Coordination of the Traditional and the Online Channels for a Short-life-cycle Product

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Abstract

The study considers a retailer that sells a short-life-cycle product and uses the online channel as a supplement to its traditional channel. The inventory of the product is finite. The sales season is split into two periods: the premium period and the discount period. The study investigates how the retailer coordinates two channels with proper pricing strategies and channel mix. We introduce a new strategy to address channel conflict, in which two channels are introduced at different periods and the retailer employs different pricing mechanisms for two channels. We also explore the effects of return rate, service difference, and the retailer's risk attitude. Numerical examples are used to compare the solutions with other channel selection and pricing strategies, and further investigate the situations when some assumptions are not satisfied. The study suggests that the benefits of introducing the online channel are contingent on the potential demand of the online channel, the inventory level of the product, and the time of introducing the online channel. The retailer could adjust price difference between two channels to reduce channel conflict. Its risk attitude also influences the pricing strategies and sales revenue. Managerial implications of the study are discussed.

Key words: decision analysis, pricing, dual channels, channel conflict, risk attitude

1. Introduction

Due to market competition and the speed of product development, developing a method of selling products with a short-life cycle is a great challenge (Zhao & Zheng, 2000). With the evolution of the Internet, many firms (for example, apparel firms) take advantage of both the traditional, bricks-and-mortar channel and the online channel with a hope to reach more customers and boost sales. However, the online channel could exert competition pressure on existing bricks-and-mortar stores, resulting in channel conflict. The introduction of the online channel may lead to the closure of bricks-and-mortar retail stores (Yue & Liu, 2006). Therefore, whether the retailer will benefit from the online channel, and how to address channel conflict, are questions calling for more studies (Yao & Liu, 2005; Dan, Xu, & Liu, 2012).

This study focuses on how a retailer that sells a perishable product can take advantage of both the traditional and the online channels. Specifically, we investigate the following research questions: (1) to mitigate channel conflict, when should the online channel be introduced and what are the pricing strategies that the retailer should use for these two channels? (2) How do return rate, service difference, and the retailer's risk attitude impact pricing, revenue, and channel conflict?

The modeling scenario of the study is drawn from the apparel industry. The apparel industry possesses seasonal and fashion characteristics (Johansen & Riis, 1995). Many apparel retailers utilize both the traditional and the online channels to reach their customers and increase their sales (Chellappa, Sin, & Siddarth, 2011). While the online channel enables retailers to reach more potential customers efficiently, introducing the online channel does not always lead to better overall profit due to channel conflict. To maximize overall profit and mitigate channel conflict, retailers often rely on strategies such as differentiating product prices of the two channels, or introducing value-added services for the traditional channel.

In this study, the sales season of the short-life-cycle product is split into two periods: the premium period and the discount period. The price of the product decreases with its time on the market. This study introduces a new strategy to coordinate the two channels and mitigate channel conflict. In this strategy, only the traditional channel is utilized in the premium period, while both the traditional and the online channels are employed in the discount period. And the two channels adopt different pricing. We further discuss the scenario where the two channels are used in both periods in Subsection 6.2.2. We find that it does not always maximize revenue to introduce the short-life-cycle

product in both the traditional and the online channels immediately. When the inventory level is small the retailer shall adopt the traditional channel alone. When the inventory is large, the retailer shall introduce the online channel in the discount period. When the inventory is extremely large, the retailer shall introduce the two channels in both periods.

The study extends prior literature in three important aspects. First, prior studies on the perishable products mainly explored dynamic pricing strategies but did not investigate the impact of the online channel (Gallego & Van Ryzin, 1994; Feng & Gallego, 1995; Zhao & Zheng, 2000; Aviv & Pazgal, 2008). This study introduces a new selling strategy for perishables in which the online channel is used as a supplement to the traditional one. Second, most studies in dual channels (Chiang, Chhajed, & Hess, 2003; Dumrongsiri et al. 2008 and Cai, Zhang, & Zhang, 2009) focused on some general products other than perishables, and used both channels in the entire sale season. This study introduces a new strategy in which the retailer utilizes only the traditional channel in the premium period but both channels in the discount period to mitigate channel conflict, considering the characteristics of perishables. Third, we develop a dynamic pricing mechanism for the traditional channel to address channel conflict. Most studies on coordination of dual channels used static pricing mechanisms (e.g., Yao & Liu, 2005; Dan, Xu, & Liu, 2012).

The rest of the paper is organized as follows. Section 2 presents the related literature. Section 3 provides the modeling scenario and assumptions. Section 4 analyzes the basic and extended models with the consideration of return rate and service difference between two channels. Section 5 discusses the impact of demand uncertainty of the online channel and the retailer's risk attitude. Section 6 presents numerical examples and simulations to compare channel-selection and pricing strategies, and further explores the situations when some assumptions are violated. Section 7 discusses managerial implications, concludes the study and discusses future research.

2. Related Literature

In the apparel industry, a retailer often incorporates consumers' price sensitivity in its pricing strategy to maximize profit. Aviv & Pazgal (2008) investigated optimal pricing strategies considering consumers' different reservation prices. Cachon & Swinney (2009) studied the impact of dynamic pricing on consumers' strategic behavior. Li, Wei, & Cai, (2012) explored how to sell new products and off-season products facing a high return rate in the apparel industry. In studying closed-loop

supply chain models for a high-tech product which is featured with a short life-cycle and volatile demand, Chuang, Wang, & Zhao (2014) presented the manufacturer's optimal production quantities and profits under the three alternative reverse channels for collecting the used product from consumers for remanufacturing. By setting different prices in the premium period and the discount period of the product, this study considers how the retailer can benefit from introducing the online channel as a supplement to the traditional channel. Broadly speaking, there are two streams of studies on channel selection related to this study.

The first stream examines the impact of introducing the online channel. Park & Keh (2003) explored three scenarios: only using the online channel, only using its bricks-and-mortar channel, and using both channels. For each scenario, they analyzed the impact of cost structure and price sensitivity on equilibrium prices and revenue. Cattani, Perdikaki, & Maruchek (2007) explored how to introduce the online channel for grocery operations and suggested that the firm's channel selection depends on product price and quality. Dumrongsiri et al. (2008) examined the impact of channel marginal cost and demand uncertainty on product pricing and supply chain structure. It showed that the firm tends to add the online channel when wholesale price and demand uncertainty are low and marginal cost of the traditional channel is high. Considering the trade-off between risk pooling effect and transportation cost, Liu, Zhou, & Zhang (2010) investigate how to assign online demands to the capacitated regional warehouses currently serving in-store demands in a multi-channel supply chain. Carrillo, Vakharia, & Wang (2014) presented a dual-channel model for a retailer with online and traditional markets, and analyzed the impact of customer environmental sensitivity on retailer channel choice as well as stocking decisions for each channel. Cao, So, & Yin (2016) added Online-to-Store channel (where customers order the product online and then pick up the product offline) and examined its impact on the demand allocations and profitability of a retailer who sells products through multiple distribution channels.

The second stream of literature primarily addresses how to solve channel conflict when a firm utilizes both the online and the traditional channels. Some literature relied on service difference between the two channels to address their conflict. Yan & Pei (2009) showed that the online channel could encourage the retailer to improve service. Ofek, Katona, & Sarvary (2011) incorporated return rate to consumers' utility function, and analyzed the impact of return rate on channel prices. Chen & Bell (2012) explored different return policies for two channels and showed that segmenting the

market based on consumers' sensitivity to return can be a profit-maximizing policy.

Some researchers also introduced product differentiation and/or price difference between two channels. Riggins (2004) developed a multi-channel model where a retailer sells a durable good to high- and low-type consumers in two different channels. Kurata, Yao, & Liu (2007) studied the optimal prices when the online channel sells national brands, and the traditional channel sells both the national brands and the store brands. Yao & Liu (2005) discussed the impact of price difference and customer service on the equilibrium prices. Marom & Seidmann (2011) distinguished the channels by prices and availability of products.

Most studies differentiated the traditional channel and the online channel either by the price or by the product (including service). In this paper we not only introduce different pricing strategies for the two channels, but also engage the two channels in different periods: the online channel will only be used to sell the product in the discount period. Based on the characteristics of short-life-cycle products, we address channel conflict in the following ways: (1) product difference—the traditional channel sells both the new and off-season product, but the online channel only sells the off-season product (or the product in the discount period); (2) pricing difference—the traditional channel employs dynamic prices for the two periods, while the online channel employs static price (only in the discount period). Furthermore, we discuss the impact of service difference, return rate, and demand uncertainty.

