1 Statistical description of firebrand size and shape
2 distribution from coniferous trees for use in Metropolis
3 Monte Carlo simulations of firebrand flight distance

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6 **Non-specialist summary** The flight of firebrands and resulting spot fires is a major hazard
7 for firefighters. This paper presents a statistical description of the size and shape of firebrands
8 generated from coniferous trees. This characterization of firebrands' size will improve modeling
9 credibility of their flight.

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13 **Abstract**

14 The process of ember/firebrand formation, lofting, wind driven transport, and resulting
15 spot fire ignition during a wildfire is still poorly understood. Lack of a tractable firebrand
16 formation model along with a detailed statistical description of the size and shape distribution
17 of typical firebrand that could be used in simulations of firebrand flight and combustion may
18 result in unrealistic outcomes. In this regard, a simple, yet quite informative, mechanical
19 failure model of the firebrand break-off process is proposed. This model suggests that the
20 previous laboratory scale firebrand generation experiments would likely provide a reasonable
21 analogue for the formation process in a full scale wildfire. In addition, geometric scaling
22 analysis is conducted and shows that the firebrand surface area scales with the firebrand

mass raised to the $2/3^{rd}$ power. This is in close agreement with measurements of firebrand from previously published data in the literature that are obtained under controlled laboratory combustion of coniferous trees of different sizes. Also, a detailed statistical characterization of the size and shape of these firebrands are presented. A nonlinear regression model on the firebrands' data led to the generation of a set of virtual firebrands. The resulting data could be used as inputs to a Monte-Carlo simulation of firebrands' transport through the velocity field induced by the interaction of a fire plume and the atmospheric boundary layer. Moreover, it is shown that the size distribution of firebrands is more dependent on the mechanics of combustion and limb failure than on a simple geometric relationship with the tree height.

Keywords: firebrands, embers, spot fires, size and mass distribution, Monte-Carlo simulation, nonlinear regression

1 Introduction

Wildfires are a major threat to people and property. In 2010 there were over 70,000 wildfires recorded in the US by the National Interagency Fire Center [NIFC \[2011\]](#), which burned over 3.4 million acres. In an average year over 1,000 homes, 1,000 outbuildings and 40 businesses are destroyed by wildfires in the US. In 2014 only San Diego County wildfires burned over 29,300 acres of land which resulted in damage or destruction of more than 55 properties. So far, this has caused the total cost of \$60 million (2014 USD) and damage estimate still continues [\[Repard, 2014\]](#). Also on June 2012 Waldo Canyon fire, the most destructive fire in Colorado fire history in terms of consumed homes after Black Forest fire [\[Parker et al., 2013\]](#), burned a total of 18,247 acres in Colorado and Manitou Springs area that led to evacuation of more than 32,000 residents and ultimately insurance claims totaling more than \$453.7 million [\[Murphy, 2013\]](#). Wildfires are becoming even costlier as more people choose to live at the wildland urban interface (WUI).

Once a wildfire starts the main goal of responders is containment and protection of people and property. In order to do this one needs to understand how wildfires spread, and how they cause home ignitions. Wildfire spread is a complex multi-physics phenomenon that depends on the available fuel (vegetation and moisture content), topography, and meteorology. Wildfires can spread by ignition due to radiant heat transfer from burning vegetation to surrounding fuel sources [\[Sardoy et al., 2008\]](#), flame impingement

on un-burned fuels [Tran et al., 1992], and fire spotting from firebrands blown ahead of the fire front [Fernandez-Pello, 2009]. Fire spread can also be strongly influenced by the terrain and atmospheric conditions [Mermoz et al., 2005]. Various models have been developed to predict fire spread from radiation and convection [Tran et al., 1992], from firebrand spotting [Albini, 1979], and by using clustered network models [Porterie et al., 2007].

There is strong evidence to suggest that firebrand showers, when firebrands are cast downwind, is a source of heat transfer [Coleman et al., 2013] and, while radiant and convective fire propagation is responsible for the destruction of forests, firebrand spotting is a major cause of home destruction at the WUI [Manzello et al., 2007; Weiser, 2009]. For instance, in large forest fires as the fire front reaches residential neighborhoods, the density of vegetation often decreases due to landscaping of the houses and communities, and wildfire management techniques such as creating designated defensible space around buildings and mechanical thinning close to WUIs [Nader, 2007; Syphard et al., 2014]. As a result the intensity of the fire decreases [Dupuy and Morvan, 2005]. Then propagation occurs mainly via either a ground fire or spotting. Spotting can also be a significant hazard to firefighters. While it is relatively easy to monitor the location of fire front in a fire, it is much harder to predict the location of spot fires created by firebrands transported by wind ahead of the fire front. Such spot fires can potentially trap firefighters, or residents who have yet to evacuate [Sullivan, 2009].

