Estimating tsunami runup with fault plane parameters

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Abstract

Forecasting the maximum runup height and inundation area caused by a tsunami event has often been done by solving a numerical model describing the physical process. This approach requires data of fault plane parameters and bathymetry. The time required to prepare and execute the numerical model could be substantial. Therefore, it might not be suitable for the purpose of warning tsunami coastal flooding. In this paper we offer an alternative method that provides analytical relationship between the runup height and the fault plane parameters and the characteristic slope of coastal bathymetry. The method uses Okada's linear elastic dislocation model (Okada 1985) to estimate the coseismic seafloor deformation and the corresponding (initial) sea surface displacement for a given set of fault plane parameters. Once the tsunami waves are generated, Carrier & Greenspan's solution (1958) is adopted to yield analytical expressions for the time histories of shoreline elevation and velocity. Two types of problems are investigated. In the first scenario, the bathymetry is modeled as a constant slope that is connected to a constant depth region, where a seismic event occurs. In the second scenario, the bathymetry is further simplified as a constant slope, on which a seismic event occurs. As far as the Carrier - Greenspan's solution for shoreline movement is concerned, the former is treated as a boundary-value-problem (BVP) and the latter as an initial-value-problem (IVP). For both problems the relationships between dimensionless runup height and dimensionless fault plane parameters are developed and discussed. In addition, since Carrier - Greenspan's approach is valid only for non-breaking waves, expressions limiting the applicability of the present solutions are obtained. Application of the present solutions is demonstrated with two examples: the 2004 Sumatra and 2010 Chile earthquakes. Acceptable agreements are obtained with the field measurements of these events. The present solutions offer a fast way to estimate the runup heights with the simplified beach geometry.

Key words: Tsunami runup heights; Wave breaking; Coseismic seafloor displacement

1. Introduction

Many numerical models have been developed to calculate the overland flows and runup heights caused by a tsunami (e.g. Wang 2009, Yamazaki et al. 2012). These models require the incident tsunami information and coastal bathymetry data. The incident tsunami wave characteristics can be obtained directly or indirectly from the rupture plane parameters associated with a co-seismic event. To prepare and execute these numerical models demand times and resources. Therefore, numerical modeling approach is excellent in preparing tsunami hazard map for coastal planning and management purpose. It is not effective, however, for tsunami warning, which requires very fast response once a tsunami is generated. The alternative method is to develop analytical expressions linking the incident tsunami waves and coastal bathymetry to the tsunami runup. If the incident tsunami waves could be further expressed in terms of the fault plane parameters and coastal bathymetry, these expressions would be very useful to provide first estimate of tsunami runup height.

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Figure 1: Incident wave propagating in the negative x direction over a constant depth followed by a uniform slope. SWL denotes the Still Water Level.

In the seminal paper by Carrier and Greenspan (1958) analytical solutions for nonlinear shallow water equations are obtained for non-breaking waves on a constant slope (it is referred to as C-G theory herein). Synolakis (1987) extended C-G's theory to the bathymetry where the constant slope is connected to a constant depth region as shown in Fig.1. Synolakis showed that the incident wave must be sufficiently small such that the hodograph transformation introduced in C-G's theory can be linearized in the vicinity of the toe of the slope. He found the analytical expression for the runup of solitary wave in the integral form. A closed form solution for solitary wave runup height was further developed by adopting the asymptotic expressions for Bessel functions inside the integral. Madsen and Schaeffer (2010) derived closed form solutions for the runup height of periodic progressive waves and transient waves whose surface elevations are described by $\text{sech}^2(\cdot)$ -profiles. To obtain these solutions the toe of the slope must be sufficiently far from the shoreline and no water velocity at the initial time is allowed.

Rybkin et al. (2014) further extended C-G's theory to investigate wave run-up in inclined channels of arbitrary cross-section. The hodograph transformation was applied to the cross-sectionally averaged nonlinear shallow water wave equations. Analytical solutions of runup of periodic waves in U-shaped bays were presented. The common characteristics of this group of work is that the runup on the slope is generated by the prescribed incident waves at the toe of the slope. Thus, these problems are classified as boundary value problems (BVP).

Following C-G's theory, Kanoglu (2004) presented runup solutions, in the integral form, for waves generated by an initial free surface displacement from the still water level on a constant slope. Kanoglu's solutions require zero initial velocity everywhere and zero initial surface elevation at the shoreline. Tinti & Tonini (2005) adopted a similar approach and solved a similar problem as Kanoglu's with the exception that the initial free surface profiles were expressed in terms of a special parametric function, allowing closed form solutions for runup. They also showed that the special parametric function can be used to mimic the seafloor displacement (hence the initial free surface displacement) of a seismic event calculated from Okada's linear elastic dislocation theory (Okada 1985). The parameters in the parametric function were determined by curve fitting to Okada's solutions, in a case-by-case base. This type of problems is classified as initial-value-problem (IVP).

In both BVP and IVP described above, the incident waves for BVP and the initial free surface profiles for IVP are not directly expressed in terms of the fault plane mechanism associated with a tsunamigenic seismic event. However, it is important to make this connection since it is desirable to quickly estimate the tsunami runup at a specific coast once the fault plane parameters are known.

The objective of this study is to derive runup solutions in terms of the fault plane parameters, such as rupture plane geometry, slip, focal depth and dip angle of a seismic event. The earthquake could occur on a constant slope or in the constant depth region that is connected to a constant slope. In this paper Okada's linear elastic dislocation model (Okada 1985) is used to estimate the coseismic seafloor deformation and the corresponding (initial) sea surface displacement for a given set of fault plane parameters. Once the tsunami waves are generated, Carrier & Greenspan's solution (1958) is adopted to find analytical expressions for the

Figure 2: Fault parameters for Okada's model to compute seafloor displacements. W : Fault plane width, L : Fault plane length, d : Fault depth, s : Fault slip, θ : Strike angle, δ : Dip angle, λ : Rake angle.

time histories of shoreline elevation and velocity. The relationships between dimensionless runup height and dimensionless fault plane parameters are developed and presented graphically. Since these solutions are valid only for non-breaking waves, the wave breaking conditions should also be expressed in terms of the fault plane parameters and the slope.

The paper is organized as follows. In Section 2 runup solutions for BVP are first presented. The wave breaking condition which limits the applicability of the solutions, is also derived. In Section 3 runup solutions and wave breaking condition are derived for IVP. Applications of both types of problems are demonstrated in Section 4. Finally, conclusions are presented in Section 5.