3. Problem Description

Consider a retailer who sells a short-life-cycle product with the traditional channel. Because of the long production life cycle, the retailer has only one ordering opportunity for the sales season (Zhao & Zheng, 2000). Therefore its inventory of the product is finite. We consider the retailer has Q units of the product in its inventory. The sales season is split into two periods, namely period 1 (the premium period) and period 2 (the discount period) (Aviv & Pazgal, 2008). In the first period, the retailer only utilizes the traditional channel (i.e., single channel mode) to sell the product. In the second period, the retailer introduces the online channel to sell the product together with the traditional channel. In other words, the product is sold via both channels in the discount period. We do not incorporate the length of each period as a model parameter, but one may reasonably assume that the length of a period can be reflected by the corresponding potential market size (a_{ij}) (Aviv & Pazgal,

2008).

3.1. Demand Functions

During the first period of the sales season, a ‘premium price’ p_{11} is applied in the traditional channel. During the second period of the season, a ‘discount price’ p_{21} is offered in the traditional channel and p_{22} in the online channel. The premium price p_{11} is announced at the beginning of period 1, and the discount prices p_{21} and p_{22} at the beginning of period 2. At the end of the sales season, the retailer clears the remaining inventory with a salvage value, and we assume that the unit loss of the product incurred by clearing is v . Because of the low cost of the online channel, we assume $p_{21} \geq p_{22}$ which is in accordance with the real world and prior studies (Yao & Liu, 2005). Fig.1 shows such a pricing policy.

add online channel

Fig.1 Dynamic pricing with dual channels

We assume that the demand in each period is price sensitive. Consumers can purchase the product from either channel. Similar to Yao & Liu (2005) and Dan, Xu, & Liu (2012), we assume the demand function of the traditional channel in the premium period is $D_{11} = a_{11} - b_1 p_{11}$. And in the discount period, the demand of the traditional channel is $D_{21} = a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})$. The demand of the online channel in the discount period is $D_{22} = a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})$. a_{ij} in the above demand equations represents the potential demand in period i channel j ($i=1$ or 2 , $j=1$ or 2). $a_{11} + a_{21} + a_{22}$ represents the total potential demand during the sales season. b_i is channel i 's marginal demand per price. θ is the sensitivity of switch-over towards price difference in the discount period. We assume that consumers make independent decisions in the two periods, and therefore do not consider the demands of period 2 (D_{21} and D_{22}) as a function of p_{11} .

The linear demand functions are adopted for two reasons. First, the linear structure well

characterizes the demand diffusion across channels (Bass, 1969; Chiang, Chhajed, & Hess, 2003), especially for the online channel in which product information is disseminated by consumers' comments (Duan, Gu, & Whinston, 2008). Second, linear models are widely used in marketing and supply chain management literature due to their parsimony and tractability for analysis (Tang & Yin, 2007; Liu, Anderson, & Cruz, 2012). Also, the results derived from linear models can also be applied for other demand forms that can be approximated or locally approximated by linear functions.

3.2 Model Assumptions

Since the product will be sold with the traditional channel in both periods and with the online channel in the discount period, the retailer needs to set the optimal prices for the traditional channel dynamically in the two periods, and the price for the online channel in the discount period only. To properly solve the model, we have a number of assumptions.

Assumption 1. The prices of the two channels satisfy $p_{11} \geq p_{21} \geq p_{22} > v$.

Assumption 2. The initial inventory Q satisfies $a_{11} < Q < a_{11} + a_{21} + a_{22}$.

Assumption 2 ensures that there is leftover inventory at the end of Period 1; however, the initial inventory Q does not exceed the total potential demand of the two channels.

Assumption 3. Consumers' price sensitivities in the premium period and in the discount period satisfy $b_1 < b_2$.

Assumption 3 reflects that consumers who purchase in the premium period are less sensitive to the price than those who purchase in the discount period.

Assumption 4. The potential demands of the two channels in the two periods satisfy $a_{11} > a_{21} > a_{22}$.

Assumption 4 means that the potential demand in the premium period is larger than that in the discount period, and the potential demand of the traditional channel is larger than that of the online channel.

Without loss of generality, Assumptions 3 and 4 are introduced to simplify mathematical analyses. Subsection 6.4 explores the scenarios when Assumption 3 and 4 are violated.

4 Model Formulation and Solution

In this section, we present basic models and their extensions. We first present the mode where the retailer only employs the traditional channel with dynamic pricing (i.e., Mode 1). Then we introduce the mode where the retailer introduces the online channel in the discount period (i.e., Mode 2). We compare the two basic models. We further incorporate the difference of return rate and service between the two channels in modeling.

4.1 Basic Models

4.1.1 Mode 1: Traditional Channel Only with Dynamic Pricing in the Two Periods

In this mode, we consider that the retailer only employs the traditional channel for both periods with dynamic pricing. The total revenue for the retailer can be expressed as follows

$$\pi(p_{11}, p_{21}) = p_{11}(a_{11} - b_1 p_{11}) + p_{21}(a_{21} - b_2 p_{21}) - v(Q - D) \quad (1)$$

s.t. $D \leq Q$, where $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21}$.

The first two terms in the equation are the revenue of the traditional channel in two periods, respectively, and the third term is the loss of the remaining inventory at the end of the sales season. The retailer's objective is to maximize its revenue. With the Lagrangian multiplier, the optimization problem (1) faced by the retailer can be written as follows:

$$\max \pi(p_{11}, p_{21}) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}(a_{21} - b_2 p_{21}) - v(Q - D) + \lambda_1(Q - D) \},$$

where $\lambda_1 \geq 0$, $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21}$.

Theorem 1. When only the traditional channel is employed for both periods with dynamic pricing,

(1) If $Q > Q^*$, the optimal prices are

$$p_{11}^* = \frac{a_{11} - b_1 v}{2 b_1},$$

$$p_{21}^* = \frac{a_{21} - b_2 v}{2 b_2},$$

(2) If $Q < Q^*$, the optimal prices are

$$p_{11}^* = \frac{a_{11}b_2 + (2a_{11} + a_{21} - 2Q)b_1}{2b_1(b_1 + b_2)},$$

$$p_{21}^* = \frac{a_{21}b_1 + (a_{11} + 2a_{21} - 2Q)b_2}{2b_2(b_1 + b_2)},$$

where $Q^* = \frac{1}{2}(a_{11} + a_{21} + vb_1 + vb_2)$.

Proof. See the Appendix.

Q^* represents a threshold point of the inventory, which is determined by the total potential demand, unit clearing loss, and price sensitivities. It is derived based on the first-order condition that the marginal profit with respect to the change of prices (p_{11} , p_{12}) equals zero because of the quasi-concavity of $\pi(p_{11}, p_{21})$ holds. When $Q=Q^*$, the optimal prices in (1) and (2) are the same, and the demand just equals to inventory; when the inventory is larger than the threshold point, the retailer will have some unsold at the end of period 2 with optimal prices of Theorem 1(1), the unsold inventory will then be cleared with a salvage price; When the inventory is less than the threshold point, the retailer will sell out the entire inventory with optimal prices of Theorem 1(2).

Proposition 1. When only the traditional channel is employed for both periods with dynamic pricing,

(1) If $Q > Q^*$, p_{11}^* increases with a_{11} , but keeps a constant with a_{21} ; p_{21}^* increases with a_{21} , but keeps a constant with a_{11} ;

(2) If $Q < Q^*$, p_{11}^* increases with a_{11} and a_{21} ; p_{21}^* increases with a_{11} and a_{21} ;

where $Q^* = \frac{1}{2}(a_{11} + a_{21} + vb_1 + vb_2)$.

Proposition 1[†] suggests that if the inventory of the product is sufficiently large, then the price in a period only be affected by its own potential demand. But if the inventory is not very large, the price increases with the potential demand in both periods.

4.1.2 Mode 2: Online Channel As a Supplement

In this mode, the retailer introduces the online channel in the discount period to maximize its revenue. First, the retailer uses the traditional channel in the premium period, and then it employs both

[†]Because the proof of most propositions is straightforward, we did not present it due to page limit.

traditional and the online channels in the discount period. Thus the retailer's payoff includes the revenue from the traditional channel in the two periods and that from the online channel in the discount period. The total revenue for the retailer can be expressed as follows:

$$\begin{aligned}\pi(p_{11}, p_{21}, p_{22}) = & p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ & + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D)\end{aligned}\quad (2)$$

s.t. $D \leq Q$, where $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}$ represents the total demand.

The first and second terms represent the revenue from the traditional channel in the two periods. The third term is the revenue from the online channel in the discount period. The last term is the loss of the remaining inventory at the end of the sales season.

With the Lagrangian multiplier, the optimization problem (2) can be written as follows:

$$\begin{aligned}\max \pi(p_{11}, p_{21}, p_{22}) = & \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] \\ & - v(Q - D) + \lambda_1(Q - D) \}\end{aligned}$$

where $\lambda_1 \geq 0$, $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}$.

