A Statistical Approach for Post-Processing Residential Building Energy Simulation Output

S. Valovcin^{a,b}, A. S. Hering^{a,*}, B. Polly^b, M. Heaney^b

^a*Department of Applied Mathematics and Statistics, Colorado School of Mines, Golden, CO, United States*

^b*Buildings and Thermal Systems Center, National Renewable Energy Laboratory, Golden, CO, United States*

Abstract

Residential building energy simulation (RBES) software plays an important role in evaluating the energy consumption and efficiency potential of homes. These physics-based models are commonly used to assess the energy performance of homes and to predict benefits of making energy-saving improvements to homes *a priori*. However, software may produce biased estimates of energy consumption for a variety of reasons, including: errors in the measurement and observation of building characteristics; differences in the assumed versus actual occupant behavior; and errors in the physical models and algorithms used in the software. In order to evaluate and improve the accuracy of RBES software, the National Renewable Energy Laboratory (NREL) has assembled a set of approximately 1,250 U.S. homes for which measured energy consumption and audit-collected household energy characteristics are available. Algorithms have also been developed that

*Corresponding author

Email addresses: svalovcin@gmail.com (S. Valovcin), ahering@mines.edu (A. S. Hering), ben.polly@nrel.gov (B. Polly), michael.heaney@nrel.gov (M. Heaney)

automatically translate the data from each home into RBES input files so that model predictions of annual electricity and natural gas consumption can be compared to measured values. To assess and improve upon the accuracy of these predictions, we first cluster the homes using weighted, independent linear combinations of these variables and then build multiple linear regressions within clusters of similar homes to model the difference between measured and predicted energy consumption based on the recorded features of the homes. The statistical post-processing techniques that we develop for RBES models have the following benefits: (1) they can identify variables and algorithms that may be causing inaccuracies in the RBES process, and (2) they can be used to adjust and improve the RBES predictions.

Keywords: Clustering, Cross-validation, Model calibration, Principal component analysis, Residential building energy simulation

1. Introduction

In 2010, the residential sector accounted for 23% of the United States' total energy consumption, which is 4.4% of the world's energy consumption [1]. Efforts to understand and model this consumption are critical to reducing it. Residential Building Energy Simulation (RBES) tools typically model the energy consumption (e.g., electricity and natural gas) of a home at hourly or sub-hourly time steps using a physics-based approach.¹ Both federally and privately funded organizations have developed such tools for use in home

¹Some RBES tools may use larger time-steps or seasonal approaches for modeling; so long as the tool is able to predict annual consumption, the statistical techniques applied in this study are relevant.

9 energy assessments that help energy efficiency practitioners and homeowners
10 evaluate the economic benefit of investing in home improvements designed
11 to reduce energy consumption.

12 One particular quantity of interest predicted by an RBES program is
13 the pre-retrofit annual energy consumption of a home. Then, the annual
14 energy consumption of a home under proposed retrofits can be projected, and
15 the estimated energy savings can be compared with the cost of the retrofit.
16 However, physics-based models can provide biased predictions of the annual
17 energy consumption of the home. The top panels of Figure 1 show a sample
18 of homes for which RBES predictions of annual energy consumption are
19 plotted against the weather-normalized measured annual energy. Natural gas
20 predictions align better with the line of perfect agreement than the electricity
21 predictions do, but the RBES model still tends to overpredict consumption
22 for this sample of homes. Both electricity and natural gas consumption can be
23 highly driven by occupant behavior, but the errors in electricity predictions
24 are thought to be even more variable due to the fact that a large fraction of
25 electric end use is almost entirely driven by occupant preferences and choices
26 (e.g., miscellaneous electric loads, appliances, and lighting). However, in this
27 study, standard occupant behavior is used in the software, and the building
28 characteristics data were generally collected for the purpose of an “asset
29 rating,” which means specific occupant behavior was not considered. Thus,
30 standard occupant behavior is assumed to provide a fair comparison of the
31 homes’ energy efficiency features.

32 An approach for adjusting predictions to more closely match the actual
33 electric energy consumption in the home would be useful, as shown in the

bottom panels of Figure 1, which will be discussed in more detail in Section 4. Deviations between observed and predicted energy consumption may occur for any of the following reasons: (1) measurement error in data collection; (2) error in utility data normalization; (3) deviations in occupant behavior; (4) coding errors in translation scripts; or (5) deficiencies in the RBES model [2].

This research focuses on the last two types of errors. First, in order for a home to be successfully simulated by an RBES tool, a specific set of simulation inputs describing the home is required. Translation scripts automatically map information in a database for a given home to create a simulation input file, as shown in Figure 2. Some inputs are computed based on characteristics observed or measured in the home. For example, duct leakage percentages are calculated when the results of a duct pressurization test that is performed during the audit are available. Assumptions, simplifications, and coding errors in the translation scripts can contribute to errors in the RBES predictions. The RBES program may also face certain limitations, like the inability to model homes with multiple hot water heaters or multiple duct systems. In such cases, assumptions and simplifications are necessary to make predictions.

Secondly, there may be errors and inaccuracies in the RBES program itself. Default assumptions made within the program, such as occupant use profiles, may not be accurate for the particular homes being analyzed. Inherent simplifications, such as the use of an isothermal, single-zone model for the conditioned space, could introduce significant error into the model predictions. Modeling algorithms, such as ground-coupling models for foun-

Figure 1: Top Row: Plot of RBES predictions of electricity (left) and natural gas (right) against weather-normalized measured annual consumption of each type of energy in MMBtu. Bottom Row: Plot of adjusted RBES predictions versus weather-normalized measured annual electricity (left) and natural gas (right) consumption for the combined clusters with a reference line of perfect agreement.

Figure 2: Outline of general RBES simulation process. The BAFDR is NREL’s Building America Field Data Repository that contains data on the homes used in this study.

59 dation heat transfer, may not accurately model the energy performance of
 60 a particular component of the home. There may even be coding errors in
 61 the RBES program itself, which can contain hundreds-of-thousands of lines
 62 of computer code.

63 Several prior studies have sought to assess various RBES models by com-
 64 paring predictions to measured energy, but most only assess accuracy on a
 65 relatively small sample of homes [2–5]. EnergyPro was found to, on aver-
 66 age, overestimate the combined electricity and natural gas savings expected
 67 from retrofits by nearly a half in a sample of 51 homes [5]. However, the
 68 Home Energy Scoring Tool (HEST) only slightly underestimated electricity
 69 ($n = 859$) and underestimated natural gas consumption in 60% of the 500
 70 homes that were examined [6]. In the HEST study, multiple linear regression
 71 (MLR) models for the difference between predicted and measured annual en-
 72 ergy consumption were built, and significant variables in these models were
 73 noted as inputs that could be investigated to identify potential model im-
 74 provements. However, the HEST model does not require many inputs, most
 75 of them are categorical, and the majority of the inputs used in the accuracy
 76 assessment were HEST inputs. For this study, we include more variables,
 77 many of which are numerical instead of categorical, and we also consider
 78 variables in the statistical analysis that are not inputs to the simulation tool

79 but perhaps could improve the accuracy of the tool if they were. All of
80 these prior studies stopped short of post-processing the RBES predictions to
81 improve them, but one analysis of the SEEM model does use building char-
82 acteristics and energy consumption data to adjust SEEM predictions of total
83 heating energy to obtain improved estimates of electric heating energy [7].
84 In that work, the adjustment is also made using a multiple linear regression
85 model with a sample size of 629 homes, but only one model is built for all
86 homes in the study, and only a small set of variables is used.

