SedFoam: A multi-dimensional Eulerian two-phase model for sediment transport and its application to momentary bed failure

Zhen Cheng^{a,*}, Tian-Jian Hsu^a, Joseph Calantoni^b

^a*Civil and Environmental Engineering, University of Delaware, Newark, DE 19716, U.S.A.*

^b*Marine Geosciences Division, Naval Research Laboratory, Stennis Space Center, MS 39529, U.S.A.*

Abstract

A multi-dimensional Eulerian two-phase model for sediment transport, called SedFoam, is presented. The model was developed under the open-source framework via the CFD toolbox OpenFOAM. With closures of particle stresses and fluid-particle interactions, the model is able to resolve processes in the concentrated region of sediment transport and hence does not require conventional bedload/suspended load assumptions. A modified $k - \epsilon$ closure was adopted for the carrier flow turbulence. The model was validated for Reynolds-averaged steady and oscillatory sheet flows and verified with empirical formulae for scour downstream of an apron. The model was used to study momentary bed failure (or plug flow) under sheet flow conditions. Model results revealed the existence of instabilities of the near-bed transport layer when momentary bed failure criteria was exceeded. These instabilities evolved into 5 – 10 cm billows and were responsible for the large transport

*Corresponding author

Email address: zcheng@udel.edu (Zhen Cheng)

rate. The instabilities were associated with a large erosion depth, which was triggered by the combination of large bed shear stresses and large horizontal pressure gradients. Further numerical experiments confirmed the conjecture by previous studies that a criterion for onset of momentary bed failure in oscillatory sheet flow was a function of both the Shields parameter and Sleath parameter.

Keywords: two-phase, Eulerian-Eulerian, multi-dimensional, sediment transport, scour, bed instability

1. Introduction

The coastal zone is one of the most economically vibrant and culturally diverse human habitats. However, it is vulnerable to sea-level rise and intense storm events. Coastal morphological evolution is caused by a wide range of coupled cross-shore and alongshore sediment transport processes driven by waves, wave-induced currents and wave-breaking turbulence (e.g., Roelvink et al., 2009). At operational forecasting scales, parameterizations of sediment transport rate associated with these hydrodynamic forcing mechanisms are needed. Typically, sediment transport rate is parameterized by the bottom shear stress (e.g., Ribberink, 1998; van Rijn, 1984a), which works reasonably well for moderate wave conditions. However, during large wave events associated with intense storms, the predicted morphological evolution is less satisfactory (Ruessink and Kuriyama, 2008) and other mechanisms that are not parameterized by bottom shear stress may become dominant.

For example, laboratory experiments (Madsen, 1974; Sleath, 1999; Liu and Sato, 2005; Holway et al., 2012) and field observations (Conley and In-

man, 1992; Foster et al., 2006) demonstrate that significant bed mobilization, called momentary bed failure or plug flow, may occur during the passage of steep waves. Although transport often occurs as sheet flow under steep waves, the timing of such large bed mobilization coincides with large horizontal pressure gradients. Moreover, momentary bed failure/plug flow can cause much larger transport than is predicted by typical sheet flow formula, and hence it is an essential process that should be included for the prediction of beach evolution during large wave events.

Our capability to resolve key mechanisms in the concentrated region of transport remains limited. Numerical models for coastal morphological evolution (Jacobsen et al., 2012; Warner et al., 2008; Lesser et al., 2004), local scour around structures (Hoffmans and Pilarczyk, 1995; Liu and Garcia, 2008) and bedform dynamics (Marieu et al., 2008; Chou and Fringer, 2010) typically separate the total sediment transport into bedload and suspended load components. Partitioning is necessary in order to efficiently model morphological evolution or to better resolve turbulence. However, bedload transport and erosion flux (pick up flux) are then parameterized semi-empirically (Ribberink, 1998; van Rijn, 1984c). Presently, these parameterizations do not explicitly include the effect of momentary bed failure.

On the other hand, the two-phase flow approach can resolve concentrated region of sediment transport by including closures of particle stresses and fluid-particle interactions in the governing equations. In the past decades, several two-phase numerical models have been developed, where the sediment phase is modeled either with an Eulerian scheme (e.g., Dong and Zhang, 1999, 2002; Hsu et al., 2004; Li et al., 2008; Amoudry and Liu, 2009; Bakhtyar et al.,

2010) or a Lagrangian scheme (e.g., Drake and Calantoni, 2001; Calantoni and Puleo, 2006; Calantoni and Thaxton, 2008; Heald et al., 2004). These two-phase models can resolve the full profiles of sediment transport without the need to divide the transport into bedload and suspended load components. For instance, one-dimensional-vertical (1DV) two-phase models are used to improve the parameterization of pickup flux in suspended load module under waves in sheet flow condition (Amoudry and Liu, 2010; Yu et al., 2012).

Due to the complexity of the model formulation, most of the existing two-phase models are based on the Reynolds-averaged approach and simplified into 1DV formulation. 1DV models cannot capture the development of inhomogeneous flow features (e.g., bedforms), flow instabilities (Charrau and Mouilleron-Arnould, 2002) and scour (Amoudry and Liu, 2009). Hence, existing two-phase models are mainly restricted to sheet flow applications where the ensemble-averaged flow field is more or less fully developed in the streamwise and spanwise directions. Also, a model developed for 1DV cannot resolve turbulence, which is inherently three-dimensional. Amoudry and Liu (2009) developed a two-dimensional (2D) two-phase model to study scour downstream of an apron, and good agreements are obtained for the development of the 2D scour characteristics. A two-phase model has been developed by Bakhtyar et al. (2010) to study surf and swash zones sediment transport under the influence of wave breaking and wave runup/rundown. The model is shown to be capable of reproducing the observed beach profile evolution in the surf and swash zones qualitatively.

Motivated by this need, we have recently developed a multi-dimensional

67 model for sediment transport that solves the complete Eulerian two-phase
 68 equations following the mathematical formulation similar to that of Hsu
 69 et al. (2004) and Yu et al. (2012). The mathematical formulation is solved
 70 numerically using the open-source CFD toolbox, OpenFOAM. OpenFOAM
 71 includes several multiphase flow modeling capabilities, such as bubbly flow
 72 (Rusche, 2002). Specifically, a two-phase flow solver in OpenFOAM called,
 73 twoPhaseEulerFoam (Rusche, 2002; Weller, 2002; Passalacqua and Fox, 2011)
 74 was adopted as the baseline. The numerical stability of the solver was im-
 75 proved and additional capabilities needed for coastal sediment transport ap-
 76 plications were implemented. This new sediment transport model is called
 77 SedFoam. Although the numerical model was written in three spatial dimen-
 78 sions, the current version of the model has only been validated/verified for
 79 2DV Reynolds-averaged condition.

80 The purpose of this study is to describe this new multi-dimensional model,
 81 SedFoam, its validation and its application to investigate momentary bed
 82 failure. The mathematical formulation of the model is discussed in Section
 83 2. The numerical methodology is discussed in Section 2.1 and 2.2. Model
 84 validation for the present 2DV Reynolds-averaged formulation is presented
 85 in Section 3. The numerical investigation of momentary bed failures/plug
 86 flows is presented in Section 4, and concluding remarks are given in Sections
 87 5, respectively.

88 **2. Model Formulation**

89 Drew (1983) derives the Eulerian two-phase flow model by averaging over
 90 carrier fluid and dispersed particles. The resulting 3D governing equations

91 for Eulerian two-phase flow can be considered as the counterpart of the clear
 92 fluid Navier-Stokes equations. Namely, the turbulent motions due to particle-
 93 particle interaction and particle-fluid interaction at the grain scale are param-
 94 eterized via fluid stress, particle stress and inter-phase momentum transfer
 95 terms in the Eulerian two-phase equations. In order to efficiently apply these
 96 equations to turbulent flow, in which turbulent motions are generated by flow
 97 shear much larger than the grain scale, additional turbulence averaging or fil-
 98 tering is necessary. Through Reynolds-averaging, Hsu et al. (2004) developed
 99 the turbulence-averaged Eulerian two-phase model and simplified the model
 100 to 1DV to study sheet flow sediment transport. In this study, we adopted
 101 similar turbulence-averaged governing equations and closures to develop the
 102 numerical model in 3D.

103 *2.1. Governing equations*

104 Assuming that there is no mass transfer between the two phases, the mass
 105 conservation equations for fluid phase and sediment phase can be written as,

$$106 \quad \frac{\partial(1-\phi)}{\partial t} + \frac{\partial(1-\phi)u_i^f}{\partial x_i} = 0, \quad (1)$$

$$107 \quad \frac{\partial\phi}{\partial t} + \frac{\partial\phi u_i^s}{\partial x_i} = 0 \quad (2)$$

109 where ϕ is the sediment volumetric concentration, u_i^f is the fluid velocity, and
 110 u_i^s is the sediment velocity. In this study, $i = 1, 2, 3$ represents streamwise,
 111 spanwise and vertical components, respectively. The momentum equations
 112 for fluid and particle phases are expressed as,

$$113 \quad \frac{\partial\rho^f(1-\phi)u_i^f}{\partial t} + \frac{\partial\rho^f(1-\phi)u_i^f u_j^f}{\partial x_j} = -\frac{\partial(1-\phi)p^f}{\partial x_i} + (1-\phi)f_i + \frac{\partial\tau_{ij}^f}{\partial x_j} \\
 114 \quad + \rho^f(1-\phi)g\delta_{i3} + M_i^{fs}, \quad (3)$$

115 and

$$116 \quad \frac{\partial \rho^s \phi u_i^s}{\partial t} + \frac{\partial \rho^s \phi u_i^s u_j^s}{\partial x_j} = -\frac{\partial \phi p^f}{\partial x_i} + \phi f_i - \frac{\partial p^s}{\partial x_i} + \frac{\partial \tau_{ij}^s}{\partial x_j} + \rho^s \phi g \delta_{i3} + M_i^{sf}, \quad (4)$$

117 where ρ^f and ρ^s are fluid and sediment density, respectively, g_i is the grav-
 118 itational acceleration and p^f is the fluid pressure. The external pressure
 119 gradient that drives the flow is written as f_i . The fluid stress τ_{ij}^f includes the
 120 grain-scale (viscous) stress and the turbulent Reynolds stress, which are cal-
 121 culated with a two-equation turbulence model. The particle normal stress p^s
 122 and shear stress τ_{ij}^s are modeled with the kinetic theory of granular flow and
 123 a phenomenological closure of frictional contact stresses. Finally, M_i^{fs} and
 124 M_i^{sf} represent the inter-phase momentum transfer between fluid and particle
 125 phase, and $M_i^{fs} = -M_i^{sf}$ follows the Newton's 3rd law.

