Hybrid Voltage Stability Assessment (VSA) for N-1 Contingency

Haoyu Yuan ^a and Fangxing Li ^{a, b}

^a: Dept. of EECS, The University of Tennessee; Knoxville, TN, 37996; USA.

^b: Contact author: fli6@utk.edu, +1-865-974-8401.

Abstract— With the growing deployment of phasor measurement units (PMUs), measurement-based voltage stability assessment (VSA) that estimates the system Thevenin equivalent has been widely discussed in recent years. This approach attracts favorable attentions mainly for its simplicity and great potential in real-time applications, as opposed to the continuation power flow (CPF) method which is model-based VSA. However, the existing measurement-based VSA deals with the normal case, but not any contingency case because there is no measurement data for a contingency case which is hypothetical. With this motivation, a hybrid VSA method is proposed based on both model-based and measurement-based techniques. First, an enhanced model-based sensitivity analysis estimating the post-contingency status is proposed, and then it is combined with the measurement-based Thevenin equivalent approach. The proposed sensitivity analysis has the feature of simplicity, which is aligned with the measurement-based Thevenin equivalent. Test results with the IEEE 14, 39, and 118-bus system show that the proposed sensitivity is highly accurate in estimating N-1 contingency system variables and the proposed hybrid VSA method for N-1 contingency captures the insecure contingencies successfully. The proposed method greatly broadens the practical application of PMUs for VSA under N-1 contingency.

Keywords— hybrid voltage stability assessment (hybrid VSA), measurement-based VSA, N-1 contingency, phasor measurement unit (PMU), Thevenin Equivalence, voltage stability index (VSI).

1. Introduction

Voltage stability is a major concern in power system operation and control [1] as a

failure in detecting voltage instability or taking proper control may lead to a system-wide blackout. Therefore, to maintain the reliability of an electric power system, it is crucial that the voltage stability margins for both normal and N-1 contingency status are assessed accurately and timely.

Continuation power flow (CPF) is a widely used model-based algorithm to estimate the steady-state voltage stability margin [2]-[4]. This method traces the voltage collapse point by solving power flows as the system load level gradually increases to indicate the load active power margin from the current operating point to the collapse point. A recent work [5] circumvents the singularity of Jacobian matrix near maximum loading by introducing a new bus type, AQ bus. CPF has been very successful in power system planning, but is time-consuming for power flow calculations and not always a good candidate for online applications especially when a significant number of contingencies are considered [6].

With the fast and broad development of phasor measurement units (PMUs), measurement-based voltage stability assessment (VSA) has attracted significant attentions in the past decade. The idea proposed in [7]-[10] uses local measurements to identify the system Thevenin Equivalent (TE) “seen” from the bus of interest. In [11], a comparative study is given on four different TE identification methods. The critical point for voltage instability is reached when the absolute value of the load impedance is equal to the Thevenin equivalent impedance [12]. Besides the impedance matching criterion, several voltage instability indicator based on local measurements have been proposed [13] [14]. In some follow-up works [15]-[20], the Thevenin equivalent model is employed for measurement-based VSA. In [15], the static polynomial ZIP load model, which includes constant impedance (Z), constant current (I) and constant power (P) components, is considered and a wide-area PMU network, rather than local measurements, is required for the proposed control scheme. In [16], a wide-area detection scheme combining load flow is proposed. In [17], the concept of coupled

single-port circuit is proposed for considering voltage stability of a load area. In [18], a modified coupled single-port circuit concept is proposed for improved performance in non-proportional load increasing direction. In [19], a double voltage source is proposed to analyze multi-feed load center. In [20], an adaptive three-bus power system equivalent is proposed to online estimate voltage stability margin for load centers supported by multiple tie lines.

Among the above works, VSA for N-1 contingency, especially the transmission contingency, has not been addressed despite its important role in daily operation and control. In [21], contingency analysis is provided with artificial neural network with the requirement of a large amount of training based on CPF results. In contrast, in our research, a hybrid VSA method is proposed to perform N-1 contingency analysis at high accuracy and efficiency, which can be suitable for online applications. This is the first goal and contribution of this paper.

For contingency analysis, the hybrid approach integrates the model-based sensitivity which gives fast estimation of the post-contingency system status. A variety of studies discussed such sensitivity analysis in the literature [22] [23] using different techniques. In [22], the outage line was kept in the system, while it was transformed to two hypothetical, dummy buses in [23]. Both approaches achieve decent results. Essentially, these sensitivity factors, a.k.a., distribution factors, are based on the Newton-Raphson Jacobian matrix [24] to represent the contingencies as changes of nodal power injection. However, this work attempts to further advance the sensitivity analysis such that the topological transformation will be mathematically proven and a succinct model will be proposed (e.g., to eliminate the extra buses in the previous work that must be added into the network). Moreover, estimated voltage angles, which are crucial for VSA but not studied/compared in the previous works, will be explored in this work. This is another important goal of this paper.

In summary, first, an enhanced and mathematically proven model for contingency

sensitivity is proposed in this paper in Section 2. Next, a hybrid VSA method, integrated with the proposed model-based sensitivity, is presented in Section 3, which is to estimate post-contingency voltage and current phasors and to further calculate the voltage stability margin via the measurement-based VSA approach. Then, the hybrid VSA method is tested in Section 4 with three IEEE test systems to demonstrate its high accuracy to provide system security information under contingencies. Finally, concluding remarks are provided in Section 5.

2. Sensitivities For Calculating Post-Contingencies Variables

In previous sensitivity analyses for transmission contingency, a line outage is typically transformed to changes of bus injections. Different transformations were introduced in the literature [22] [23], as previously mentioned. In this section, a new transformation is proposed with a rigorous mathematical proof, and the outage sensitivity factors based on the transformation are developed.