87 Post-processing physics-based models began with numerical weather pre-
88 diction models in the early 1970's [8, 9] and has been shown to improve
89 forecasts made by these models by correcting biases and adapting forecasts
90 to local conditions. Since then, variations of Model Output Statistics (MOS)
91 have been applied to many atmospheric variables, including temperature,
92 ozone, wind speeds, and precipitation (e.g., [10–14]). The general principle
93 is that the physical model captures the nonlinearities of the large-scale sys-
94 tem while the statistical post-processing calibrates the model output to local
95 conditions. Using a large suite of homes with measured energy consumption
96 that have been simulated using an RBES program, we apply such an ap-
97 proach to the RBES predictions and can correct the predictions for bias and
98 reduce variability in the predictions. As a by-product, we can also identify
99 variables that may be miscoded in the translation scripts or that may be
100 related to inaccuracies in the RBES model.

101 Using physical characteristics recorded for each home, variables calcu-
102 lated with the translation scripts, and RBES model input variables, MLR
103 models are built for the difference between the measured and predicted an-

104 nual electricity and natural gas consumption. Homes are first put into similar
105 groups using a statistical clustering technique, and MLR models built within
106 each cluster are shown to have better predictive accuracy than models built
107 using all homes. Clustering offers a way to construct similar groups for ob-
108 servations with large dimensionality, and at least two prior building energy
109 studies have also demonstrated improved results when clustering is applied
110 first [15, 16]. MLR models for the groups of homes allow us to pinpoint areas
111 of improvement in the RBES process that are specific to the homes in each
112 group.

113 The remainder of this paper is organized as follows: Section 2 describes
114 the RBES model, translation scripts, and the homes used in this analysis.
115 Section 3 describes the methodology, including the cluster analysis on prin-
116 cipal components, MLR models, and leave-one-out cross-validation. The
117 results are illustrated in Section 4, and Section 5 concludes with a discussion
118 of the practical impact of this work and future questions of interest.

119 **2. Experimental Data**

120 The methodology and results in this work are based on the energy con-
121 sumption and recorded features of a set of approximately 1,250 homes in
122 the National Renewable Energy Laboratory’s (NREL’s) Building America
123 Field Data Repository (BAFDR). The electricity and natural gas consump-
124 tion of a subset of these homes is predicted using NREL’s Building Energy
125 Optimization Software (BEoptTM/ EnergyPlus) and is the particular RBES
126 model that we use to obtain energy predictions. However, the methodology
127 described herein could apply to any RBES tool. Descriptions of the BAFDR

homes, their recorded features, and the BEopt simulation tool are given in this section. This section also includes a summary of the translation process, which takes the original information recorded for each home and converts it to BEopt inputs.

2.1. BAFDR Data

The data used in this study is a subset of homes from NREL’s Building America Field Data Repository (BAFDR), which is a database containing energy audit information and measured energy use for approximately 1,250 homes (at the time this study was conducted) that have been collected from 8 different sources, which are listed in Table 1 of the Appendix. The variables recorded for each home include information on actual energy consumption,² location, weather, and household characteristics. The household characteristics cover a broad range of variables that are used to describe features of each home related to its geometry, wall construction, attic and roof construction, foundation, windows, infiltration, ducts, ventilation, appliances, space conditioning, and water heating.

²Unless the measured energy consumption data were originally received in a weather-normalized format, utility billing data were weather normalized using the ASHRAE Inverse Modeling Toolkit (IMT) software program. Three parameter change-point heating models were used with natural gas billing data, and both three and five parameter heating and cooling change-point models were used with electricity billing data. For cases with insufficient goodness-of-fit ($R^2 < 0.7$), the model coefficients were not used, and an annual average energy use was calculated from the utility billing data. See “ASHRAE Research Project 1050-RP: Development of a Toolkit for Calculating Linear, Change-Point Linear and Multiple-Linear Inverse Building Energy Analysis Models” [17], which includes background information about IMT and the models.

144 2.2. RBES Process

145 The BEopt (Building Energy Optimization) software has been developed
146 by NREL in support of the U. S. Department of Energy Building Amer-
147 ica program. BEopt evaluates residential building designs and identifies
148 cost-optimal efficiency packages at various levels of whole-house energy sav-
149 ings along the path to zero net energy, which is when a home consumes as
150 much energy as it produces annually. Both new construction and existing
151 home retrofits can be analyzed through evaluation of single building designs,
152 parametric sweeps, and cost-based optimizations. BEopt provides detailed
153 simulation-based analysis using specific house characteristics, such as size,
154 architecture, occupancy, vintage, location, and utility rates using existing,
155 established simulation engines (currently DOE-2.2 or EnergyPlus). For this
156 study, BEopt 2.0 Beta is used with the EnergyPlus v 7.1 simulation engine
157 (DOE 2013). This version of BEopt is not capable of and is not intended
158 to be used for simulating the energy consumption of certain homes in the
159 BAFDR, such as complex multi-family buildings and homes with multiple
160 duct systems.

161 At the time this study was completed, translation algorithms and asso-
162 ciated computer scripts were being developed at NREL that automatically
163 translate BAFDR data to create BEopt extensible markup language (XML)
164 input files for each home. A development version of these translation scripts
165 was used in this study for the secondary purpose of testing and improving the
166 translation algorithms. Automated simulation approaches are gaining trac-
167 tion as data transfer standards like Home Performance XML (HPXML) are
168 becoming more widely used. Large datasets that are stored in a standard file

169 format, such as the BAFDR, will allow industry software developers to apply
170 empirical testing and software corrections using the approaches presented in
171 this study.

172 The development version of the translation algorithms were written to
173 handle the most common features of homes in the BAFDR, such as wood-
174 frame walls, but were not capable of processing uncommon technologies like
175 SIP walls or geothermal heat pumps. The homes used in this project are
176 the 997 single-family, detached homes in the BAFDR that were successfully
177 translated and whose electricity consumption could be simulated in BEopt.
178 Note, a subset of 709 of these homes had non-zero measured natural gas us-
179 age and were also simulated in BEopt. While BEopt’s user interface allows
180 homes to be described using predefined options, the household characteristics
181 collected from homes in the BAFDR are generally not described using these
182 options. Instead, custom BEopt options were developed for these homes
183 based on the information given in the BAFDR, such as the duct and in-
184 filtration variables. Furthermore, there is no information on occupants in
185 the BAFDR database, so the homes were simulated assuming “typical occu-
186 pant behavior,” which refers to the standardized occupancy defined in the
187 Building America House Simulation Protocols [18].