Theorem 2. When the retailer sells the product with the traditional channel in both periods and the online channel in the discount period,

(1) If $Q \geq Q^*$, the optimal prices are

$$\begin{aligned}p_{11}^* &= \frac{a_{11} - vb_1}{2b_1} \\ p_{21}^* &= \frac{\theta(a_{21} + a_{22}) + b_2 a_{21} - vb_2^2 - 2\theta vb_2}{2b_2(b_2 + 2\theta)} \\ p_{22}^* &= \frac{\theta(a_{21} + a_{22}) + b_2 a_{22} - vb_2^2 - 2\theta vb_2}{2b_2(b_2 + 2\theta)}\end{aligned}\quad (3)$$

(2) If $Q < Q^*$, the optimal prices are

$$\begin{aligned}p_{11}^* &= \frac{2a_{11}b_2 + b_1(2a_{11} - 2Q + a_{21} + a_{22})}{2b_1(b_1 + 2b_2)} \\ p_{21}^* &= \frac{b_2^2[a_{11} + 3a_{21} + a_{22} - 2Q] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q] + \theta b_1(a_{21} + a_{22}) + b_1 b_2 a_{21}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)} \\ p_{22}^* &= \frac{b_2^2[a_{11} + 3a_{22} + a_{21} - 2Q] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q] + \theta b_1(a_{21} + a_{22}) + b_1 b_2 a_{22}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)}\end{aligned}\quad (4)$$

where $Q^* = \frac{1}{2}(a_{11} + a_{21} + a_{22} + vb_1 + 2vb_2)$.

Proof. See the Appendix.

Similar to Theorem 1, Q^* is a function of the total potential demand, unit clearing loss, and price sensitivities. It is derived based on the first-order condition that the marginal profit

with respect to the change of prices (p_{11}, p_{12}, p_{22}) equals to zero. Substituting the optimal prices (Eq. (3) and (4)) into the total demand expression, we can obtain that when Q is not very large, the retailer chooses to sell out the entire inventory by adjusting the prices. Otherwise, the prices are constant with Q , and the retailer's revenue decreases with Q when Q exceeds a threshold point Q^* . Therefore, the optimal order quantity should be less than the threshold point. For a fixed inventory, the retailer tends to sell out the entire inventory when the potential demand or the unit loss incurred by the remaining inventory is large.

Proposition 2. When the retailer sells the product with the traditional channel in both periods and the online channel in the discount period,

- (1) If $Q > \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]$, the optimal prices keep constant with Q ;
- (2) If $Q < \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]$, the optimal prices decrease with Q .

Proposition 3. When the retailer sells the product with the traditional channel in both periods and the online channel in the discount period,

- (1) p_{11}^* keeps constant with θ ;
- (2) p_{21}^* decreases and p_{22}^* increases with θ .

Proof. See the Appendix.

Proposition 3 suggests that the retailer should decrease the price difference between the online and traditional channels to increase the revenue as consumers' sensitivity of switch-over towards price difference increases.

Proposition 4. Regarding the price difference of the traditional channel in the two periods:

- (1) Mode 2 (dual-channel) has a larger price difference between the two periods than Mode 1 (single-channel).
- (2) Mode 2 has the same price as Mode 1 in the premium period, but it has a lower price than Mode 1 in the discount period when $Q > \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]$.

Proof. See the Appendix. Proposition 4 illustrates the price change of the traditional channel when the online channel is introduced in the discount period.

Proposition 5. When the retailer sells the product with the traditional channel in both periods and the online channel in the discount period,

(1) If $Q > Q^*$, p_{11}^* increases with a_{11} , but keeps a constant with a_{21} and a_{22} ; p_{21}^* and p_{22}^* increases with a_{21} and a_{22} , but keeps a constant with a_{11} ;

(2) If $Q < Q^*$, p_{11}^* , p_{21}^* and p_{22}^* increases with a_{11} , a_{21} and a_{22} ;

where $Q^* = \frac{1}{2}(a_{11} + a_{21} + a_{22} + vb_1 + 2vb_2)$.

Proposition 5 is in the same line as suggested in Proposition 1.

4.2 Extended Models

4.2.1 Return Rate

Due to asymmetric information, product returns occur when the product does not fit consumer preference. Compared with the traditional channel, the return rate of the online channel is much higher. According to Grimaldi(2013), the return rate of the traditional channel is about 8.7%, while that of the online channel is 18%-35%. Therefore, this subsection focuses on the effect of the online channel's return rate on pricing.

We assume that the return rate of the online channel is τ in the discount period and the loss incurred by each returned product is denoted as v_1 . For the convenience of analysis, we treat the return rate of the traditional channel to be zero. Please note that the analysis results can still be applied when the return rate of the traditional channel is non-zero, but the interpretation of τ will be the difference of the return rates between the two channels.

We now have the retailer's revenue function as follows:

$$\begin{aligned} \pi(p_{11}, p_{21}, p_{22}) = & p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] \\ & - \tau v_1[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) \end{aligned} \quad (5)$$

$$\text{s.t. } D \leq Q, \text{ where } D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}.$$

The first and second terms are the revenue from the traditional channel in the two periods. The third term is the revenue from the online channel in the discount period. The fourth term is the loss due to the return, and the last term is the loss due to the leftover inventory. With the Lagrangian multiplier, the optimization problem (5) can be written as follows:

$$\max \pi(p_{11}, p_{21}, p_{22}) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - \tau v_1[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) + \lambda_1(Q - D) \}$$

where $\lambda_1 \geq 0$ and $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}$.

Theorem 3. Given the return rate of the online channel τ ,

(1) If $Q \geq Q^*$, the optimal prices are

$$\begin{aligned} p_{11}^* &= \frac{a_{11} - v b_1}{2 b_1} \\ p_{21}^* &= \frac{b_2 a_{21} + \theta(a_{21} + a_{22})}{2 b_2 (b_2 + 2\theta)} - \frac{v}{2} \\ p_{22}^* &= \frac{\tau v_1 - v}{2} + \frac{a_{21} + a_{22}}{4 b_2} - \frac{a_{21} - a_{22}}{4(b_2 + 2\theta)}. \end{aligned}$$

(2) If $Q < Q^*$, the optimal prices are

$$\begin{aligned} p_{11}^* &= \frac{2a_{11}b_2 - b_1b_2\tau v_1 + b_1(2a_{11} - 2Q + a_{21} + a_{22})}{2b_1(b_1 + 2b_2)} \\ p_{21}^* &= \frac{b_2^2[a_{11} + 3a_{21} + a_{22} - 2Q - 2\theta\tau v_1 - b_2\tau v_1] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q] + \theta b_1(a_{21} + a_{22}) + b_1b_2a_{21}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)} \\ p_{22}^* &= \frac{b_2^2[a_{11} + 3a_{21} + a_{22} - 2Q + 2\theta\tau v_1 + b_1\tau v_1 + b_2\tau v_1] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q + 2b_1\tau v_1] + \theta b_1(a_{21} + a_{22}) + b_1b_2a_{22}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)}, \end{aligned}$$

where $Q^* = \frac{1}{2}(a_{11} + a_{21} + a_{22} - \tau v_1 b_2 + v b_1 + 2v b_2)$.

Proof. The proof is similar to Theorem 2. And the derivation of Q^* is similar to that in Theorem 2.

Proposition 6. Given the return rate of the online channel τ ,

(1) If $Q \geq Q^*$, p_{11}^*, p_{21}^* keep constant with τ , but p_{22}^* increases with τ .

(2) If $Q < Q^*$, p_{11}^*, p_{21}^* decrease with τ , but p_{22}^* increases with τ .

Proposition 6 means that the prices of the traditional channel are not affected by the return rate of the online channel when the inventory is less than the threshold point Q^* . Otherwise, the prices of the traditional channel decrease to induce more consumers to purchase the products via the traditional channel. And the threshold point of inventory, Q^* decreases with τ . From Theorem 3, we can see that the retailer tends to keep the remaining inventory when the return rate is large instead of selling out the entire inventory.

4.2.2 Service Difference

Service quality is an important factor affecting consumers' purchasing decisions. The

traditional channel could offer customer services and experience that cannot be provided by the online channel. For example, consumers could try on shoes and clothes in a bricks-and-mortar store, experiencing the product personally. Consumers may also enjoy after-sale services that are offered in a bricks-and-mortar store (Wagner, 2002). Consumers who value such services are more likely to purchase from the traditional channel (Yao & Liu, 2005).

The demand function of the traditional channel in the premium period is $D_{11} = a_{11} - b_1 p_{11}$. In the discount period, we assume that the demand function of the traditional channel is $D_{21} = a_{21} - b_2 p_{21} + \theta_1(p_{22} - p_{21}) + \theta_2 S$; the demand function of the online channel is $D_{22} = a_{22} - b_2 p_{22} + \theta_1(p_{21} - p_{22}) - \theta_2 S$, where S represents the service difference, and θ_2 represents consumers' sensitivity of switch-over towards the service difference.