2.1. Proposed transformation and proof of equivalence

As shown in Figure 1, System A is a system in normal state, with a line flow S_{mn} from bus m to bus n and S_{nm} from bus n to bus m . The rest of the system is not shown for simplicity. System A^c is the post contingency case of System A after the line mn outage where the superscript c stands for contingency. System B has the same settings as System A , except that System B does not have branch mn . Also, the bus injection at bus m is $S_m^{inj} - S_{mn}$ and the bus injection at bus n is $S_n^{inj} - S_{nm}$, where S_m^{inj} and S_n^{inj} are the bus injections at bus m and n , respectively, in System A . System B^c is the post contingency status case of System B when S_{mn} and S_{nm} are added to Bus m and n , respectively.

The proposed transformation is to transform System A to System B , then use System B to predict System B^c by changing the bus injections at buses m and n , and finally transform System B^c to System A^c which represents the post-contingency status of interest. In other literatures, similar transformations were raised but not proven. Though achieving acceptable

accuracy, these transformations are not necessarily rigorous till proven. The Proof of the equivalence between Systems A and System B as well as that that between Systems A^c and B^c is given next.

For a power system, the nodal power equations are:

$$\Delta P_i = P_i^{inj} - V_i \sum_{j=1}^{NB} V_j Y_{ij} \cos(\theta_i - \theta_j - \phi_{ij}) = 0 \quad (1)$$

$$\Delta Q_i = Q_i^{inj} - V_i \sum_{j=1}^{NB} V_j Y_{ij} \sin(\theta_i - \theta_j - \phi_{ij}) = 0 \quad (2)$$

where

$i = 1 \sim NB$ where NB is the number of buses in the system;

V_i (or V_j) is the voltage magnitude of bus i (or j);

θ_i (or θ_j) is the voltage angle at bus i (or j);

P_i^{inj} (or Q_i^{inj}) is the real (or reactive) injection at bus i ; and

Y_{ij} (or ϕ_{ij}) is the magnitude (or angle) of the element in the system admittance matrix.

Assume N_{PQ} is the number of PQ buses and N_{PV} is the number of PV buses in the system. Therefore, there will be $2N_{PQ} + N_{PV}$ unknown variables: voltage angles for $N_{PQ} + N_{PV}$ buses and voltage magnitude for N_{PQ} buses. Let the unknown variables be vector $\bar{x}$. Let $\bar{f}(\bar{x}) = \bar{0}$ represent (1) of $N_{PQ} + N_{PV}$ buses and (2) of N_{PQ} buses. To solve the load flow is to solve

$$\bar{f}(\bar{x}) = \bar{0} \quad (3)$$

Assume $\bar{x}_A$ is the load flow solution for System A such that $\bar{f}_A(\bar{x}_A) = \bar{0}$, and $\bar{x}_B$ is the load flow solution for System B such that $\bar{f}_B(\bar{x}_B) = \bar{0}$. Though both System A and B have more than one solution for (3), there is only one feasible solution around 1.0 p.u. voltage magnitude. To prove equivalence is to show $\bar{x}_A = \bar{x}_B$. Since $\bar{x}_A$ and $\bar{x}_B$ are the feasible solution, it is to show $\bar{f}_B(\bar{x}_A) = \bar{0}$. This is the key of the rigorous proof to be presented next.

Assume that the symbols introduced in (1) and (2) represent the values in System A . Underscores are used to represent the values in System B .

Differences between Systems A and B are as follows:

- a) the different nodal injections at buses m and n ; and
- b) the branch mn .

The first difference will only matter corresponding to (1) and (2) for buses m and n . The second difference will only affect Y_{mn} , Y_{nm} , Y_{mm} , and Y_{nn} in the admittance matrix, which also only appear in (1) and (2) for buses m and n . To sum up, only the equations related to the injections at buses m and n are different for $\bar{f}_A(\bar{x}_A) = \bar{0}$ and $\bar{f}_B(\bar{x}_B) = \bar{0}$. Therefore, $\bar{f}_B(\bar{x}_A) = \bar{0}$ holds for the equations other than those related to the injections at buses m and n . Thus, we only need to prove that $\bar{x}_A$ also satisfies the equations related to the injections at buses m and n .

For the admittance matrix Y of System A , the element $Y_{ij} = G_{ij} + jB_{ij}$ is given by

$$Y_{ij} = -y_{ij}, i \neq j \quad (4)$$

$$Y_{ii} = \sum_j y_{ij} + y_{ig} \quad (5)$$

$$y_{ig} = j \sum_{Y_{ij} \neq 0} \frac{b_{ijg}}{2} \quad (6)$$

where $y_{ij} = g_{ij} + jb_{ij}$ is the admittance of branch ij ; y_{ig} is the ground admittance of bus i ; and $j \cdot b_{ijg}/2$ is the shunt admittance of line ij .

Again, underscores are used to represent the values in System B values. For The elements of the admittance matrix Y , the element is given by

$$\underline{Y}_{ij} = Y_{ij}, ij \notin \{mm, nn, mn, nm\} \quad (7)$$

$$\underline{Y}_{mn} = \underline{Y}_{nm} = 0 \quad (8)$$

$$\underline{Y}_{mm} = Y_{mm} - y_{mn} - jb_{mng} / 2 \quad (9)$$

$$\underline{Y}_{nn} = Y_{nn} - y_{mn} - jb_{mng} / 2 \quad (10)$$

The line flow between buses m and n is given by

$$P_{mn} = V_m V_n Y_{mn} \cos(\theta_m - \theta_n - \phi_{mn}) + V_m^2 g_{mn} \quad (11)$$

$$Q_{mn} = V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \phi_{mn}) - V_m^2 (b_{mn} + b_{mng} / 2) \quad (12)$$

The types of buses m and n will lead to different number of equations. First, consider the situation that both m and n are PQ buses. We may rewrite the bus injection equations for buses m and n in System B . With detailed proof shown in (A.1) in the appendix, the real power injection equation is given by:

$$\underline{\Delta P}_m(\bar{x}_A) = 0 \quad (13)$$

With detailed proof shown in (A.2) in the appendix, the reactive power injection equation is given by:

$$\underline{\Delta Q}_m(\bar{x}_A) = 0 \quad (14)$$

Substituting m with n and n with m in (A.1) and (A.2), we have

$$\underline{\Delta P}_n(\bar{x}_A) = 0 \quad (15)$$

$$\underline{\Delta Q}_n(\bar{x}_A) = 0 \quad (16)$$

If we combine (13) and (16), $\bar{f}_B(\bar{x}_A) = \bar{0}$ is true. Thus, System A and System B are mutually equivalent from the viewpoint of steady-state power flows.