188 Given that there are several steps to the simulation process, the variables
189 used in this study come from multiple points in this process, as depicted in
190 Figure 2. Specifically, the 105 household characteristics we use come from (i)
191 the BAFDR data, (ii) intermediate steps in the translation scripts, and (iii)
192 the BEopt input files. While approximately 60% of the variables are straight
193 from the BEopt input file, most of the geometry related variables come from

194 the BAFDR (20%), and all of the variables related to U-values are from
195 the translation scripts (20%). Variables are included in the analysis if they
196 directly represent or are derived from collected information about the home.
197 For a complete list of variables used, their types, definitions, and summary
198 statistics, see Appendix A.

199 Studies that use many variables describing features of the same home
200 often find that redundancies in the variables can cause numerical instabilities
201 [7]. For example, both the BAFDR and BEopt have different representations
202 of floor area. Called multicollinearity, this phenomenon occurs when one
203 variable is a linear combination of the remaining variables in the set. In this
204 project, the presence of multicollinearity presents the problem of overstating
205 similarities and differences between homes in the cluster analysis, and it can
206 have a large impact on fitting multiple linear regression (MLR) models, as
207 it causes the estimates of the model coefficients to be unstable with large
208 standard errors. Multicollinearity can be detected using variance inflation
209 factors (VIFs) that describe the percentage of variability in one particular
210 variable that can be described as a linear function of the remaining variables.
211 The rule of thumb is that variables with VIFs larger than 10 are removed
212 [19].

213 Given that many of the variables present in the BAFDR, intermediate
214 set, and BEopt inputs are related, the VIFs are used to identify redundant
215 variables before subsequent analysis is performed. The subset of variables is
216 chosen by removing variables one at a time based on large VIFs and expert
217 opinion. This process is iterated until all but one variable, Heating Degree
218 Days, have a VIF less than 10. The VIF for Heating Degree Days is just

greater than 10, but since it is considered to be important from a building energy standpoint, it is retained. Thus, a subset of 49 of the original 105 variables that are nearly independent is used in this analysis.

The top panels of Figure 1 show the RBES predicted versus measured annual energy consumption for the electricity and natural gas consumption of the homes, respectively. The observations that fall below the line of perfect agreement are homes for which the simulation process underpredicts energy consumption, and the observations above the line are homes that are overpredicted by the simulation process. We define Δ Electricity to be the difference between a set of RBES model predictions and measured electricity (RBES Predicted – Measured), and Δ Natural Gas is defined similarly. Table 1 gives the mean and standard deviation of these differences and of the measured electricity and natural gas. From this, we see that across all homes, electricity is overpredicted by 0.8% on average, and natural gas is overpredicted by 10.1% on average. While across all homes, the electricity predictions do not appear strongly biased, within each group (which will be described in Section 4.1), larger discrepancies appear.

3. Methodology

In this section, we first describe the statistical clustering technique that is applied based on the set of 49 explanatory variables to divide the homes into similar groups. The principal components of the 49 variables are first computed, then two clustering methods are applied, and multiple methods for choosing the number of clusters are tested. Then, the creation of multiple linear regression (MLR) models for each of the Δ energy responses is briefly

Table 1: Mean and standard deviation of the measured annual energy and deviations between RBES predicted and measured annual energy given in MMBtu.

Grouping	Measured Electricity		Δ Electricity		
	Mean	SD	Mean	SD	Bias
All Homes	35.41	14.85	0.29	12.77	0.8%
Warm Cluster	40.37	15.00	-1.96	13.50	-4.9%
Cold Cluster	30.63	14.85	2.46	11.63	8.0%

Grouping	Measured Natural Gas		Δ Natural Gas		
	Mean	SD	Mean	SD	Bias
All Homes	87.00	37.25	9.56	32.04	10.1%
Warm Cluster	62.31	31.34	3.62	30.70	4.2%
Cold Cluster	95.02	35.48	11.48	32.27	18.4%

described along with a cross-validation technique for adjusting the RBES predictions based on the fitted MLR models.

3.1. Clustering Analysis

Three main steps in performing the clustering analysis are described here. First, we obtain principal components (PCs) of the 49 explanatory variables. Even though the explanatory variables were checked for strong multicollinearity, some dependence among them remains, and the PCs produced are independent linear combinations of the original variables. Next, we apply two clustering algorithms to find similar groupings of homes. Since clustering algorithms may be sensitive to assumptions or initial conditions, two such

253 algorithms are tested to check for the robustness of the identified clusters.
 254 Finally, we apply three methods to select the number of clusters. The number
 255 of clusters chosen may also vary depending on which criterion is minimized,
 256 so using three different criteria will aid in validating the final choice.

257 3.1.1. Principal Component Analysis

Principal Component Analysis (PCA) is a multivariate statistical technique that transforms the original set of p variables into a new set of p variables through an eigendecomposition of the covariance or correlation matrix. Given that the variables in the BAFDR data set have very different units, we use the correlation matrix for the PCA. Let $\mathbf{X}' = [X_1, X_2, \dots, X_p]$ be a set of p random variables. To compute the correlation between two quantitative variables, Pearson's product-moment correlation coefficient is used, which is given as

$$\rho_{ij} = \frac{\text{cov}(X_i, X_j)}{\sigma_{X_i} \sigma_{X_j}},$$

258 where $\text{cov}(X_i, X_j)$ is the covariance of X_i and X_j , and σ_{X_i} and σ_{X_j} are the
 259 standard deviations of X_i and X_j , respectively. Pearson's correlation coefficient is chosen since it is mathematically equivalent to the phi coefficient,
 260 which we use to measure the association between two binary variables [20],
 261 and the point-biserial correlation coefficient, which we use to measure association between continuous and binary variables [21].
 262
 263

264 The correlation matrix, $\boldsymbol{\rho}$, of $\mathbf{X}$ has an eigenvalue-eigenvector decomposition with pairs $\{(\lambda_1, \mathbf{e}_1), (\lambda_2, \mathbf{e}_2), \dots, (\lambda_p, \mathbf{e}_p)\}$, where $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p \geq$
 265 0. Then, the k^{th} PC is given by
 266

$$Y_k = \mathbf{e}_k' \mathbf{X} = e_{k1}X_1 + e_{k2}X_2 + \dots + e_{kp}X_p, \quad (1)$$

for $k = 1, 2, \dots, p$. The variance of Y_k is λ_k , and when the eigenvalues, $\{\lambda_1, \lambda_2, \dots, \lambda_p\}$, are distinct, the PCs are independent and have maximum variance with decreasing magnitude [22]. For example, the first PC, Y_1 , has the largest variance of λ_1 , followed by the second PC with the second largest variance of λ_2 , and so on. PCA is commonly used to reduce the dimensionality of a dataset by representing a large set of p variables with a smaller set of PCs by truncating those PCs that do not describe a large percentage of the variability in $\mathbf{X}$.