Firebrand transport is a complex process involving firebrand lofting into the atmosphere by buoyant plumes (continuous release of buoyant fluid [Morton, 1965]) and thermals (finite release of buoyant fluid [Turner, 1969]) formed above the fire, downwind transport by atmospheric winds [Albini, 1983b], and ignition upon landing. During all these steps, lofted firebrands combust through pyrolysis and charring [Tse and Fernandez-Pello, 1998]. A schematic diagram is shown in figure 1. Spotting is, also, highly stochastic. The lofting height will depend on the size and shape of the individual firebrands and the buoyancy of the fire plume. Turbulence characteristics of the plume, also, affects the induced velocity field through which lofting occurs. Karimpour and Kaye [2012] show that ignoring turbulence fluctuations underestimates the aerodynamic forces that are being exerted on firebrands. The downwind transport distance will depend on the velocity and turbulence characteristics of the wind and the physical properties of the firebrands. The combustion rate will depend on the size, shape, density, and chemical properties of the firebrand. Finally, the potential for creating a spot fire upon landing will depend on the remaining fuel and heat content of the firebrand on impact, and the available fuel sources at the point of impact. On

Figure 1: Schematic diagram of the spotting process from firebrand formation and break-off through lofting, wind driven transport, and spot fire ignition.

81 this note, the formation of firebrands from burning vegetation is a complex phenomenon that is influenced
 82 by the size of the tree due to self-similarity characteristics of the vegetative structures [Mandelbrot, 1983;
 83 Barr and Ezekoye, 2013], the fire intensity, and the wind speed which varies with height. Even the
 84 interaction of the wind with a non-combusting tree is complex. As the wind speed increases, the frontal
 85 area of the tree reduces as the leaves and branches align with the wind direction. The drag coefficient
 86 of the tree decreases with increasing wind speed, and the total drag scales linearly with the wind speed
 87 [Guan et al., 2000; Kane and Smiley, 2006; Wilson et al., 2010].

88 To date, several studies have been conducted to examine the generation of firebrands from individual
 89 burning trees in the absence of a wind field [Manzello et al., 2007,?, 2008, 2009]. For the tested coniferous
 90 trees it is observed that the firebrands were predominantly cylindrical in shape, see details of experiments
 91 in Manzello et al. [2007, 2009]. There appears to be no controlled laboratory data in the literature on
 92 firebrand generation rates from collections of trees, or from trees in a wind field. Parameterization of the
 93 shape of firebrands is often in terms of mass, surface area, and shape, though Huang et al. [2007] did a

94 series of experiments to parametrize disk shaped firebrands in terms of their mass and Stokes equivalent
95 diameter. In addition, [Barr and Ezekoye \[2013\]](#) have developed a model for predicting the size distribution
96 of brands being lofted from a fractal tree that is based on mechanical breakage models which are coupled
97 to a thermal decomposition model.

98 Once the firebrands are formed they are lofted into the atmosphere and blown down wind by the
99 ambient wind field. There are a number of models in the literature on lofting and wind driven transport
100 which primarily build on the work of [Albini \[1979, 1981, 1983b,a\]](#) who presented a range of models for the
101 lofting and transport of firebrands. These models have been extended to look at lofting in more detail
102 [[Woycheese et al., 1998a](#)], bent line-fire plumes [[Sardoy et al., 2008](#)], and various criteria for transition
103 from lofting to transport [[Woycheese et al., 1998b](#)]. More detailed reviews of these models are available in
104 the literature; see, [[Woycheese and Pagni, 1999](#); [Koo et al., 2010](#)]. These models are often coupled with
105 combustion models to account for the change in firebrand mass and volume during flight. Combustion
106 occurs on the firebrand surface and, therefore, the rate of combustion depends on the shape and surface
107 area of the firebrand.

108 All firebrand transport models are based on the wind borne debris flight equations of [Tachikawa \[2012,](#)
109 [1988\]](#) who presented equations for the flight of solid objects in terms of their mass and shape. Different
110 equations have been developed for compact debris (approximately spherical), rod-like debris, and plate-
111 like debris [[Richards et al., 2002](#)]. On this note, a combustion model for different firebrand shapes is
112 investigated by [Baum and Atreya \[2014\]](#).

113 Ignition of spot fires caused by landing firebrands depends on the state of the fuel bed the firebrands
114 land on and the time of exposure to radiant and convective heat fluxes [[Boonmee and Quintiere, 2002](#)].
115 Spot fire ignition therefore depends on the size, number, temperature, and heat capacity of the landing
116 firebrands, and the available fuel at ground level. This process has been investigated through a series of
117 experiments by [Manzello et al. \[2008\]](#).

118 In summary, the lofting height, flight distance, combustion rate, and spot fire ignition potential of an
119 firebrand are strongly dependent on the firebrand shape and mass. One approach to fully understand
120 the risk of spot fire formation during a wildfire is to run the models discussed above for a broad range of
121 statistically appropriate firebrand sizes and shapes using Monte Carlo type simulations similar to those
122 conducted to understand the risk of wind borne debris in severe storms by [Karimpour and Kaye \[2012\]](#).
123 However, a detailed characterization of the formation, size, and shape of firebrands is lacking within

the literature. The main objective of this study is to begin to fill this gap by presenting simple physical models for firebrand formation and break-off and characterizing firebrand geometric properties by detailed statistical analysis of available data on firebrand size and mass from coniferous trees of various heights.

The remainder of the paper is structured as follows. In Section (2) a simple model and scaling analysis of firebrand geometry is presented followed in Section (3) a mechanical model for firebrand break-off that can be applied to both laboratory test burns and to real wildfire events. In Section (4) preliminary statistical analysis of available firebrand geometric data is presented relating firebrand surface area, mass, and aspect ratio. These results are used in Section (5) to develop a statistical description of the firebrand size and shape and to present a method for developing statistically appropriate sets of virtual firebrands for use in Monte Carlo simulations. The results are discussed and conclusions are drawn in Section (6).