If bus m or bus n is a PV bus, then Eq. (2) considering the reactive power injection for that bus cannot be included in the formation of $\bar{f}(\bar{x}) = \bar{0}$. If one of the two buses is the slack bus, both equations for real and reactive power injection will not be included in $\bar{f}(\bar{x}) = \bar{0}$. However, the change of bus types will not affect (1), (2), and (4)-(12), which will not affect the conclusion that System A and System B are equivalent. As a matter of fact, it only changes the number of unknown variables and equations.

Therefore, it is concluded that System A and System B are equivalent regardless of the types of buses m and n .

For System A^c and System B^c , it is seen from Figure 1 that they have the identical topology and bus injections, so they must be equivalent. Besides, if we set line flow mn to zero, Systems A and B become A^c and B^c , respectively.

2.2. Proposed Sensitivity Based On The Transformation

A commonly used approach to solve nonlinear load flow equations in (3) is the Newton-Raphson method. The Jacobian matrix in each iteration is given as follows [24]

$$J = \frac{\partial f(x)}{\partial x} \quad (17)$$

Based on the Jacobian matrix, the sensitivity can be formulated as [24]

$$\Delta \bar{x} = J^{-1} \Delta \bar{f}(\bar{x}) \quad (18)$$

where $\Delta \bar{f}(\bar{x})$ is the vector of nodal injection change which is numerally equal to the outage

line flow, and $\Delta \bar{x}$ is the change of system variables.

There are two types of outages, generation/load outage and line outage. For the first type, generation or load outages can be directly represented by nodal injection change. Thus, it is not discussed in this paper. While for the line outage, various transformations have been explored to represent the line outages in nodal injection change.

Since the equivalence of System A and System B has been proven, it is natural to transform A to B by adding the line flow to nodal injection and remove the outage line. Then, by adding nodal change to System B , sensitivity based on (18) can predict the status of System B^c , which is equivalent to system A^c , the post-contingency case of System A .

2.3. Piece-wise-linear Sensitivity Considering Q Limit Violations

Practically, in power flow model, most PV buses have upper and lower limits for reactive power. After a severe contingency, there may be violations of Q limits at some buses, say Bus x . If this happens, the PV bus should be considered a PQ bus with the same scheduled P and the scheduled Q set at the violated Q limit. If only the sensitivity of the initial condition is used, Bus x will be treated as a PV bus in the post-contingency state. The voltage magnitude will be the same as before. However, after the PV-PQ transition, the voltage magnitude of that bus will drop for lack of reactive supply. This will cause significant error. Therefore, a piece-wise-linear sensitivity model is proposed next to deal with the PV to PQ transition should be introduced.

First, a new index called Status Change Coefficient Factor, F , is introduced. When a contingency happens, the reactive power of a PV bus may violate either the upper or lower limit. As shown in Figure 2, Q_0 is the reactive generation at a bus at normal state and Q_c is the predicted reactive generation using the sensitivity factors derived in this iteration. Q_{upper} and Q_{lower} are the upper and lower limits, respectively, for reactive power generation. The F factor is defined as

$$F = \begin{cases} \frac{Q_{upper} - Q_0}{Q_c - Q_0}, & Q \text{ upper limit is violated} \\ 1, & Q_{lower} \leq Q_c \leq Q_{upper} \\ \frac{Q_0 - Q_{lower}}{Q_0 - Q_c}, & Q \text{ lower limit is violated} \end{cases} \quad (19)$$

It is seen from Figure 2 that when the reactive power limits are violated at a specific bus, F for that bus will be between 0 and 1. Assume that $\Delta f(x)$, the hypothetical nodal injection, and Q are linearly related. Then F can be used to identify when the Q limit is first violated at that bus. The smallest F of all buses, say F_j , approximates the point where the first PV bus in the system violates its limit when $F_j \cdot \Delta f(x)$ is applied to the system.

Therefore, the result in the range of $[F_j \cdot \Delta f(x), \Delta f(x)]$ (i.e., beyond the point where the first violation occurs) is no longer acceptable. Thus, the original sensitivity can be applied only in the range of $[0, F_j \cdot \Delta f(x)]$ in which the specific bus is considered a PV bus. In the range of $[F_j \cdot \Delta f(x), \Delta f(x)]$, the bus is converted from a PV bus to PQ, therefore, the system Jacobian matrix needs to be modified accordingly. After modifying the Jacobian and finding the new sensitivity factors, we may apply the remaining amount of bus injection change, $(1-F_j) \cdot \Delta f(x)$. Then, if we still have another Q violation at a second bus after converting the first PV bus to PQ, then we will have to convert the second PV bus to PQ. Thus, a total of three segments will be identified in the overall range of $[0, \Delta f(x)]$. This process continues until no Q violation is found, after the scheduled $\Delta f(x)$ is fully applied to the system. The algorithm is illustrated in Figure 3.