3.1.2. Clustering Methods

Cluster analysis is an exploratory technique used to find “natural” groupings in multidimensional data. The groupings are determined based on a measure of dissimilarity computed between each pair of homes, and no assumptions are made about the number of groups or the group structure [22]. We cluster observations using the PCs since the PCs are independent, linear combinations of the original variables that can be weighted by their corresponding eigenvalue. In other words, PCs that describe more of the variability in $\mathbf{X}$ (i.e., those with a higher value of λ_k) should be allowed to influence the dissimilarity of two homes more than a PC that explains very little of the variability in $\mathbf{X}$. Given that the PCs are numeric, Euclidean distances can be used to measure dissimilarity between pairs of homes, and the distances can be weighted by the corresponding importance of each PC. The weighted Euclidean distance between two $p \times 1$ vectors of principal components, $\mathbf{y}_k$ and $\mathbf{y}_j$, is given by

$$d_{ij} = \sqrt{\sum_{m=1}^p w_m (y_{km} - y_{jm})^2}, \quad (2)$$

290 where w_m is equal to $\frac{\lambda_m}{\sum_{m=1}^p \lambda_m}$.

291 For each pair of homes in a set of n homes, d_{ij} is computed, and these
292 distances are organized into an $n \times n$ matrix, and many methods for clustering
293 observations based on such a matrix have been developed. We apply Ward’s
294 method of hierarchical clustering that is based on minimizing the within
295 cluster variance, calculated using the error sum of squares (ESS) [23]. As
296 with all agglomerative clustering methods, each cluster is initially made up
297 of one observation. At each step, the ESS is computed for the union of every
298 possible pair of clusters, and the pair with the smallest increase in ESS is
299 joined. To check the robustness of the clusters that are formed, we also apply
300 the non-hierarchical technique of fuzzy clustering [24]. For a preselected
301 number of clusters, fuzzy clustering assigns each observation a probability
302 of belonging to each cluster, known as the “strength of membership,” by
303 minimizing an objective function based on the dissimilarities.

304 3.1.3. *Choosing the Number of Clusters*

305 The ideal number of clusters is not known, and many methods exist for
306 choosing the number of clusters. We consider the following three common
307 approaches: the Xie-Beni Index [25], C Index [26], and Dunn Index [27]. Each
308 is based on some comparison of a criteria (such as squared deviations between
309 pairs of homes) between those observations within a cluster and observations
310 assigned to different clusters. Each criteria is computed for different numbers
311 of clusters for each clustering technique, and the number of clusters for which
312 the minimum (or maximum) occurs is chosen. If all three criteria choose a
313 similar optimal number of clusters, then we will have greater confidence in
314 the choice of number of clusters.

3.2. MLR Models

Multiple linear regression analysis is a technique used to assess the relationship of a response variable to a set of explanatory variables. The goal of regression analysis is to fit an empirical model that explains the effects of the explanatory variables on the response [19]. Using the original 49 explanatory variables, we will create models for the following two responses:

- Δ electricity (RBES predicted – measured) and
- Δ natural gas (RBES predicted – measured)

both for all homes simultaneously and for the subsets of homes within each identified cluster. We use the original variables in the models instead of the PCs because they are more interpretable, and the presence of particular explanatory variables may point to errors in the translation scripts or areas of potential improvement in the RBES tool. These regression models will indicate how much of the remaining variability in the Δ variables can be explained by characteristics of the home and will point to those variables that are significant in explaining the difference between predicted and measured energy consumption.

To build each model, all explanatory variables are initially included, and variables are removed one at a time, beginning with the variable with the largest individual p -value for significance. This backward elimination process is continued until all remaining variables are significant with a p -value less than 0.05, or all variables have been removed from the model. Once this model is built, the fundamental assumptions of an MLR model are validated. Techniques for checking these assumptions are described in any introductory

339 MLR text, such as [19, 28], and since we did not find any violations of the
 340 assumptions in any of our fitted models, we do not mention this issue again.
 341 After assessing and comparing the fitted MLR models, we also assess their
 342 predictive accuracy and describe how the MLR predicted values can be used
 343 to improve the RBES predictions.

344 3.2.1. *Leave-one-out Cross-Validation*

345 A common technique used to assess the predictive capability of a method
 346 is leave-one-out cross-validation (CV) in which one observation is removed
 347 from the modeling process, and a model fit with the remaining observations is
 348 used to predict the removed one. While this method can be computationally
 349 expensive, it is commonly used when the sample size is small [29]. Given
 350 our relatively small sample size and the even smaller sample size within each
 351 cluster, a leave-one-out CV technique is employed to compare the predictive
 352 capabilities of the overall model based on all observations versus the models
 353 built within each cluster.

354 First, we assume that the removal of one observation will not significantly
 355 impact the correlation matrix, and therefore, the PCs will not change. If the
 356 PCs are unchanged, then the dissimilarities based on the eigenvalues and
 357 their associated PCs will remain the same. Finally, we also assume that
 358 removing one home will not affect the number of clusters chosen. The basic
 359 steps for this application of leave-one-out CV are:

- 360 1. Remove observation i , where $i = 1, 2, \dots, n$.
- 361 2. Recluster the remaining $n - 1$ homes using Ward's method into the
 362 predetermined number of clusters. (Fuzzy clustering produced similar
 363 results.)

- 364 3. Assign observation i to cluster C by calculating the distance between
365 its variable values and the mean of each cluster and choosing the cluster
366 with the smallest distance.
- 367 4. Select the variables (via backward selection) and estimate the coeffi-
368 cients for an MLR model for one of the two responses using all of the
369 observations in C except for observation i .
- 370 5. Calculate the predicted value, $\hat{y}_{-i}$, for obs i based on this model.
- 371 6. Repeat steps 1 through 5 for each observation.

372 Then, the CV mean-squared error (MSE) is calculated as follows:

$$CV_{MSE} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_{-i})^2, \quad (3)$$

373 where y_i is the observed value of the response for observation i .

374 3.2.2. *Adjusting RBES Model Output*

375 Finally, the left-out predictions, $\hat{y}_{-i}$, described in the previous section are
376 independent predictions of how much the RBES tool annual energy predic-
377 tions will be in error relative to the weather-normalized measured energy use.
378 We claim that these are “independent” predictions since the observation is
379 not used in determining the clusters, is only assigned to a cluster after the
380 clusters have been formed, and is not used to estimate the parameters in
381 the MLR model. Of course, we do assume that the withheld home is similar
382 enough to the remaining homes that it will fit well within one of the clus-
383 ters and will not require extrapolation of the MLR model to predict its Δ
384 response. Then, we can obtain an adjusted RBES prediction, $RBES_{adj,i}$, by
385 subtracting $\hat{y}_{-i}$ from the original RBES prediction, $RBES_i$. If the RBES pre-
386 diction error is expected to be positive, subtracting the positive error from

the RBES prediction should reduce the prediction error (and vice versa for a prediction error that is expected to be negative). We can compute these adjusted prediction errors, $\Delta_{adj,i}$, as follows:

$$\begin{aligned}\Delta_{adj,i} &= RBES_{adj,i} - M_i \\ &= (RBES_i - \hat{y}_{-i}) - M_i,\end{aligned}\tag{4}$$

where M_i is the measured annual energy consumption for home i . We hope to see a reduction in the bias and in the variability of the $\Delta_{adj,i}$ values over the Δ 's in Table 1.