2. Runup for tsunami waves generated in a constant depth region

In this section solutions for the runup of tsunami waves generated by a submarine earthquake in the constant depth region away from the toe of a slope (see Fig. 1) are presented. The definition sketch of the fault plane is depicted in Fig. 2.

2.1. Tsunami waves generated by a seismic event

The rupture mechanism during a seismic event in subduction zones is complex. The corresponding tsunami wave generation mechanism is equally challenging. The simplest conceptual model for tsunami generation can be described as follows. Tsunami waves are generated mainly by vertical seafloor displacements, which can be estimated by using a linear elastic dislocation model, such as the model of Okada (1985). Okada's model provides analytical solutions for the seafloor deformation in terms of a set of fault plane parameters, including the length (L), and width (W) of the rupture plane, slip (s), depth (d), dip angle (δ), strike angle (θ) and rake angle (λ) of the fault plane, which are defined in Fig. 2. Okada's solution adopts many simplifications, including that the earth can be modeled as a semi-infinite half-space, bounded by a horizontal seafloor surface, and that earth's properties are homogeneous and isotropic. Once the seafloor deformation is determined, it is further assumed that the sea water surface responds instantaneously and its shape mimics the seafloor deformation. The latter simplification is reasonable since the rupture speed of fault plane is one or two orders of magnitude larger than the propagation speed of tsunami waves, and the duration of rupture for a main seismic event is usually much smaller than the typical tsunami wave period. Finally, since the compressibility of water is weak, the entire water column is being pushed up or pulled down by the vertical seafloor displacement during the earthquake.

In the present study we examine only the situation in which (1) the length of the rupture plane " L " is much longer than the width, (2) the fault is parallel to the toe of the sloping beach, and (3) the rake angle λ is 90° . Assumption (2) is reasonable because the orientation of coast is frequently sculpted by the same tectonic mechanism generating seismic events. Assumption (3) is a conservative one, since the maximum

Figure 3: (Upper panel) Seafloor displacement profile, η_0 , obtained from Okada's solution with fault parameters $W=100\text{km}$, $d=40\text{km}$, $s=10\text{m}$, $\delta = 20^\circ$, $\lambda = 90^\circ$. (Lower panel) Spectrum of the progressive waves obtained by the Fourier expansion with $N = 67$.

vertical displacement is generated when the dip direction $\lambda = 90^\circ$. With the additional assumption (1), the runup problem becomes 2D (in the vertical and on-offshore plane). The present study is, therefore, oriented for near-field tsunamis. For far-field tsunamis, the directional spreading of tsunami waves needs to be considered and the present approximation is not directly applicable, which need to consider the spreading effects. However, 2D solutions tend to be more conservative than 3D solutions. Finally, the near coast bathymetry is represented by a constant slope so that C-G's theory can be adopted.

Okada's solution is lengthy and will not be repeated here. A typical profile of the seafloor displacement or water surface elevation η_0 , is shown in the upper panel of Fig. 3. Mega-thrust earthquake generated tsunami waves typically have amplitudes of a meter or so in open ocean, while water depth is in the order of magnitude of a few kilometers. The corresponding wavelength is typically tens to hundred of kilometers, which is much larger than water depth. Consequently, tsunami waves in open ocean behave as small amplitude long waves (see An and Liu (2014)). For simplicity, in the present study the water depth in ocean basin is further assumed to be a constant (see Fig. 1). According to the 1D linear shallow water theory, once the initial water surface elevation profile $\eta_0(x)$ is determined from Okada's model (e.g., upper panel of Fig. 3), tsunami waves, propagating in a constant depth, h , can be expressed as:

$$\eta(x, t) = \frac{1}{2} [\eta_0(x + ct) + \eta_0(x - ct)], \quad (1)$$

in which c is the wave celerity, $c = \sqrt{gh}$, and g is the gravitational acceleration. Thus, tsunami waves arrive at the toe of the slope with the same shape as the initial profile, but with a half of the initial amplitude. Additionally, because the wave is linear, the Fourier series expansion can be used to express the free surface

100 elevation at the toe of the slope as:

$$\eta_{toe} = \sum_{n=1}^N \frac{A_n}{2} \cos[\Omega_n(t - t_n)], \quad (2)$$

where $A_n/2$ denotes the wave amplitude of n^{th} harmonic of the Okada solution arriving at the toe, Ω_n is the wave frequency, t_n is the phase lag, and N is the number of wave components considered. We remark here that the angular frequency of each wave component, Ω_n , can be expressed in terms of the celerity and the wavelength, L_n , obtained in the Fourier series expansion of the seafloor deformation at the source, as:

$$\Omega_n = \frac{2\pi\sqrt{gh}}{L_n}. \quad (3)$$

Okada's solution is a non-periodic function approximated with periodic functions in the Fourier expansion. In order to obtain a meaningful approximation it is necessary to specify a largest wavelength L_1 containing the relevant information of Okada's solution. The Fourier representation shows a spurious periodic behavior with wavelength L_1 . Thus, the approximation is only valid along one wavelength L_1 . For this study we specify L_1 as the segment containing the maximum uplift and subsidence and where seafloor displacements are greater than 0.2% of the maximum uplift. The number of harmonics N used are based on the accuracy desired. In this study N is specified for each case such that the relative error between the seafloor displacement represented by the Fourier expansion is less than 2%. Comparisons are made at locations where the seafloor displacement is larger than 0.2% of the maximum uplift, so the relative error is not computed where the surface displacement is zero. It is important to mention that the expressions to compute the runup, presented in the following sections, involve convolutions in which the runup at a given time depends only on the properties of the wave at the shoreline during prior times. Therefore spurious waves occurring because of the periodic nature of the Fourier expansion do not alter the runup solution of the leading and truth wave.

The lower panel of Fig. 3 shows the amplitude spectrum of the Okada's solution displayed in the upper panel. A total of 67 harmonics are required to achieve the mentioned accuracy. Notice that although the amplitudes associated with larger wave numbers are small, these harmonics are responsible for the steep slopes of the seafloor displacement. Hence, in the cases where the fault depth is shallower more harmonics are required in the Fourier expansion because Okada's solution presents a sharp peak in the seafloor displacement profile.