3. Hybrid Voltage Stability Assessment

Presently, model-based contingency analysis such as continuation power flow (CPF) for voltage stability assessment (VSA) is widely applied to predict voltage stability margin. If applied online, though capable of foreseeing contingencies which may cause voltage instability, such approach requires significant amount of computational resources. Practically, in some utilities, only selected contingencies are analysed on a 15 minutes base.

An alternative approach, measurement based VSA, assesses voltage stability directly based on measurement data. The core technique is to approximate the Thevenin equivalent of the external system based on real-time measurement data and then predict the stability margin by comparing the load bus with its external system.

The 2-bus Thevenin equivalent circuit is shown in Figure 4. The 4 unknown variables are the magnitudes and angles of Thevenin voltage and impedance. A given measurement set at a time stamp will provide two equations using Kirchhoff's law. Therefore, with the assumption that the Thevenin equivalent remains constant in a time frame, two or more sets of measurements will be sufficient in estimating the parameters using Least-Square method.

After the Thevenin equivalent is identified, voltage stability margin can be estimated based on the characteristics of the two-bus circuit. The Voltage Stability Index (VSI) in [15] is employed here to indicate the stability margin. The VSI is formulated in (20). When the VSI decreases to 1, the system approaches to the instable point (i.e., the nose point or the bifurcation point) in typical PV curves.

$$VSI = \left| \frac{V_k}{\Delta V_k} \right| \quad (20)$$

Since such approach only uses measurement as input instead of information of the complex system model, the computational burden is small which is desirable for real-time applications. However, due to the lack of system information under N-1 contingency, the capability of this approach has been limited to normal state.

With the contingency sensitivity analysis derived in the previous section, a hybrid VSA method for N-1 contingency is proposed based on a combination of the model-based contingency sensitivity and the measurement-based Thevenin equivalent. Since there is no multiple power flow runs (as opposed to the CPF approach), the hybrid approach is computationally efficient.

The hybrid VSA is shown in Figure 5. In the flow chart: 1) subscript i stands for the

bus ID; 2) superscript c_j stands for the j^{th} contingency; 3) N is the total number of contingencies; 4) subscript th stands for Thevenin equivalence; and 5) VSI stands for voltage stability index, i.e., an index which represents the system voltage stability margin.

The left most branch in Figure 5 is the same as the measurement-based approach in calculating the VSI for bus i under normal case (i.e., N-0). The remaining N branches in the right are parallel processes to calculate the VSI for N single contingency cases.

In each of the parallel branch, the algorithm first calculates the proposed sensitivity factors based on the measurement, i.e., the present system status, and then predicts the post-contingency status. These first two steps are model-based approaches utilizing the system data. After that, it calculates the Thevenin equivalent and provides the VSI for the post-contingency system. These two steps are measurement-based except that the input is not directly from measurement but from predicted post-contingency values.

Note, this method assumes that the system is observable at all studied buses which is aligned with previous works [15] [16]. That is because when calculating the sensitivity, the system status must be known to form the Jacobian matrix.

As long as the prediction of post-contingency cases achieves excellent accuracy, the VSIs for N-1 contingency can provide good references for the system stability margin. Moreover, since both of the prediction process and the measurement-based approach are computationally efficient and highly parallelizable, the whole approach can speed up the process of N-1 contingency analysis and has great potential for online implementation.

4. Case Studies

In this section, first, the proposed sensitivity calculation is tested with the IEEE 14-bus system [25] for normal operating condition and is compared with the results from [22] and [23]. The IEEE 14-bus system is chosen for the sensitivity test because it is also the test system used in [22] and [23]. Details such as the bus voltage magnitudes and angles are

provided for two severe contingencies under the base case load level. Then, the proposed sensitivity is further tested on the IEEE 14, 39, and 118-bus systems [25] for all possible line outages in the load range between the normal condition and the collapsing point. Finally, the hybrid VSA is tested on the IEEE 14, 39, and 118-bus systems to demonstrate its ability in online monitoring of the stability margins of N-1 contingencies, when a load increase scenario is studied.

4.1. Test of the proposed sensitivity on IEEE 14-bus system

The IEEE 14-bus system which is the test system in [22] and [23] is used to test the sensitivity in this subsection in order to compare the proposed sensitivity calculation with the previous methods. For this system, two contingencies, line 2-4 outage and line 5-6 outage, under normal loading condition are considered.

The first contingency, line 2-4 outage, will not cause any reactive (Q) violation which means it is not a severe contingency. The results from the proposed sensitivity model, as well as the results using two previous methods in [22] and [23], are presented in Tables 1 and 2 for comparison.

Table 1 shows the predicted voltage magnitudes. The column ‘Exact Value’ lists the exact post-contingency voltage magnitude by re-running the post-contingency power flow. Although all three methods achieve acceptable results, the proposed method performs even better than the other two in both average errors and variances.

Table 2 shows the voltage phase angle of the predicted results. Similarly, the column ‘Exact Value’ comes from the post-contingency power flow. From the table, the proposed method and the method in [23] are very accurate in predicting the angles with an average error of 0.43% and 0.40%, respectively. However, method in [22] is not so accurate at an average error of 5.43%.

From the above results, the proposed method predicts the post-contingency status at a very high accuracy. The performance of the proposed method is very close to the method in [23] when there is no Q limit violation.

The second contingency, line 5-6 outage, is considered a severe contingency because this line has a large volume of power flow under normal case and its outage causes Q limit violation at Bus 6. The approach shown in Figure 3 is employed here to deal the Q limit violation.

Note, since Ref. [22] does not address this Q violation, so it cannot be compared with the proposed sensitivity approach. Also, the approach in [23] is improved and enhanced in this study. Although, rigorously speaking, it is not completely fair to compare the proposed approach with the one in [23], the comparison is carried out to illustrate the advantage of the proposed method. Details are elaborated next.