4. Results and Discussion

Two clusters are indicated in the cluster analysis, and these are described first. Then, the MLR models that are built within each cluster and for all of the observations for each response are reported and discussed. We demonstrate that splitting the observations into two clusters produces better results than treating all of the observations simultaneously, and more insight is gained into improvements that may be made to the RBES process. Finally, we show that the MLR leave-one-out predictions can adjust the RBES model output to remove the bias and reduce variability.

4.1. Clustering Analysis

Figure 3 shows the dendrogram for Ward's method of hierarchical clustering using weighted Euclidean distances based on the PCs. A dendrogram is a visual display used in hierarchical clustering, where the branches represent the clusters, and the height is the value of the metric used to measure

Figure 3: Dendrogram for Ward’s method using weighted Euclidean distances of the principal components.

407 “distance between clusters” when two clusters are joined. These clusters are
 408 well-formed; in other words, no homes remain in their own cluster until the
 409 last steps, and while the homes along the bottom of the plot are indistin-
 410 guishable, there is a clear separation between one and two clusters. The
 411 clusters identified by the fuzzy method are also well defined.

412 All three criteria for choosing the number of clusters indicate that two
 413 clusters are optimal for both Ward’s method and fuzzy clustering. For Ward’s
 414 method, there are 520 homes in the first cluster and 477 homes in the second.
 415 Similarly, there are 507 homes in the first cluster and 490 homes in the second
 416 for fuzzy clustering. The clusters are very stable regardless of whether Ward
 417 or fuzzy clustering is used since only thirteen homes change clusters. Going
 418 forward, we use the clusters formed by Ward’s method for the regression
 419 analysis, since Ward’s method is less computationally expensive.

420 We label the two clusters as follows: even though the coastal region of
 421 Oregon is considered to be a marine climate, to simplify discussion the homes
 422 in the first cluster are labeled as “Warm,” and the homes in the second cluster

are labeled as “Cold.” The location of the homes at the city level are shown in Figure 4. Cities with red circles indicate that the homes are located in the first Warm cluster, while cities with blue circles indicate that the homes are located in the second Cold cluster. Moreover, the more homes located in a particular city, the darker the color. As shown in Table 1, electricity consumption is underpredicted by 4.9% on average for Warm cluster homes and overpredicted by 8% on average for Cold cluster homes. Natural gas consumption is overpredicted by 4.2% on average for Warm cluster homes and 18.4% on average for Cold cluster homes.

Clustering is a purely exploratory tool. To proceed, we can either choose to (1) build separate MLR models of the Δ responses for the homes within each cluster; (2) build a single MLR model ignoring the clusters; or (3) build a single MLR model that attempts to incorporate the split in homes revealed by the clustering directly into the model. The “Heating Degree Day” (HDD) variable should suitably distinguish between the homes in each group, so for the third approach, we build a single MLR model for all homes, and we begin the model-building process not only with the original 49 variables, but also including the 48 interaction terms between HDD and each of the remaining variables. Then, we apply backward selection until all remaining variables in the model are significant. This approach did not produce a better model than the first or second types of models, so we do not report on its results. We simply mention this third approach to demonstrate that trying to sensibly split this complex dataset only on the basis of HDD could not replicate the information contained in the cluster analysis.

Figure 4: Plot of the location of homes at city level for both clusters. Cities with red circles indicate that the homes are located in the first Warm cluster, while cities with blue circles indicate that the homes are located in the second Cold cluster. The blue city in Oregon contains homes in both clusters.

4.2. MLR Models

In this section, we first describe the fit of the MLR models both overall and within each cluster and analyze the types of variables included in each model. Then, we show that the predictive ability of the separate Warm and Cold cluster models is superior to the overall model, which clearly demonstrates the benefit of two clusters over none. Finally, we show the improvements in the RBES predictions after adjusting each home’s energy usage based on the MLR models.

4.2.1. Fitted MLR Models

Of the 997 and 709 homes used for the electricity and natural gas analyses, 121 are missing information on measured energy; 9 are not successfully simulated in BEopt; and 1 home does not have measured energy and is also

Table 2: Table of Adjusted R^2 values for both responses and the number of homes, n , present in each regression.

Grouping	Δ Electricity		Δ Natural Gas	
	n	Adj. R^2	n	Adj. R^2
All Homes	840	16.8%	554	44.6%
Both Clusters	859	18.4%	556	44.0%
Warm Cluster	419	29.2%	142	47.3%
Cold Cluster	440	8.2%	414	42.9%

not successfully simulated. The full sets of homes were used in the clustering analysis but not in the subsequent MLRs. Therefore, the MLRs are built using $n = 866$ homes in the BAFDR dataset for the Δ electricity response and using $n = 578$ homes with non-zero natural gas usage for the Δ natural gas response. Some missing explanatory variable values cause the actual number of homes used in the fit to differ from these sample sizes, as shown in Table 2. Table 2 also lists the adjusted R^2 values from each model for both responses. The Warm cluster has the largest adjusted R^2 value for both Δ electricity and Δ natural gas. A weighted average of the Warm and Cold clusters based on the number of observations in each one is used to compute an adjusted R^2 for “Both Clusters,” and taken together, the clusters explain more of the variability in Δ electricity than the model built on all homes, and they explain roughly the same amount of variability in Δ natural gas as the overall model.

Tables 3 and 4 list the variables included in each of the overall, Warm cluster, and Cold cluster models along with each one’s associated p -value for

475 the Δ electricity and Δ natural gas responses, respectively. The variables are
476 sorted into two columns, one for the variables with positive coefficients in the
477 MLR and one for the variables with negative coefficients. Within each col-
478 umn, the variables are sorted with those that are most significant at the top.
479 Variables with a positive estimated coefficient lead in the direction of over-
480 predicting electricity consumption, while variables with a negative estimated
481 coefficient lead in the direction of underpredicting electricity consumption.
482 For example, Age of Home has a positive coefficient, so the older the home,
483 the more likely it becomes that the Δ electricity will be positive (i.e., RBES
484 overpredicts electricity) as well.

485 For the Δ electricity response in Table 3, we note important groups of
486 variables. No Garage, Conditioned Floor Area, and East and South Window
487 Area are all geometry related variables and are processed to be used in BEopt.
488 Since BEopt requires a 3-dimensional closed geometry, this may indicate
489 that the RBES process could be improved in this area (e.g., improve the
490 translation algorithms used to create the 3-D geometry from the audit data).
491 Furthermore, Above Grade Wall U-value and Dishwasher Energy Factor (EF)
492 are from the intermediate steps in the translation scripts, so information for
493 these variables has potentially been lost in that process or could be processed
494 more effectively.

