“Buoyancy” in granular Medium: How deep can an object sink in sand?

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Abstract

The behavior of granular matter is different from either fluids or solids. One may not be able to answer even a naive question such as how deep an object can sink in sand. Answers to the depth of footprints on sand beach and its dependence on grain size have never been seriously studied before, and may deserve a closer look and better understanding. Laying a ball of fixed size onto granules, we have measured the sinking depth (SD) of the ball into granules of different sizes and studied the dependence of SD on the sizes of the ball and granules. We find that the SD is very sensitive to the size of granules and the variation of SD on granule size is not monotonic. The maximum SD occurs at $r \approx \frac{1}{20}R$, where r and R are the radii of granules and the ball, respectively. This ratio does not depend on the density of the ball and the volume fraction of granules. An empirical formula of SD on densities and sizes of the ball and granules are obtained based on the experimental results.

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1. Introduction

Granular materials have recently become a popular subject of studies [1][2][3] because of their scientific and technological importance. Our understanding of properties of granular materials is far from complete, such as
5 jamming transition in static granular assemblies[4], a static assembly of non-cohesive, spherical particles in contact [1][5]. Particles within this system are under stress, supporting the weight of the material above them in addition to any applied load. It is known that stresses in a granular assembly are distributed in a highly inhomogeneous manner, along networks containing
10 the largest inter-particle forces known as force chains [6][7]. The assembly will behave like a solid when the force chains are strong enough to hold the assembly together and will flow like a fluid when these force chains are broken under stress[8]. Obviously, properties of these contact force chains crucially determine the bulk properties of the assembly, especially its load bearing
15 capability [9].

The formation of the force chains is the result of the inherently non-linear nature of inter-particle friction forces and the particle's nearly hard-sphere interaction [10][11]. The forces between granules are characterized by their contacts as they vanish identically when the granules are not in
20 contact[12][13][3][14]. An important characteristic of the force chains is that their distribution in the granular assembly will be modified in response to an external stress[11][15]. When under stress, the particles start deforming. New contacts are created, and additional force chains are created. For that

reason, a quantitative understanding of the force chains will be needed to
 25 understand the response of the granular assembly to an external stress. However, detailed studies of changes in these contacts and their corresponding forces are proved to be difficult because forces measurement inside a granular assembly is still not possible even if the stress pattern has been observed in a two-dimensional granular material under compression [16][17].

30 One of the most important finding of these studies is that the force chains support most of the external load, effectively shielding large regions of the material [18]. Presumably, the characteristics of the force chains can be inferred from the external response of the granular assembly to a well-defined external stress without the need of measurements in the interior of the assembly if a reasonable model can be used[11][19][20][22][23][24]. For example,
 35 when an intruder is lowered into a granular bed, properties of the contact force between the intruder and the bed will be related to the sinking depth of the intruder, which is the response of the bed [21][25]. In this letter, we report results of such an experiment. Properties of contact forces F in granular
 40 beds with beads of size r and density ρ_r are probed by experiments in which the sinking distance d of an intruder of size R and density ρ_R is measured as a function of r when the intruder is lowered into the beds under its own weight. It is found that d is not monotonic and its maximum sinking can be understood as the balance between the changes in F and the number of
 45 contacts as r is varied. The ratio r/R and ρ_r/ρ_R are found to be important parameters, and the contact forces are highly nonlinear in nature.

2. Experimental setup

Our experimental setup is very simple. As is shown in the inset of Figure 1(a), a cylindrical container of an inner diameter $72mm$ and a height $50mm$ is filled with beads. Directly above the granular bed is an intruder ball, which is suspended from a vertical translation stage so that the position of the intruder can be controlled. The granular beds studied are all disordered three-dimensional, compact random packing of smooth glass beads (spheres) with 21 different radii in a range of $r = 0.02mm - 1.8mm$ with a size dispersion of about 20%. The density of the glass beads is $\rho_r = 2.5g/cm^3$. Eight radii of intruder balls (stainless steel balls used in ball bearing) are used: $1.50mm$, $2.55mm$, $3.15mm$, $4.00mm$, $5.00mm$, $5.75mm$, $7.50mm$ and $10.30mm$. Density of the intruder balls is $\rho_R = 7.8g/cm^3$. In order to maintain a constant bulk packing density of the beds for various bead sizes, we always keep the volume of the granules and the total weight constant. The total mass in the experiment is kept a constant of $370g$. The volume of the granular beds is then kept constant by various operations such as shaking, removal and addition of beads. In the experiments, the volume fraction is kept as 0.61 ± 0.01 . During the compaction of the granular bed in the procedure above, the surface of the bed is always leveled off. The intruder ball is suspended from a fine cotton cord and lowered onto the surface of the granular packing as slowly as possible. The response of the beds to an intruder lowered into the beds under gravity is measured by the maximum distance that the intruder can travel after the intruder is in contact with the bed. This distance is defined as the SD of the intruder. The initial (contact with the bed) and final positions of the intruder are measured by a traveling