2.2. Shoreline elevation and shoreline velocity

Following Madsen & Schaeffer (2010), the shoreline elevation, $Se(\tau)$, and the shoreline velocity, $U(\tau)$, for the incident tsunami waves generated by Okada's model, can be computed as the superposition of incident sinusoidal wave components. Thus,

$$Se(\tau) = \sum_{n=1}^N \frac{\sqrt{2h}\pi}{(L_n\gamma)^{1/2}} A_n \cos \left[\frac{2\pi\sqrt{gh}}{L_n} \left(\tau - t_n - 2\frac{h/\gamma}{\sqrt{gh}} \right) + \frac{\pi}{4} \right] - \frac{U(\tau)^2}{2g}, \quad (4)$$

$$U(\tau) = \sum_{n=1}^N \frac{2\pi^2\sqrt{2gh}}{(L_n\gamma)^{3/2}} A_n \sin \left[\frac{2\pi\sqrt{gh}}{L_n} \left(\tau - t_n - 2\frac{h/\gamma}{\sqrt{gh}} \right) + \frac{\pi}{4} \right], \quad (5)$$

where γ is the constant slope of the bed near the shore, h is the water depth in the constant depth zone, and τ is an auxiliary variable defined as $\tau = t - U/(g\gamma)$ at the shoreline. The maximum value of the shoreline elevation is, by definition, the runup,

$$R = \max \{Se(\tau)\}. \quad (6)$$

The time histories of the shoreline elevation and shoreline velocity are shown in Fig.4, resulted from the initial free surface profile in Fig. 3. The fault plane parameters, slope and constant water depth are

Figure 4: Time histories of shoreline elevation and shoreline velocity with fault plane parameters $W = 100$ km, $d = 40$ km, $s = 10$ m, $\delta = 20^\circ$, $\lambda = 90^\circ$, $\gamma = 0.03$, $h = 3000$ m.

indicated in the figure caption. The shoreline velocity vanishes when the shoreline elevation reaches its local maximum and minimum.

It should be pointed out that Madsen & Schaeffer employed an asymptotic expression for the Bessel functions that requires that the distance from the toe of the slope to the shoreline be larger than half the characteristic wavelength of the incident wave (Madsen & Schaeffer (2010), p.36).

The solutions given in Eq.4 and Eq.5 are valid only for non-breaking waves. Therefore, a breaking criterion must be established to bound the applicability of the solution. Madsen & Schaeffer (2010) defined the wave breaking as the moment when the temporal gradient of the shoreline velocity becomes infinite. At wave breaking the determinant of the Jacobian associated with the coordinate transformation vanishes and the solutions for $Se(\tau)$ and $U(\tau)$ become multivalued in space and time. Thus, the following expression becomes the breaking criterion:

$$\frac{1}{\gamma g} \frac{\partial U(\tau)}{\partial \tau} \rightarrow -1. \quad (7)$$

Using Eq.5 in the equation above yields:

$$\frac{2^{5/2} h^{3/2} \pi^3}{\gamma^{5/2}} \sum_{n=1}^N \frac{A_n}{L_n^{5/2}} \cdot \cos \left[\Omega_n \left(\tau - t_n - 2 \frac{h/\gamma}{\sqrt{gh}} \right) + \frac{\pi}{4} \right] \rightarrow -1. \quad (8)$$

Notice that the breaking criterion can be satisfied during the runup or the rundown phases. For the N-shape waves with a leading negative wave (see Fig. 3), the breaking criterion is reached during the second rundown phase when the largest $-\frac{\partial U}{\partial \tau}$ occurs (see Fig.4), invalidating the analytical solution.

Additional discussions on the wave breaking criterion in terms of fault plane parameters are given in the following section.

2.3. Normalized runup and wave breaking criterion

Since the runup occurs at the time that the shoreline velocity U vanishes (see Fig.4), the expression for runup is a summation of cosine functions with different amplitudes and phases. The resulting runup

Figure 5: The dimensionless runup, $\frac{R}{s} \sqrt{\frac{\gamma d}{h}}$, for tsunami waves generated in a constant depth, as a function of W/d with different dip angle δ . Each symbol represents one numerical realization with a given set of parameters. Solid lines denote the 8th degree polynomials fitting the data with same dip angle. $\diamond$: $\delta = 40^\circ$, $+$: $\delta = 35^\circ$, $\triangle$: $\delta = 30^\circ$, $\bullet$: $\delta = 25^\circ$, $\square$: $\delta = 20^\circ$, x : $\delta = 15^\circ$, $\circ$: $\delta = 10^\circ$

expression obtained from Eq.4 can be manipulated into the following form so that the phases of the cosine functions are independent of the slope:

$$R = \max \left\{ \sum_{n=1}^N \frac{\sqrt{2h\pi} A_n}{\sqrt{L_n \gamma}} \cdot \cos [\Omega'_n (\tau' - t'_n)] \right\}, \quad (9)$$

where

$$\Omega'_n = \frac{\Omega_n}{\sqrt{gh}} = \frac{2\pi}{L_n}, \quad \tau' = \tau - 2 \frac{h/\gamma}{\sqrt{gh}}, \quad t'_n = t_n - \frac{\pi}{4} \Omega'_n. \quad (10)$$

The seafloor displacement is linearly proportional to the fault slip (s) in Okada's model (U_2 in Equation 26 of Okada (1985)) and the linearity is preserved in the Fourier expansion. Hence, runup is also expected to be linearly proportional to fault slip. A dimensional analysis is performed by considering all physical variables: R , s , h , γ , W , d and δ . As suggested by Eq. 9 a dimensionless relation can be expressed as:

$$\frac{R}{s} \sqrt{\frac{d}{l_b}} = F_1 \left(\frac{W}{d}, \delta \right), \quad (11)$$

where $l_b = h/\gamma$ denotes the distances between the stillwater shoreline and the toe of the beach. $F_1(W/d, \delta)$ is a function that relates the three dimensionless parameters involved, and that can be obtained by computing the runup using Eq.9 with different sets of fault and slope parameters. It is anticipated that a fixed value of δ all the results for $\frac{R}{s} \sqrt{d/l_b}$ collapse into one curve when they are plotted against W/d . Fig. 5 shows such a family of curves with different values of δ . The 8th degree polynomial has been used to fit the data. The expressions for the best fitted curves are given in Appendix A.

It is notice that for very small W/d values, the dimensionless runup approaches zero. In the region $W/d < 1$ the dimensionless runup is insensitive to the dip angle δ . For the region where $W/d > 1$ the dimensionless runup increases for larger dip angle since the vertical component of slip is also larger (See Fig. 2). Finally, the dimensionless runup solutions approach infinity as $(W \sin \delta)/d \rightarrow 2$, which represents the condition where the rupture plane intersects the seafloor.