495 Number of Bedrooms and Age of Home appear in both cluster models and
496 in the overall model for Δ electricity. In this RBES process, miscellaneous
497 electric loads are calculated based on the number of bedrooms and floor area
498 of a home, so the presence of Number of Bedrooms in these models could
499 be a proxy for occupant behavior, and its presence in these models suggests

500 that miscellaneous electric loads may not be completely accounted for in the
501 calculation. In addition, the quantitative Age of Home is not used directly
502 in the RBES simulation process, so one possible explanation for its presence
503 in these models is the effect of RBES defaults on older homes. Particular
504 characteristics, like heating thermostat set points, are defaulted in BEopt
505 because the BAFDR data were collected for asset analyses, and therefore no
506 information on occupant behavior is known. Since the defaults are based
507 on benchmark values given by the Building America House Simulation Pro-
508 tocols, it is possible that they are not representative for older homes. For
509 example, older homes are typically less energy efficient than newer homes,
510 which could lead to a lower heating set points than the benchmark value since
511 they would be more expensive to heat. This possible error in the assumed
512 heating set point would contribute to overprediction.

513 Similar to the Δ electricity models, there are several geometry related
514 and duct and ventilation variables present in the Δ natural gas models listed
515 in Table 4, like Cathedral Ceiling Area, East and South Window Area, Con-
516 ditioned Floor Area, Duct Number of Returns, and Duct Location. Since
517 translating these variables from the original data set into inputs to be used
518 in BEopt is a particularly difficult process, some information from these
519 variables is likely lost or distorted. In particular, Living Space ACH50 is
520 a significant variable in all models for the Δ natural gas response, and one
521 explanation for its presence is that there was a calculation error (that was
522 subsequently identified and corrected) in the translation scripts. The cor-
523 rection was related to leakage area, which is calculated using Living Space
524 ACH50.

Table 3: List of variables that lead to increasing (left column) and decreasing (right column) Δ electricity in the model for all homes, the Warm cluster, and the Cold cluster and their p -values.

Positive Coefficient		Negative Coefficient	
Variables	p -value	Variables	p -value
<i>Model with All Homes</i>			
Age of Home	3.64e-05	Number of Bedrooms	8.88e-06
AH Leak RA	0.0014	Enclosed Crawlspace Perimeter	5.70e-05
Roof Color: Dark	0.0025	Duct Location: Garage	0.0014
Unconditioned Duct R-value	0.0068	Left Window U-value	0.0031
Ventilation Type: Balanced	0.0130	No Garage	0.0035
Conditioned Floor Area	0.0221	East Window Area	0.0040
		House Vent Fan Power	0.0068
		Dishwasher EF	0.0140
		No Furnace	0.0396
<i>Model for Warm Cluster</i>			
Above Grade Wall U-value	1.74e-05	Number of Bedrooms	1.30e-06
Age of Home	2.54e-05	Duct Location: Garage	8.55e-06
South Window Area	0.0001	Left Window U-value	0.0003
Unconditioned Duct R-value	0.0012	No Garage	0.0005
Right Window U-value	0.0034	Water Heater Tank Volume	0.0020
AH Leak RA	0.0043	Dishwasher EF	0.0022
		Living Space ACH50	0.0234
		Duct Number of Returns	0.0236
		Duct Location: Unfinished Bsmt	0.0443
		No Furnace	0.0457
<i>Model for Cold Cluster</i>			
Age of Home	0.0016	East Window Area	0.0035
Ventilation Type: Balanced	0.0057	Azimuth: 0	0.0054
Roof Color: Dark	0.0112	Number of Bedrooms	0.0171
Conditioned Floor Area	0.0142	House Vent Fan Power	0.0197
Water Heater Energy Factor	0.0223		

525 Above Grade Wall U-value is the most significant variable in both the
526 overall and Cold cluster models. As the Above Grade Wall U-value increases,
527 the average Δ natural gas also increases. Above Grade Wall U-value is an
528 intermediate variable calculated in the translation scripts and is not used
529 directly in BEopt, so it is possible that the variables created from Above
530 Grade Wall U-value for use in BEopt mishandle the information present
531 in this variable. Or, for example, there could be inherent issues with the
532 hard-wired assumptions and modeling algorithms in the simulation tool for
533 poorly-insulated wall assemblies (walls with high U-values).

534 Finally, several variables appear in the model for the Warm cluster or the
535 model for the Cold cluster that do not appear in the model with all homes for
536 both of the Δ responses. These variables provide additional information as
537 to where improvements may be made in the RBES process for both groups of
538 homes when simulating electricity or natural gas consumption. For example,
539 an “Enclosed Crawlspace Perimeter” is significant in explaining the Δ natural
540 gas for Cold cluster homes but is not significant in the model for all homes.
541 This could indicate the presence of translation or modeling errors for enclosed
542 crawlspaces that are most pronounced in colder climates (e.g., crawlspace
543 insulation assumptions or ground-coupling models). These results indicate
544 that while similar variables appear in all models for each of the Δ responses,
545 when the homes are broken into clusters, additional information is gained on
546 which areas may need improvement in the simulation process for homes of
547 these particular types.

Table 4: List of variables that lead to increasing (left column) and decreasing (right column) Δ natural gas consumption in the models for all homes, the Warm cluster, and the Cold cluster and their p -values.

Positive Coefficient		Negative Coefficient	
Variables	p -value	Variables	p -value
<i>Model with All Homes</i>			
Above Grade Wall U-value	4.44e-13	East Window Area	0.0002
Heating Degree Days	1.16e-12	South Window Area	0.0019
Cathedral Ceiling Area	2.44e-11	Water Heater Energy Factor	0.0028
Duct Location: Unfinished Attic	1.23e-08		
Living Space ACH50	5.03e-06		
Conditioned Floor Area	0.0006		
Age of Home	0.0033		
Unfinished Attic U-value	0.0047		
Duct Number of Returns	0.0075		
AH Leak RA	0.0164		
<i>Model for Warm Cluster</i>			
Living Space ACH50	7.67e-08	Azimuth: 0	0.0266
Unfinished Attic U-value	1.09e-05		
Fixed Heating Capacity	9.35e-05		
Cathedral Ceiling Area	0.0009		
<i>Model for Cold Cluster</i>			
Above Grade Wall U-value	9.54e-11	East Window Area	0.0001
Cathedral Ceiling Area	5.21e-09	South Window Area	0.0012
Living Space ACH50	6.04e-07		
Duct Location: Unfinished Attic	3.06e-05		
Conditioned Floor Area	0.0008		
Duct Number of Returns	0.0015		
Heating Degree Days	0.0069		
Use Ceiling Fan	0.0180		
Dishwasher EF	0.0219		
Left Window Percent of Total	0.0228		
Enclosed Crawlspace Perimeter	0.0473		

Table 5: CV_{MSE} for all responses with the largest 5% of squared errors removed.