126 2.2. Closure Models

127 2.2.1. Inter-phase Momentum Transfer

128 In the two-phase flow formulation, the momentum exchange of the fluid
 129 phase and particle phase are coupled through the inter-phase momentum
 130 transfer terms (Maxey and Riley, 1983). For simplicity, only the dominant
 131 terms, namely the drag force and the buoyancy force are included in the
 132 present study. Thus the momentum exchange has the following form,

$$133 \quad M_i^{fs} = -\phi \beta (u_i^f - u_i^s) + \beta \frac{\nu^{ft}}{\sigma_c} \frac{\partial \phi}{\partial x_i} + p^f \frac{\partial (1 - \phi)}{\partial x_i}. \quad (5)$$

134 The first two terms on the right-hand-side (RHS) are due to drag force with
 135 β as the drag parameter. The first term represents averaged drag force due
 136 to mean velocity difference between fluid and particle phases. The second

term is the fluid turbulent suspension term, which results from the correlation of sediment concentration and fluid velocity fluctuations. The gradient transport assumption was adopted here with a closure for ν^{ft} representing the turbulent viscosity, which was calculated with a turbulence closure (see Section 2.2.2), and σ_c is the Schmidt number (see Table 1). The Schmidt number is a ratio of fluid turbulent viscosity to the sediment diffusivity, and according to previous studies, values smaller than unity were reported (Coleman, 1970; van Rijn, 1984b; Amoudry et al., 2005; Revil-Baudard et al., 2015). However, there does not seem to be a universally agreed value or relationship for Schmidt number for different sediment properties and flow conditions. In the present model, the Schmidt number is chosen to be 1 for simplicity, and its effect on the model results will be briefly discussed (see Section 3.1). The last term on the RHS of Eq. (5) represents the averaged buoyancy force. The correlation term between sediment concentration and fluid pressure is ignored in the turbulence averaged momentum equations. However, in the closure of eddy viscosity, the effect is included via buoyancy terms in the $k - \epsilon$ equations.

For the closure of the drag parameter β , we follow Ding and Gidaspow (1990), who combined the model of Ergun (1952) for dense sediment concentration ($\phi \geq 0.2$) and the model of Wen and Yu (1966) for lower sediment concentration ($\phi < 0.2$),

$$\beta = \begin{cases} \frac{150\phi\nu^f\rho^f}{(1-\phi)d^2} + \frac{1.75\rho^f|u^f - u^s|}{d}, & \phi \geq 0.2 \\ \frac{0.75C_d\rho^f|u^f - u^s|(1-\phi)^{-1.65}}{d}, & \phi < 0.2 \end{cases}, \quad (6)$$

where d is the sediment diameter, and the drag coefficient C_d is calculated

160 as:

$$161 \quad C_d = \begin{cases} \frac{24(1 + 0.15Re_p^{0.687})}{Re_p}, & Re_p \leq 1000 \\ 0.44, & Re_p > 1000 \end{cases}, \quad (7)$$

162 in which, $Re_p = (1 - \phi) |u^f - u^s| d / \nu^f$ is the particle Reynolds number, and
 163 ν^f is the fluid molecular viscosity.

164 2.2.2. Fluid turbulence Model

165 Because the present model equations were obtained by averaging over
 166 turbulence, the fluid stresses consist of a large-scale component R_{ij}^{ft} (i.e.,
 167 Reynolds stress) and a grain-scale component r_{ij}^f , which includes the viscous
 168 stress and an additional effect due to fluid-particle interaction at grain scale.
 169 In the present study, this later effect is ignored and only viscous stress is
 170 considered. Thus, the total fluid stress is written as,

$$171 \quad \tau_{ij}^f = R_{ij}^{ft} + r_{ij}^f = \rho^f (1 - \phi) \left[(\nu^{ft} + \nu^f) \left(\frac{\partial u_i^f}{\partial x_j} + \frac{\partial u_j^f}{\partial x_i} - \frac{2}{3} \frac{\partial u_k^f}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} k^f \delta_{ij} \right]. \quad (8)$$

172 The turbulent eddy viscosity ν^{ft} is calculated by the fluid turbulence kinetic
 173 energy k^f and turbulent dissipation rate ϵ^f ,

$$174 \quad \nu^{ft} = C_\mu \frac{(k^f)^2}{\epsilon^f}, \quad (9)$$

175 where C_μ is an empirical coefficient (see Table 1). The turbulent kinetic
 176 energy k^f (TKE) is calculated by its balance equation appropriate for sand
 177 particles in water (Yu et al., 2010),

$$178 \quad \frac{\partial k^f}{\partial t} + u_j^f \frac{\partial k^f}{\partial x_j} = \frac{R_{ij}^{ft}}{\rho^f} \frac{\partial u_i^f}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\left(\nu^f + \frac{\nu^{ft}}{\sigma_k} \right) \frac{\partial k^f}{\partial x_j} \right] - \epsilon^f - \frac{2\beta(1 - \alpha)\phi k^f}{\rho^f(1 - \phi)} \\ 179 \quad - \frac{1}{(1 - \phi)} \frac{\nu^{ft}}{\sigma_c} \frac{\partial \phi}{\partial x_j} (s - 1) g_j. \quad (10)$$

180 Except for the last two terms on the RHS, the rest of terms in the present k^f
 181 equation are similar to those in the clear fluid TKE equation. The last term in
 182 Eq. (10) represents the buoyancy effect. For typical sediment concentration
 183 with an upward decaying profile, the buoyancy term represents the well-
 184 known sediment-induced density stratification that can attenuate turbulence.
 185 The fourth term on the RHS represents attenuation of TKE due to drag. The
 186 parameter α is proposed to characterize the degree that particles follow the
 187 fluid velocity fluctuations and it can be quantified by the Stokes number
 188 $St = t_p/t_l$ (Balachandar and Eaton, 2010), where $t_p = \rho^s/\beta$ is the particle
 189 response time, and $t_l = k^f/(6\epsilon^f)$ is the characteristic time scale of energetic
 190 eddies. Considering a given particle encounters a turbulent eddy on its path,
 191 if the particle has very small inertia ($St \ll 1$), it can closely follow the
 192 eddy motion. However, if $St \sim O(1)$, the particle cannot completely follow
 193 the eddy motion and may cause disturbance to the eddy. Finally, if $St \gg 1$,
 194 the particle trajectory is hardly affected by the eddy motion. With these
 195 observations, Danon et al. (1977) and Chen and Wood (1985) proposed an
 196 exponential function for α ,

$$197 \quad \alpha = e^{-B \cdot St}, \quad (11)$$

198 where B is an empirical coefficient. It is evident from the model that the
 199 fourth term in Eq. (10) represents a dissipation mechanism for the turbu-
 200 lent kinetic energy. It is also possible that the presence of particle provides
 201 a generation mechanism of flow turbulence. However, this is observed for
 202 particles with very large Stokes number. For sand transport in water consid-
 203 ered in this study, this turbulence generation mechanism is unlikely and it is
 204 neglected.

205 Finally, the balance equation for the rate of turbulent kinetic energy
 206 dissipation ϵ^f is written as,

$$\begin{aligned}
 207 \quad \frac{\partial \epsilon^f}{\partial t} + u_j^f \frac{\partial \epsilon^f}{\partial x_j} = & C_{1\epsilon} \frac{\epsilon^f}{k^f} \frac{R_{ij}^{ft}}{\rho^f} \frac{\partial u_i^f}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\left(\nu^f + \frac{\nu^{ft}}{\sigma_\epsilon} \right) \frac{\partial \epsilon^f}{\partial x_j} \right] - C_{2\epsilon} \frac{\epsilon^f}{k^f} \epsilon^f \\
 208 \quad & - C_{3\epsilon} \frac{\epsilon^f}{k^f} \frac{2\beta(1-\alpha)\phi k^f}{\rho^f(1-\phi)} - C_{4\epsilon} \frac{\epsilon^f}{k^f} \frac{1}{(1-\phi)} \frac{\nu^{ft}}{\sigma_c} \frac{\partial \phi}{\partial x_j} (s-1) g_j. \quad (12)
 \end{aligned}$$

209 As discussed in Hsu et al. (2004), due to lack of comprehensive experimental
 210 data, the coefficients associated with the present two-equation closure are
 211 adopted from their clear fluid counterpart. The coefficient $C_{3\epsilon}$ in Eq. (12)
 212 is chosen to be 1.2. For the coefficient associated with the buoyancy term,
 213 $C_{4\epsilon} = 0$ is used in stably stratified condition, while it is set to 1 for unstably
 214 stratified condition. Table 1 summarizes the coefficients used for the present
 215 turbulence closure. The coefficient B in Eq. (11) is left as the only free
 model parameter to be calibrated with measured data.