2 Firebrand geometry

There is currently a substantial amount of data published in the literature on the size and shape of firebrands from various pine trees (Douglas pine and Korean pine) [Manzello et al., 2007, 2008, 2009]. These papers present the results of a series of experiments in which individual trees were set alight and allowed to burn under controlled laboratory conditions. During the burning, the firebrands that fell from the trees were extinguished and then weighed and measured. Results were presented for the weight distribution, size distribution and surface area. A total of 1337 firebrands were collected and measured from five different tree burning tests. The results suggest that the mass distribution is significantly skewed toward low mass firebrands (80% of firebrands had a mass of 0.2 g or less). Through these studies, it is also shown that the firebrand surface area is scaled on the firebrand mass, although the exact nature of the scaling is not discussed.

Based on the observations of [Manzello et al., 2007, 2008, 2009] from burning coniferous trees in no wind condition, the generic shape of a firebrand can be characterized as a long thin cylinder of length L , diameter D and density ρ (assumed constant), see figure 2. The volume $\forall$ and surface area s are given by

$$\forall = \frac{\pi D^2 L}{4} \quad \text{and} \quad s = \pi D L + \frac{\pi D^2}{2} \quad (1)$$

149 respectively. The mass of the firebrand is $m = \rho\forall$ and the aspect ratio is denoted by

$$\eta = \frac{L}{D}. \quad (2)$$

150 If one assumes that in most of the firebrands $L \gg D$ or $\eta \gg 1$, then the effective surface area can be
 151 approximated by

$$s \approx \pi DL. \quad (3)$$

The mass and surface area of an firebrand can be written in terms of the firebrand length and the

Figure 2: Schematic of a cylindrical firebrand showing the length, diameter, volume, and approximate surface area.

152

153 firebrand aspect ratio,

$$m = \rho \frac{\pi L^3}{4\eta^2} \quad \text{and} \quad s = \frac{\pi L^2}{\eta} + \frac{\pi L^2}{\eta^2}. \quad (4)$$

154 Provided the aspect ratio is relatively large ($\eta \gg 1$), then the equation for the firebrand surface area (4)
 155 is dominated by the first of the two terms. Ignoring the second term one can write

$$L = \sqrt{\frac{s\eta}{\pi}} = \left(\frac{4m\eta^2}{\rho\pi} \right)^{1/3} \quad (5)$$

156 which can be simplified to

$$\eta = \frac{L}{D} = \frac{s^3 \rho^2}{16\pi m^2}. \quad (6)$$

157

158 Therefore, given mass and surface area information, it is possible to approximate the firebrand aspect
 159 ratio by expression (6).

Assuming for a moment that the aspect ratio of all the firebrands (generated from coniferous trees in no wind condition) is the same, then both the mass and surface area will only be functions of the firebrand length. The mass will scale on the L^3 and the surface area on L^2 . Therefore, the cube root of the mass and square root of the surface area will scale linearly on the firebrand length. That is,

$$m^{1/3} \sim L \sim s^{1/2}, \quad (7)$$

and therefore,

$$s \sim m^{2/3}. \quad (8)$$

The same approximate scaling result can be gained from dimensional analysis as the dimensions of volume and surface area are $[V] = L^3$ and $[S] = L^2$, respectively. For a given density, one non-dimensional group (Π) can be formed. Possible forms for the Π include

$$\Pi_1 = \frac{s}{m^{2/3}} \quad \text{or} \quad \Pi_2 = \frac{L}{D} = \eta. \quad (9)$$

In order to test this scaling relationship the mass and surface area data in the literature [Manzello et al., 2007, 2008, 2009] is re-plotted on a log-log scale in figure 3. Also, shown is a line with a slope of 2/3 indicating the power law relationship derived in (8) and (9). Specifics of the experimental data can be found on table 1. Figure 3, clearly illustrates two points. First, the broad trend in the data

Table 1: Characteristics of the isolated burnt trees in the absence of wind. In the table, abbreviations M.C. and N.A. stand for Moisture Content and Not Available, respectively.

Vegetation Type	Source	Height (m)	Girth (m)	M.C. at Ignition
Douglas-fir	Manzello et al. [2007]	2.6	1.5	(10-50)% $\pm$ 10%
Douglas-fir		5.2	3.0	(10-50)% $\pm$ 10%
Korean Pine	Manzello et al. [2009]	3.6	N.A.	(10-80)% $\pm$ 15%
Korean Pine	Manzello et al. [2008]	4.0	N.A.	(10-80)% $\pm$ 15%

strongly agrees with the scaling proposed in (8) and (9). Second, there is substantial variability around this underlying trend. This variability is because the assumption of a fixed aspect ratio which is clearly not correct. Variation in the aspect ratio of the firebrands will result in variation about the power law trend shown in figure 3.

The data plotted in figure 3 is derived from four separate experiments in which conifers of different

Figure 3: log-log scale plot of measured firebrand surface area versus mass for different tree sizes. Also shown is a line (not fitted) of slope $\alpha = 2/3$ to illustrate the power law scaling relationship given in (8). Data is excerpted from [Manzello et al. \[2007, 2008, 2009\]](#) studies.

heights were burnt. The dependence of firebrand size and shape on tree height is shown in figures 4 and 5. The mean and standard deviation of the firebrand mass, surface area, aspect ratio, length, and diameter are plotted versus the height of the tree from which they fell. While there is significant variation in these parameters from tree to tree there is no strong correlation between them and the height of the tree from which they fell.