Grouping	Δ Electricity	Δ Natural Gas
All Homes	99.64	470.63
Both Clusters	97.59	408.68
Warm Cluster	102.85	329.79
Cold Cluster	92.69	433.47

548 4.2.2. Leave-One-Out Cross-Validation

549 To demonstrate the utility of constructing models within each cluster as
550 opposed to one overall model, we compare the predictive capabilities of these
551 models using leave-one-out CV. Table 5 gives the CV_{MSE} computed using the
552 algorithm described in Section 3.2.1. A few observations that have unusually
553 high squared errors are observations that already have unusually high or low
554 response values. Therefore, the largest 5% of CV squared errors are removed.
555 In Table 5, the CV_{MSE} values for all homes, both clusters, the Warm cluster,
556 and the Cold cluster with the largest 5% of the squared errors removed are
557 given, where “Both Clusters” denotes the weighted average of the Warm
558 and Cold clusters. For both responses, the weighted average CV_{MSE} of the
559 Warm and Cold clusters is smaller than the CV_{MSE} for all homes. Therefore,
560 splitting the homes into two clusters produces models with better predictive
561 capabilities than does a single model using all homes. We also see that Δ
562 electricity is more predictable for Cold cluster homes, and Δ natural gas is
563 more predictable for Warm cluster homes. This may be due to the fact that
564 electricity is more likely to be used for cooling, so there is less electricity
565 consumption in Cold cluster homes (and vice versa for natural gas).

566 4.2.3. Model Output Adjustments

567 Using the cross-validation predictions, we can now adjust the RBES pre-
568 dicted energy consumption of a home by its predicted Δ (the fitted value of
569 the model) and use the adjusted predictions to calculate the adjusted Δ for
570 electricity and natural gas consumption, as shown in Equation (4). Table
571 6 shows the mean and standard deviation of the adjusted Δ electricity and
572 adjusted Δ natural gas responses for all models. Compared with the sum-
573 maries of the original Δ responses in Table 1, the adjusted Δ responses are
574 now centered near zero with smaller standard deviations. The Δ electricity
575 standard deviations have been reduced for the All Home, Warm Cluster, and
576 Cold Cluster models by 7%, 13%, and 3%, respectively. The variability in
577 the natural gas errors have been reduced more substantially with reductions
578 in standard deviations for the All Home, Warm Cluster, and Cold Cluster
579 models of 27%, 24%, and 26%, respectively.

580 Visually, the bottom panels of Figure 1 show the adjusted predicted en-
581 ergy consumption based on the cluster models plotted against the measured
582 energy consumption for both electricity and natural gas. The most striking
583 results are that (1) the substantial bias in the electricity predictions has been
584 corrected, since the points align with the line of perfect agreement; and (2)
585 the variability in the natural gas predictions has been significantly reduced,
586 as shown by the reduced scatter around the line of perfect agreement.

587 5. Conclusions

588 The goal of this work was to efficiently use available information on a set of
589 homes to build statistical models that would be used to improve the accuracy

Table 6: Mean and standard deviation of the Δ responses when predictions are adjusted by the fitted value of the MLR models.

Model	Adj. Δ Electricity		Adj. Δ Natural Gas	
	Mean	SD	Mean	SD
All Homes	0.006	11.86	-0.028	23.48
Both Clusters	0.014	11.51	-0.020	23.76
Warm Cluster	0.016	11.75	-0.028	23.28
Cold Cluster	0.011	11.30	-0.017	23.95

of annual energy consumption predictions of an RBES simulation tool. To achieve this goal, we (1) removed redundant information in the variable set; (2) clustered the homes into two groups; and (3) created MLR models both for all homes and within each cluster. Creating models based on these two clusters produced better predictions of the error in annual electricity and natural gas use than building a single model based on all homes, as shown by the higher adjusted R^2 values and lower cross-validation MSE's. However, the adjusted predictions are very similar regardless of whether the clusters are used or not. These models also point to areas of potential improvement both in the RBES tool and in the translation algorithms. Differences in variables selected for the two clusters provide insight on factors that could be improved in the simulation process for homes in those groups. In this example, the MLR model even identified a variable that was incorrectly coded in the translation script.

Ultimately, the fitted MLR models are used to adjust the RBES predic-

605 tions independently for each home, and the reductions in bias and variability
606 are substantial. Even without grouping the homes, the adjusted predictions
607 are much improved, so the approach described here can be applied to post-
608 process the predictions of any RBES tool. When removing one home at a
609 time to assign it to a cluster, fit the MLR model, and adjust its prediction, we
610 are assuming that the removed home is similar to the homes in the BAFDR
611 database. The BAFDR database that is used in this work is not representa-
612 tive of all detached single-family homes in the United States, so the clusters
613 and models should be updated as new homes are added to the BAFDR. How-
614 ever, ideally, when a new home is simulated in an RBES program, it would
615 be placed in the group to which its characteristics are the most similar, and
616 the model for that group would be used to adjust the home’s RBES annual
617 energy consumption prediction.

618 We have described a method for post-processing RBES predictions to
619 improve accuracy, but we also recognize the value of the MLR analysis in
620 identifying potential improvements to the RBES process. As a general ap-
621 proach, significant variables identified in the MLR should be investigated and
622 an effort should be made to correct errors in the translation process and/or
623 simulation tool before developing the “final” MLR post-processing models.
624 Fixing these errors should generally reduce the magnitude of the corrections
625 made from post-processing and should also improve the predictive capability
626 of the RBES tool when analyzing scenarios such as energy saving upgrades.
627 In this study, we used a developmental version of the translation scripts and
628 only created the post-processing models to demonstrate how they could be
629 applied to improve predictions; we plan to use the MLR results to improve

630 the RBES process and then generate updated post-processing models using
631 a larger set of homes from the BAFDR. Potential future work related to this
632 study includes: (a) finding ways to post-process RBES predictions of energy
633 savings for retrofit measures; (b) automating the variable selection in the
634 MLR models with the LASSO method [30]; and (c) automating the removal
635 of collinear variables with a decision tree algorithm [31]. Automation of the
636 methods applied in this study would allow them to be included in industry
637 software testing procedures, which would increase the amount of actionable
638 information test takers have to improve the accuracy of their RBES tools
639 and associated predictions.

640 **Acknowledgements**

641 This work was supported by the U.S. Department of Energy under Contract
642 No. DE-AC36-08GO28308 with the National Renewable Energy Laboratory.
643 The project was completed under the Alliance Partner University Program.
644 The authors would like to thank David Roberts for support and feedback
645 on this project and Joseph Robertson for his assistance in preparing the
646 BEopt and BAFDR datasets for analysis. Finally, the authors would like to
647 thank those who have contributed data to the Building America Field Data
648 Repository and to the anonymous referees whose comments have improved
649 the content and presentation of this work.

650 **References**

- 651 [1] Annual energy review 2010, 2011. URL: [http://www.eia.gov/](http://www.eia.gov/totalenergy/data/annual/pdf/aer.pdf)
652 [totalenergy/data/annual/pdf/aer.pdf](http://www.eia.gov/totalenergy/data/annual/pdf/aer.pdf).