Table 1: List of coefficients in fluid turbulence closure.

C_μ	$C_{1\epsilon}$	$C_{2\epsilon}$	$C_{3\epsilon}$	$C_{4\epsilon}$	σ_k	σ_ϵ	σ_c
0.09	1.44	1.92	1.2	0 or 1.0	1.0	1.3	1.0

216

217 2.2.3. Particle Stress Closure

218 In this study, the closure of particle stresses includes two components.
 219 For low to moderate sediment concentration, inter-granular interaction is
 220 assumed to be dominated by binary collisions and a closure based on ki-
 221 netic theory is adopted. For large sediment concentration, binary collision
 222 becomes unlikely and inter-granular interaction is dominated by enduring

223 contact/frictional forces among particles (Tardos, 1997). For the present sed-
 224 iment transport application where the immobile bed and concentrated region
 225 of transport are also simulated (Hsu et al., 2004), we assume that particle
 226 pressure and particle stress both consist of a collisional-kinetic component
 227 (super-script ‘sc’) and a frictional component (super-script ‘sf’) (Johnson and
 228 Jackson, 1987),

$$229 \quad p^s = p^{sc} + p^{sf}, \quad (13)$$

$$230 \quad \tau_{ij}^s = \tau_{ij}^{sc} + \tau_{ij}^{sf}. \quad (14)$$

232 The closure of the collisional-kinetic component is based on the kinetic
 233 theory of granular flow where particle stress and particle pressure are quan-
 234 tified by the granular temperature, Θ , which is defined as one-third of the
 235 kinetic energy of particle velocity fluctuation. Similar to fluid TKE, bal-
 236 ance equation of granular temperature can be derived, but a closure model is
 237 needed. In this study, we adopt the model of kinetic theory of granular flow.
 238 The model is originally developed for dry granular flow consists of smooth,
 239 slightly inelastic, spherical particles (Jenkins and Savage, 1983; Lun and Sav-
 240 age, 1987). Here, we implement the model suggested by Ding and Gidaspow
 241 (1990), which takes into account of the fluid phase. The balance equation
 242 for granular temperature is written as,

$$243 \quad \frac{3}{2} \left[\frac{\partial \phi \rho^s \Theta}{\partial t} + \frac{\partial \phi \rho^s u_j^s \Theta}{\partial x_j} \right] = \left(-p^{sc} \delta_{ij} + \tau_{ij}^{sc} \right) \frac{\partial u_i^s}{\partial x_j} - \frac{\partial q_j}{\partial x_j} - \gamma_s + J_{int}, \quad (15)$$

244 where the terms on the RHS are the production of granular temperature, the
 245 flux of granular temperature q_j , the energy dissipation rate due to inelastic
 246 collision γ_s and the production (or dissipation) due to the interaction with
 247 the carrier fluid phase J_{int} . Following Ding and Gidaspow (1990), closure of

248 particle pressure is written as,

$$249 \quad p^{sc} = \rho^s \phi [1 + 2(1 + e)\phi g_{s0}] \Theta, \quad (16)$$

250 where e is the coefficient of restitution during collision. The radial distribu-
 251 tion function g_{s0} is introduced to describe the crowiness of particle, which
 252 can be calculated as (Carnahan and Starling, 1969),

$$253 \quad g_{s0} = \frac{2 - \phi}{2(1 - \phi)^3}. \quad (17)$$

254 The radial distribution function g_{s0} is a sharp increasing function of sediment
 255 concentration, ϕ , representing more frequent collisions, which further causes
 256 larger particle pressure. It has been demonstrated that the formula of Car-
 257 nahan and Starling (1969) under-predicts g_{s0} when concentration $\phi > 0.57$
 258 (Ding and Gidaspow, 1990; Sinclair and Jackson, 1989). However, it is noted
 259 that in the present study when $\phi > 0.57$, inter-granular interactions are
 260 dominated by enduring contact/frictional component of the stress. In fact,
 261 granular temperature modeled by the kinetic theory reduces to nearly zero
 262 when particle concentration is approaching $\phi = 0.57$. Therefore, we still
 263 adopt the radial distribution function of Carnahan and Starling (1969) for
 264 simplicity.

265 Following Gidaspow (1994), the particle stress is calculated as,

$$266 \quad \tau_{ij}^{sc} = \mu^{sc} \left[\frac{\partial u_i^s}{\partial x_j} + \frac{\partial u_j^s}{\partial x_i} \right] + \left(\lambda - \frac{2}{3} \mu^{sc} \right) \frac{\partial u_k^s}{\partial x_k} \delta_{ij}. \quad (18)$$

267 Through the kinetic theory, the particle shear viscosity is calculated as a
 268 function of granular temperature and radial distribution function,

$$269 \quad \mu^{sc} = \rho^s d \sqrt{\Theta} \left[\frac{4}{5} \frac{\phi^2 g_{s0} (1 + e)}{\sqrt{\pi}} + \frac{\sqrt{\pi} g_{s0} (1 + e) (3e - 1) \phi^2}{15(3 - e)} + \frac{\sqrt{\pi} \phi}{6(3 - e)} \right]. \quad (19)$$

270 Similarly, the bulk viscosity is calculated as,

$$271 \quad \lambda = \frac{4}{3}\phi^2\rho^s dg_{s0}(1+e)\sqrt{\frac{\Theta}{\pi}}. \quad (20)$$

272 It is worth mentioning that the sediment collisional stress closure adopted in
 273 this study differs from that used in Hsu et al. (2004). In the present model,
 274 the kinetic contribution of particle stress is included, thus the sediment stress
 275 closure can be extended to very dilute condition without an artificial cut-off
 276 as discussed in Hsu et al. (2004). The modification was essential for numerical
 277 stability of the model when extending to multi-dimensions. Meanwhile, the
 278 sediment collisional stress closure adopted here takes into account of the
 279 effect inelastic collisions, which is more appropriate for sand transport in
 280 water. The closure of granular temperature flux is assumed to be analogous
 281 to the Fourier's law of conduction,

$$282 \quad q_j = -\kappa^{sc}\frac{\partial\Theta}{\partial x_j}, \quad (21)$$

283 where the κ^{sc} is the conductivity of granular temperature, calculated as,

$$284 \quad \kappa^{sc} = \rho^s d \sqrt{\Theta} \left[\frac{2\phi^2 g_{s0}(1+e)}{\sqrt{\pi}} + \frac{9\sqrt{\pi} g_{s0}(1+e)^2(2e-1)\phi^2}{2(49-33e)} + \frac{5\sqrt{\pi}\phi}{2(49-33e)} \right]. \quad (22)$$

285 The dissipation rate due to inelastic collision is calculated based on that
 286 proposed by Ding and Gidaspow (1990),

$$287 \quad \gamma_s = 3(1-e^2)\phi^2\rho^s g_{s0}\Theta \left[\frac{4}{d} \left(\frac{\Theta}{\pi} \right)^{1/2} - \frac{\partial u_j^s}{\partial x_j} \right]. \quad (23)$$

288 Due to the presence of carrier fluid phase, carrier flow turbulence can also
 289 induce particle fluctuations. Following Hsu et al. (2004), the fluid-particle
 290 interaction term is given as,

$$291 \quad J_{int} = \phi\beta(2\alpha k^f - 3\Theta). \quad (24)$$

292 When the volumetric concentration of particles becomes close to random
 293 loose packing ($\phi \approx 0.57$), particles are constantly in contact with one another,
 294 and most of the particle phase energy is dissipated by friction between slid-
 295 ing particles, rather than inelastic collision (Tardos, 1997). In this condition,
 296 the normal stress acts to support the particles and the sediment bed be-
 297 haves like a solid. In other words, when the sediment concentration exceeds
 298 certain threshold value ϕ_f , sediment transport is in the stage of enduring
 299 contact, kinetic theory is no longer valid, and a frictional stress model needs
 300 to be incorporated. In the 1DV model of Hsu et al. (2004), the location of
 301 immobile bed was dynamically tracked using a Hertz contact law combined
 302 with a Coulomb criteria while modeling the sediment phase transport above
 303 the instantaneous immobile bed. Our experience suggested that extending
 304 this formulation to multi-dimensions was not practical. Consequently, in the
 305 present model, we adopt a different approach aimed at modeling the full
 306 transport of sediment phase into the immobile bed. The particle stresses
 307 due to the enduring contact in the concentrated regions are modeled with a
 308 phenomenological closure that ensures a solid-like behavior when sediment
 309 concentration is near 0.6 without the need to track the location of the im-
 310 mobile bed. Following Johnson and Jackson (1987), the enduring contact
 311 component of particle pressure is calculated as,

$$312 \quad p^{sf} = \begin{cases} 0, \phi < \phi_f \\ F \frac{(\phi - \phi_f)^m}{(\phi_{max} - \phi)^n}, \phi \geq \phi_f, \end{cases} \quad (25)$$

313 where $\phi_f = 0.57$, $\phi_{max} = 0.635$ and F , m and n are empirical coefficients to
 314 be discussed next. The particle stress due to frictional contact is calculated

315 by,

$$316 \quad \tau_{ij}^{sf} = -2\mu^{sf} S_{ij}^s, \quad (26)$$

317 where μ^{sf} is the frictional viscosity and S_{ij}^s is the deviatoric part of strain
318 rate tensor of sediment phase,

$$319 \quad S_{ij}^s = \frac{1}{2} \left(\frac{\partial u_i^s}{\partial x_j} + \frac{\partial u_j^s}{\partial x_i} \right) - \frac{1}{3} \frac{\partial u_k^s}{\partial x_k} \delta_{ij}. \quad (27)$$

320 Srivastava and Sundaresan (2003) combine the frictional normal stress
321 from Johnson and Jackson's model and the frictional viscosity from Schaeffer
322 (1987) model, and suggest the friction viscosity to be calculated by,

$$323 \quad \mu^{sf} = \frac{\sqrt{2} p^{sf} \sin(\theta_f)}{2 \sqrt{S_{ij}^s \cdot S_{ij}^s}}, \quad (28)$$

324 where θ_f is the angle of repose and is taken to be $\theta_f = 28^\circ$ for sand.