Before making any further comparison between the proposed method and the one in [23], it should be mentioned that the exact results of post-contingency status used in the two methods are not the same because of the way the outage is modeled. In [23], with the consideration of line outage, only the series impedance of that branch is taken out. While in this research, both the series impedance and the shunt admittance are taken out when an outage occurs. Since the shunt branch is part of the transmission PI (π) model, it is more reasonable to take out the shunt admittance with the series impedance to model a contingency event.

Consequently, in the prediction of the post-contingency variables, the load injection change used in [23] only considers the line flow through the series impedance. In contrast, the proposed method considers the flow change through the series impedance as well as the shunt admittance, which is more reasonable. Thus, the approaches in [23] and this work are comparable, loosely speaking.

The predicted voltages for line 5-6 outage are shown in Table III. The result from the algorithm considering Q limits is labelled with “P.W.L.” and that from ignoring Q limits is labelled with “Linear”. Obviously, Table 3 shows the advantage of P.W.L. approach when Q limits are violated.

Ref. [23] presents only the predicted voltage magnitudes which reports an average error of 1.58% compared with the exact value. The proposed scheme has improved the accuracy by 50% in this case with an average error of 0.82%. Ref. [23] provides the predicted reactive power generation and line flow whose accuracy is closely related to the accuracy of voltage angles. Thus, the comparison of the predicted reactive power is shown in Tables 4 and 5. The numerical results show that the proposed method is more accurate than the previous methods. This is reasonable because this work provides a more reasonable outage model than those in the literature.

4.2. Extensive Test of the sensitivity with three systems in broad load ranges for all N-1 contingencies

The previous subsection provides a detailed study at a particular load level (i.e., the base case load level) for two contingencies for the IEEE 14-bus system, as well as a comparison with the results from the literature. Next, in this subsection, the proposed sensitivity is extensively tested on three different systems (i.e., the IEEE 14, 39, and 118-bus systems) for all branch outages except those causing islands like a branch connecting a generation bus only. Also, the tests are carried out in a broad load range between the base case load and the maximum load at the collapse point.

For each system, the PV curve for normal condition, namely the N-0 case, is first calculated using continuation power flow (CPF). The power flow solutions on the PV curve can be viewed to mimic the measurements from PMUs. Then, the proposed sensitivity model is combined with the N-0 voltages to estimate the N-1 voltages for a specific branch outage,

and this can be applied to draw the N-1 PV curve once various load levels are considered.

Since this extensive test involves a large amount of data which are not suitable to be shown in tables, two types of N-1 PV curves, one *estimated* from the sensitivity and the N-0 voltages and the other *calculated* based on the actual N-1 CPF runs, are depicted and compared in figures.

The test results for the IEEE 14-bus system are shown in Figure 6. The PV curves for Bus 14, which is the critical/weakest load bus, are chosen. The solid lines are the estimated values using the proposed sensitivity and N-0 voltages, while the lines with point markers are the benchmark PV curves calculated using CPF under N-1 contingency. Different contingencies are distinguished using different colors. The X-axis represents the loading factor which can be multiplied with the base load level to obtain the actual load. In other words, a loading factor of 1.0 represents the base case. A total number of 17 line outages are considered but only the top 10 most severe outages are shown in Figure 6 due to the space limit. For better illustration, the PV curves start from 2.0 loading factor of the base case load. Based on the figure, the solid curves overlap with the curves with point marks except at the very end close to the nose points. The errors at the end are larger than at other sections in the curves. This is reasonable because the increasing line flow tends to give less accurate results due to the linear estimation based on sensitivity, when the loading factor grows. Nevertheless, such error is still very small, generally less than 2% in terms of the power at the collapse point, which is clearly shown in Figure 6. Thus, the results are considered very accurate.

Figure 7 demonstrates the test results for the IEEE 39-bus system. The PV curves for Bus 7, which is the critical/weakest load bus, are chosen. Similarly, the solid lines are from the sensitivity-based estimation, while the lines with point markers are from an actual CPF calculation under N-1 contingency. Due to the space limit, only the top 10 most severe outages are included. For better illustration, the PV curves start from 1.5 loading factor of the

base case load. Note, the excluded portion of the curves between 1 and 1.5 times of the base load shows perfect overlap between the dashed and solid curves. As shown in Figure 7, the sensitivity also performs very well for the IEEE 39-bus system at all N-1 contingencies between the base case and the maximum load levels.

Test results for the IEEE 118-bus system are presented in Figure 8. The critical/weakest load bus is Bus 44, and there are a total number of 177 line outages examined. Again, only the top 10 most severe outages are presented in Figure 8 for easy illustration. Similarly, the solid lines are estimated results based on the proposed sensitivity, while the lines with point markers are from the actual CPF calculation under N-1 contingency. Similar observations can be concluded that the sensitivity-based estimation gives very accurate N-1 PV curves.

4.3. Test of the Hybrid VSA

The previous subsection 4.2 demonstrates the validity of the proposed sensitivity at three systems under N-1 contingencies with a broad range of load levels, which is between the base case load level and the maximum load level at the nose point. In this subsection, the model-based sensitivity will be combined with the measurement-based approach to evaluate the system's voltage stability margin under various N-1 contingencies. Here, the voltage stability index (VSI) introduced in equation (20) is employed to estimate the N-1 stability margin, which is defined as the distance in % between the base case load and the load at the VSI-based nose point. Note, under measurement based techniques, the VSI-based nose point is defined using the VSI value. That is, VSI is greater than 1.0 and the system is N-1 secure if the system operates at the upper portion of the PV curve for a specific N-1 contingency; while the VSI decreases to 1.0 once the system operating point reaches the nose point and therefore unsecure for that contingency.

In the case study, power flow solutions of the system N-0 status are used as if they are

the measurements from the PMUs [17] [18]. Noise or incorrect measurements from the PMUs are not considered here which is also commonly assumed in many measurement-based research works such as in [17] [18].