The non-monotonic behavior of the runup as a function of W/d can be understood by examining Eq. 4. The runup is inversely proportional to the squared root of the wavelength and linearly proportional to the amplitude. For a sufficiently deep rupture plane (i.e. a large d), as the fault width increases both wave amplitude and wavelength in Okada's solution increase. However, the rates of increase are different. This causes the particular behavior of the runup curves. For shallow rupture planes as $(W \sin \delta)/d \rightarrow 2$,

Figure 6: Wave breaking criterion for incident tsunami waves generated in constant water depth. Symbols represent numerical results using different fault plane parameters and beach slope. The solid line denote the best fit curve for $0.25 < (W \sin \delta)/d < 2.0$.

the rupture plane is close to the seafloor, generating a finite amplitude peak with a short characteristic wavelength, which results in large runup approaching infinity.

Finally it is remarked here that, for shallow fault cases, where W/d values become large, the characteristic wavelength of the seafloor deformation profile may become shorter than water depth. Therefore, frequency dispersion effects may become important, which has been ignored in the present study.

The wave breaking criterion has been established in Eq. 7 when the left hand side quantity is equal to -1. In the dimensionless form it can be expressed as

$$\left(\frac{d\gamma}{2h}\right)^{5/2} \frac{h}{\pi^3 s} = G_1 \left(\frac{W}{d}, \delta\right). \quad (12)$$

Function G_1 can be obtained by using different sets of fault plane and slope parameters in the breaking criterion. It turns out that G_1 depends only on a single dimensionless parameter, $(W \sin \delta)/d$ and is presented in Fig. 6. Wave breaking occurs for the set of fault and slope parameters located below the curve. Note that G_1 approaches infinity as $(W \sin \delta)/d \rightarrow 2$ and approaches zero for small values of $(W \sin \delta)/d$. It is clear that tsunami waves tend to break for larger slip, smaller beach slope, and shallow rupture depth.

As shown in Fig. 6, when values of $(W \sin \delta)/d$ are greater than 0.25, the function G_1 is satisfactorily represented by a power fitting curve as:

$$G_1 \left(\frac{W \sin \delta}{d}\right) = 0.0436 \left(2 - \frac{W \sin \delta}{d}\right)^{-2.486} \quad (13)$$

We remark here that when $(W \sin \delta)/d < 0.25$ the generated tsunami waves are small and are of no practical interest. However, some results are presented in Fig. 6 to show the behavior of G_1 for completeness.

It is reiterated here that the normalized runup curves shown in Fig. 5 are valid only in the region for non-breaking zone (above the curve) shown in Fig. 6. Examples are given in section 4.

3. Runup for tsunami waves generated on a slope

When tsunamis are generated by a seismic event on the slope, the runup solution is formulated as an IVP in which the initial water surface profile is prescribed above the rupture plane. Fig. 7 shows the definition sketch of the problem setup, where we introduce a new variable l_d as the distance between the stillwater

Figure 7: Initial wave profile corresponding to an submarine earthquake on the slope, with γ being the slope. l_d represents the distance between the still water shoreline and the upper edge of the rupture plane. R is the runup. SWL indicates the still water level.

shoreline and the location of the upper edge of the fault plane. Analytical solutions for shoreline movement and flow velocity of long waves, generated by a prescribed initial water surface waveform, have been given by Kanoglu (2004). In this section we combine the solutions of Okada's dislocation model and Kanoglu's (2004) results for runup and thus, runup can be predicted using the fault plane parameters and slope.

3.1. Shoreline elevation due to an initial free surface disturbance on a slope

We follow Kanoglu's (2004) approach and use Madsen & Schaeffer's (2010) coordinate system in the following analysis. The shoreline elevation $Se(\tau)$ and the shoreline velocity $U(\tau)$ can be expressed in terms of the pre-determined initial water surface profile in the physical space, $\eta(x, t = 0)$:

$$Se(\tau) = \frac{\gamma}{2} \int_0^\infty \xi \Phi(\xi) d\xi - \frac{\gamma}{2} \tau^2 \Phi(0) - \frac{\gamma}{2} \int_0^\tau \tau \sqrt{\tau^2 - \xi^2} \frac{d\Phi(\xi)}{d\xi} d\xi - \frac{1}{2g} U^2(\tau), \quad (14)$$

$$U(\tau) = \tau \Phi(0) + \frac{1}{2} \int_0^\tau \frac{2\tau^2 - \xi^2}{\sqrt{\tau^2 - \xi^2}} \frac{d\Phi(\xi)}{d\xi} d\xi. \quad (15)$$

In the equations above, the auxiliary variables τ and σ are function of x and t by (Madsen & Schaeffer 2004)

$$x = \frac{g\gamma}{4} \sigma^2 - \frac{\eta}{\gamma}, \quad t = \frac{u}{g\gamma} + \tau. \quad (16)$$

Assuming that the initial shoreline elevation and initial velocity are very small, the following approximations have been employed in Kanoglu's analysis:

$$x \approx \frac{g\gamma}{4} \sigma^2, \quad t \approx \tau, \quad (17)$$

$$\Phi(\sigma) \approx -\frac{2}{\gamma\sigma} \frac{\partial \eta\left(\frac{g\gamma}{4}\sigma^2, 0\right)}{\partial \sigma}. \quad (18)$$

Okada's solutions are used to calculate the initial water surface profile, which can be expressed in Fourier series as follows:

$$\eta(x, 0) = \sum_{n=1}^N A_n \cos[k_n x - \varphi_n], \quad (19)$$

where A_n is the amplitude and k_n is the wave number of the n^{th} component. The phase shift φ_n is defined as

$$\varphi_n = k_n(l_d - W \cos \delta) - \phi_n, \quad (20)$$

in which ϕ_n is the phase shift obtained from the Fourier decomposition of Okada's solution, and $k_n(l_d - W \cos \delta)$ is due to the coordinate translation between Kanoglu's and Okada's. The shoreline elevation and velocity, Eq. 14 and Eq. 15, can be rewritten in the Fourier series form as:

$$S_e(\tau) = \sum_{n=1}^N A_n \pi \beta \sin(\varphi_n) \left[C(\beta) \cos\left(\frac{\pi}{2} \beta^2\right) + S(\beta) \sin\left(\frac{\pi}{2} \beta^2\right) \right] \\ + A_n \cos(\varphi_n) \left\{ 1 - \pi \beta \left[C(\beta) \sin\left(\frac{\pi}{2} \beta^2\right) - S(\beta) \cos\left(\frac{\pi}{2} \beta^2\right) \right] \right\} - \frac{1}{2g} U^2(\tau), \quad (21)$$