The model has two main features when we incorporate the service difference between the two channels. First, the demand switch-over caused by the price difference θ_1 is different from the service difference θ_2 . The price difference between the traditional channel and the online channel is caused by not only service, but also channel convenient and consumers' acceptance of the online channel (Cattani, Perdikaki, & Maruchek 2007; Fruchter & Tapiero, 2005; Tsay & Agrawal, 2004). To analyze the effect of the service difference on demand switch-over, the service difference is singled out. Second, please note that $D_{21} + D_{22} = a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}$, the total demand in Period 2 is affected only by prices but not by service. The service difference only affects demand switch-over between these two channels. Then we have the retailer's revenue function:

$$\begin{aligned} \pi(p_{11}, p_{21}, p_{22}) &= p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta_1(p_{22} - p_{21}) + \theta_2 S] \\ &\quad + p_{22}[a_{22} - b_2 p_{22} + \theta_1(p_{22} - p_{21}) - \theta_2 S] - v(Q - D_{11} - D_{21} - D_{22}) \\ \text{s.t. } D_{11} + D_{21} + D_{22} &\leq Q. \end{aligned}$$

The first and second terms are the traditional channel's revenue in the two periods. The third term is the online channel's revenue in the discount period. The last term is the loss due to the leftover inventory.

Theorem 4. With the service difference between two channels,

- (1) If $Q \geq Q^*$, the optimal prices are

$$\begin{aligned}
p_{11}^* &= \frac{a_{11} - vb_1}{2b_1} \\
p_{21}^* &= \frac{\theta_1(a_{21} + a_{22}) - 2\theta_1vb_2 - vb_2^2 + b_2(a_{21} + \theta_2S)}{2b_2(b_2 + 2\theta_1)} \\
p_{22}^* &= \frac{\theta_1(a_{21} + a_{22}) - 2\theta_1vb_2 - vb_2^2 + b_2(a_{22} - \theta_2S)}{2b_2(b_2 + 2\theta_1)} ;
\end{aligned}$$

(2) If $Q < Q^*$, the optimal prices are,

$$\begin{aligned}
p_{11}^* &= \frac{2a_{11}b_2 + b_1(2a_{11} - 2Q + a_{21} + a_{22})}{2b_1(b_1 + 2b_2)} \\
p_{21}^* &= \frac{b_2^2(a_{11} + 3a_{21} + a_{22} - 2Q) + \theta_1b_2[2a_{11} + 4(a_{21} + a_{22} - Q)] + \theta_1b_1(a_{21} + a_{22}) + b_1b_2a_{21}}{2b_2(b_2 + 2\theta_1)(b_1 + 2b_2)} + \frac{S\theta_2}{2(b_2 + 2\theta_1)} \\
p_{22}^* &= \frac{b_2^2(a_{11} + 3a_{22} + a_{21} - 2Q) + \theta_1b_2[2a_{11} + 4(a_{21} + a_{22} - Q)] + \theta_1b_1(a_{21} + a_{22}) + b_1b_2a_{22}}{2b_2(b_2 + 2\theta_1)(b_1 + 2b_2)} - \frac{S\theta_2}{2(b_2 + 2\theta_1)}
\end{aligned}$$

where $Q^* = \frac{a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)}{2}$.

Proof. The proof is similar to Theorem 2.

Proposition 7. With the service difference between two channels, p_{11}^* keeps constant with S ; p_{21}^* increases with S , and p_{21}^* decreases with S .

Proposition 8. With the service difference between two channels, p_{11}^* keeps constant with θ_2 ; p_{21}^* increases with, and p_{21}^* decreases with θ_2 .

Propositions 7 and 8 show that the price of the traditional channel in the premium period is not affected by the service difference. And the price difference between the traditional and the online channels in the discount period decreases when consumers are sensitive to the service difference. The threshold point of inventory ($\frac{a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)}{2}$ in Theorem 4) keeps constant with S .

5. The Retailer's Risk Attitude

The demand of the online channel could be affected by some non-product or service-related factors such as Internet accessibility and consumers' acceptance that may be difficult for the retailer to observe (Chiang, Chhajed, & Hess, 2003). Therefore, in this subsection, we assume that the online channel demand is a random variable, and examine how the retailer's risk attitude affects its pricing strategy. Prior studies suggested risk attitude as an important factor affecting a firm's decision (Borgonovo & Peccati, 2009).

We assume the demand function of the online channel in the discount period as:

$$D_{22} = \tilde{a}_{22} - b_2 p_{22} + \theta(p_{21} - p_{22}),$$

where $\tilde{a}_{22}$ is a random variable, it represents the potential demand of the online channel in Period 2.

We assume $\tilde{a}_{22} = a_{22} + \varepsilon$, where a_{22} is the mean of the potential intrinsic demand, ε follows a normal distribution, and $E(\varepsilon) = 0, Var(\varepsilon) = \sigma^2$.

Since the online demand in the discount period is stochastic, we should consider the risk attitude of the retailer into its pricing decision. The preference theory provides a framework for incorporating decision makers' risk propensity in their decision process. The valuation measure is known in the preference theory as "certainty equivalent". It is defined as a certain value for an uncertain event which a decision maker is willing to accept (Raiffa, 1968; Holloway, 1979). One form of the utility function dominant in both theoretical and applied work in areas of decision theory and finance is the exponential utility function $U(\pi) = -\exp(-\pi / R)$, where R is the retailer's risk tolerance level and π is revenue. A value of $R < \infty$ implies risk-averse behavior. When R approaches ∞ , risk-neutral behavior is implied (Howard, 1988). Correspondingly, the certainty equivalent of π in the basic model is expressed by the following equation: $E(U) = E(\pi) - \frac{Var(\pi)}{2R}$, where R is the retailer's risk tolerance level, $Var(\pi)$ is the variance of revenue π . We have $Var(\pi) = (p_{22} + v)^2 \sigma^2$, when $Q > D$; $Var(\pi) = p_{22}^2 \sigma^2$, when $Q \leq D$. Therefore, the optimization problem is to maximize the expected utility $E(U)$, as follows:

$$E(U) = p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) - \frac{Var(\pi)}{2R}$$

s.t. $D \leq Q$, where $D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}$ represents the total demand.

Theorem 5. When the retailer sells the product with the traditional channel in both periods and the online channel in the discount period,

(1) If $Q \geq Q^*$, the optimal prices are

$$\begin{aligned}
p_{11}^* &= \frac{a_{11} - vb_1}{2b_1} \\
p_{21}^* &= \frac{\theta(a_{21} + a_{22})R + b_2a_{21}R - vb_2^2R - 2\theta vb_2R}{2b_2(b_2 + 2\theta)R + \sigma^2(b_2 + \theta)} + \frac{\sigma^2(a_{21} - 2\theta v - b_2v)}{4b_2(b_2 + 2\theta)R + 2\sigma^2(b_2 + \theta)}; \\
p_{22}^* &= \frac{[\theta(a_{21} + a_{22}) + b_2a_{22}]R - \sigma^2v(\theta + b_2)/2}{2b_2(b_2 + 2\theta)R + \sigma^2(b_2 + \theta)} - \frac{v}{2}
\end{aligned}$$

(2) If $Q < Q^*$, the optimal prices are

$$\begin{aligned}
p_{11}^* &= \frac{b_2^2[a_{11}\sigma^2 + 4Ra_{11}(b_1 + b_2 + 2\theta) + 2Rb_1(-2Q + a_{21} + a_{22})] + \sigma^2[b_1b_2(-2Q + 2a_{11} + a_{21} + 2a_{22}) + 2b_1\theta(-Q + a_{11} + a_{21} + a_{22})] + 4Rb_1b_2\theta(-2Q + 2a_{11} + a_{21} + a_{22})}{2b_1[2b_2^2R(2b_2 + b_1 + 4\theta) + \sigma^2(b_1b_2 + b_1\theta + b_2^2) + 4Rb_1b_2\theta]} \\
p_{21}^* &= \frac{b_2[(\sigma^2 + 4R\theta)(a_{11} + 2a_{21} + 2a_{22} - 2Q) + 2Ra_{21}b_1] + 2b_2^2R(a_{11} + 3a_{21} + a_{22} - 2Q) + 2Rb_1\theta(a_{21} + a_{22}) + a_{21}b_1\sigma^2}{2[2Rb_2^2(2b_2 + b_1 + 4\theta) + \sigma^2(b_2^2 + b_1b_2 + b_1\theta) + 4Rb_1b_2\theta]} \\
p_{22}^* &= \frac{b_2^2R(a_{11} + 3a_{22} + a_{21} - 2Q) + 2b_2R\theta(a_{11} + 2a_{21} + 2a_{22} - 2Q) + Rb_1\theta(a_{21} + a_{22}) + a_{22}b_1b_2R}{2Rb_2^2(2b_2 + b_1 + 4\theta) + \sigma^2(b_2^2 + b_1b_2 + b_1\theta) + 4Rb_1b_2\theta}
\end{aligned}$$

where

$$Q^* = \frac{2Rb_2(b_2 + 2\theta)(a_{11} + a_{21} + a_{22} + b_1v + 2b_2v) + \sigma^2[(a_{11} + a_{21} + a_{22})(b_2 + \theta) + b_2v(b_1 + 4\theta + 3b_2) + \theta(b_1v + a_{21} + a_{22})]}{2[2Rb_2(b_2 + 2\theta) + \sigma^2(b_2 + \theta)]}.$$

Proof. See the Appendix.