3 Firebrand formation model

The failure of trees, when the energy of distortion, whether due to uprooting or bending moments on the branches, exceeds the yield energy of the tree material, is a very complex problem [[Vogel, 1996](#)]. Tree crowns have highly complex fractal geometries [[Zeide and Pfeifer, 1991](#); [Zhang et al., 2007](#)] and a canopy frontal area that changes with the wind speed [[Spatz and Bruechert, 2000](#)]. There is also evidence that the drag coefficient of a tree crown varies with wind speed [[Wilson et al., 2010](#); [Kane and Smiley, 2006](#)]. Trees also exhibit complex dynamic responses to wind loads that can lead to a broad range of failure

Figure 4: Plots of, from top left to right and bottom, the mean firebrand (a) mass (μ_m), (b) surface area (μ_s), (c) aspect ratio (μ_η), (d) length (μ_L), and (e) diameter (μ_D) versus tree height.

mechanisms [James et al., 2006]. A full predictive model for firebrand formation through tree failure is beyond the scope of this paper. Instead, a simple mechanical model for firebrand formation is presented

Figure 5: Plots of, from top left to right and bottom, the standard deviation of firebrand (a) mass (σ_m), (b) surface area (σ_s), (c) aspect ratio (σ_η), (d) length (σ_L), and (e) diameter (σ_D) versus tree height.

191 in order to investigate possible relationships between the aspect ratio, mass, surface area, length and
 192 diameter of firebrands that form and the height of the tree from which they form.

193 The model assumes that firebrands form when a twig or branch snaps off due to the shear stress
 194 associated with internal bending moment exceeding the capacity of the material to resist it. The model
 195 for firebrand formation will, therefore, focus on firebrands formed from relatively long thin cylinders
 196 such as those observed in the experiments of [Manzello et al., 2007, 2008, 2009]. This is also consistent
 197 with Koo et al. [2012] who mentions that a large fraction of firebrands generated from vegetation or
 198 burning structures can be represented as cylinders. The model does not examine the formation of disk
 199 like firebrands from bark being stripped from tree trunks or from components of burning man-made
 200 structures [Suzuki et al., 2013].

201 Consider a cylindrical twig or branch attached at one end to a larger branch such that the attachment
 202 can be regarded as built in connection. Any loading on the firebrand due to either firebrand weight or
 203 aerodynamic drag, will result in a bending moment at the support. For a given firebrand diameter and
 204 set of material properties, the maximum bending moment that the connection can resist before failing is
 205 controlled by the maximum shear stress the material can support

$$\sigma_{max} = \frac{M_0 D}{2I} \quad (10)$$

206 where M_0 is the bending moment at the connection, D is the firebrand diameter, and

$$I = \frac{\pi D^4}{64} \quad (11)$$

207 is the second moment of area of the cylinders circular cross section about its centerline.

208 There are two possible sources of bending moment that can result in break off; weight (see figure 6-left)
 209 and drag which can be due to either the vertical air flow induced by the heat release from the fire or from
 210 the horizontal airflow from ambient winds and induced by the thermal plume that forms above the fire
 211 (see figure 6-right).

212 When the loading is due primarily to the weight ($W = wL$) of the firebrand, the bending moment at
 213 the connection is

$$M_0 = \frac{wL^2}{2} = \frac{WL}{2} = \frac{\rho_{wood} g \pi D^2 L^2}{8}. \quad (12)$$

Figure 6: Schematic diagram of a cylindrical firebrand supported on the left with (a) bending moment due to the distributed load w =weight per unit length (b) bending moment due to the distributed aerodynamic drag per unit length f_D .

Therefore, the critical shear stress at which the firebrand will break off is given by

$$\sigma_{max} = \frac{4\rho_{wood}gL^2}{D} = 4\rho_{wood}gL\eta. \quad (13)$$

Assuming that the maximum allowable shear stress is somewhat constant then, when firebrand break off is due to firebrand self weight, then the firebrand aspect ratio is inversely proportional to the length of the firebrand. This relationship is clearly not seen in the experimental data of [Manzello et al., 2007, 2008, 2009] as shown in figure 7.

When the loading is due to aerodynamic drag the bending moment is given by

$$M_0 = \frac{1}{4}\rho_{air}C_DU^2L^2D \quad (14)$$

where C_D is the drag coefficient for a cylinder. The resulting critical shear stress condition is given by

$$\sigma_{max} = \frac{8\rho_{air}C_DU^2\eta^2}{\pi}. \quad (15)$$

Assuming again that the material property σ_{max} is somewhat constant then, for a given wind speed U , the firebrand aspect ratio will be a constant. In reality vertical drag, horizontal drag, and weight will all act simultaneously and the resulting break off condition will be

$$\sigma_{max} = \sqrt{\left(\frac{8\rho_{air}C_DU_h^2\eta^2}{\pi}\right)^2 + \left(\frac{8\rho_{air}C_DU_v^2\eta^2}{\pi} - 4\rho_{wood}gL\eta\right)^2}. \quad (16)$$

where U_v and U_h are the vertical and horizontal components of the air velocity.