microscope. The measured d will be a function of the dimension of the container for the granular bed if the container is not large enough to eliminate boundary effects. The container used in our experiment is tested to be large
75 enough by comparing measured d with a larger container. All measurements reported in this report are averaged over ten times to minimize statistical error. It should be pointed out that the intruder ball should be large and heavy enough when compared to the weight of the beads because there is a threshold value of SD for one kind of bed (Fig. 3). Also the intruder
80 should not be small to comparable with the size of the beads, otherwise big measurement-error will result.

3. Results and discussion

Figure 1(a) is the measured SD , shown as d , for an intruder in granular beds as a function of bead sizes r with different intruder radius R . (Due to
85 too many sets of data in the figure, the error bars are not shown. Error bars are shown in Figure 2.) When r is small, d increases with r . After reaching a maximum d_{max} , d begins to decrease at larger r . A remarkable feature of Figure 1(a) is that the measured d is not a monotonic function of the bead sizes. There is a specific value of bead size r^* at which d is maximum or the
90 bed seems to be weakest.

When we re-plot the ratio of d/R versus r/R in Figure 1(b), the curves of different R almost overlap. This indicates d (at a fixed r) and d_{max} increases linearly with R , and that d_{max} is found at a characteristic $\lambda = r/R \approx 0.05$. In other words, the sinking depth of an intruder reaches a maximum when its
95 size is about 20 times of the size of the beads. In our experimental condition,

Figure 1: (a) Sinking depths of intruder balls versus bead size r , where x-axis is the bead radius r and y-axis is the SD of the intruder ball. The curves from top to bottom correspond to SD of ball radius $R = 10.30, 7.50, 5.75, 5.00, 4.00, 3.15, 2.55$ and 1.50 mm, respectively. Inset is the schematic of the setup. (b) Curves of the ratio d/R versus r/R , the symbols are same as in Fig. 1(a). Inset shows a linear dependence of the r^* on R , where r^* is the granule radius when the maximum SD occurs.

d_{max} is about $0.6R$. In order to confirm d_{max} and d change consistently with r , the SD is measured as a function of r at two different ball densities $10.6g/cm^3$ and $5.4g/cm^3$ and under two different packing density $\phi = 0.57$ and $\phi = 0.61$ as shown in Figure 2(a) and 2(b). The results show that for intruder balls of the same size R , the ratio of d_{max}/d is basically not a function of r when changing the ball density or the bead packing density. In other words, the geometric dependent factor of SD can be universal to all ball densities and granular random packings. It can also be seen from Figure 2(b) that the measured d is a decreasing function of the bulk density. This

Figure 2: (a) $d-r$ dependence at intruder size $R = 9.5\text{mm}$, $d_1(r)r$ and $d_2(r)$ correspond to curves of ball densities 10.6g/cm^3 and 5.4g/cm^3 , respectively. Inset is the plot of $\frac{d_1(r)}{d_2(r)}$ as a function of r , where $\frac{d_1(r)}{d_2(r)} \approx 1.62$. (b) $d-r$ curves of the same intruder ($R=10.3\text{mm}$) at packing density $\phi = 0.57$ and $\phi = 0.61$. Inset shows the ratio of $\frac{d_3(r)}{d_4(r)}$ to r . The average value of $\frac{d_3(r)}{d_4(r)}$ is approximately a constant i.e. 1.32

105 is expected, as the strength of the bed should increase with its bulk density. However, the value of r^* does not seem to be sensitive to the values of the bulk density. Since the general feature of Figure 2(b) is independent of the bulk densities, results reported below are all from experiments with a fixed volume fraction of 0.61 ± 0.01 . With the same ρ_R , ρ_r , ϕ , and using the
 110 data $d(r)$ in Fig. 1(a), we can find that the SD is approximately a linear function of R , and thereby can be expressed as

$$d(rR) = D(r)[R - B(r)] \quad (1)$$

When $R \leq B(r)$, $d(rR) = 0$. $B(r)$ is the threshold of the size of the intruder ball in a chosen granular bed of size r . The slope $D(r)$ and intercept $B(r)$ are plotted as a function of r in Figure 3(a) and 3(b). $D(r)$ is fitted by

115 $D(r) = ar^\alpha(1 - br)$, we obtain $a = 1.2$, $b = 0.4$, $\alpha = 0.4$. $B(r)$ can be fitted by $B(r) = k(r - e)$, where $k = 3.2$ and $e = 0.09mm$. When $r \leq e$, $B(r) = 0$.