Given the complexity of analytical expressions given in Theorem 5, we will use a numerical example to explore some properties of the optimal solutions. Let $\sigma = 10$, $Q = 850$, $a_{11} = 600$, $a_{21} = 520$, $a_{22} = 280$, $b_1 = 1$, $b_2 = 2$, $\theta = 4$, $v = 20$, and $R \in [20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220]$. From Fig. 2, we can see that the optimal prices are determined by Theorem 5(2) when $R \leq 140$; and the optimal prices are determined by Theorem 5(1) when $R \geq 160$.

Fig.2 Channel price as a function of R

The prices of the two channels in the discount period decrease with the presence of demand uncertainty of the online channel, but increase with the retailer's risk tolerance level. When the retailer

is much risk averse ($R \leq 140$), it prefers to set lower prices in both periods than those without demand uncertainty. As its risk tolerance increases ($R \geq 160$), it tends to set higher prices to obtain the maximum revenue rather than selling out the entire inventory.

Specifically, when the retailer is extremely risk averse (when $R < 30$), the price of the traditional channel in the premium period is higher and the price of the online channel in the discount period is lower than that without demand uncertainty. In other words, the retailer will keep the traditional channel profitable, but reduce its online price to deal with online demand uncertainty. When the retailer's risk tolerance increases ($40 \leq R \leq 140$), the retailer adjusts both the premium price and the discount price of the two channels to deal with demand uncertainty. If the retailer's risk tolerance is very high ($R \geq 160$), it only changes prices in the discount period to deal with online demand uncertainty.

Fig.3 Channel price as a function of the standard deviation σ

Fig.3 presents how the prices change with the standard deviation of online demand. When the standard deviation is small ($0 \leq \sigma \leq 10$ in this numerical example), the premium and the discount prices of both channels (p_{11} , p_{21} , p_{22}) decrease with the standard deviation. In other words, the retailer decreases the premium and the discount prices to deal with the increasing uncertainty in this situation. When the standard deviation of the online demand is large ($10 \leq \sigma \leq 50$), the discount price of the online channel (p_{22}) decreases, the premium price of the traditional channel (p_{11}) increases, and the discount price of the traditional channel (p_{21}) keeps constant. In other words, the retailer increases the premium price of the traditional channel and decreases the discount price of the online channel to deal with the increasing uncertainty in this situation.

The revenue change with demand uncertainty is given in Appendix Figs.A1 and A2. The expected revenue with demand uncertainty is lower than the revenue with deterministic demand, and the expected revenue increases with the retailer's risk tolerance but decreases with the standard deviation.

6. Numerical Examples

In this section, we explore some scenarios to verify the results obtained in the above sections and offer more insights. The general model parameters are set as follows: in the premium period the potential demand $a_{11} = 600$, and the price sensitivity $b_1 = 1$. In the discount period, the potential demand of the traditional and the online channels are $a_{21} = 520$ and $a_{22} = 280$, respectively, and the price sensitivity is $b_2 = 2$. The sensitivity of switch-over towards price difference is $\theta = 4$. The unit loss caused by the remaining inventory is $v = 20$. These values satisfy Assumptions 2, 3, and 4.

6.1. Comparison of Channel Selection Solutions

6.1.1. Comparison of Single Channel and Dual Channels

This subsection focuses on analyzing the effect of the online channel's potential demand on product prices and retail revenue with the two basic modes mentioned in Section 4.1. The parameters are set as follows: $Q = 800$, $a_{22} \in [0, 50, 100, 150, 200, 250, 300]$. When $a_{22} = 0$, it is equivalent to Mode 1.

Table 1 Comparison of Optimal Solutions

	p_{11}	p_{21}	p_{22}	D_{11}	D_{21}	D_{22}	π
Single Channel	290	120	--	310	280	--	119300
Dual-Channel a_{22}							
50	290	70	50	310	280	40	109270
100	290	78	57	310	280	70	112930
150	290	80	60	310	280	100	116970
200	290	88	72	310	280	120	121380
250	290	90	80	310	280	140	126170
300	290	98	87	310	280	170	131330

From Table 1, we can see that introducing the online channel in the discount period could decrease the leftover inventory, but it also makes the price of the traditional channel lower in the

discount period. The results indicate the occurrence of channel conflict. If the potential demand of the online channel is small, the retailer has less revenue with dual channels than with single channel. If the potential demand of the online channel is large, the retailer has more revenue with dual channels. The price of the traditional channel in the discount period increases with the potential demand of the online channel. Therefore, introducing the online channel could decrease the unsold inventory effectively. However, whether the dual-channel mode benefits the retailer depends on the potential demand of the online channel.

6.2.2. Comparison of Partial and Complete Dual Channels

This subsection examines the scenario in which the online channel is introduced in both periods (complete dual channels). We compare the optimal prices of this scenario with those of the scenario when the online channel only introduced in the discount period (partial dual channels) as we discussed above. We assume that the potential demand of the online channel in the premium period, $a_{12} = 300$, other parameters are the same as in the general setting.

Table 2 Optimal Prices and Revenues with Partial Dual Channels

Q	p_{11}	p_{21}	p_{22}	D_{11}	D_{21}	D_{22}	π
600	320	126	114	280	220	100	128720
550	330	136	124	270	200	80	126220
500	340	146	134	260	180	60	122720
450	350	156	144	250	160	40	118220
400	360	166	154	240	140	20	112720
350	370	176	164	230	120	0	106220
300	380	186	174	220	100	0	98720

Table 3 Optimal Prices and Revenues with Complete Dual Channels

Q	p_{11}	p_{12}	p_{21}	p_{22}	D_{11}	D_{12}	D_{21}	D_{22}	π
600	275	258	148	136	258	108	177	57	132800
550	283	267	156	144	250	100	160	40	128220
500	292	275	164	152	242	92	143	23	122800
450	300	283	173	161	233	83	127	7	116550
400	308	292	181	169	225	75	110	0	109470
350	317	300	189	177	217	67	93	0	101550
300	325	308	198	186	208	58	77	0	92803

The results in Tables 2 and 3 suggest that the complete-dual-channels is profitable when inventory is sufficiently large. When the inventory is not very large, the partial dual channels are more

profitable. If the inventory is small (i.e., $Q < 400$) the single-channel solution is more profitable than other channel solutions. Introducing the online channel in the premium period decreases the premium price of the traditional channel but increases the prices of both channels in the discount period. The results suggest channel conflict. When $Q = 400$, the two channels only co-exist in Period 1 in Table 3. By comparing Tables 2 and 3, we can see that introducing the online channel in the discount period (Table 2) results in more revenue than using the online channel in the premium period (Table 3).

In summary, the retailer shall adopt the traditional channel alone when the inventory level is very small. When the inventory is large, the retailer could introduce the online channel in the discount period. When the inventory is extremely large, the retailer will benefit from introducing the two channels in both periods. The results are summarized in Table 4. $Q_i (i=1,2)$ in Table 4 are thresholds determined by the channel potential demand $a_{ij} (i,j=1,2)$, the marginal demand per price $b_i (i=1,2)$, the sensitivity of switch-over towards price difference $\theta_i (i=1,2)$, and the unit loss of the product incurred by clearing v .

Table 4 Optimal Channels Choices with Inventory Level Q

Inventory level Q	Premium period	Discount period
$Q < Q_1$	Traditional channel	Traditional channel
$Q_1 < Q < Q_2$	Traditional channel	Traditional +online channels
$Q > Q_2$	Traditional +online channels	Traditional +online channels

6.3. Channel Conflict

This subsection focuses on the effect of dynamic pricing on channel conflict. We present following three scenarios: (1) the traditional channel employs static pricing in the two periods (i.e., single-static); (2) extending scenario 1, we introduce the online channel in the discount period (i.e., dual-static); (3) the traditional channel employs dynamic pricing in the two periods, and the online channel is introduced in the discount period (i.e., dual-dynamic). The inventory $Q \in [600, 800]$ and other parameters are the same as in the general setting.