Figure 7: Plots of firebrand aspect ratio versus length for each of the four tree heights tested by [Manzello et al., 2007, 2008, 2009].

The geometric results for the three cases of weight, drag, and weight and drag controlled break off are clearly overly simplistic. In a real combustion event the material properties of the wood change and the material becomes heterogeneous due to charring of the surface. As such, the distribution of shear stress within the cylinder will not be linear and the maximum allowable shear stress will likely decrease over time. As such, the critical condition is also likely to be a function of combustion time.

230 However, despite their simplicity, the models presented above do conform to a couple of intuitive
 231 results regarding firebrand formation. For example, for a given set of conditions, the models indicate that
 232 larger aspect ratio firebrands (longer and thinner) are more susceptible to break-off. Further, during the
 233 combustion of a twig or branch the length and diameter of the cylinder will decrease over time. Assuming
 234 that the wood burns at a constant rate over the entire surface, the cylinder length and diameter will
 235 decrease at the same rate and the aspect ratio of the cylinder will increase over time. Therefore, the
 236 maximum shear stress at the connection will increase over time until the critical condition for break off is
 237 achieved. Again, this result is consistent with observations showing that firebrands break off continuously
 238 over the course of a combustion event as more branches burn down to reach break off criteria. It is,
 239 therefore, worth examining some of the implications of these simple models. We consider two cases. One
 240 is a single tree being burnt in a laboratory with no ambient wind. In this case $U_h = 0$ and the break off
 241 criteria is

$$\sigma_{max} = \left| \frac{8\rho_{air}C_D U_v^2 \eta^2}{\pi} - 4\rho_{wood}gL\eta \right|. \quad (17)$$

242 Dividing both sides by $8\rho_{air}C_D D U_v^2 \eta^2 / \pi$ allows (17) to be written as

$$\hat{\sigma}_{max} = \left| 1 - \frac{\Gamma}{\eta} \right|. \quad (18)$$

243 where

$$\hat{\sigma}_{max} = \frac{\pi \sigma_{max}}{8\rho_{air}C_D D U_v^2 \eta^2} \quad (19)$$

244 is a non-dimensional maximum shear stress and

$$\Gamma = \frac{\pi \rho_{wood}gL}{2\rho_{air}C_D U_v^2} \quad (20)$$

245 is a non-dimensional parameter that quantifies the relative importance of firebrand weight and vertical
 246 drag on the bending moment at the firebrand connection. If $\Gamma/\eta > 1$ then the firebrand weight is the
 247 main contributor to break off. Whereas, if $\Gamma/\eta < 1$ then the vertical drag controls break off. For a given
 248 value of Γ , this implies that firebrands with aspect ratio $\eta < \Gamma$ will tend to fall off due to their weight
 249 whereas longer thinner firebrands with $\eta > \Gamma$ will be broken off due to vertical drag. The updraft velocity
 250 will depend on the intensity of the fire, often parameterized in terms of the heat release rate (HRR) of
 251 the fire. Following the result for the near source fire plume velocity presented by [Baum and McCaffrey](#)

[1989] the peak vertical velocity is given by

$$U_{max} = 2.45 \left(\frac{g^2 HRR}{\rho_{air} C_p T_0} \right)^{1/5}. \quad (21)$$

where C_p is the specific heat of air and T_0 is the absolute ambient air temperature (see Bhutia et al. [2010]). Numerical simulations of individual tree burning experiments indicated mass loss rates of up to 2 kg/s indicating a heat release rate of 34 MW (see Mell et al. [2010]). This gives a peak vertical velocity of approximately 4 m/s. Substituting this value into (20) along with $\rho_{wood} = 450 \text{ kg/m}^3$, $\rho_{air} = 1.23 \text{ kg/m}^3$, $C_D = 1$ [Munson et al., 2010], and using the average firebrand length measured of approximately 10 cm gives $\Gamma = 35$. This suggests that firebrands with aspect ratios larger than 35 will break off due to vertical air flow and be lofted. For smaller aspect ratios the firebrands would break off due to their own self weight. For the trees tested the majority of the firebrands measured had aspect ratios greater than 35 indicating that vertical drag is likely the dominant loading that leads to firebrand break off. This is consistent with the lofting of firebrands that was observed during the tree burning experiments. See for example figure 1 of Manzello et al. [2007].

In the formulation presented the vertical air flow is a function of the HRR which in turn should be a function of the tree height as the HRR should depend on the amount of available fuel. Therefore, the HRR should scale with the surface area of the tree (SA_T). For regular shaped objects $SA_T \sim H^2$. However, trees have fractal geometries [Zeide and Pfeifer, 1991; Zhang et al., 2007] and as such the tree surface area scales on

$$SA_T \sim H^\delta \quad (22)$$

where $\delta > 2$ is the fractal dimension of the tree. Measurements of conifers suggest that δ is in the range 2.2 – 2.8 depending on the exact tree species. Making a first order steady combustion assumption that the heat release rate is linearly proportional to the surface area available for combustion ($HRR \sim SA_T$) and substituting (22) and (21) into (20) indicates that

$$\Gamma \sim H^{-2\delta/5}. \quad (23)$$

That is, Γ is roughly inversely proportional to the tree height. This would imply that single tree burns of larger trees could produce smaller aspect ratio firebrands. This is not observed in the data shown in figure 4 though it should be noted that there is significant variability in the experimental data and this

276 variability could hide any underlying relationship between tree height and firebrand aspect ratio. That
277 said, for the three larger tree sizes the average firebrand aspect ratio does clearly decrease with increasing
278 tree height.