325 This frictional shear viscosity model has the capability to capture the
326 transition of solid-like feature to fluid-like feature of the sediment bed. In
327 the original model of Srivastava and Sundaresan (2003), a correction of Θ/d^2
328 is added to the denominator of Eq. (28) for numerical stability. However,
329 we found that in the present sediment transport application, the shear rate
330 in the concentrated region of transport was rather small. Therefore, the cor-
331 rection of Θ/d^2 can introduce large error. In this study, this extra term was
332 replaced by a small number of 10^{-20} to ensure numerical stability. In sed-
333 iment transport, the frictional components of particle pressure and particle
334 stress play a definite role to ensure the existence of an immobile sediment
335 bed and above which, a low mobility layer of enduring contact can be mod-
336 eled (Hsu et al., 2004). Hence, the empirical coefficients in Eq. (25) were
337 calibrated to ensure a stable sediment bed can be established. The following

338 values are adopted,

$$339 \quad F = 0.05, m = 3, n = 5. \quad (29)$$

340 With this new frictional stress model and choice of empirical coefficients,
341 the sediment concentration becomes close but rarely exceeds the maximum
342 packing limit of 0.635. Notice that in the original model of Johnson and
343 Jackson (1987) applied to debris flow, these coefficients are suggested to be
344 $F = 0.05$, $m = 2$ and $n = 5$. Hence, the present model applied to sediment
345 transport in a bottom boundary layer also gives similar values.

346 *2.3. Numerical Implementation*

347 The numerical implementation of the present model utilizes the open-
348 source CFD toolbox OpenFOAM, which provides a variety of finite volume
349 method libraries and solvers. OpenFOAM includes several single phase
350 and multiphase flow solvers. Specifically, the mathematical formulations
351 presented above were implemented by modifying a two-phase flow solver,
352 twoPhaseEulerFoam (Rusche, 2002; Weller, 2002; Passalacqua and Fox, 2011).

353 The solution procedure is outlined in the Fig. 1, and the sequence of
354 solution is summarized as follow: At beginning of the numerical simula-
355 tion, the sediment concentration, fluid and sediment velocity, fluid turbu-
356 lence and granular temperature are initialized in a smooth manner. In each
357 computational cycle, the sediment concentration is first updated, followed by
358 the drag coefficient, β . The fluid turbulence and granular temperatures are
359 then solved, and the fluid viscosity and sediment stresses are updated. For
360 the velocity-pressure coupling procedure, the standard PIMPLE approach,
361 which is a merged PISO-SIMPLE algorithm, is used after the version 2.1.0 of

Fig. 1: Flow chart of solution procedure

362 OpenFOAM. The intermediate velocities are first solved using the momen-
 363 tum equation without the pressure gradient term. To ensure the Newton's
 364 third law in the momentum equations of the fluid phase and sediment phase,
 365 the drag force is calculated explicitly using the velocities at the previous
 366 time step. The pressure equation is constructed and solved by considering
 367 the continuity equations for both phases. In the present model, collocated

368 variables are solved at cell centers, and the method of Rhie and Chow (1983)
369 is adopted for solving the pressure equations to prevent velocity-pressure de-
370 coupling and oscillation in the pressure field. After solving the pressure, the
371 fluid and sediment velocities are updated, and the resulting fluid phase and
372 particle phase velocities both satisfy their mass conservation. The velocities
373 are then used to update the volumetric sediment concentrations, turbulence
374 quantities, fluid stresses, granular temperature and particle stresses.

375 The advection terms in the governing equations are solved using a second-
376 order TVD scheme, based on the Sweby limiter (Sweby, 1984). Two special
377 versions of this scheme are available in OpenFOAM: (1) `limitedLinearV`,
378 which is specifically for vectors, and the limiter is applied independently
379 in each direction; (2) `limitedLinear01`, which ensures the scalar variable is
380 bounded between 0 and 1, is adopted to solve the volumetric concentration
381 ϕ . In this manner, the convective terms are treated implicitly after being
382 linearized using the value of the velocity flux at the previous iteration. The
383 central difference schemes are used for diffusion terms, and the standard
384 Crank-Nicolson scheme is used for time integration. The time step is updated
385 every cycle to satisfy the Courant–Friedrichs–Lewy (CFL) condition and it
386 is adjusted in a smooth fashion to ensure numerical stability. More details
387 about the numerical implementations can be found in Cheng and Hsu (2014).

388 **3. Model Validation**

389 The model was first calibrated/validated with laboratory flume data of
390 Sumer et al. (1996) for sand transport in steady channel flow under the
391 sheet flow condition. Then, the calibrated model was further validated with

392 measured sheet flow sediment concentration and flow velocity for 1DV/2DV
 393 oscillatory flow (O’Donoghue and Wright, 2004) and 2DV scour development
 394 downstream of an apron (Amoudry and Liu, 2009).

395 3.1. Steady Sheet Flow

396 The intensity of sediment transport is often quantified by the bed shear
 397 stress τ_b . The non-dimensional bed shear stress, called the Shields param-
 398 eter, is defined as, $\theta = \tau_b/(\rho^s - \rho^f)gd$. Compared with Hsu et al. (2004),
 399 the present model adopts a new turbulence modulation parameterization for
 400 α in the $k - \epsilon$ equations as well as in the granular temperature equation.
 401 The improved α parameterization model is expected to be more robust in
 402 predicting sediment suspension over a wider range of sediment size and den-
 403 sity (Kranenburg et al., 2014). We first calibrate the model with fine sand
 404 ($d=0.13$ mm; $\rho^s = 2650$ kg/m³) in the sheet flow condition driven by steady
 405 flows in a channel (Sumer et al., 1996). Specifically, the model coefficient B
 406 in Eq. (11) associated with turbulence modulation due to sediment is cal-
 407 ibrated and demonstrated to work well for Shields parameter ranging from
 408 1.0 to 2.2.

Table 2: A summary of the fine sand (grain diameter $d = 0.13$ mm, specific gravity $s = 2.65$) experiments reported in Sumer et al. (1996) used here for model calibration.

Shields parameter θ	τ_b (Pa)	h (cm)	f_x (Pa/m)
1.10	2.31	10.5	52
1.68	3.54	10.8	82
2.20	4.63	11.2	110

409 As shown in Table 2, the three different Shields parameters modeled here

410 cover a range from $\theta = 1.1$ to 2.2. In the numerical simulation, both the flow
 411 depth and bottom shear stress were kept the same as Sumer et al. (1996).
 412 The flow was driven by a prescribed constant streamwise pressure gradient,
 413 which was determined by matching the targeted bottom shear stress at the
 414 bed. Since sheet flow is typically assumed to be statistically homogeneous in
 415 the streamwise and spanwise directions, 1DV simulations were carried out,
 416 i.e., L_x and L_y in the streamwise and spanwise directions are reduced to one
 417 grid cell (see Fig. 2). Periodic boundary conditions were used in the stream-
 418 wise direction. For 2D simulation in the OpenFOAM framework, a dummy
 419 boundary condition called ‘empty’ can be used in the spanwise direction.
 420 Wall boundaries were imposed on both the top and bottom boundary of the
 421 computational domain. The total domain height is set to be $L_z = 0.15$ m with
 422 an initial sediment bed depth of $h_b = 0.05$ m. We use 600 uniformly spaced
 423 grid points in the vertical direction, and the grid resolution is $\Delta z = 0.25$ mm.

424 When using $B = 0.22$, we obtained good agreement with measured sedi-
 425 ment concentration profiles for Shields parameter in the range of $\theta = 1.1$ to
 426 2.2 for fine sand (Fig. 3(a),(e),(i)). Although the fluid and sediment veloc-
 427 ities were not measured, the model results for the streamwise fluid velocity
 428 were shown in Fig. 3 (b),(f),(j). Due to the small particle size considered
 429 here, the difference between the fluid phase and sediment phase velocity was
 430 within 0.01 m/s and hence the sediment phase velocities are not shown here
 431 for brevity. The model also provided profiles of fluid turbulent kinetic energy
 432 and granular temperature (see Fig. 3 (c),(g),(k)). The velocity fluctuation
 433 energies were both zero in the bed, while the fluid turbulent kinetic energy
 434 increased rapidly to its peak value several millimeters above the bed due to

435 large flow shear. Similar behavior can be observed for granular temperature.
 436 The fluid shear stress and particle shear stress are shown in Fig. 3 (d),(h),(l).
 437 It can be observed that the particle shear stress increased rapidly in the sheet
 438 layer and peaked at the immobile bed. On the other hand, the fluid shear
 439 stress dropped sharply in the sheet flow layer, and became close to zero at
 440 the immobile bed. The summation of particle shear stress and fluid (mostly
 441 turbulent) shear stress gave the expected linear profile of total shear stress
 442 in a steady channel flow. Note that the B value was calibrated with a con-
 443 stant Schmidt number $Sc = 1$. Simulations showed that the B value was
 444 also sensitive to the choice of Schmidt number. In order to produce similar
 445 sediment concentration profiles, the B needed to be increased to $B = 0.6$
 446 when a Schmidt number of $Sc = 0.5$ was used. Here, the Schmidt number is
 447 assumed to be $Sc = 1$, and the calibrated value of $B = 0.22$ will be used for
 448 the remainder of this paper.