Figs. 4 and 5 show that when the inventory exceeds a certain threshold point Q^* , the prices and the revenues tend to be fixed. In the scenario of the single channel with static pricing, the threshold point $Q^* < 600$. In other scenarios the threshold point $Q^* = 700$. In addition, Fig. 4 indicates that the premium price with the dual-static mode is lower than that with the single-static mode. That is, introducing the online channel decreases the price of the traditional channel when the retailer employs

static pricing for the traditional channel; however, when the retailer employs dynamic pricing for the traditional channel, the premium price with the dual-dynamic mode is much larger than that with the dual-static mode and the single-static mode, and the discount price is much lower than that with the dual-static mode. By implementing dynamic pricing, the price difference becomes larger, and retailer's revenue increases compared with static price (Fig. 5).

Fig.4 Channel price as a function of Q

Fig.5 Total revenue as a function of Q

Fig.5 shows the total revenue decreases after introducing the online channel when static pricing is used for the traditional channel. Figs. A3 and A4 in Appendix A illustrate the change of the

revenues of the two channels respectively. The results suggest the existence of channel conflict. But when dynamic pricing is used for the traditional channel, both the revenue of the traditional channel and the total revenue increase. In other words, dynamic pricing mitigates channel conflict.

6.4. Assumption Violation

This subsection explores the changes of optimal solutions when Assumptions 3 and 4 are not satisfied.

Example 1

In this example, we present the change of optimal solutions when Assumption 4 is violated which assumes that the potential demands in the two periods with two channels satisfy $a_{11} > a_{21} > a_{22}$. Similar to Yao & Liu (2005), assuming the total potential demand in the discount period is fixed, we discuss how the prices change with the proportion of the online demand. Let α denote the proportion of the online demand in period 2. The total potential demands of the traditional channel and the online channel in the discount period are $a_{21} + a_{22} = 600$. When $\alpha < 0.5$, $a_{11} > a_{21} > a_{22}$; or else $a_{11} > a_{22} > a_{21}$. Other parameters are set as follows: $Q = 700, a_{11} = 600, b_1 = 1, b_2 = 2, \theta = 4$, and $v = 20$. Table 5 shows that prices of the traditional channel decrease, while the price of online channel increases, when the potential demand of online channel increases. The total revenue reaches the minimum when $\alpha = 0.5$. In other words, channel conflict is at its maximum when the potential demands of the two channels are equal.

Table 5 Optimal Prices and Revenues

α	p_{11}	p_{21}	p_{22}	D_{11}	D_{21}	D_{22}	Revenue
0.1	290	77	53	310	290	50	113880
0.2	290	74	56	310	260	80	112620
0.3	290	71	59	310	230	110	111720
0.4	290	68	62	310	200	140	111180
0.5	290	65	65	310	170	170	111000
0.6	290	62	68	310	140	200	111180
0.7	290	59	71	310	110	230	111720
0.8	290	56	74	310	80	260	112620
0.9	290	53	77	310	50	290	113880

Example 2

In this example, we present the change of optimal solutions when Assumption 3 is violated which assumes consumers' price sensitivities in the premium period and in the discount period satisfy $b_1 < b_2$. Let $b_1 = 1$, and $b_2 \in [0.5, 1.5]$. Consumers become more sensitive to price as b_2 increases. Other parameters are set as follows: $a_{21} = 520$, $a_{22} = 280$, $\theta = 4$ and $v = 20$. Table 6 suggests that the prices of the two channels and sales revenues decrease as b_2 increases. When the price sensitivity in Period 1 is much larger than that in Period 2 ($b_1 > b_2 \in [0.5, 0.6]$), the prices in Period 1 are lower than that in period 2. In other words, Period 1 is the discount period and Period 2 becomes the premium period. However, p_{21} is smaller than p_{11} when $b_2 \in [0.7, 1)$. This is because the prices are not only affected by price sensitivities, but also affected by the potential demands in the two periods. When the difference of price sensitivities in the two periods is small and the total potential demand in Period 2 is larger than that in Period 1, the price in Period 2 with small price sensitivity may be even lower than that in Period 1 with large price sensitivity.

Table 6 Optimal Prices and Revenues

b_2	p_{11}	p_{21}	p_{22}	D_{11}	D_{21}	D_{22}	Revenue
0.5	300	407	393	300	260	140	250850
0.6	300	340	326	300	260	140	224170
0.7	300	293	279	300	260	140	205110
0.8	300	257	243	300	260	140	190820
0.9	300	229	215	300	260	140	179700
1	300	207	193	300	260	140	170800
1.1	300	188	175	300	260	140	163520
1.2	300	173	160	300	260	140	157450
1.3	300	160	147	300	260	140	152310
1.4	300	149	136	300	260	140	147910
1.5	300	140	127	300	260	140	144090

Example 3

This example examines the change of optimal solutions when both Assumption 3 and 4 are violated. Assume that the potential demand, $a_{11} = 280$, $a_{21} = 520$, $a_{22} = 600$, $b_1 = 2$, and $b_2 = 1$. Similar to Example 2, Period 2 becomes the premium period with higher price and strong demand, and Period 1 is the discount period. In Period 2 (the premium period), both channels are used. And the

online channel is the main channel. And in Period 1 (the discount period) only the traditional channel is used. Thus this case is totally different from the modeling background we discussed in this paper. It could be corresponding to when the retailer sells the product whose value increases over time on the market, such as when selling the air tickets, and the retailer mainly uses the online channel to reach consumers.

From Table 7, we can see that the optimal prices in Period 2 are higher than that of Period 1. And the price of the online channel is higher than that of the traditional channel which is due to a large potential demand of the online channel. The prices in the two periods decrease with the inventory level Q . The prices of the traditional channel in the two periods are identical when the inventory exceeds a threshold point. In other words, when the supplement channel (in this case, the traditional channel) is introduced in both periods and the inventory is very large, the prices of the supplement channel in both periods should keep constant.

Table 7 Optimal Prices and Revenues

Q	p_{11}	p_{21}	p_{22}	D_{11}	D_{21}	D_{22}	Revenue
700	140	210	210	0	330	370	147090
750	140	180	190	0	360	390	138840
800	140	140	160	0	380	420	128090
850	131	131	157	18	494	338	120164
900	118	118	145	43	507	350	115789
950	106	106	132	68	519	363	110164
1000	93	93	120	93	532	375	103289

7. Discussion and Conclusion

The online channel plays different roles for different industries. This study focuses on the sales of a short-life-cycle product as in the apparel industry using the online channels as a supplement to the traditional channel. We consider a retailer can employ both the traditional and the online channels, but mostly relies on the traditional channel. We explore such a channel mix: the retailer uses the traditional channel in the premium period, and both channels in the discount period.

The use of dynamic pricing for the traditional channel captures the nature of short-life cycle products. Such a strategy is consistent with prior studies about perishable products (Zhao & Zheng, 2000; Aviv & Pazgal, 2008). Further, the dynamic pricing mechanism can also mitigate channel conflict effectively. If the retailer uses static pricing for the traditional channel in the two periods, then sales revenue will decrease when the online channel is introduced in Period 2 (Fig. 5 and Figs. A3 and

A4in Appendix).

Our results also suggest that price and service differences between these two channels will not change the optimal price in the premium period when the online channel is introduced in the discount period. When the product inventory is small, the optimal price in the premium period increases (Theorems 1-4). In other words, the introduction of the online channel in the discount period will not decrease the premium price of the traditional channel, but rather increase the premium price when the inventory is not very large. Thus, choosing the right time to introduce the online channel could improve the retailer's revenue and mitigate channel conflict.

The study further shows that whether dual channels benefit the retailer is contingent on the potential demand of the online channel and the product inventory (Tables 1-3). The retailer should adopt single channel when the demand of the online channel or the inventory is very small. Otherwise, the retailer will benefit from using both channels. And the online channel should be added first in the discount period, and then expanded to the premium period if needed.

The retailer's expected revenue with demand uncertainty is lower than the revenue with deterministic demand. The revenue increases with retailer's risk tolerance but decreases with the standard deviation of the online channel demand (Appendix Figs. A1 and A2). The retailer adjusts the premium price and the online channel price to sell out the entire inventory when it is risk-averse. The optimal prices in both periods decrease with the retailer's risk tolerance. When the retailer's risk tolerance level approaches infinite, only prices in the discount period decrease to deal with online uncertainty (Theorem 5 and Fig. 2).

In summary, there are three significant implications for practitioners. First, the retailer could mitigate channel conflict by choosing the right time to introduce the online channel. Secondly, the retailer could mitigate channel conflict by differentiating the prices between the two channels. Third, introducing the online channel could effectively reduce the unsold inventory. However, whether the dual-channel solution is beneficial to the retailer depends on the potential demand of the online channel and the product inventory.