$$U(\tau) = \sum_{n=1}^N \frac{A_n k_n g \tau}{2} \left\{ -\sin \varphi_n \right. \\ \left. + [\pi \beta \sin \varphi_n + 1/\beta \cos \varphi_n] \left[C(\beta) \sin\left(\frac{\pi}{2} \beta^2\right) - S(\beta) \cos\left(\frac{\pi}{2} \beta^2\right) \right] \right. \\ \left. + [\pi \beta \cos \varphi_n + 1/\beta \sin \varphi_n] \left[C(\beta) \cos\left(\frac{\pi}{2} \beta^2\right) + S(\beta) \sin\left(\frac{\pi}{2} \beta^2\right) \right] \right\}, \quad (22)$$

where $\beta = \sqrt{\tau^2 g k_n \gamma / 2\pi}$. $C(\cdot)$ and $S(\cdot)$ are Fresnel cosine and sine function, respectively, and are defined as:

$$S(a) = \int_0^a \sin(\pi v^2 / 2) dv; \quad C(a) = \int_0^a \cos(\pi v^2 / 2) dv, \quad (23)$$

which can be approximately computed following Thompson (1997). [As an illustration the time histories of the shoreline elevation and velocity](#) are shown in Fig. 8, using the initial water surface profile given in Fig. 3 with $l_d = 250$ km. The runup, $R = \max \{Se(\tau)\}$, can then be determined. Okada's solutions show that the location (x -coordinate) of the maximum seafloor uplift is approximately the same as that of the upper edge of the rupture plane.

As in the BVP solution given in the previous section, only non-breaking waves can be considered in the present theory. Therefore, a wave breaking criterion must be imposed. Again, wave breaking occurs at the time and location where the determinant of the Jacobian associated with the coordinate transformation vanishes and $Se(\tau)$ and $U(\tau)$ become multi-valued in space and time.

Then the breaking criterion can be defined by:

$$\frac{1}{g\gamma} \frac{\partial U}{\partial \tau} \rightarrow -1, \quad (24)$$

or

$$\frac{1}{g\gamma} \frac{\partial}{\partial \tau} \left[\tau \Phi(0) - \frac{1}{2} \int_0^{-\tau} \frac{2\tau^2 - \xi^2}{\sqrt{\tau^2 - \xi^2}} \frac{d\Phi(\xi)}{d\xi} d\xi \right] \rightarrow -1. \quad (25)$$

3.2. Normalized runup and breaking criterion

Following the same concept introduced in Section 2.3, a relation between the fault plane parameters and runup is sought after for the IVP. Similar to Eq. 11, a dimensionless runup expression, $\frac{R}{s} \sqrt{\frac{d}{l_d}}$, is expected to be a function of W/d , the dip angle δ , and $\frac{l_d}{W}$. We remark here that the distance $l_b = h/\gamma$ for the BVP is equivalent to l_d introduced here for the IVP. However, l_d is an independent physical parameter, measuring the distance between the still water shoreline the location of the upper edge of the fault plane. Thus,

$$\frac{R}{s} \sqrt{\frac{d}{l_d}} = F'_2 \left(\frac{W}{d}, \delta, \frac{l_d}{W} \right). \quad (26)$$

Figure 8: Time histories of the shoreline elevation and velocity with the following fault plane parameters $W = 100$ km, $d = 40$ km, $s = 10$ m, $\delta = 20^\circ$, $\lambda = 90^\circ$, and $l_d = 250$ km. The slope is $\gamma = 0.03$.

l_d/W is an additional dimensionless parameter for the IVP. Using 300 sets of fault plane parameters and slope, we found that all the results for dimensionless runup collapse into a single curve vs W/d for fixed values of δ and l_d/W . Moreover, we observed that the dimensionless runup becomes insensitive to l_d/W when l_d/W values become large. For example, the maximum relative difference between the dimensionless runup values using $l_d/W = 3$ and $l_d/W = 4$ is less than 4%, when the same fault plane and slope parameters are used. Thus, it is reasonable to approximate Eq. (3.13) as:

$$\frac{R}{s} \sqrt{\frac{d}{l_d}} \approx F_2 \left(\frac{W}{d}, \delta \right) \text{ for } \frac{l_d}{W} > 4. \quad (27)$$

This runup relationship for the IVP is equivalent to Eq. 11 for the BVP as the distance l_d in the above equation is equivalent to $l_b = h/\gamma$ used in Eq. 11. The dimensionless runup values for $l_d/W > 4$ are plotted in Fig. 9 as a family of curves depending only on W/d and δ . The expressions for the best fitted curves are given in Appendix A. We remark here that although the behaviors of the dimensionless runup are very similar for BVPs and IVPs (i.e., Fig. 5 and Fig. 9, respectively), there are not identical.

Tinti & Tonini (2005) obtained analytical solutions for runup generated by Okada's seafloor displacement on a slope. Their analytical solution can be obtained only when the Okada's solutions is represented by special parametric functions; values of the parameters are specified on the case-by-case base, and therefore the solution is not general. However, their solution method does not require that the initial shoreline elevation and velocity vanish. Thus, Tinti & Tonini's (2005) solution allows the fault plane be close to the shoreline, which is prohibited in the present approach. To check the severity of this limiting approximation in the present (and Kanoglu's) theory, experiments were conducted to compare the runup obtained by both approaches. Tinti & Tonini (2005) showed 3 cases (identified as cases 1, 2 and 3 in their paper), in which the initial shoreline elevation experiences an uplift, no deformation and a subsidence, respectively. We adopted the same sets of parameters and calculated the shoreline motions using Eq.21 and Eq.22, respectively. Despite of the violation in the initial condition, the results obtained from the present approach agree well with those from Tinti & Tonini (2005). The relative differences in the runup values are less than 3%.

The collapsed curves shown in Fig. 9 are limited to scenarios where $l_d/W > 4$. In the cases where $l_d/W < 4$, the dimensionless runup will depend more strongly on l_d/W value and can be directly calculated from

Figure 9: The dimensionless runup, $\frac{R}{s} \sqrt{\frac{d}{l_d}}$, corresponds to the initial free surface profile generated by Okada's model on a constant slope and $l_d/W > 4$. Each symbol represents one numerical realization with a set of parameters. Solid lines denote the 8 - th degree polynomials fitting the data. $\diamond$: $\delta = 40^\circ$, $+$: $\delta = 35^\circ$, $\triangle$: $\delta = 30^\circ$, $\bullet$: $\delta = 25^\circ$, $\square$: $\delta = 20^\circ$, $\times$: $\delta = 15^\circ$, $\circ$: $\delta = 10^\circ$.

