation, $\vec{u}(115^\circ, 245^\circ, d)$ for the spray measured in the same pendent sprinkler as Fig. 2.

Figure 4: Cumulative volume fraction, $F_v(d, 115^\circ, 245^\circ)$, and spray probability density function, $f_v(d, 115^\circ, 245^\circ)$ with $d_{v50} = 0.57\text{mm}$ and $\Gamma = 2.2$.

Figure 5: Volume flux from drops with diameter (a) greater than 0.6 mm and (b) less than 0.6 mm for a pendent type sprinkler head . The sum of these volume fluxes is the total volume flux, seen in Fig. 2 [7].

Figure 6: Lagrangian particles representing the initial spray; red $d > 1.5\text{mm}$, blue $0.5 < d < 1.5\text{mm}$, and white $d < 0.5\text{mm}$; and rays are scaled with velocity.

Figure 7: Effective sheet breakup surface (a) without Lagrangian particles and (b) with Lagrangian particles; red $d > 1.5\text{mm}$, blue $0.5 < d < 1.5\text{mm}$, and white $d < 0.5\text{mm}$; and rays are scaled with velocity.