A limitation of this study is that it assumes the demands in the two periods are independent. There is a possibility that the lower premium price will exhaust more demand in the premium period, and decrease the demand in the discount period. When the premium price is very high, customers may delay their purchase to the discount period. Hence the potential demand in the discount period

increases. If we model the potential demand in period 2 is affected by the premium price:

$$D_{21} = a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21}) + klp_{11}, \quad D_{22} = a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22}) + k(1-l)p_{11}, \quad D_{11} = a_{11} - b_1 p_{11}$$

, where k represents the demand switch-over toward price from period 1 to period 2, and $k < b_1$, then

the total demand of the two periods is $D = a_{11} + a_{21} + a_{22} - (b_1 - k)p_{11} - b_2 p_{21} - b_2 p_{22}$. Because of

mathematical complexity, we are not able to give analytical solutions. Given that the total demand in

the basic model is $D = a_{11} + a_{21} + a_{22} - b_1 p_{11} - b_2 p_{21} - b_2 p_{22}$, and that in the new model is

$$D = a_{11} + a_{21} + a_{22} - (b_1 - k)p_{11} - b_2 p_{21} - b_2 p_{22}, \quad \text{we cannot perform meaningful comparisons}$$

between the optimal solutions of the two models, so future studies could investigate this direction.

As consumers often compare utility before deciding where to purchase, future studies could investigate the effect of consumers' strategic choices on channel conflict and dynamic pricing. In addition, since the apparel industry suffers from the problem of maintaining high inventory, future research could explore the optimal inventory when the retailer employs dual channels. Further, as the model parameters may vary from industry to industry, it would be a good idea to obtain more accurate and realistic parameter values to give better empirical implications.

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Appendix

Proof of Theorem 1.

$$\max \pi(p_{11}, p_{21}) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{12}(a_{21} - b_2 p_{21}) - v(Q - D) + \lambda_1(Q - D) \}$$

where $\lambda_1 \geq 0, D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21}$.

(1) when $\lambda_1 = 0$, which implies $Q \geq D$, the optimization problem is

$$\max \pi(p_{11}, p_{21}) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{12}(a_{21} - b_2 p_{21}) - v(Q - D) \}$$

with Assumptions 1-4, we consider the first order conditions,

$$\begin{aligned} \frac{\partial \pi}{\partial p_{11}} &= a_{11} - 2b_1 p_{11} - b_1 v = 0, \\ \frac{\partial \pi}{\partial p_{21}} &= a_{21} - 2b_2 p_{21} - b_2 v = 0, \end{aligned}$$

then obtain the optimal prices

$$\begin{aligned} p_{11}^* &= \frac{a_{11} - vb_1}{2b_1}, \\ p_{21}^* &= \frac{a_{21} - vb_2}{2b_2}, \end{aligned}$$

Substituting p_{11}^*, p_{21}^* into D ,

$$D = a_{11} - b_1 p_{11}^* + a_{21} - b_2 p_{21}^* = a_{11} - \frac{b_1(a_{11} - vb_1)}{2b_1} + a_{21} - \frac{b_2(a_{21} - vb_2)}{2b_2} = \frac{a_{11} + a_{21} + vb_1 + vb_2}{2} \text{ then we}$$

have $D = \frac{1}{2}(a_{11} + a_{21} + vb_1 + vb_2)$. In this case, $Q \geq D$, that is, $Q \geq Q^*$, where $Q^* = \frac{1}{2}(a_{11} + a_{21} + vb_1 + vb_2)$.

(2) when $\lambda_1 > 0$, which implies $Q = D$, then the optimization problem is

$$\max \pi(p_{11}, p_{21}) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{12}(a_{21} - b_2 p_{21}) - v(Q - D) + \lambda_1(Q - D) \} \text{ with}$$

Assumptions 1-4, we consider the first order conditions,

$$\begin{aligned}\frac{\partial \pi}{\partial p_{11}} &= a_{11} + b_1 \lambda_1 - 2b_1 p_{11} - b_1 v = 0, \\ \frac{\partial \pi}{\partial p_{21}} &= a_{21} + b_2 \lambda_1 - 2b_2 p_{21} - b_2 v = 0, \\ \frac{\partial \pi}{\partial \lambda_1} &= Q - a_{11} - a_{21} + b_1 p_{11} + b_2 p_{21} = 0,\end{aligned}$$

then obtain the optimal solutions

$$\begin{aligned}p_{11}^* &= \frac{a_{11}b_2 + (2a_{11} + a_{21} - 2Q)b_1}{2b_1(b_1 + b_2)} \\ p_{21}^* &= \frac{a_{21}b_1 + (a_{11} + 2a_{21} - 2Q)b_2}{2b_2(b_1 + b_2)}. \\ \lambda_1 &= \frac{a_{11} + a_{21} - 2Q}{b_1 + 2b_2}\end{aligned}$$

And $\lambda_1 = \frac{a_{11} + a_{21} - 2Q}{b_1 + 2b_2} > 0$ is equivalent to $Q < \frac{1}{2}(a_{11} + a_{12} + vb_1 + vb_2)$, that is $Q < Q^*$, where

$$Q^* = \frac{1}{2}(a_{11} + a_{21} + vb_1 + vb_2).$$

Proof of Theorem 2.

$$\begin{aligned}\max \pi(p_{11}, p_{21}, p_{22}) &= \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ &\quad + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) + \lambda_1(Q - D) \end{aligned}$$

$$\text{where } \lambda_1 \geq 0, D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}.$$

(1) when $\lambda_1 = 0$, which implies $Q \geq D$, the optimization problem is

$$\begin{aligned}\max \pi(p_{11}, p_{21}, p_{22}) &= \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ &\quad + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) \end{aligned}$$

With the first order conditions, $\frac{\partial \pi}{\partial p_{11}} = 0, \frac{\partial \pi}{\partial p_{21}} = 0, \frac{\partial \pi}{\partial p_{22}} = 0$, using Matlab 6.5, we obtain the optimal

prices,

$$\begin{aligned}
p_{11}^* &= \frac{a_{11} - vb_1}{2b_1} \\
p_{21}^* &= \frac{\theta(a_{21} + a_{22}) + b_2 a_{21} - vb_2^2 - 2\theta vb_2}{2b_2(b_2 + 2\theta)}, \\
p_{22}^* &= \frac{\theta(a_{21} + a_{22}) + b_2 a_{22} - vb_2^2 - 2\theta vb_2}{2b_2(b_2 + 2\theta)}
\end{aligned}$$

Substituting $p_{11}^*, p_{12}^*, p_{22}^*$ into D_{11}, D_{12}, D_{22} , we have $D = D_{11} + D_{21} + D_{22}$

$$\begin{aligned}
&= a_{11} - b_1 p_{11}^* + a_{21} - b_2 p_{21}^* + \theta(p_{22}^* - p_{21}^*) + a_{22} - b_2 p_{22}^* + \theta(p_{21}^* - p_{22}^*) \\
&= \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]
\end{aligned}$$

The restriction condition $Q \geq D$ is equivalent to $Q \geq Q^*$, where

$$Q^* = \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)].$$

(2) When $\lambda_1 > 0$, which implies $Q = D$, then the optimization problem is

$$\begin{aligned}
\max \pi(p_{11}, p_{21}, p_{22}) &= \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\
&\quad + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) + \lambda_1(Q - D) \}
\end{aligned}$$

Let $\frac{\partial \pi}{\partial p_{11}} = 0, \frac{\partial \pi}{\partial p_{21}} = 0, \frac{\partial \pi}{\partial p_{22}} = 0, \frac{\partial \pi}{\partial \lambda_1} = 0$, using Matlab 6.5, the optimal solutions are

$$\begin{aligned}
p_{11}^* &= \frac{2a_1 b_2 + b_1(2a_{11} - 2Q + a_{21} + a_{22})}{2b_1(b_1 + 2b_2)}, \\
p_{21}^* &= \frac{b_2^2[a_{11} + 3a_{21} + a_{22} - 2Q] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q] + \theta b_1(a_{21} + a_{22}) + b_1 b_2 a_{21}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)}, \\
p_{22}^* &= \frac{b_2^2[a_{11} + 3a_{22} + a_{21} - 2Q] + \theta b_2[2a_{11} + 4a_{21} + 4a_{22} - 4Q] + \theta b_1(a_{21} + a_{22}) + b_1 b_2 a_{22}}{2b_2(b_2 + 2\theta)(b_1 + 2b_2)}
\end{aligned}$$

$$\lambda_1 = \frac{a_{11} + a_{21} + a_{22} - 2Q + b_1 v + 2b_2 v}{b_1 + 2b_2},$$

Because $\lambda_1 > 0$, it is equivalent to $Q < Q^*$, where

$$Q^* = \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)].$$

Proof of Proposition 3.