Eq.21. It is, however, useful to get some sense of the strength of this dependency. [Analytical experiments](#) were carried out to determine the regions where the errors for the calculated dimensionless runup relative to that estimated from Fig. 9 are greater or less than 10%. In Fig.10 such a plot is presented in which the dividing lines for relative errors equal to 10% are plotted for different dip angle, δ , l_d/W , and W/d . We observe that the l_d/W value decreases monotonically with increasing W/d value. It is pointed out that the dimensionless runup value estimated by the curves shown in Fig. 9 are always larger than the values calculated from Eq.21. Results from Fig. 9 are more conservative estimations.

The breaking criterion, Eq. 25, is computed using 70 sets of rupture plane parameters, where $l_d/W > 4$. The results are plotted in Fig. 11. The results appear to collapse into one curve, representing the wave breaking criterion. The solid line is a best fit curve, which can be expressed as:

$$\left(\frac{d}{2l_d}\right)^{5/2} \frac{\gamma l_d}{\pi^3 s} = G_2 \left(\frac{W \sin \delta}{d}\right) = 0.0449 \left(2 - \frac{W \sin \delta}{d}\right)^{-2.54} \quad (28)$$

We have also analyzed the breaking condition for $1 < l_d/W < 4$. As shown in Fig. 12, the breaking limit does not depend strongly on l_d/W within this range. In the following section, examples using the presented solution are given.

4. Two examples of application

In this section two examples of application of the runup solutions are presented. They are associated with the 2004 Sumatra earthquake (Choi et al. 2006) and the 2010 Chile earthquake (Fritz et al. 2011), respectively. The areas of study are located in the vicinity of Banda Aceh, Indonesia and at Constitucion, Chile. The bathymetry data are obtained from the ETOPO database, Smith and Sandwell (1997), and are simplified by fitting a constant slope. Two sets of fault plane parameters are employed for each example. The first set is based on the scalar seismic moment reported in the National Earthquake Information Center catalog (NEIC, Sipkin et al. 2000) with an assumed earth rigidity of 40 GPa. The second set uses direct estimations of the fault plane parameters based on inversion models (the model of Hirata et al. 2006 for Sumatra and the model of Vigny et al. 2011 for Chile). The parameters adopted in the analytical solutions are given in Table 1. Based on the bathymetry and the location of the rupture planes, the solutions of the BVP shown in section 2 is used to estimate the runup for the Sumatra case, while the IVP solutions presented in section 3 is used for the Chile case.

Figure 10: Values of l_d/W at which the dimensionless runup estimated by Fig. 9 has an error of 10% due to the proximity of the fault to the shoreline, as a function of W/d and the dip angle. $\diamond: \delta = 40^\circ$, $\triangle: \delta = 30^\circ$, $\square: \delta = 20^\circ$, $\circ: \delta = 10^\circ$.

Figure 11: Relation G_2 between adimensional numbers for breaking criterion for negative leading incident waves in the IVP and $\frac{l_d}{W} > 4$.

Figure 12: Breaking condition for scenarios with $l_d/W = 1, 2$ and 4 .

Figure 13: The left upper panel is a map showing the rupture area of the 2004 Sumatra earthquake. The left lower panel displays the on-offshore bathymetry profile. The light line denotes the seafloor location according to ETOPO database, while the dark line represents the simplified bathymetry. The right upper panel shows a map for the rupture area of the 2010 Chile Earthquake. The on-offshore bathymetry for the area of study is depicted in the lower right panel. In the lower panels, the light lines denote the seafloor location according to ETOPO database, while the dark lines represent the simplified bathymetry.

Example	2004 Sumatra		2010 Chile	
	(1) Moment based (□)	(2) Inversion model (■)	(3) Moment based (○)	(4) Inversion model (●)
	$h=1500\text{m}$	$h=1500\text{m}$	$l_d=100\text{km}$	$l_d=100\text{km}$
Beach slope, γ	0.029	0.029	0.031	0.031
Dip Angle, δ ($^\circ$)	10	10	20	15
Fault depth, d (km)	30	23	30	25
Width, W (km)	150	150	100	100
Length, L (km)	1,200	-	450	-
Seismic moment Nm	7.36×10^{22}	9.86×10^{22}	1.81×10^{22}	1.76×10^{22}
Slip, s (m)	10.22	18.13	10.05	10.00
W/d	5.00	6.52	3.33	4.00
F_i	0.58	0.61	0.64	0.61
G_i	0.214	0.062	0.086	0.055
Computed Runup, R (m)	7.79	16.71	11.67	12.15

Table 1: Rupture and beach parameters for two examples: 2004 Sumatra and 2010 Chile. In column (1) and (3) data are obtained from NEIC. The fault depth used is referred to the hypocenter. In column (2) data are obtained from Hirata et al. (2006). The slip is calculated by averaging 3 fault patches in front of Banda Aceh. In column (4) information is obtained from Vigny et al. (2011). The slip is the average of the slips in front of Constitucion, Chile

To estimate the runup, the ratio W/d is first computed. Using the dip angle value for each case, the dimensionless runups F_1 and F_2 can be found from the pre-prepared curves, which have been shown in Fig. 5 and Fig. 9, respectively. For clarity the curve for $\delta = 10^\circ$ in Fig. 5 is reproduced in Fig. 14 and the curves for $\delta = 15^\circ$ and 20° in Fig. 9 are replotted in Fig. 15. The corresponding dimensionless runup values can be readily obtained from these figures.

Finally, it is necessary to ensure that tsunami waves do not break in these examples so that the solutions are valid. As shown in Fig.16 and Fig.17, for the prescribed rupture and beach parameters the generated tsunami waves are all in the non-breaking zone.

For the Sumatra case, the runup values estimated by using different sources for fault parameters differ considerably. This can be explained mainly by the significant slip variation along the rupture area presented in the model of Hirata et al. (2006), which results in highest fault slip in front of Banda Aceh, despite the fact that the scalar seismic moment is only 30% larger (see Table 1). Comparisons with fields measurements show agreement with the averaged measured runup of 15-20 m. Nevertheless our solutions underestimate the maximum runup of 40 m reported by Choi et al. (2006) in a post-tsunami survey. This is not entirely surprising since the methodology proposed in this study does not capture local topographical effects, which create a wide variation in the runup height distribution along the coast.

The runup estimations for the 2010 Chilean tsunami waves, using two different sets of rupture parameters, have a small difference, of 5%. Again, the averaged measured runup of 10-15 m in the region is close to the estimated values listed in Table 1, and the solutions underestimates the maximum runup in the survey report (Fritz et al. (2011)).