449 *3.2. Oscillatory Sheet Flow*

450 O'Donoghue and Wright (2004) measured sediment concentration and
 451 flow velocity in the concentrated region of transport under a range of oscil-
 452 latory sheet flow conditions. The experimental results have been used by
 453 many researchers to validate their models (e.g., Li et al., 2008; Chen et al.,
 454 2011; Yu et al., 2012). The calibrated model coefficient $B = 0.22$ was used
 455 here to validate sheet flow of medium sand ($d = 0.28$ mm) under oscillatory
 456 flow condition.

457 In the numerical model, the oscillatory flow is driven by a mean stream-
 458 wise pressure gradient. Outside the wave boundary layer, the shear stress

459 vanishes, and the momentum equation reduces to,

$$460 \quad f_x = \rho^f \frac{\partial U_0}{\partial t}, \quad (30)$$

461 where U_0 is the free stream velocity. Following O'Donoghue and Wright
462 (2004), the asymmetric flow was forced by the second-order Stokes wave
463 motion,

$$464 \quad U_0(t) = U_1 \sin(\omega t + \sigma_0) - U_2 \cos(2\omega t + 2\sigma_0), \quad (31)$$

465 where U_1 and U_2 are velocity amplitudes for each harmonic, $\omega = 2\pi/T$ is the
466 oscillatory frequency, T is the oscillatory period and σ_0 is the a phase lag,
467 which is chosen such that the free-stream velocity is zero at $t = 0$. The flow
468 asymmetry is defined as, $a = U_{max}/(U_{max} - U_{min})$, in which U_{max} and U_{min}
469 are maximum and minimum free-stream velocities, respectively. Here, we
470 consider the asymmetric flow case with wave period $T = 7.5$ s and maximum
471 velocity $U_{max} = 1.5$ m/s with $a = 0.63$ (MA7515 in O'Donoghue and Wright
472 2004). A 1DV simulation was first carried out, and the domain height is
473 $L_z = 0.3$ m, with an initial sediment bed depth $h_b = 0.05$ m. A total of 240
474 grid points are used in the vertical direction with finer grid resolutions of
475 $\Delta z = 0.5$ mm around the initial bed level and the grid resolution increases
476 gradually away from the initial bed level to a maximum grid size of $\Delta z =$
477 2 mm near the top boundary.

478 The concentration profiles at the four representative instants are shown
479 in Fig. 4(b-d) and the corresponding phase of each panel is indicated in
480 Fig. 4(a). Using the calibrated value of $B = 0.22$ without further tuning,
481 the agreements between the model results and measured data were generally
482 good. During the flow reversal (see Fig. 4 (b) and (d)), only a small amount

483 of sediment was suspended and a sharp concentration gradient near the bed
 484 was observed. On the contrary, during the positive flow peak (see Fig. 4 (c)),
 485 significantly more sediment was suspended with a notably thicker suspension
 486 layer. Suspended sediment during the negative flow peak (see Fig. 4 (d))
 487 was less than those during the positive flow peak and the model results also
 488 matched well with the measured data.

489 The agreement between the measured sediment concentration and model
 490 results can also be examined via the time series of sediment concentration
 491 at four representative locations near the bed (Fig. 5). In the pick up layer
 492 (e.g., $z = -0.95$ mm, see Fig. 5(b)), namely the region below the initially
 493 undisturbed bed ($z < 0$), sediment concentration decreased from about 0.6
 494 (concentration of immobile bed) to about 0.25 as free-stream approaches flow
 495 peak. Sediment concentration then increased to about 0.6 as flow became
 496 weaker and approaches flow reversal. A similar feature repeated again during
 497 the wave trough although the sediment concentration only decreased to about
 498 0.33 due to less energetic flow. Similar agreements were observed at $z =$
 499 -0.35 mm (see Fig. 5(c)). The model was clearly able to capture the main
 500 characteristic in the pick up layer. In the suspension layer, namely the region
 501 above the initially undisturbed bed ($z > 0$), the temporal evolution with
 502 respect to the pick-up layer reverses. The behavior was qualitatively captured
 503 by the model (see Fig. 5(e)). It is noted that the model cannot predict the
 504 burst events during the flow reversal (see $t = 2 - 3$ s in Fig. 5(e)), and the
 505 burst events may be due to the turbulence-sediment interactions, which may
 506 require a turbulence-resolving model to resolve this phenomenon. However,
 507 the model was able to reproduce the nearly constant sediment concentration

508 occurring around the initially undisturbed bed level ($z \approx 0$, see Fig. 5(d)).

509 The erosion depth is defined as the distance between the initial bed level
510 to the location where the particle velocity is sufficiently small ($u^s < 5$ mm/s).
511 The modeled temporal evolution of erosion depth is similar to the measured
512 data (Fig. 6), although it was also observed that the model tends to over-
513 predict the erosion depth during positive or negative flow peak by about
514 0.5 mm. On the contrary, during flow reversal, the model under-predicted
515 the erosion depth by about 1 mm. We like to also point out that a difference
516 of 0.5 to 1 mm is roughly 2-3 grain diameters, and also it is similar to the
517 measurement error and our numerical grid size.

518 The model results of velocity profiles at four different instants are shown
519 in Fig. 7 (solid curves). During the experiment of O'Donoghue and Wright
520 (2004), concurrent measurement of flow velocity was not performed for medium
521 sand cases. However, velocity profiles of another case under the same wave
522 condition for mixed grain sizes with the same d_{50} (X2A7515) were reported
523 and they were also presented in Fig. 7 (symbols). Overall, the model pre-
524 dicted velocity profiles were in good agreement with the measured data.
525 However, the numerical model slightly over-predicted the flow velocity dur-
526 ing positive flow peak (see panel (c)). During the negative wave phase, the
527 agreement was very good (see panel (d), (e)). The same simulation for case
528 MA7515 was also carried out for 2DV domain with $L_x = 0.4$ m. The re-
529 sulting flow field was completely uniform in the streamwise direction and the
530 modeled sediment concentration and flow velocity were almost identical to
531 those obtained from the 1DV simulation (not shown).

532 3.3. Scour Downstream of an Apron

533 To further validate the present model for more complicated 2DV flow
534 conditions, we simulated the scour development downstream of an apron.
535 Following the numerical study of Amoudry and Liu (2009), the scour problem
536 was simplified and sketched in Fig. 8. The flow depth is chosen to be $h_0 =$
537 0.15 m, and the initial bed depth is $h_b = 0.05$ m. The length of the domain
538 downstream of the apron is chosen to be $L_x = 1$ m. Stretched grid are used in
539 both the streamwise and vertical directions. In the vertical direction, 300 grid
540 points are used with uniform grid size of $\Delta z = 0.5$ mm near and below the
541 initial bed level. The vertical grid size then increases away from the initial bed
542 with $\Delta z_{max} \approx 1.1$ mm near the top boundary. In the streamwise direction,
543 finer resolution is used around the apron with $\Delta x_{min} = 0.5$ mm. The grid size
544 then increases gradually away from the apron with the maximum grid size
545 of $\Delta x_{max} = 2.5$ mm near the downstream end of the domain. A symmetric
546 boundary condition is imposed on the top boundary, and at the step wall and
547 bottom wall, wall boundaries are applied for both the fluid and the sediment
548 velocities and sediment concentration. At the downstream boundary, an
549 outlet boundary condition is used for the velocities of both phases. When the
550 flow is going outward, a zero gradient is imposed for the velocities. Otherwise,
551 it is set to zero. As for the fluid pressure, when the flow is going outward,
552 a constant pressure value is imposed. Otherwise, a zero gradient boundary
553 condition is used. For the inlet boundary (left boundary of the domain), a
554 zero gradient boundary condition is used for the pressure, and a rough-wall
555 log-law velocity profile is imposed for both phases,

$$556 \quad \frac{u}{u_*} = \frac{1}{\kappa} \ln \left(\frac{30z}{K_s} \right), \quad (32)$$

557 where $u_* = 3.69$ cm/s is the frictional velocity, $\kappa = 0.41$ is the Karman
 558 constant, and $K_s = 2.5d$ is the bed roughness with the grain size specified
 559 as $d = 0.25$ mm. With sediment specific gravity specified as $s = 2.65$, the
 560 Shields parameter was estimated to be $\theta = 0.34$. The domain is initialized
 561 with a flat bed having bed concentration of 0.61. For the initial fluid and
 562 sediment velocities above the bed, they are set to be identical to the inlet
 563 velocity, and within the bed, both fluid and sediment velocities are set to be
 564 zero.

565 Fig. 9 shows three snapshots of sediment concentration contour and veloc-
 566 ity profiles at four different locations down stream of the apron. At $t = 5$ s,
 567 the development of a scour hole near the inlet can be identified (see Fig.
 568 9(a)). The scour angle and scour depth were both small at this instant. Sev-
 569 eral small bedforms were observed downstream of the scour hole. At $t = 15$ s
 570 and $t = 45$ s, the scour hole became sufficiently large and a return flow was
 571 observed within the scour hole (see the velocity profiles in Fig. 9(b),(c)). At
 572 $t = 15$ s, the maximum scour depth was located at $x = 0.1$ m. At $t = 45$ s,
 573 the maximum scour depth occurred further downstream at $x = 0.17$ m. Bed-
 574 forms were also observed to migrate away from the inlet.