- 653 [2] B. Polly, N. Kruis, D. Roberts, Assessing and Improving the Accuracy of
654 Energy Analysis for Residential Buildings, Technical Report, National
655 Renewable Energy Laboratory, 2011.
- 656 [3] K. Lomas, H. Eppel, C. Martin, D. Bloomfield, Empirical validation of
657 building energy simulation programs, *Energy and Buildings* 26 (1997)
658 253–275.
- 659 [4] L. Tronchin, K. Fabbri, Energy performance building evaluation in
660 Mediterranean countries: Comparison between software simulations and
661 operating rating simulation, *Energy and Buildings* 40 (2008) 1176–1187.
- 662 [5] R. Brown, Modeled vs. Actual Energy Savings for Energy Upgrade Cal-
663 ifornia Home Retrofits, Technical Report PIR-08-018, BKi, Oakland,
664 CA, 2012.
- 665 [6] D. Roberts, N. Merket, B. Polly, M. Heaney, S. Casey, J. Robertson,
666 Assessment of the U.S. Department of Energy’s Home Energy Scor-
667 ing Tool, Technical Report TP-5500-54074, National Renewable Energy
668 Laboratory, Golden, CO, 2012.
- 669 [7] SEEM 94 Calibration to Single Family RBSA Data, Technical Report,
670 Regional Technical Forum, Portland, OR, 2013. URL: [http://rtf.
671 nwccouncil.org/subcommittees/seem/Meetings/meetings.htm](http://rtf.nwccouncil.org/subcommittees/seem/Meetings/meetings.htm).
- 672 [8] H. R. Glahn, D. A. Lowry, The use of model output statistics (MOS) in
673 objective weather forecasting, *Journal of Applied Meteorology* 11 (1972)
674 1203–1211.

- 675 [9] W. H. Klein, H. R. Glahn, Forecasting local weather by means of model
676 output statistics, *Bulletin of the American Meteorological Society* 55
677 (1974) 1217–1227.
- 678 [10] E. Eccel, L. Ghielmi, P. Granitto, R. Barbiero, F. Grazzini, D. Ce-
679 sari, Prediction of minimum temperatures in an Alpine region by linear
680 and non-linear post-processing of meteorological models, *Nonlinear Pro-
681 cesses in Geophysics* 14 (2007) 211–222.
- 682 [11] T. Gneiting, A. E. Raftery, A. H. Westveld, T. Goldman, Calibrated
683 probabilistic forecasting using ensemble model output statistics and min-
684 imum CRPS estimation, *Monthly Weather Review* 133 (2004) 1098–
685 1118.
- 686 [12] J. Chenevez, C. O. Jensen, Operational ozone forecasts for the region
687 of Copenhagen by the Danish Meteorological Institute, *Atmospheric
688 Environment* 35 (2001) 4567–4580.
- 689 [13] J. M. Sloughter, T. Gneiting, A. E. Raftery, Probabilistic wind speed
690 forecasting using ensembles and Bayesian model averaging, *Journal of
691 the American Statistical Association* 105 (2010) 25–35.
- 692 [14] H. Paeth, N. M. J. Hall, M. A. Gaertner, M. D. Alonso, S. Moumouni,
693 J. Polcher, P. M. Ruti, A. H. Fink, T. L. M. Gosset, A. T. Gaye, D. P.
694 Rowell, W. Moufouma-Okia, D. Jacob, B. Rockel, F. Giorgi, M. Rum-
695 mukainen, Progress in regional downscaling of West African precipita-
696 tion, *Atmospheric Science Letters* 12 (2011) 75–82.

- 697 [15] M. Santamouris, G. Mihalakakou, P. Patargias, N. Gaitani,
698 K. Sfakianaki, M. Papaglastra, C. Pavlou, P. Doukas, E. Primikiri,
699 V. Geros, M. Assimakopoulos, R. Mitoula, S. Zerefos, Using intelli-
700 gent clustering techniques to classify the energy performance of school
701 buildings, *Energy and Buildings* 39 (2007) 45–51.
- 702 [16] S. Petcharat, S. Chungpaibulpatana, P. Rakkwamsuk, Assessment of
703 potential energy saving using cluster analysis: A case study of lighting
704 systems in buildings, *Energy and Buildings* 52 (2012).
- 705 [17] K. Kissock, J. Haberl, D. Ciaridge, Development of a toolkit for cal-
706 culating linear, change-point linear and multiple-linear inverse build-
707 ing energy analysis models, Technical Report 1050-RP, ASHRAE,
708 2002. URL: [http://repository.tamu.edu/bitstream/handle/1969.](http://repository.tamu.edu/bitstream/handle/1969.1/2847/ESL-TR-02-11-02.pdf?sequence=1)
709 [1/2847/ESL-TR-02-11-02.pdf?sequence=1](http://repository.tamu.edu/bitstream/handle/1969.1/2847/ESL-TR-02-11-02.pdf?sequence=1).
- 710 [18] R. Hendron, C. Engebrecht, Building America House Simulation Proto-
711 cols, Technical Report TP-550-49426, National Renewable Energy Lab-
712 oratory, Golden, CO, 2010.
- 713 [19] S. Sheather, A Modern Approach to Regression with R, Springer Sci-
714 ence+Business, New York, 2010.
- 715 [20] J. Guilford, Psychometric Methods, McGrawHill Book Company, Inc,
716 New York, NY, 1942.
- 717 [21] G. Glass, K. Hopkins, Statistical Methods in Education and Psychology,
718 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1984.

- 719 [22] R. Johnson, D. Wichern, Applied Multivariate Statistical Analysis, 6th
720 ed., Pearson Education, Upper Saddle River, NJ, 2007.
- 721 [23] J. Ward, Hierarchical grouping to optimize an objective function, Jour-
722 nal of the American Statistical Association 48 (1963) 236–244.
- 723 [24] A. Izenman, Modern Multivariate Statistical techniques, Springer Sci-
724 ence+Business, New York, NY, 2008.
- 725 [25] X. L. Xie, G. Beni, Validity measure for fuzzy clustering, IEEE Trans-
726 actions on Pattern Analysis and Machine Intelligence 3 (1991) 841–847.
- 727 [26] L. Hubert, J. Schultz, Quadratic assignments as a general data-analysis
728 strategy, British Journal of Mathematical and Statistical Psychology 29
729 (1976) 190–241.
- 730 [27] J. Dunn, Well separated clusters and optimal fuzzy partitions, Journal
731 of Cybernetics 4 (1974) 95–104.
- 732 [28] J. Neter, M. Kutner, W. Wasserman, C. Nachtsheim, Applied Linear
733 Statistical Models, 4th ed., McGraw-Hill, Boston, MA, 1996.
- 734 [29] D. Wilks, Statistical Methods in the Atmospheric Sciences, 3rd ed.,
735 Elsevier Inc., Oxford, UK, 2011.
- 736 [30] R. Tibshirani, Regression shrinkage and selection via the lasso, Journal
737 of the Royal Statistical Society, Series B 58 (1996) 267–288.
- 738 [31] L. Breiman, J. Friedman, R. Olshen, C. Stone, Classification and Re-
739 gression Trees, Wadsworth, Boca Raton, FL, 1984.