5. Conclusions

In this paper we have presented closed form expressions relating tsunami runup to the fault plane and slope parameters. Tsunamis could be generated at an offshore location and propagate towards the continental shelf, or on the continental shelf. The former is treated as a BVP and the latter an IVP. From the analytical solutions, runup values are obtained using a large set of fault plane parameters and slope. It is shown that for the BVP a dimensionless runup, $\frac{R}{s}\sqrt{d/l_b}$ with $l_b = h/\gamma$, depends on W/d and δ , where W is the fault plane width, d is the fault depth, s is the fault slip, δ is the dip angle, h is the constant water

Figure 14: Computation of runup for Sumatra tsunami. $F_1 = \frac{R}{s} \sqrt{\frac{\gamma d}{h}}$. The symbols are defined in Table 1.

Figure 15: Computation of runup for Chile tsunami. $F_2 = \frac{R}{s} \sqrt{\frac{d}{l_d}}$. Symbols (○) and (●) are defined in Table 1.

Figure 16: Verification of breaking condition for Sumatra tsunami. The vertical axis is defined as $G_1 = (d\gamma/2)^{5/2} (\pi^3 h^{3/2} s)^{-1}$. The symbols are defined in Table 1.

Figure 17: Verification of breaking condition for Chile tsunami. The vertical axis is defined as $G_2 = \gamma (d/2)^{5/2} (\pi^3 l_d^{3/2} s)^{-1}$. The symbols are defined in Table 1.

depth, and γ is the slope angle. In Fig. 5 a family of curves are presented relating the dimensionless runup to W/d for a fixed δ . Note that in the analysis we have employed the rake angle, $\lambda = 90^\circ$. Thus, this figure can be used to calculate the runup once the fault plane parameters and bathymetry slope are prescribed.

Similar analyses were performed for the IVP. The dimensionless runup, $\frac{R}{s} \sqrt{d/l_d}$, is found to be a function of W/d , the dip angle δ , and l_d/W , where l_d is defined as distance from the still water shoreline to the location of the upper edge of the fault plane. However, the dimensionless runup becomes independent of l_d/W for $l_d/W > 4$. Fig. 9 represents the solutions for runup as a family of curves varying in W/d ; each curve is for a fixed δ value. Since the runup results are derived for non-breaking waves, the breaking conditions for BVP and IVP are shown in Fig. 6 and Fig. 10, respectively.

Two examples are given to illustrate the application of the results for estimating the runup for the 2004 Sumatra and 2010 Chile earthquake. In both examples, we obtain acceptable agreement between the estimations and the averaged runup measured along the coasts. It is also clear that the fault parameters published in different data sources could have high variability, which will lead to different estimations. It is also evident that the maximum runup heights collected in the post-tsunami surveys are always higher than those estimated by the present approach. The survey results include the local topographical effects, which are not considered in the theory. The present approach provides a reliable and fast estimations of the runup once the fault plane parameters are known.

In conclusion, we have developed a methodology that can be used to estimate the runup generated by a seismic event once the fault plane parameters and bathymetry slope are known. [The methodology can be developed into a useful tool in the tsunami flooding warning system.](#)

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Appendix A. Polynomials of function F

Based on the analytical solutions derived in this paper, we present the 8th degree polynomials which best fit the numerical results shown in Fig. 5 and Fig. 9 for the BVP and IVP, respectively.

Appendix B. Kanoglu's solution for Fourier components

Eq.14 and Eq.15 are the dimensional versions of Kanoglu's solution. It is important to point out that the dimensional and dimensionless forms for the shoreline velocity differ in sign, since τ have different signs for positive time (see Eq.2.7 to Eq.2.8 in Kanoglu (2004)). Following the decomposition of Eq.19, the initial condition given by Φ is decomposed as well:

$$\Phi(\sigma) = -\frac{2}{\gamma\sigma} \frac{d\eta(\frac{g\gamma}{4}\sigma^2, 0)}{d\sigma} = \sum_{n=1}^N A_n k_n g \sin(k_n g \gamma \sigma^2 / 4 - \varphi_n), \quad (\text{B.1})$$

and its first derivative

$$\frac{d\Phi(\sigma)}{d\sigma} = \sum_{n=1}^N \frac{A_n k_n^2 g^2 \gamma \sigma}{2} \cos(k_n g \gamma \sigma^2 / 4 - \varphi_n). \quad (\text{B.2})$$

The summation terms of Eq.B1 and Eq.B2 are used to solve Eq.14 and Eq.15. Then, solutions are superposed to obtain the result of the shoreline elevation and velocity. It is important to mention how the integrals of Eq.14 and Eq.15 are computed. The first term at the right hand side of Eq.14 is solved by a change of variables. The other integrals involved (3rd term in Eq.14 and 2nd term in Eq.15) are determined and expressed in terms of Fresnel cosine and sine functions. Gathering the solutions, the shoreline elevation and velocity are obtained by:

$F_1(x) = \sum_{i=1}^N a_i (W/d)^i$	
Dip angle δ ($^\circ$)	Boundary Value Problem
10	$a_8 = 1.9135 \times 10^{-6}; a_7 = -7.9261 \times 10^{-5}; a_6 = 1.318 \times 10^{-3};$ $a_5 = -1.0961 \times 10^{-2}; a_4 = 4.4208 \times 10^{-2}; a_3 = -4.4204 \times 10^{-2};$ $a_2 = -2.543 \times 10^{-1}; a_1 = 7.8774 \times 10^{-1}; a_0 = -9.9099 \times 10^{-4}$
15	$a_8 = 4.2266 \times 10^{-5}; a_7 = -1.2461 \times 10^{-3}; a_6 = 1.5223 \times 10^{-2};$ $a_5 = -9.8506 \times 10^{-2}; a_4 = 3.5431 \times 10^{-1}; a_3 = -6.4867 \times 10^{-1};$ $a_2 = 3.2340 \times 10^{-1}; a_1 = 6.0756 \times 10^{-1}; a_0 = -1.9879 \times 10^{-4}$
20	$a_8 = 2.2229 \times 10^{-4}; a_7 = -4.8172 \times 10^{-3}; a_6 = 4.4106 \times 10^{-2};$ $a_5 = -2.2085 \times 10^{-1}; a_4 = 6.4548 \times 10^{-1}; a_3 = -1.0361;$ $a_2 = 5.8775 \times 10^{-1}; a_1 = 5.5258 \times 10^{-1}; a_0 = -4.8065 \times 10^{-5}$
25	$a_8 = 8.844 \times 10^{-4}; a_7 = -1.457 \times 10^{-2}; a_6 = 1.0075 \times 10^{-1};$ $a_5 = -3.8451 \times 10^{-1}; a_4 = 8.9104 \times 10^{-1}; a_3 = -1.2222;$ $a_2 = 6.5948 \times 10^{-1}; a_1 = 5.4892 \times 10^{-1}; a_0 = -1.4116 \times 10^{-4}$
30	$a_8 = 2.8623 \times 10^{-3}; a_7 = -3.8888 \times 10^{-2}; a_6 = 2.1813 \times 10^{-1};$ $a_5 = -6.6339 \times 10^{-1}; a_4 = 1.2257; a_3 = -1.411;$ $a_2 = 7.1103 \times 10^{-1}; a_1 = 5.4426 \times 10^{-1}; a_0 = -2.197 \times 10^{-4}$
35	$a_8 = 9.2956 \times 10^{-3}; a_7 = -1.1336 \times 10^{-1}; a_6 = 5.6529 \times 10^{-1};$ $a_5 = -1.4939; a_4 = 2.3038; a_3 = -2.1559;$ $a_2 = 9.7362 \times 10^{-1}; a_1 = 5.0463 \times 10^{-1}; a_0 = -2.0477 \times 10^{-4}$
40	$a_8 = 4.183 \times 10^{-2}; a_7 = -4.6312 \times 10^{-1}; a_6 = 2.0859;$ $a_5 = -4.9131; a_4 = 6.5205; a_3 = -4.9413;$ $a_2 = 1.8706; a_1 = 3.9201 \times 10^{-1}; a_0 = -2.3854 \times 10^{-5}$