575 According to several experimental/observational studies (e.g., Breusers,
 576 1967; Breusers and Raudkivi, 1991), it is clear that the development of scour
 577 hole is rapid at the initial stage, and eventually an equilibrium state can be
 578 established. An empirical formulation was suggested to describe the initial
 579 rapid development of scour depth (Breusers, 1967),

$$\frac{\delta_s}{h_0} = \left(\frac{t}{T_s} \right)^{n_s}, \quad (33)$$

581 where T_s is a characteristic timescale, and the exponent n_s characterizes the

speed of the scour depth growth at the initial stage. Notice that Eq. (33) only describes the initial development of the scour depth, and the equilibrium scour depth may not be determined from this empirical formula. The equilibrium scour shape is generally independent of the flow velocity and grain size if the Shields parameter is sufficiently larger than the critical Shields parameter. On the other hand, the development of upstream scour angle α_s can reach equilibrium rapidly. An empirical formula to describe the development of scour angle is written as (Breusers, 1967),

$$\alpha_s = \alpha_{s0} \left(1 - e^{-t/T_{\alpha s}} \right), \quad (34)$$

where $T_{\alpha s}$ is the equilibrium timescale of scour angle development and α_{s0} is the equilibrium scour angle. In Fig. 10, the modeled evolution of these two key scour quantities were compared with the empirical formulas (33) and (34). By using the least-square fit to the modeled scour depth, we observe that the model also reproduced the power law for the initial development of scour depth similar to the empirical formula with $n_s = 0.54$ and $T_s = 1100$ s. Similar to the model of Amoudry and Liu (2009), our model results gave a value of n_s at the high range of the reported value but still can be considered reasonable. For instance, Buchko et al. (1988) reported n_s as high as 0.6. The best fit value for the modeled scour angle was $\alpha_{s0} = -14.55^\circ$, and $T_{\alpha s} = 15.2$ s. Modeled α_{s0} was within the reported range of observation, $-16^\circ < \alpha_{s0} < -9.5^\circ$ (Breusers, 1967).

It is worth mentioning that the predicted scour evolution coefficients are sensitive to many factors, especially the initial condition. Model experiments suggested that the sediment concentration of the initial bed has an impact on the evolution of the scour depth. It was found that a uniform bed con-

607 centration rendered a rather fast scour process as compared with the initial
 608 condition where the bed concentration was obtained directly from the steady
 609 state of 1DV results. We consider the latter to be more realistic and the
 610 results reported here were based on the established bed condition from the
 611 1DV model. Meanwhile, the evolution of scour was also sensitive to the B
 612 value in the turbulence modulation model (see Eq. (11)). With a lower
 613 value of B ($= 0.15$), numerical experiments showed that the scour process
 614 was slightly faster ($T_{\alpha s} = 14$ s), and the characteristic scour angle was slightly
 615 larger ($\alpha_{s0} = -15.56^\circ$). The development of scour depth was also faster with
 616 $T_s = 785$ s and $n_s = 0.6$. In addition, since the bedload transport dominated
 617 the scour process, the frictional stress model may also play a role. It was
 618 found that the scour process was slightly slower (i.e., both $T_{\alpha s}$ and T_s were
 619 larger) when the coefficient m was reduced from $m = 3$ to $m = 2$ or the
 620 angle of repose θ_f was increased from 28° to 32° . However, the differences
 621 were within 15%.

622 **4. Momentary bed failure in sheet flow**

623 Several field observations (Foster et al., 2006; Conley and Inman, 1992)
 624 and laboratory experiments (Sleath, 1999; Flores and Sleath, 1998) on wave-
 625 induced sediment transport revealed that large horizontal pressure gradients
 626 induced by the passage of steep near-breaking or broken waves can destabi-
 627 lize the sand bed and drive momentary bed (shear) failure. Comparing with
 628 the well-known sheet flow condition, the observed large transport (or erosion
 629 depth) cannot be solely explained by bottom shear stress (or Shields param-
 630 eter) and hence the resulting transport rate is postulated to be enhanced

631 by horizontal pressure gradient. This mechanism is often called plug flow
 632 (Sleath, 1999) or momentary bed failure (Madsen, 1974).

633 In the oscillating water tunnel, the horizontal pressure gradient can be
 634 expressed by free-stream flow acceleration. In order to quantify the effect of
 635 the horizontal pressure gradient, a non-dimensional parameter is used, which
 636 is often called the Sleath parameter (Sleath, 1999),

$$637 \quad S = \frac{|\partial p^f / \partial x|_{max}}{\rho^f (s - 1)g} = \frac{U_0 \omega}{(s - 1)g}. \quad (35)$$

638 The last expression converts the horizontal pressure gradient to the flow ac-
 639 celeration, which is only appropriate for the conditions in an oscillatory water
 640 tunnel. Madsen (1974) and Sleath (1999) both derived similar criteria to de-
 641 scribe the initiation of momentary bed failure using a simple force balance.
 642 These criteria give a rather large threshold value of Sleath parameter, typi-
 643 cally around 0.33. Later, Foster et al. (2006) reported the first field evidence
 644 of momentary bed failure (or plug flow) in Duck, NC and demonstrated its
 645 occurrence at a much lower threshold value. They argue that in general the
 646 initiation of momentary bed failure must also depend on the Shields param-
 647 eter. Hence, a combined sheet flow-plug flow mode, which is responsible
 648 for a large amount of sediment transport, may occur more commonly than
 649 previously believed. More recently, Holway et al. (2012) reported an oscilla-
 650 tory water tunnel experiment using bimodal nylon particles. Due to the low
 651 density of nylon particles ($\rho^s = 1170 \text{ kg/m}^3$), the resulting Sleath parameter
 652 exceeded 0.36 and momentary bed failure occurred. In addition to the large
 653 transport rate obtained due to momentary bed failure, they observed wave-
 654 like instabilities and billows during the transport. These wave-like instabili-
 655 ties may play a major role in the plug flow dynamics and more quantitative

656 investigation is needed.

657 In order to study the role of horizontal pressure gradient in driving mo-
658 mentary bed failure in sheet flow conditions, 2DV numerical investigations
659 were performed. In Section 3.2, the numerical model was validated with
660 medium sand ($d = 0.28$ mm) for a second-order Stokes wave motionan of
661 O’Donoghue and Wright (2004). Here, we performed an additional simula-
662 tion using the same medium sand for sinusoidal wave conditions (O’Donoghue
663 and Wright 2004; Case 1 in Table 3). Model results were first validated with
664 the measured data and then a series of numerical experiments with a wide
665 range of Sleath parameter and Shields parameter for two different grain sizes
666 were carried out to investigate momentary bed failure in sheet flow conditions
667 (see Table 3).

668 To ensure the domain was tall enough to cover the wave boundary layer in
669 different flow conditions, the domain height was set to be $L_z = 0.5$ m, which
670 is larger than that used in Section 3.2. An initial sediment bed depth of
671 $h_b = 0.075$ m is specified. Similar vertical spatial discretization is used with
672 minimum Δz of 0.25 mm (maximum Δz of 4 mm), and the total number
673 of grid points in the z-direction is $N_z = 595$. The streamwise length of
674 the domain is chosen to be $L_x = 0.4$ m, and a uniform grid resolution of
675 $\Delta x = 2$ mm with $N_x = 200$ are used.

676 *4.1. 1DV and 2DV sheet flow simulations*

677 Case 1 was a sinusoidal wave with $T=5$ s and $U_m=1.5$ m/s (O’Donoghue
678 and Wright, 2004). The Sleath parameter and Shields parameters for Case 1
679 and other cases investigated in this section are summarized in Table 3. The

680 Shields parameter, θ , is estimated based on peak flow velocity,

$$681 \quad \theta = \frac{0.5f_w U_m^2}{(s-1)gd}, \quad (36)$$

682 where f_w is the wave friction factor, and it is calculated by (Swart, 1974),

$$683 \quad f_w = \exp \left[5.213 \left(\frac{2.5d}{A} \right)^{0.194} - 5.977 \right], \quad (37)$$

684 where $A = U_m T / 2\pi$ is the semi-excursion length.

685 Although the simulation was carried out in 2DV, the modeled flow veloc-
686 ity and concentration fields remained nearly homogeneous in the streamwise
687 direction (not shown). In Fig. 11, modeled sediment concentration profiles
688 (averaged over the x-direction) at four instants during the positive wave phase
689 were compared with the measured data of O'Donoghue and Wright (2004).
690 The agreement was generally good. However, we can also observe that in this
691 case, the model over-predicted the suspended sediment concentration during
692 the flow peak (see Fig. 11c). The discrepancy may be due to the param-
693 eterization in Eq. (11) of the degree that particles follow the fluid velocity
694 fluctuations. In this study, the empirical coefficient B associated with this
695 parameterization was calibrated for steady sheet flow (Sumer et al., 1996) of
696 fine sand with $d = 0.13$ mm. The same value was used here for oscillatory
697 flow of medium sand. Meanwhile, another identical simulation was carried
698 out in 1DV, and the model results from the 1DV simulation (also not shown)
699 was nearly identical to the 2DV results. In summary, the estimated Shields
700 parameter for Case 1 is 2.16, while the Sleath parameter is $S = 0.12$, which
701 is much smaller than the critical value of 0.33. Hence, a regular sheet flow
702 with complete homogeneity in the streamwise direction are expected. 2DV

703 model results indeed reproduced homogeneous flow in the streamwise direc-
 704 tion, Case 2 (see Table 3) with wave period reduced to 2.1 s and hence the
 705 Sleath parameter increased to $S = 0.28$ produced similar sheet flow results.