For the IEEE 14-bus system, the stability margins from hybrid VSA (using VSI-based nose point criterion) and actual N-1 CPF are presented in Figure 9. Similarly, the results of the IEEE 39-bus system are shown in Figure 10, and the results of the 30 most severe contingencies (due to space limit) for the IEEE 118-bus system are shown in Figure 11. According to these three figures, the stability margins calculated by hybrid VSA based on N-0 voltages and the proposed sensitivity calculation are very close to the actual margins obtained from the N-1 CPF. The average errors in estimating the system stability margin for the three test systems for all outages (not limited to 30 most severe contingencies for the 118-bus system) are presented in Table 6. It is shown that the VSI-based hybrid VSA is able to estimate the stability margin very accurately.

Further simulation is carried out in order to better illustrate the effectiveness of hybrid VSA using a load increasing scenario. In this study, the load of the whole system increases continuously with small randomly embedded noises for more than 1000 seconds. The voltage magnitude of Bus 44 for this scenario is shown in Figure 12 (a) where the voltage decreases as the load increases. With PMUs installed, the voltage for N-0 can be monitored by the system operator in the control room. With the proposed hybrid VSA, the system operator will be able to keep track of the voltages under N-1 as shown in Figure 12 (b), as well as the VSI for those contingencies shown in Figure 12 (c) in real time. Once again, only the top 30 most severe contingencies are shown due to space limit and for easy illustration. By combining the model-based sensitivity with the measurement-based VSA, the hybrid VSA using N-1 VSI provides clear information about how far the system is away from voltages instability under N-1 contingencies, as shown in Figure 12(c). Since the N-1 VSI is obtained in real time based

on the measured N-0 voltages and the model-based sensitivity matrices which can be calculated in real time because it is only related to topology (obtained offline) and the present operating point (from measurement). Thus, this will significantly increase the situation awareness by identifying the most critical contingency and monitoring the security margin.

Another observation is that according to Figure 12 (c), at the end of the simulation the VSI of the blue curve (line 8-5 outage) reaches 1.0, which means the system reaches an insecure state. However, this contingency is not the most severe one in terms of voltage drop, and the most severe voltage drop is from the contingency of line 38-65 outage which is shown in Figure 12(b). Thus, Figures 12(b) and (c) clearly verify that the most critical contingency in terms of voltage security may not be the contingency causing the largest voltage drop.

5. Conclusions

In this paper, a hybrid voltage stability assessment (VSA) method is proposed for N-1 contingency. With the proposed model-based contingency sensitivity and the measurement-based VSA technique, the hybrid VSA method solves the challenge of VSA under contingency cases. This broadens the application of PMUs in actual practices.

The model-based sensitivity calculation method can be used to estimate the post-contingency system status such as voltages. The transformation in deriving the sensitivity is mathematically proven in the paper. Test results on the IEEE 14-bus system show that the proposed method considerably reduces the estimation errors, especially when reactive power limits are violated. This is the advantage of the proposed method over the methods in the literature.

Then, extensive case studies of the proposed sensitivity on IEEE 14, 39, and 118-bus systems for all line outages demonstrate that it is suitable and accurate for estimating post-contingency status for VSA applications. The proposed hybrid VSA method, which is based

on the estimated post-contingency system status, shows very good accuracy in estimating the voltage security margin for the N-1 contingency on all three test systems. Finally, a load increasing scenario simulated on the IEEE 118 system illustrates the ability that the hybrid VSA can assist the system operators with monitoring the stability margin for N-1 contingencies.

Since the proposed hybrid VSA utilizes the measured N-0 voltages and the proposed sensitivity calculation methods, calculation of voltage stability margin under N-1 contingencies will be easy and fast. This addresses the challenge that the N-1 voltages cannot be measured and also makes the proposed approach very promising in future online applications.

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7. Appendix

The real power injection equation is given by:

$$\begin{aligned}
 \underline{\Delta P_m}(\bar{x}_A) &= \underline{P_m^{inj}} - V_m \sum_{j=1}^{NB} V_j \underline{Y_{mj}} \cos(\theta_m - \theta_j - \underline{\phi_{mj}}) \\
 &= \underline{P_m^{inj}} - P_{mn} - V_m \sum_{j=1, j \neq m, n}^{NB} V_j \underline{Y_{mj}} \cos(\theta_m - \theta_j - \underline{\phi_{mj}}) - V_m^2 \underline{Y_{mm}} \cos(\underline{\phi_{mm}}) \\
 &\quad - V_m V_n \underline{Y_{mn}} \cos(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
 &= \underline{P_m^{inj}} - P_{mn} - V_m \sum_{j=1}^{NB} V_j \underline{Y_{mj}} \cos(\theta_m - \theta_j - \underline{\phi_{mj}}) + V_m^2 \underline{Y_{mm}} \cos(\underline{\phi_{mm}}) + V_m V_n \underline{Y_{mn}} \cos(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
 &\quad - V_m^2 \underline{Y_{mm}} \cos(\underline{\phi_{mm}}) - V_m V_n \underline{Y_{mn}} \cos(\theta_m - \theta_n - \underline{\phi_{mn}})
 \end{aligned}$$

$$\begin{aligned}
(1) &= -P_{mn} + V_m^2 Y_{mm} \cos(\phi_{mm}) + V_m V_n Y_{mn} \cos(\theta_m - \theta_n - \phi_{mn}) - V_m^2 \underline{Y_{mm}} \cos(\underline{\phi_{mm}}) \\
&\quad - V_m V_n \underline{Y_{mn}} \cos(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
\frac{Y_{mn}}{=0} &= -P_{mn} + V_m^2 (G_{mm} - \underline{G_{mm}}) + V_m V_n Y_{mn} \cos(\theta_m - \theta_n - \phi_{mn}) \\
(9) &= -P_{mn} + V_m^2 (g_{mn}) + V_m V_n Y_{mn} \cos(\theta_m - \theta_n - \phi_{mn}) \\
(11) &= 0
\end{aligned} \tag{A.1}$$