279 The second case considered is of a larger scale forest fire with hundreds of trees burning. In this case
280 the fire *HRR* will be orders of magnitude larger. There will also be a substantial horizontal wind field
281 due to ambient wind and entrainment of ambient air into the fire plume. Assuming that the *HRR* is 500
282 times that of a single tree then the peak vertical wind speed will be 14 m/s (about 50 km/h). In the
283 absence of substantial ambient wind, this would lead to a much smaller value of $\Gamma \approx 3$. As such, wind
284 driven break off will dominate firebrand formation relative to single tree burns and the weight term in
285 (16) can be ignored.

286 There are clearly many assumptions in the proposed model. Firstly, the model is entirely deterministic
287 whereas there is clearly considerable variation in the size and shape of the firebrands formed during either
288 a single tree burn or a forest fire. Variability could be introduced into the model as many of the input
289 parameters will either vary over time (wind speed), vary within the tree (initial twig size), or vary during
290 burning (wood density, maximum shear stress). Variability in firebrand size and shape will be due to
291 all of these. One would, therefore, not expect the model to be quantitatively predictive. However, given
292 appropriate average flow and material properties, the model can be qualitatively predictive.

293 The main qualitative result of the proposed model is that the majority of the firebrands formed during
294 the single tree burn experiments had a large enough aspect ratio that they were most likely formed by
295 vertical drag forces from the fire updraft. This is the same firebrand formation mechanism one would
296 expect from larger scale wildfires. As such, the laboratory experiments of [Manzello et al., 2007, 2008,
297 2009] likely provide a reasonable analog for the formation of firebrands from branches and twigs in full
298 scale wildfires.

299 4 Firebrand characterization

300 To adequately and accurately describe the size and shape of cylindrical firebrands two parameters, other
301 than density that is assumed constant, are needed. For instance, length and mass, or surface area
302 and length. The goal of the remainder of the paper is to develop a method for numerically generating
303 statistically appropriate virtual firebrands for use in Monte Carlo type simulations of firebrand flight. As

such, the describing pair of parameters would ideally satisfy two conditions. First, the two parameters should be unrelated, that is, the information in one variable is not simply explained by a linear regression on the other variable. Otherwise the independent sampling procedure in Monte-Carlo simulations would be altered. Second, they should be well described by a standard statistical distribution function in order to generate proper random values from the sample space that the distribution provides.

Clearly firebrand mass and firebrand surface area are related and correlated (see figure 3) and so a third parameter needs to be considered. The most obvious and easily calculated parameter is the firebrand aspect ratio (η) given by (6). A useful measure of relation between aspect ratio, surface area and mass is the coefficient of determination, i.e. R-squared value. The pair of variables with the smallest R^2 value should be used as the describing pair. Values of R^2 for each experimentally available set of pairs of η , m and s are shown on log-log plots in figures 8-9. The lower the R^2 value the less the two variables are linearly related. That is, the variability of one variable cannot be captured using a linear regression model to the other one. According to the reported R^2 -values, η and s are the least linearly related. Nonetheless, it is possible to observe a strong relationship with significantly low R^2 , if the relationship is strongly non-linear. To determine the significance of the linear relation between the pairs, the Pearson and Spearman correlations of η with m and s are calculated and shown on figures 8-9. The data in figures 8-9 indicates that there is very little if any statistically significant correlation between η and s . Spearman values show evidence of weak nonlinear correlation. However, between m and η , stronger and more significant correlations can be observed. In order to confirm η and s as the describing pair, it would be useful to collapse all the data for the different trees together and observe the overall trend. On this matter, the calculated ηs of all firebrands, regardless of tree burn, are plotted against their corresponding values of m and s on figure 10. For the combined data set, the Spearman correlation between η and s is 0.076 with significance level of 0.006 and R^2 -value of 0.0002. Whereas, for m and η , these values are -0.264 , 0.000 and 0.004 respectively. This indicates that η and s are much less related and correlated than m and η .

The second criteria for determining the describing pair to be used a Monte Carlo simulation is that each of the describing pair parameters be well represented by a standard probability distribution function. The main requirement of the probability distribution function to be fitted is that it not allow negative values for either η or s . A set of quantile-quantile plots of η , s , and m are shown on figure 11, for a log-normal distribution. The plots indicate that the log-normal distribution provides the best description

Figure 8: (Left) log-log plots of calculated firebrand aspect ratio versus corresponding measured surface area for each tree in experimental data. (Right) log-log plots of calculated firebrand aspect ratio versus corresponding measured mass for each tree in experimental data. Pearson and Spearman correlation coefficients along with their significance level are shown above of each pair. P-values less than 0.05 show statistically significant correlations.

334 for η followed by s while m is the parameter least well represented by a log-normal distribution.

Figure 9: (Left) log-log plots of calculated firebrand aspect ratio versus corresponding measured surface area for each tree in experimental data. (Right) log-log plots of calculated firebrand aspect ratio versus corresponding measured mass for each tree in experimental data. Pearson and Spearman correlation coefficients along with their significance level are shown above of each pair. P-values less than 0.05 show statistically significant correlations.