706 4.2. Combined sheet flow and plug flow

707 When the oscillatory period was reduced to $T = 1.8$ s (see Case 3 in Table
 708 3), the resulting Shields parameter is 2.8. Although sheet flow is expected,
 709 model results were no longer homogeneous in the streamwise direction. Snap-
 710 shots of the 2DV sediment concentration field (Fig. 12) showed billows as-
 711 sociated with instabilities of the near bed transport layer. The billows had
 712 a streamwise length scale of $5 \sim 10$ cm. Because the billows enhance mix-
 713 ing, the sheet flow layer thickness is increased considerably to about 3 cm.
 714 In Case 3, the Sleath parameter is increased to $S = 0.32$ due to a shorter
 715 oscillatory period and hence a larger flow acceleration (horizontal pressure
 716 gradient). At such large value of Sleath parameter, momentary bed failure
 717 or plug flow is expected. The present model indicated that the occurrence of
 718 momentary bed failure or plug flow is associated with the formation of the
 719 observed instability and billows near the bed.

720 Although qualitative, the instability and billows observed in the model re-
 721 sults seem to be consistent with several field and laboratory observations. For
 722 example, Conley and Inman (1992) described their observation of a carpet of
 723 tufts (about $O(6)$ cm, see their Fig. 5) under the crest of near-breaking waves
 724 in the surf zone. Madsen (1974) reported divers' observations of sediment
 725 bed seemingly explode just prior to the passage of the crest of near-breaking
 726 waves. As mentioned previously, similar wave-like instabilities under com-
 727 bined sheet flow and plug flow conditions were observed during a more recent

728 experiment performed in a small oscillatory flow tunnel at the Naval Research
 729 Laboratory (Holway et al., 2012). Video of sheet flow transport with a bed
 730 composed of a bimodal distribution of nylon cylinders ($\rho^s = 1170 \text{ kg/m}^3$)
 731 having diameters of 0.5 mm (red) and 1.0 mm (white) demonstrated the
 732 streamwise sorting of surficial grains. Sequence of four (top view) images
 733 shown in Fig. 13 demonstrated the development of wave-like instabilities.
 734 Although the primary billows were more or less spanwise homogeneous, there
 735 were notable smaller 3D features suggesting that instabilities grew rapidly
 736 into turbulent features. We note that in the present 2DV Reynolds-averaged
 737 simulation, smaller 3D turbulent features are parameterized by turbulent
 738 viscosity/diffusivity and only primary 2D billows can be directly resolved.

739 More quantitative analysis and comparison between 1DV and 2DV simu-
 740 lations were performed to elucidate the effects of momentary bed failure and
 741 billows on the transport dynamics. Since the instability and billows of the
 742 near bed transport layer are inhomogeneous in the streamwise direction, the
 743 1DV model is not adequate to simulate the process. Therefore, we carried
 744 out the same simulation of Case 3 but in a 1DV domain to understand the
 745 role of instabilities and billows on the resulting transport. In Fig. 14, the
 746 streamwise-averaged profiles of sediment concentration, fluid velocity and
 747 sediment flux under flow peak for Case 3 are presented along with those cal-
 748 culated by the 1DV model. The differences between 1DV and 2DV results
 749 for Case 3 were evident. In the 2DV results, a large dense mobile layer from
 750 $z = -0.019 \sim 0.01 \text{ m}$ was observed, and more sediment was mobilized, which
 751 gave considerably large sediment flux (see Fig. 14(c)). On the other hand,
 752 in the 1DV results the region between $z = -0.019 \sim -0.002 \text{ m}$ was immo-

753 bile and the overall mobile layer thickness and sediment flux were several
754 times smaller. Laboratory observation and linear stability analysis for light
755 particle transport in steady channel flow (Charru and Mouilleron-Arnould,
756 2002; Schaflinger et al., 1995) suggest that instability of particle transport
757 layer may occur when the mobile layer thickness is sufficiently large. Their
758 findings are consistent with the model results presented here.

759 Fig. 15 further contrasts the erosion depth for Case 1 and Case 3. For
760 Case 1, the model result was also compared with the measured data of
761 O'Donoghue and Wright (2004) in Fig. 15(b). The measured maximum
762 erosion depth occurred during the middle of the deceleration but the pre-
763 dicted peak value occurred just slightly after the peak flow. The model
764 slightly under-predicted the maximum erosion depth by about 0.5 mm but
765 the general agreement was good. For Case 3 where the momentary bed
766 failure occurred (Fig. 15(c)), the temporal evolution and the magnitude of
767 erosion depth were distinctly different from that of Case 1. We observed that
768 the erosion depth reached its maximum immediately after the flow reversal.
769 Clearly, the rapid increase of erosion depth responded directly to the peak
770 horizontal pressure gradient (or free-stream acceleration) at the flow reversal.
771 Similar features have been observed by several other researchers during the
772 occurrence of plug flow (Foster et al., 2006; Liu and Sato, 2005; Sleath, 1999;
773 Flores and Sleath, 1998). It is also evident that the erosion depth in Case
774 3 ($\delta_m \approx 22$ mm) was considerably larger than that of Case 1 ($\delta_m \approx 6$ mm).
775 Clearly, the this major difference in erosion depth is because the Sleath pa-
776 rameter is sufficiently large, although the Shields parameters in Case 1 and
777 Case 3 were similar (see Table 3).

778 4.3. Discussion

779 Sediment transport rate is the most important physical quantity to be
 780 evaluated. The present model can provide total sediment transport rate
 781 without the bedload/suspended load assumptions. To quantify how momen-
 782 tary bed failure can enhance the transport rate, additional simulations were
 783 carried out. A total number of 24 cases were presented in this study (see
 784 Table 3). Two different grain diameters ($d=0.28$ mm and $d=0.2$ mm) were
 785 considered and these 24 simulations covered Shields parameter in the range
 786 of 1.62 to 6.12 and Sleath parameter in the range of 0.12 to 0.39.

787 To compare sediment transport rate under sinusoidal wave motion, the
 788 half-wave averaged sediment transport rate $Q_{1/2}$ is defined as,

$$789 \quad Q_{1/2} = \frac{2}{T} \int_0^{T/2} \int_0^{L_z} u^s \phi dz dt. \quad (38)$$

790 The nondimensional half-wave averaged sediment transport rate is then de-
 791 fined as,

$$792 \quad \Psi = \frac{Q_{1/2}}{\sqrt{(s-1)gd^3}}. \quad (39)$$

793 The normalized half-wave averaged transport rate calculated from the 1DV
 794 and 2DV models are both shown in Table 3. As long as the momentary
 795 bed failure (or plug flow) did not occur, the predicted sediment transport
 796 rate between the 1DV and 2DV results were very similar. However, when
 797 momentary bed failure/plug flow was observed, the resulting transport rate
 798 was significantly enhanced in the 2DV results. It is clear that the instabilities
 799 and billows of the transport layer observed in the 2DV simulation played a
 800 major role in the enhancement of the sediment transport rate.

801 The 2DV results of the normalized half-wave averaged transport rate for
802 the 24 cases in Table 3 are plotted in Fig. 16. For typical sheet flow transport,
803 the Shields parameter can be used to parameterize the half-wave averaged
804 nondimensional sediment transport rate using a power law:

$$805 \quad \Psi = M_0(\theta - \theta_c)^{N_0}. \quad (40)$$

806 For the cases without momentary bed failure (pure sheet flow), the best fit
807 values are $M_0 = 5.78$ and $N_0 = 1.65$, which are quite similar to commonly
808 used sheet flow transport formulae (Ribberink, 1998). However, when mo-
809 mentary bed failure in sheet flow occurred, the 2DV results predicted the
810 transport rates that were considerably ($10 \sim 20$ times) larger than the pure
811 sheet flow cases (see Fig. 16(a)). It is expected that the sediment transport
812 rate for momentary bed failure in sheet flow may not be solely related to the
813 Shields parameter (Nielsen, 2006), and the critical Shields parameter may
814 become irrelevant. As an attempt to describe the sediment transport rate
815 in momentary bed failure, we further fit the sediment transport rate as a
816 function of Shields parameter θ regardless of critical Shields parameter θ_c
817 (Hanes and Bowen, 1985),

$$818 \quad \Psi = M_1\theta^{N_1}. \quad (41)$$

819 The fitted curve from Eq. 41 is shown in Fig. 16(b). For the cases with
820 momentary bed failure, we obtain $M_1 = 20.36$ and $N_1 = 2.5$. Clearly, the
821 resulting non-dimensional transport rate increases rapidly with the Shields
822 parameter. In general, we can expect that the transport rate also depends
823 on the Sleath parameter.

824 It was previously found that plug flows typically occurred after the sheet

825 flow threshold was exceeded (Foster et al., 2006). Hence, the threshold crite-
826 ria for momentary bed failure, parameterized by S_{cr} , must also depend on the
827 Shields parameter. This conjecture is confirmed by the present model (Fig.
828 17). Model experiments with a range of θ and S show that momentary bed
829 failure/plug flow were observed at S_{cr} as large as 0.32 in mild sheet flow con-
830 ditions ($\theta = 1.5$). However, in energetic sheet flow with high θ , momentary
831 bed failure is observed at S_{cr} as low as 0.1 – 0.2.

832 By rearranging the force balance on the block of sediment given in Sleath
833 (1999), the criterion of bed mobilization is proposed as, $|K \cdot \theta + S| > C$, where
834 K and C are coefficients, which may be related to the bed friction coefficient
835 and bed concentration (Foster et al., 2006; Frank et al., 2015). According
836 to the present model results with two different grain sizes ($d = 0.2$ mm and
837 0.28 mm), we obtain the dashed curve shown in Fig. 17 and the momentary
838 bed failure occurs when,

$$839 \quad |0.053\theta + S| > 0.426. \quad (42)$$

840 5. Conclusion

841 A multi-dimensional two-phase Eulerian model for sediment transport,
842 called SedFoam, was developed based on the open-source CFD toolbox,
843 OpenFOAM. The mathematical formulations follow the prior two-phase flow
844 approach for sediment transport of Hsu et al. (2004), and several modifica-
845 tions to the numerical solver twoPhaseEulerFoam are introduced to make
846 the model more robust for sediment transport applications. The model was
847 validated for sheet flow in steady channel flow, oscillatory flow, and scour
848 downstream of an apron.