The reactive power injection equation is given by:

$$\begin{aligned}
\Delta Q_m(\bar{x}_A) &= \underline{Q_m^{inj}} - V_m \sum_{j=1}^{NB} V_j \underline{Y_{mj}} \sin(\theta_m - \theta_j - \underline{\phi_{mj}}) \\
&= \underline{Q_m^{inj}} - Q_{mn} - V_m \sum_{j=1, j \neq m, n}^{NB} V_j Y_{mj} \sin(\theta_m - \theta_j - \phi_{mj}) - V_m^2 \underline{Y_{mm}} \sin(\underline{\phi_{mm}}) \\
&\quad - V_m V_n \underline{Y_{mn}} \sin(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
&= \underline{Q_m^{inj}} - Q_{mn} - V_m \sum_{j=1}^{NB} V_j Y_{mj} \sin(\theta_m - \theta_j - \phi_{mj}) + V_m^2 Y_{mm} \sin(-\phi_{mm}) + V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \phi_{mn}) \\
&\quad - V_m^2 \underline{Y_{mm}} \sin(-\underline{\phi_{mm}}) - V_m V_n \underline{Y_{mn}} \sin(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
(2) &= -Q_{mn} + V_m^2 Y_{mm} \sin(-\phi_{mm}) + V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \phi_{mn}) - V_m^2 \underline{Y_{mm}} \sin(-\underline{\phi_{mm}}) \\
&\quad - V_m V_n \underline{Y_{mn}} \sin(\theta_m - \theta_n - \underline{\phi_{mn}}) \\
\frac{Y_{mn}}{=0} &= -Q_{mn} + V_m^2 (-B_{mm} - \underline{B_{mm}}) + V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \phi_{mn}) \\
(9) &= -Q_{mn} - V_m^2 (b_{mn} + b_{mng}/2) + V_m V_n Y_{mn} \sin(\theta_m - \theta_n - \phi_{mn}) \\
(12) &= 0
\end{aligned} \tag{A.2}$$

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Figure 1: Proposed transformation

Figure 2: Status change coefficient factor F

Figure 3: Algorithm considering Q limits

Figure 4: VSI for hybrid VSA

Figure 5: Flow chart of the proposed hybrid VSA

Figure 6: Voltage magnitude for N-1 contingencies of the IEEE 14-bus system

Figure 7: Voltage magnitude for N-1 contingencies of the IEEE 39-bus system

Figure 8: Voltage magnitude for N-1 contingencies of the IEEE 118-bus system

Figure 9: Stability margin for N-1 contingencies of the IEEE 14-bus system

Figure 10: Stability margin for N-1 contingencies of the IEEE 39-bus system

Figure 11: Stability margin for N-1 contingencies of the IEEE 118-bus system

Figure 12: Illustration of hybrid VSA under N-1 contingency

Figure 4: VSI for hybrid VSA.

Figure 5: Flow chart of the proposed hybrid VSA

Figure 6: Voltage magnitude for N-1 contingencies of the IEEE 14-bus system

Figure 7: Voltage magnitude for N-1 contingencies of the IEEE 39-bus system

Figure 8: Voltage magnitude for N-1 contingencies of the IEEE 118-bus system

Figure 9: Stability margin for N-1 contingencies of the IEEE 14-bus system

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Figure 11: Stability margin for N-1 contingencies of the IEEE 118-bus system

Figure 12: Illustration of hybrid VSA under N-1 contingency

Table 1: Comparison of Predicted Voltage Magnitude

Bus No	Exact Value	Method in [18]		Method in [19]		Proposed method	
		Predicted	Error (%)	Predicted	Error (%)	Predicted	Error (%)
1	1.0600	1.0600	--	1.0600	--	1.0600	--
2	1.0450	1.0450	--	1.0450	--	1.0450	--
3	1.0100	1.0100	--	1.0100	--	1.0100	--
4	1.0071	1.0093	0.22	1.0079	0.08	1.0070	0.01
5	1.0112	1.0136	0.23	1.0120	0.08	1.0115	0.03
6	1.0700	1.0700	--	1.0700	--	1.0700	--
7	1.0564	1.0575	0.11	1.0567	0.03	1.0563	0.00
8	1.0900	1.0900	--	1.0900	--	1.0900	--
9	1.0504	1.0518	0.13	1.0508	0.04	1.0504	0.00
10	1.0463	1.0475	0.11	1.0466	0.03	1.0463	0.00
11	1.0544	1.0551	0.06	1.0546	0.02	1.0544	0.00
12	1.0548	1.0549	0.01	1.0549	0.00	1.0548	0.00
13	1.0495	1.0497	0.02	1.0495	0.01	1.0495	0.00
14	1.0319	1.0328	0.08	1.0322	0.02	1.0319	0.00
Ave. of Error (%)		0.11		0.03		0.00	
Variance		7×10^{-7}		8×10^{-8}		7×10^{-9}	

Note: Bus1 is the slack bus, and Buses 2, 3, and 6 are PV buses. Thus, the error is not applicable to these buses.