If $Q > \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]$,

$$\frac{\partial p_{11}^*}{\partial \theta} = 0, \frac{\partial p_{21}^*}{\partial \theta} = \frac{-(a_{21} - a_{22})}{4b_2^2(b_2 + 2\theta)^2}, \frac{\partial p_{22}^*}{\partial \theta} = \frac{a_{21} - a_{22}}{4b_2^2(b_2 + 2\theta)^2}.$$

If $Q < \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)]$, the derivatives are the same. From Assumption 4,

$a_{21} > a_{22}$, then we can prove Proposition 3.

Proof of Proposition 4.

(1) From Theorem 1, we can get the price difference of the traditional channel in Mode 1

$$\text{is } \Delta_1 = p_{11}^* - p_{21}^* = \frac{a_{11}b_2 - a_{21}b_1}{2b_1b_2} \text{ for both } Q \geq Q^* \text{ and } Q < Q^*;$$

From Theorem 2, we can get the price difference of the traditional channel in Mode 2 is

$$\Delta_2 = p_{11}^* - p_{21}^* = \frac{a_{11}b_2(b_2 + 2\theta) - a_{21}b_1b_2 - (a_{21} + a_{22})b_1\theta}{2b_1b_2(2\theta + b_2)} \text{ for both } Q \geq Q^* \text{ and } Q < Q^*;$$

$$\text{then } \Delta_1 - \Delta_2 = -\frac{(a_{21} - a_{22})\theta}{2b_2(b_2 + 2\theta)} < 0,$$

because $a_{21} > a_{22}$, hence Proposition 4(1) is obtained.

(2) It is obvious that the premium price are equal when

$$Q > \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)];$$

$$\text{and when } Q > \frac{1}{2}[a_{11} + a_{21} + a_{22} + v(b_1 + 2b_2)], p_{21}^*_{(\text{Mode 2})} - p_{21}^*_{(\text{Mode 1})} = -\frac{(a_{21} - a_{22})\theta}{2b_2(b_2 + 2\theta)} < 0,$$

hence Proposition 4(2) is obtained.

Proof of Theorem 5.

The optimization problem is as follows:

$$\begin{aligned} \max E(U) = \max \{ & p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ & + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) - \frac{Var(\pi)}{2R} \} \end{aligned}$$

$$\text{s.t. } D \leq Q, \quad D = a_{11} - b_1 p_{11} + a_{21} - b_2 p_{21} + a_{22} - b_2 p_{22}.$$

Using Lagrangian multiplier,

$$\begin{aligned} \max E(U) = \max \{ & p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ & + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) - \frac{Var(\pi)}{2R} + \lambda_1(Q - D) \} \end{aligned}$$

(1) When $\lambda_1 = 0$, which implies $Q \geq D$, the optimization problem is

$$\begin{aligned} \max E(U) = \max \{ & p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})] \\ & + p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) - \frac{(p_{22}\sigma + v\sigma)^2}{2R} \} \end{aligned}$$

With the first order conditions $\frac{\partial \pi}{\partial p_{11}} = 0, \frac{\partial \pi}{\partial p_{21}} = 0, \frac{\partial \pi}{\partial p_{22}} = 0$, using Matlab 6.5, we obtain the optimal

prices,

$$\begin{aligned} p_{11}^* &= \frac{a_{11} - vb_1}{2b_1} \\ p_{21}^* &= \frac{\theta(a_{21} + a_{22})R + b_2 a_{21} R - vb_2^2 R - 2\theta vb_2 R}{2b_2(b_2 + 2\theta)R + \sigma^2(b_2 + \theta)} + \frac{\sigma^2(a_{21} - 2\theta v - b_2 v)}{4b_2(b_2 + 2\theta)R + 2\sigma^2(b_2 + \theta)}; \\ p_{22}^* &= \frac{[\theta(a_{21} + a_{22}) + b_2 a_{22}]R - \sigma^2 v(\theta + b_2)/2}{2b_2(b_2 + 2\theta)R + \sigma^2(b_2 + \theta)} - \frac{v}{2} \end{aligned}$$

Substituting $p_{11}^*, p_{12}^*, p_{22}^*$ into D_{11}, D_{12}, D_{22} , we have

$$\begin{aligned} D = D_{11} + D_{21} + D_{22} &= a_{11} - b_1 p_{11}^* + a_{21} - b_2 p_{21}^* + \theta(p_{22}^* - p_{21}^*) + a_{22} - b_2 p_{22}^* + \theta(p_{21}^* - p_{22}^*) = Q^*, \text{ where} \\ Q^* &= \frac{2Rb_2(b_2 + 2\theta)(a_{11} + a_{21} + a_{22} + b_1 v + 2b_2 v) + \sigma^2[(a_{11} + a_{21} + a_{22})(b_2 + \theta) + b_2 v(b_1 + 4\theta + 3b_2) + \theta(b_1 v + a_{21} + a_{22})]}{2[2Rb_2(b_2 + 2\theta) + \sigma^2(b_2 + \theta)]}. \end{aligned}$$

The restriction condition $Q \geq D$ is equivalent to $Q \geq Q^*$.

(2) When $\lambda_1 > 0$, which implies $Q = D$, the problem can be described as follows:

$$\max E(U) = \max \{ p_{11}(a_{11} - b_1 p_{11}) + p_{21}[a_{21} - b_2 p_{21} + \theta(p_{22} - p_{21})]$$

$$+ p_{22}[a_{22} - b_2 p_{22} + \theta(p_{21} - p_{22})] - v(Q - D) - \frac{p_{22}^2 \sigma^2}{2R} + \lambda_1(Q - D) \},$$

With the first order conditions, $\frac{\partial \pi}{\partial p_{11}} = 0, \frac{\partial \pi}{\partial p_{21}} = 0, \frac{\partial \pi}{\partial p_{22}} = 0, \frac{\partial \pi}{\partial \lambda_1} = 0$, we can obtain the optimal prices

using Matlab 6.5,

$$p_{11}^* = \frac{b_2^2[a_{11}\sigma^2 + 4Ra_{11}(b_1 + b_2 + 2\theta) + 2Rb_1(-2Q + a_{21} + a_{22})] + \sigma^2[b_1b_2(-2Q + 2a_{11} + a_{21} + 2a_{22}) + 2b_1\theta(-Q + a_{11} + a_{21} + a_{22})] + 4Rb_1b_2\theta(-2Q + 2a_{11} + a_{21} + a_{22})}{2b_1[2b_2^2R(2b_2 + b_1 + 4\theta) + \sigma^2(b_1b_2 + b_1\theta + b_2^2) + 4Rb_1b_2\theta]}$$

$$p_{21}^* = \frac{b_2[(\sigma^2 + 4R\theta)(a_{11} + 2a_{21} + 2a_{22} - 2Q) + 2Ra_{21}b_1] + 2b_2^2R(a_{11} + 3a_{21} + a_{22} - 2Q) + 2Rb_1\theta(a_{21} + a_{22}) + a_{21}b_1\sigma^2}{2[2Rb_2^2(2b_2 + b_1 + 4\theta) + \sigma^2(b_2^2 + b_1b_2 + b_1\theta) + 4Rb_1b_2\theta]}$$

$$p_{22}^* = \frac{b_2^2R(a_{11} + 3a_{22} + a_{21} - 2Q) + 2b_2R\theta(a_{11} + 2a_{21} + 2a_{22} - 2Q) + Rb_1\theta(a_{21} + a_{22}) + a_{22}b_1b_2R}{2Rb_2^2(2b_2 + b_1 + 4\theta) + \sigma^2(b_2^2 + b_1b_2 + b_1\theta) + 4Rb_1b_2\theta}$$

similar to Theorem 1,2, $\lambda_1 > 0$ is equivalent to $Q \leq Q^*$ where

$$Q^* = \frac{2Rb_2(b_2 + 2\theta)(a_{11} + a_{21} + a_{22} + b_1v + 2b_2v) + \sigma^2[(a_{11} + a_{21} + a_{22})(b_2 + \theta) + b_2v(b_1 + 4\theta + 3b_2) + \theta(b_1v + a_{21} + a_{22})]}{2[2Rb_2(b_2 + 2\theta) + \sigma^2(b_2 + \theta)]}.$$

Figures

Fig.A1 The retailer's revenue as a function of R

Fig.A2 The retailer's revenue as a function of the standard deviation σ

Figs.A1 and A2 show that the expected revenue with an uncertainty online demand is lower than the revenue with a deterministic online demand. And the expected revenue increases with the retailer's risk tolerance but decreases with the standard deviation.

Fig.A3Revenue of the traditional channels as a function of Q

Fig.A4 Revenue of the online channel as a function of Q

Figs.A3 and A4 illustrate the change of the revenues of the traditional and the online channels, respectively. When the retailer uses static price for the traditional channel, the revenue of the traditional channel decreases with the introduction of the online channel. The results suggests the existence of channel conflict. But when dynamic pricing is used for the traditional channel, the revenue of the traditional channel and the total revenue increase but the revenue of the online channel decreases. In other words, dynamic pricing mitigates channel conflict.