849 The validated model was further used to study momentary bed failure
850 (plug flow) in sheet flow when the horizontal pressure gradient (or flow ac-
851 celeration in U-tube) is sufficiently large. Consistent with limited field and
852 laboratory observation, model results indicate that momentary bed failure is
853 associated with the rapid growth of large erosion depth during flow reversal.
854 Moreover, large erosion depth further leads to instabilities and billows of the
855 near bed transport layer and the resulting transport rate is significantly en-
856 hanced. The criteria of momentary bed failure/plug flow was investigated
857 with a range of Shields parameter and Sleath parameter. Our numerical re-
858 sult was consistent with the empirical formulation proposed by Foster et al.
859 (2006) that the threshold value for the initiation of plug flow is lower when
860 the Shields parameter is larger. More detailed experimental investigations,
861 especially regarding the generation of billows via instabilities and their role on
862 the resulting transport rate are warranted. Due to the turbulence-averaging
863 approach, the present model results are sensitive to the closure coefficients
864 associated with the parameterization on turbulence-sediment interactions.
865 The present study calibrated this empirical coefficient with steady sheet flow
866 data of Sumer et al. (1996). However, when applying the model to oscilla-
867 tory sheet flow, the model tended to over-predict the suspended sediment
868 concentration during peak flow. To further improve the model and carry out
869 fundamental investigations of turbulence-sediment interactions in sediment
870 transport, a 3D turbulence-resolving simulation is needed. Such model ex-
871 tension will also allow direct resolving the evolution and dissipation of billows
872 associated with the momentary bed failure.

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884 Surface Dynamics Modeling System (CSDMS) model repository maintained
885 by GitHub: <https://github.com/csdms-contrib/twophaseeulersedfoam>

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Fig. 2: Sketch of the domain setup.

Fig. 3: (a) Comparison of concentration profiles between numerical results (solid curve) and experiment data (circle symbols); (b) fluid velocity profile; (c) fluid turbulent kinetic energy (solid curve) and granular temperature (dashed curve) and (d) fluid shear stress (solid curve), sediment shear stress (dashed curve) and total shear stress (dash-dot curve) for steady sheet flow of at $\theta = 1.10$ (Sumer et al., 1996). The corresponding results for $\theta = 1.68$ ($\theta = 2.2$) are shown in (e), (f), (g) and (h) ((i), (j), (k) and (l)), respectively. The bed location $z = 0$ is defined as initial bed level.

Fig. 4: (a) Time series of free-stream velocity, in which four representative instants are indicated: (b) $t = 0$; (c) $t = 1.6$ s; (d) $t = 3.2$ s and (e) $t = 5.36$ s; The corresponding comparison of sediment concentration profiles between the numerical results (solid curve) and measured data (symbols) from O'Donoghue and Wright (2004) are presented in panel (b), (c), (d) and (e), respectively.

Fig. 5: (a) Time series of free stream velocity driven by Stokes second-order wave motion (MA7515, O'Donoghue and Wright (2004)); The comparisons of concentration time series for sheet flow of medium sand at four different vertical locations are shown in (b) $z = -0.95$ mm; (c) $z = -0.35$ mm; (d) $z = -0.15$ mm and (e) $z = 0.85$ mm.

Fig. 6: (a) Time series of free-stream velocity, and (b) Comparison of erosion depth between numerical results (solid curve) and experiment data (symbols) of MA7515 in O'Donoghue and Wright (2004).

Fig. 7: (a) Time series of free-stream velocity for MA7515 in O'Donoghue and Wright (2004), in which four representative instants are indicated: (b) $t = 0$; (c) $t = 1.6$ s; (d) $t = 3.2$ s and (e) $t = 5.36$ s; The corresponding comparison of fluid velocity profiles between the numerical results (solid curve) and measured data (symbols) at these four instants are shown in panel (b), (c), (d) and (e).

Fig. 8: Sketch of scour downstream of an apron.

Fig. 9: Velocity profiles at different locations downstream of the apron at (a) $t = 5$ s, (b) $t = 15$ s, and (c) $t = 45$ s; The domain is zoomed in vertical direction to focus on the near-bed processes, and the background is the concentration contour.

Fig. 10: (a) Evolution of maximum scour depth for model results (symbols) and the empirical formula of Eq. (33) with $T_s = 1100$ s and $n_s = 0.54$ (solid line); (b) Evolution of upstream scour angle between model results (dashed curve) and Eq. (34) with $T_{\alpha_s} = 15.2$ s and $\alpha_{s0} = -14.55^\circ$ (solid curve).

Table 3: Flow conditions

Cases	d (mm)	T (s)	U_m (m/s)	θ	S	bed instability	Ψ_{1D}	Ψ_{2D}
1	0.28	5	1.5	2.16	0.12	no	19.2	19.29
2	0.28	2.1	1.5	2.7	0.28	no	21.53	21.96
3	0.28	1.8	1.5	2.82	0.32	yes	17.87	256.5
4	0.28	5	2.1	3.91	0.16	no	38.59	38.6
5	0.28	5	2.7	6.12	0.21	yes	95.8	1170.8
6	0.28	1	1	1.73	0.39	yes	8.15	67.64
7	0.28	1.2	1	1.62	0.32	no	7.01	7.14
8	0.28	1.2	1.2	2.19	0.39	yes	11.23	117.7
9	0.28	1.5	1.2	2.04	0.31	no	10.22	11.3
10	0.28	2.4	1.8	3.57	0.29	yes	27.38	568.3
11	0.28	3	2	4.05	0.26	yes	34.43	500.3
12	0.28	4	2	3.77	0.19	no	33.86	34.59
13	0.2	5	1.5	2.79	0.12	no	39.23	39.27
14	0.2	2.1	1.5	3.45	0.28	yes	34.40	521.24
15	0.2	1.8	1.5	3.59	0.32	yes	32.24	639.1
16	0.2	5	2.1	5.09	0.16	yes	97.65	3049.3
17	0.2	1	1	2.15	0.39	yes	11.21	103.41
18	0.2	1.2	1	2.03	0.32	yes	11.17	119.4
19	0.2	1.2	1.2	2.76	0.39	yes	17.96	217.9
20	0.2	1.5	1.2	2.58	0.31	yes	18.34	181.29
21	0.2	1.2	0.9	1.7	0.29	no	9.57	9.55
22	0.2	3	1.5	3.14	0.19	no	39.49	39.81
23	0.2	5	1.8	3.86	0.14	no	65.43	65.51
24	0.2	6	2	4.76	0.13	no	87.99	88.05

Fig. 11: (a) Time series of free-stream velocity, in which four representative instants are indicated: (b) $t = 0$; (c) $t = 1.25$ s; (d) $t = 1.88$ s and (e) $t = 2.5$ s; The corresponding comparison of sediment concentration profiles between the numerical results (solid curve) and measured data (symbols) at these four instants for Case 1 (see Table 3) are shown in panel (b), (c), (d) and (e).

Fig. 12: Snapshot of concentration field at (a) flow peak ($t = 0.45$ s) and (b) flow reversal ($t = 0.9$ s) for Case 3 (see Table 3); white arrows denotes the plane-averaged velocity vectors.

Fig. 13: Sequence of four images (top to bottom) highlighting the development of instabilities and billows under combined sheet flow and plug flow conditions in a small oscillatory flow tunnel during half a wave cycle (Holway et al., 2012). The streamwise length (spanwise width) of the field of view in each snapshot is 64 cm (25 cm).

Fig. 14: Comparison of (a) sediment concentration profiles, (b) fluid velocity profiles and (c) sediment flux profiles of 1DV (dashed curves) and 2DV (solid curves) for Case 3 at flow peak.

Fig. 15: Time series of free-stream velocity (panel (a)) and numerical results of the erosion depth (solid curves) for (b) Case 1 and (c) Case 3; In panel (b), the circle symbols represent measured data from O'Donoghue and Wright (2004). In panel (c), the dash-dot curve is the 1DV result.

Fig. 16: Panel (a) shows the normalized half-wave-averaged transport rate in terms of Shields parameter θ . $\times$: 2D pure sheet flow of $d = 0.28$ mm; $\circ$: 2D sheet flow with momentary bed failure of $d = 0.28$ mm; $*$: 2D pure sheet flow of $d = 0.2$ mm; $\square$: 2D sheet flow with momentary bed failure of $d = 0.2$ mm; The solid curve shows the best-fit power law (Eq. 40) for pure sheet flow predicted by the present model ($M_0 = 5.78$, $N_0 = 1.65$) where $\theta_c = 0.05$ is the critical shields parameter. By using Eq. 41, the fitted curve for the sediment transport rate for momentary bed failure (solid curve) is shown in panel (b), which corresponds to $M_1 = 20.36$ and $N_1 = 2.5$ in Eq. 41.

Fig. 17: Schematic map of sheet flow with momentary bed failure in terms of Sleath parameter S and Shields parameter θ . $\times$: sheet flows of $d = 0.28$ mm; $\circ$: momentary bed failure of 0.28 mm; $\triangle$: sheet flows of $d = 0.2$ mm; $\square$: momentary bed failure of 0.2 mm; dashed curve: proposed criteria for the occurrence of momentary bed failure (Eq. (42)).