Table A.2: Coefficients of the 8th order polynomials for the dimensionless runup for the BVP.

$$\begin{aligned}
S_e(\tau) = \sum_{n=1}^N A_n \sin(\varphi_n) \tau \sqrt{\frac{\pi k_n g \gamma}{2}} & \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) \right. \\
& \left. + S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) \right] \\
& + A_n \cos(\varphi_n) \left\{ 1 - \tau \sqrt{\frac{\pi k_n g \gamma}{2}} \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) \right. \right. \\
& \left. \left. - S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) \right] \right\} - \frac{1}{2g} U^2(\tau), \quad (\text{B.3})
\end{aligned}$$

$F_2(W/d) = \sum_{i=1}^N a_i(W/d)^i$	
Dip angle δ ($^\circ$)	Initial Value Problem
10	$a_8 = 2.3495 \times 10^{-6}; a_7 = -8.7984 \times 10^{-5}; a_6 = 1.3017 \times 10^{-3};$ $a_5 = -9.2753 \times 10^{-3}; a_4 = 2.7497 \times 10^{-2}; a_3 = 2.7238 \times 10^{-2};$ $a_2 = -3.9220 \times 10^{-1}; a_1 = 8.7755 \times 10^{-1}; a_0 = -6.4306 \times 10^{-5}$
15	$a_8 = 2.8216 \times 10^{-5}; a_7 = -8.5925 \times 10^{-4}; a_6 = 1.0743 \times 10^{-2};$ $a_5 = -7.0078 \times 10^{-2}; a_4 = 2.4692 \times 10^{-1}; a_3 = -4.0677 \times 10^{-1};$ $a_2 = 2.6427 \times 10^{-2}; a_1 = 7.4799 \times 10^{-1}; a_0 = -2.0625 \times 10^{-5}$
20	$a_8 = 2.5706 \times 10^{-4}; a_7 = -5.7448 \times 10^{-3}; a_6 = 5.3359 \times 10^{-2};$ $a_5 = -2.6484 \times 10^{-1}; a_4 = 7.4426 \times 10^{-1}; a_3 = -1.1039;$ $a_2 = 5.0873 \times 10^{-1}; a_1 = 6.3830 \times 10^{-1}; a_0 = 7.7011 \times 10^{-5}$
25	$a_8 = 2.9018 \times 10^{-4}; a_7 = -5.4498 \times 10^{-3}; a_6 = 4.4459 \times 10^{-2};$ $a_5 = -2.0251 \times 10^{-1}; a_4 = 5.4442 \times 10^{-1}; a_3 = -7.8722 \times 10^{-1};$ $a_2 = 2.8324 \times 10^{-1}; a_1 = 6.9685 \times 10^{-1}; a_0 = 2.6086 \times 10^{-5}$
30	$a_8 = 1.9022 \times 10^{-3}; a_7 = -2.829 \times 10^{-2}; a_6 = 1.7599 \times 10^{-1};$ $a_5 = -5.9367 \times 10^{-1}; a_4 = 1.1766; a_3 = -1.3247;$ $a_2 = 5.0459 \times 10^{-1}; a_1 = 6.6114 \times 10^{-1}; a_0 = 3.3054 \times 10^{-5}$
35	$a_8 = 4.0577 \times 10^{-3}; a_7 = -5.4099 \times 10^{-2}; a_6 = 2.9971 \times 10^{-1};$ $a_5 = -8.8937 \times 10^{-1}; a_4 = 1.5309; a_3 = -1.5075;$ $a_2 = 5.3582 \times 10^{-1}; a_1 = 6.5225 \times 10^{-1}; a_0 = 3.1005 \times 10^{-5}$
40	$a_8 = 1.5334 \times 10^{-2}; a_7 = -1.8071 \times 10^{-1}; a_6 = 8.7712 \times 10^{-1};$ $a_5 = -2.2493; a_4 = 3.2657; a_3 = -2.6539;$ $a_2 = 8.9125 \times 10^{-1}; a_1 = 5.9765 \times 10^{-1}; a_0 = 3.5662 \times 10^{-5}$

Table A.3: Coefficients of the 8th order polynomials for the dimensionless runup for the IVP.

$$\begin{aligned}
U(\tau) = & \sum_{n=1}^N A_n k_n g \sin(\varphi_n) \\
& \left\{ -\frac{1}{2}\tau + \frac{\tau^2}{2} \sqrt{\frac{k_n g \gamma \pi}{2}} \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) - S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) \right] \right. \\
& \left. + \sqrt{\frac{\pi}{2k_n g \gamma}} \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) + S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) \right] \right\} \\
& + A_n k_n g \cos(\varphi_n) \left\{ \frac{\tau^2}{2} \sqrt{\frac{k_n g \gamma \pi}{2}} \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) \right. \right. \\
& \left. \left. + S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) \right] \right. \\
& \left. + \sqrt{\frac{\pi}{2k_n g \gamma}} \left[C \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \sin(\tau^2 k_n g \gamma / 4) - S \left(\sqrt{\frac{\tau^2 k_n g \gamma}{2\pi}} \right) \cos(\tau^2 k_n g \gamma / 4) \right] \right\}. \quad (\text{B.4})
\end{aligned}$$

Arranging adequately and defining $\beta = \sqrt{\frac{\tau^2 g k_n \gamma}{2\pi}}$, Eq.21 and Eq.22 are obtained.

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