Figure 10: (Left) log-log plot of calculated firebrand aspect ratio versus corresponding measured surface area in experimental data. (Right) log-log plots of calculated firebrand aspect ratio versus corresponding measured mass in experimental data. Pearson and Spearman correlation coefficients along with their significance level are shown above of each pair. P-values less than 0.05 show statistically significant correlations.

5 Statistical description of firebrands

As η and s are, at best, only weakly related and are also the two parameters that are best described by the log-normal distribution function, they are adopted as the describing pairs for the Metropolis Monte-Carlo simulation analysis. Further, η is chosen as the independent (predictor) variable for a nonlinear regression model as it is the parameter best represented by the log-normal PDF. The regression model is used to estimate corresponding values of s .

The relationship between η and s is unknown. Therefore, the computational domain is mapped to a logarithmic domain such that the nonlinear regression problem is transformed to a linear regression problem and both variable's distribution's are approximately normal. The surface area simulation assumptions are as follows. There is a normally distributed subpopulation of s for each value of η and these subpopulations all have a common variance, i.e. $s \mid \eta \sim N(\mu_{s|\eta}, \sigma_e^2)$. The means of the subpopulations fall on a straight-line function $\mu_{s|\eta} = \hat{\Phi} + \hat{\Psi}\eta$ and that $\hat{s} = \hat{\Phi} + \hat{\Psi}\eta$ estimates the mean response for s . The selection of an observation from any of the subpopulations is independent of the selection of any

Figure 11: Long-normal quantile-quantile plots of the calculated aspect ratio, measured surface area and mass for all trees. Plots are ordered with respect to tree height, 2.4 (m), 3.6 (m), 4.0 (m), 4.5 (m), from left to right. Also shown is the red dashed line with the slope of one.

other observation. The η values are adopted randomly from the best fitted normal distribution and are assumed to be fixed through the procedure.

The rest of the simulation procedure is as follows. Assuming a linear relation between $\ln(\eta)$ and $\ln(s)$, simple regression is applied, using the least squares method. Virtual aspect ratios, η_v , equal to the number of samples collected in the previously published tree burn experiments are generated, randomly, from the fitted normal distribution. Corresponding predicted values of s are calculated utilizing linear regression with associated standard error to the mean predicted value ($\sigma_{\hat{s}}$), according to the following expression

$$\sigma_{\hat{s}} = \hat{\sigma}_e \left(1 + \frac{1}{n} + \frac{(\eta_v - \bar{\eta}_v)^2}{\sum_{i=1}^n (\eta_i - \bar{\eta}_v)^2} \right)^{1/2} \quad (24)$$

where $\hat{\sigma}_e$ is the common standard deviation of subpopulations, $\bar{\eta}_v$ is the average value of virtual aspect

ratios, η_i is the value of experimental aspect ratio and n is the number of firebrand samples from each individual tree experiment.

Having calculated the corresponding surface area values of the virtual aspect ratios, the results are transformed back to the non-logarithmic domain. Figure 12, shows the simulated describing pairs along with the corresponding experimental data. The estimated describing pair was then used to calculate each

Figure 12: Comparison of the generated describing pairs with corresponding experimental data for each individual tree.

firebrands' mass using (6). The resulting mass from Metropolis Monte-Carlo simulations is plotted against

estimated surface area and compared with the corresponding experimental data on figure 13. There is

Figure 13: Comparison of the simulated mass versus surface area with corresponding experimental data for each individual tree.

very good agreement between the results of the simulations and experimental data. In this regard, the synthetic firebrand data from all trees is superimposed and plotted next to the experimental data and shown in figure 14. The similarity of the results corroborates the proposed modeling approach. In order to find out if the simulated mass and experimental mass of firebrands are identically distributed a series of graphical techniques and statistical test are conducted and the empirical cumulative density functions

Figure 14: Log-log scale plots of firebrand surface area versus mass for 1337 synthetic firebrands and the 1337 firebrands from laboratory measurements of [Manzello et al. \[2007, 2008, 2009\]](#)

(CDF) of mass and surface area versus the corresponding CDFs of the simulated data are shown on figure 15. From figure 15, it appears that the simulated firebrand mass and experimental data are coming from the same continuous distribution. To confirm this, a series of tests are performed for each tree.

Through the logarithmic domain a null hypothesis is designated such that the experimental and estimated masses for each individual tree are from the same continuous distribution. The test of this hypothesis is known as the two-sample Kolmogorov-Smirnov (K-S) test. The results are presented in table 2 where a zero value of h_0 indicates that the null hypothesis is not rejected. As can be seen, only in the data set from four meter Korean Pine is the null hypothesis rejected. Hence, it can be inferred that the estimated mass values possess the same continuous distribution as the corresponding experimental ones for three out of four trees. One possible explanation for the test result is that the surface area (s) is approximated in the calculations such that the contribution of the two ends of the firebrand is neglected. This may introduce errors in firebrands with low aspect ratios and may affect the mass distribution obtained, in particular at tails of the distribution.

If the simulated and experimental data are from the same continuous distribution, then the mean of experimental mass should be equal to the mean of simulated mass. Therefore, a null hypothesis was created such that the two distribution means are equal. A test of this hypothesis is the well known two

Table 2: Results of various statistical tests on the firebrand mass (m) from each individual tree in logarithmic domain. Status of the decision on null hypothesis is denoted by h_0 where the value of one rejects the corresponding null hypothesis of the test. DOF stands for the degree of freedom of the test and p -value shows the probability of the test result. The significance level is set to 0.05 for all tests.