Figure 3: a) Dots are the slopes of $d(R)$. The line is a fitting of $D(r)$ by a function $r^\alpha(1 - br)$, where $\alpha = 0.4$ and $b = 0.4$. (b) Plot of intercepts $B(r)$. The $B(r)$ is fitted by a line $k(r - e)$, where k and e are fitting parameters, $k = 3.2$ and $e = 0.09mm$.

The dependence of SD on the ball density is also investigated. Filling a hollow brass shell ($R = 9.5mm$) with material of different densities, we have made balls of densities 0.9, 2.6, 5.4, 7.3 and $10.6g/cm^3$. Keeping the granular
120 packing density $\phi = 0.61$, we measure the d_{max} as a function of ball density. We find the d_{max} at $r/R \approx 0.05$. The $d_{max}(\rho_R/\rho_r)$ curve is shown in Figure 4. It is seen that d_{max} is not a linear function of ρ_R/ρ_r . Fitting the curve with function $d_{max} = \acute{A}(\rho_R/\rho_r - c)^\beta$, we obtain $\acute{A} = 2.85$, $c = 0.05$ $\beta = 0.7$. Since d_{max} and d follows the same dependence on ball density and volume
125 fraction, we can write

$$d \approx \left(\frac{\rho_R}{\rho_r} - c\right)^\beta \quad (2)$$

c is the threshold density ratio at which the ball begins to sink. When

$\rho_R/\rho_r \leq c$, $d = 0$. Therefore, combining (1) and (2) we obtain

$$d = ARr^\alpha(1 - br)[1 - \frac{k(r - e)}{R}](\frac{\rho_R}{\rho_r} - c)^\beta \quad (3)$$

where A is a parameter related to ϕ . We find that A decreases nonlinearly with increasing ϕ . The ϕ dependent of d will be discussed in a separate article (our future work). In above equation when $r \leq e$:

$$d = ARr^\alpha(1 - br)(\frac{\rho_R}{\rho_r} - c)^\beta \quad (4)$$

And when $R \leq k(r - e)$ or $\rho_R/\rho_r \leq c$, then :

$$d = 0 \quad (5)$$

This result is different from the fluid case. Any shearing stress can induce flow of the fluid. The submerged depth of a floating object d (when d/R is small) is proportional to $R(\rho_s/\rho_l)^{1/2}$ [14][26]; where ρ_s and ρ_l are the densities of the object and the fluid, respectively. In granular media, however, d is proportional to $(\rho_R/\rho_r c)^{0.75}$, and there exist thresholds $k(re)/R$ and c only above which particles yield to external forces. A small or light ball does not sink into sand. Why does the sinking depth in granules changes non-monotonically with the size of the granules? We believe it is caused by two competing factors. One is the dependence of the number of ball-granule contact points on the sizes r and R . With the same submerged area, the number of contacts decreases as the radius of the granules increases. The reducing number of contacts reduces the supporting the ball obtains and causes the sinking depth to increase with the increasing r . The other factor is the formed force chain in granules, which is larger with greater r and prevent ball from sinking. When the granule size is nearly the same as

Figure 4: Plot of $d_{\max}$ versus ρ_R/ρ_r with $\rho_r = 2.5g/cm^3$ and $R = 9.5mm$. Fitting the data with function $d_{\max} = \acute{A}(\rho_R/\rho_r - c)^\beta$, we obtain $\acute{A} = 2.85$, $c = 0.05$, $\beta = 0.7$.

the ball (assume the density of the two are the same), the sinking depth approaches zero.

4. Conclusion

150 In this work we found that the sinking depth of an intruder ball in granular media does not change monotonically with the size of the granules. There exists a maximum $d_{\max}$ at a ratio $\lambda = r/R \approx 1/20$, and λ does not change with the ball mass (density) or the granular random packing. We have obtained an empirical formula of d as a function of r , R and ρ_R/ρ_r . Our results
155 demonstrate that the “buoyancy” of granular matter is different from that of

solids or fluids. The sinking depth in granules does not only depend on the interaction forces, it also depends on the size of the granules. For instance, when a car or a tank moves in the desert its sinking depth depends on the sand size.

160 Our experimental work could not investigate the effect of coefficient of friction and the shape of grains. Such types of studies are hard to carry out experimentally; however, further research using DEM simulation is needed for better understanding of the mechanisms of this phenomenon.

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