Table 2: Comparison of Predicted Voltage Angles

Bus No	Exact Value (degree)	Method in [18]		Method in [19]		Proposed method	
		Predict (degree)	Error (%)	Predict (degree)	Error (%)	Predict (degree)	Error (%)
1	0.0000	0.0000	--	0.0000	0.00	0.0000	--
2	-4.5038	-4.6946	4.24	-4.4792	0.55	-4.4805	0.52
3	-14.131	-13.601	3.75	-14.069	0.44	-14.075	0.40
4	-13.234	-12.092	8.62	-13.160	0.55	-13.148	0.65
5	-10.765	-9.9934	7.17	-10.713	0.49	-10.706	0.55
6	-16.593	-15.683	5.49	-16.526	0.41	-16.524	0.42
7	-16.145	-15.062	6.71	-16.074	0.44	-16.064	0.50
8	-16.145	-15.062	6.71	-16.074	0.44	-16.064	0.50
9	-17.646	-16.594	5.96	-17.576	0.39	-17.567	0.44
10	-17.745	-16.718	5.78	-17.676	0.39	-17.668	0.43
11	-17.300	-16.331	5.60	-17.232	0.39	-17.227	0.42
12	-17.475	-16.554	5.27	-17.407	0.39	-17.405	0.40
13	-17.575	-16.645	5.29	-17.508	0.38	-17.505	0.40
14	-18.622	-17.622	5.37	-18.553	0.37	-18.547	0.40
Avg. of Error (%)		5.43		0.40		0.43	
Variance		2×10^{-4}		4×10^{-7}		5×10^{-7}	

Table 3: Predicted Voltage for Line 5-6 Outage

Bus No	Magnitude (p.u.)			Angle (degree)		
	Exact	Linear	P.W.L.	Exact	Linear	P.W.L.
1	1.0600	1.0600	1.0600	0.0000	0.0000	0.0000
2	1.0450	1.0450	1.0450	-5.1355	-5.0588	-5.0946
3	1.0100	1.0100	1.0100	-13.187	-13.025	-13.107
4	1.0140	1.0206	1.0161	-10.977	-10.897	-10.911

5	1.0243	1.0291	1.0259	-8.6054	-8.5375	-8.5540
6	1.0073	1.0700	1.0195	-27.009	-26.628	-26.383
7	1.0534	1.0693	1.0577	-17.281	-16.911	-17.064
8	1.0900	1.0900	1.0900	-17.281	-16.911	-17.064
9	1.0443	1.0735	1.0515	-20.569	-20.004	-20.257
10	1.0305	1.0671	1.0393	-21.976	-21.450	-21.626
11	1.0159	1.0668	1.0271	-24.555	-24.126	-24.096
12	0.9937	1.0554	1.0061	-27.415	-26.961	-26.799
13	0.9945	1.0544	1.0068	-27.003	-26.527	-26.417
14	1.0029	1.0478	1.0134	-24.368	-23.814	-23.949
Avg. of Error		3.69%	0.82%		1.73%	1.43%
Variance		5×10^{-4}	2×10^{-5}		4×10^{-5}	4×10^{-5}

Table 4: Comparison of Predicted Var Generation

Gen No	Method in [19]				Proposed Method			
	Exact (p.u.)	Pred. (p.u.)	Error		Exact (p.u.)	Pred. (p.u.)	Error	
			p.u.	%			p.u.	%
1	-20.16	-20.94	0.78	3.87	-19.73	-20.11	0.39	1.93
2	42.82	38.29	4.53	10.5	44.21	41.82	2.39	5.41
3	25.37	23.02	2.35	9.26	27.15	25.91	1.24	4.57
6	24.00	24.00	0.00	0.00	24.00	24.00	0.00	0.00
8	22.25	17.19	5.06	22.7	22.65	20.01	2.65	11.7
Avg. Error			2.54	9.29			1.33	4.71
Variance			4.97	0.007			1.38	0.002

Table 5: Comparison of Predicted Reactive Line Flow

Branch No	Method in [19]				Proposed method			
	Exact (p.u.)	Pred. (p.u.)	Error (p.u.)	Error (%)	Exact (p.u.)	Pred. (p.u.)	Error (p.u.)	Error (%)
1-2	-21.46	-20.95	0.51	2.38	-21.48	-21.19	0.29	1.35
1-5	1.30	0.02	1.28	98.46	1.75	1.08	0.67	38.29
2-3	3.30	3.35	0.05	1.52	3.29	3.33	0.03	1.22
2-4	-1.48	-3.58	2.10	141.89	-0.65	-1.75	1.09	169.23
2-5	-1.22	-2.80	1.58	129.51	-0.68	-1.50	0.82	120.59
3-4	3.77	1.66	2.11	55.97	5.53	4.42	1.11	20.07
4-5	8.42	10.16	1.74	20.67	8.70	9.76	1.06	12.18
4-7	-4.83	-7.87	3.04	62.94	-5.06	-6.27	1.21	23.91
4-9	3.22	0.54	2.68	83.23	3.16	2.08	1.07	34.18
6-11	5.39	5.37	0.02	0.37	5.39	5.33	0.06	1.11
6-12	3.26	3.26	0.00	0.00	3.26	3.20	0.06	1.84
6-13	7.85	7.86	0.01	0.13	7.85	7.71	0.14	1.78
7-8	-21.52	-16.75	4.77	22.17	-21.89	-19.42	2.48	11.28
7-9	10.26	4.26	6.00	58.48	10.38	7.50	2.88	27.75
9-10	4.90	1.62	3.28	66.94	4.91	3.44	1.47	29.94
9-14	3.88	1.74	2.14	55.15	3.89	3.05	0.84	21.59
10-11	-1.76	-4.41	2.65	150.57	-1.76	-2.64	0.87	50.00
12-13	1.59	1.62	0.03	1.89	1.59	1.53	0.06	3.77
13-14	3.53	3.79	0.26	7.37	3.53	3.68	0.16	4.25
Avg.			1.80	50.51			0.44	30.23
Var.			2.74	0.26			0.63	0.19

Table 6: Average errors of stability margins

	<i>IEEE 14-bus system</i>	<i>IEEE 39-bus system</i>	<i>IEEE 118-bus system</i>
Average error	0.63%	0.23%	0.21%