Tree type	K-S test			T-test			F-test		
	h_0	h -statistic	p -value	h_0	DOF	p -value	h_0	DOF	p -value
Douglas-fir, 2.4 (m)	0	0.1005	0.220	0	416	0.399	0	208	0.761
Korean pine, 3.6 (m)	0	0.0652	0.197	0	1072	0.843	0	536	0.804
Korean pine, 4.0 (m)	1	0.1234	0.005	0	760	0.530	0	380	0.474
Douglas-fir, 4.5 (m)	0	0.1095	0.151	0	418	0.293	0	209	0.277

Figure 15: (Left) Empirical cumulative density function of mass from all experiments against the Metropolis Monte-Carlo simulations. Statistics of the Kolmogorov-Smirnov test is 0.0688 with p -value of 0.0034 under the null hypothesis which is rejected. (Right) Empirical cumulative density function of surface area from all experiments against the predicted values by regression analysis. Statistics of the Kolmogorov-Smirnov test is 0.0419 with p -value of 0.1872 under the null hypothesis that is accepted.

sample T-test. The results for each individual tree data set are shown on table 2 in which DOF represents the degree of freedom of the test. Results on table 2 indicate that the means of the estimated masses are not statistically significantly different from the means of experimental data. Lastly, if the distributions are equal, then their variances should be equal. Thus, a null hypothesis is created such that the distributions' variances are equal. The test of this hypothesis is the two-sample F-test of variance. According to the

Figure 16: Plots of (Left) the average length scales of each firebrand data set and (Right) the coefficient of variation for the length scale versus height. Where, σ is the standard deviation and μ is the mean value.

presented results on table 2, the variance of the simulated data is not statistically significantly different from the corresponding experimental data.

Moreover, experimental mass and surface area are subjected to Leave-One-Out Cross Validation (LOOCV) to further investigate the credibility of the power-law model against the linear model, although it is computationally expensive. It is worth mentioning that the training and test sets are drawn from the same population with fixed values. In this regard, 0.51% and 0.60% are the mean square root cross validation errors for the power-law and linear models, respectively. From the Cross-Validation results, it can be inferred that the power-law model better describes the relationship between mass and surface area than the linear model.

The synthetic data set presented in figure 14 was based on nonlinear regression analysis of firebrands from four trees [Manzello et al., 2007, 2008, 2009]. However, this approach assumes that the size distribution of the firebrands is independent of the size of the tree. To test this hypothesis volume and surface area characteristic length scales, $L_V = \forall^{1/3}$ and $L_S = s^{1/2}$, were calculated for each firebrand in each data set. For each data set, the mean and coefficient of variation of each of these two length scales were calculated and plotted against the tree height. The data, shown in figure 16, shows only a slight dependence of the firebrand size on the tree height. This is a somewhat surprising result given

the fractal nature of many tree crowns [Zeide and Pfeifer, 1991]. One might expect the branch and twig dimensions of a given tree to scale on the tree height and there to be a linear relationship between the tree height and the characteristic length scale of the firebrands produced. This does not appear to be the case given the results in figure 16. Pearson correlation coefficients were calculated between the tree height and each of the length scales obtained from the experimental data and found to be 0.18 for L_V and 0.40 for L_S . the Spearman correlation coefficient is 0.2 for both of the length scales. These are not particularly high. Therefore, more tree burns over a broader range of tree heights are needed to fully understand the relationship.

6 Summary and conclusions

A simple mechanical firebrand break off model is presented for the formation of cylindrical firebrands similar to those collected from full scale tree burn experiments Manzello et al. [2007, 2008, 2009]. The firebrand formation model suggests that the majority of firebrands that would form by a single tree burn experiment in isolation would have large enough aspect ratio such that they are most likely to be formed by a vertical drag force from the fire updraft. This implies that the size distribution of firebrands produced by a tree burning in a large wildfire might be the same as those produced under the controlled laboratory conditions. Further, geometric scaling analysis of firebrands indicated that the firebrand surface area scales on the firebrand mass raised to the $2/3^{rd}$ power. This was in very good agreement with firebrands from the previously published data cited above.

This model and analysis served as the motivation to statistically analyze the available firebrands' data. Results of the thorough analysis showed that, for the firebrands formed, the aspect ratio and surface area are only weakly correlated. The collection of firebrands can be well described by the log-normally distributed surface area and aspect ratio pair. Further, the results of statistical analysis led to generation of a set of virtual firebrands utilizing a nonlinear regression model on the experimental data. The synthesized firebrands compared well with the experimental data on a plot of firebrand mass versus surface area. The resulting statistical description of firebrand size and shape can be used as input in Monte-Carlo simulations of firebrand transport models to provide more realistic spot fire location predictions.

Moreover, the size of each firebrand was characterized in terms of two length scales, the cube root of

434 volume and the square root of the surface area. Interestingly, the average of each of these length scales
435 for the different tree burns did not correlate with the tree height. This implies that the size distribution
436 of firebrands is more dependent on the mechanics of combustion and limb failure than on a straight
437 geometric relationship with the tree height.

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