Super Efficiency Evaluation using a Common Platform on a Cooperative Game

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Abstract: Classical DEA models (such as CCR and BCC models) compute efficiencies of decision-making units (DMUs) based on a common efficient frontier but with no capability of differentiating efficient units. The super efficiency model was developed to rank efficient DMUs based a new efficient frontier comprised by all other DMUs. This may lead to a multi-platform problem that different efficient DMUs are evaluated based on different efficient frontiers and the resulted super efficiencies of efficient DMUs are not comparable. This paper addresses the multi-platform problem from the perspective of a cooperative game. Efficient DMUs are regarded as players and subsets of these efficient DMUs as coalitions. The effect of a coalition on a specific efficient DMU is defined as the DMU's efficiency change proportion (ECP) based on the traditional DEA models when the coalition is removed from the reference set. Basing on the ECP, we define a characteristic function as the sum of all efficient DMUs' ECPs, and prove that this function is super-additive. Then, the Shapley value is introduced as a solution of this cooperative game and applied to rank efficient DMUs. The proposed approach is demonstrated by two numerical examples. Finally, we extend the proposed approach in this paper to the VRS assumption.

Key words: Data envelopment analysis (DEA); Super efficiency; Common platform; Cooperative game; Efficiency change proportion (ECP).

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1. Introduction

Originating from the work of Farrell (1957) and Debreu (1951), data envelopment analysis (DEA) developed by Charnes, Cooper, & Rhodes (1978) is a popular nonparametric method to evaluate performance of various peer decision-making units (DMUs). Now, DEA has been widely applied to measuring performance in many areas such as universities (Lim & Zhu, 2013), banks (Staub, Souza, & Tabak, 2010; Wanke, Barros, & Emrouznejad, 2016), reallocation of emission permits (Wu, Du, Liang, & Zhou, 2013), and health care (Pulina, Detotto, & Paba, 2010; Shwartz, Burgess, & Zhu, 2016). Its popularity can be attributed to no requirements specific production functional form and prior knowledge of DMUs' input and output weights (Cooper, Seiford, & Tone, 2007; Cooper, Seiford, & Zhu, 2011). In our opinion, another reason of its popularity is that DMUs are evaluated fairly under a common frontier comprised of all DMUs, and such a frontier constitutes a common platform for all DMUs. Therefore, efficiencies obtained by classical DEA models are comparable and can be used to rank DMUs. However, classical DEA models fail to rank (weakly) efficient DMUs¹, for all efficient DMUs' efficiencies are ones.

A number of novel approaches are proposed to rank efficient DMUs (Angulo & Lins, 2002; Adler, Friedman, & Stern, 2002). For example, super efficiency, introduced by Peterson and Anderson (1993), is one of the important approaches for ranking efficient DMUs. In this approach, each efficient DMU is evaluated by an efficient frontier comprised by all other DMUs. If a DMU is strongly efficient, it may be beyond of envelopment of such the frontier. Therefore, the efficient DMU's super efficiency may be bigger than one. In this case, super efficiency is a popular approach used to rank efficient DMUs. And it has been widely applied in many areas such as fixed cost allocation (Li, Yang, Liang, & Hua, 2009), manufacturing firm evaluation (Düzakın & Düzakın, 2007), and bank evaluation (Avkiran, 2011). However, this model brings at least two problems. The first is that the model may be infeasible based on the variable return to scale (VRS) assumption (Baker & Talluri, 1997; Seiford & Zhu, 1999; Cook, Liang, Zha, & Zhu, 2008; Johnson & McGinnis, 2009; Lee, Chu, & Zhu, 2011; Chen & Liang, 2011; Lee & Zhu, 2012; Chen, 2013). The second is the multi-platform problem. Super efficiency model evaluates each efficient DMU by a specified efficient frontier comprised of other DMUs, and such the frontier may vary when different efficient DMUs are evaluated. Therefore, super efficiency evaluates these efficient DMUs based on different standards,

¹In this paper, "efficient DMUs" refer to DMUs that are on the efficient frontier. Therefore, for "radial" DEA models, the set of "efficient DMUs" not only includes strongly Pareto-efficient DMUs but also weakly efficient DMUs. For non-radial DEA models, such as the slack-based measure model, "efficient DMUs" only refer to Pareto-efficient DMUs.

and super efficiencies of efficient DMUs may be not comparable. For details, readers can refer to Section 3. Similarly, Banker & Chang (2006) argued that the super efficiency model is useful for outlier identification but not good at ranking efficient units. Banker, Chang, & Zheng (2015) found that the reason of unsatisfactorily comes from the “left corner” of DMUs.

Based on the philosophy of super efficiency, a novel approach of changing the reference set is proposed to discriminate efficient DMUs (Chen & Deng, 2011; Jahanshahloo, Junior, Lotfi, & Akbarian, 2007; Hibi & Sueyoshi, 1999). This approach regards each efficient DMU as a candidate. For a given candidate, its effect is defined as the sum of the efficiency changes of other DMUs when the candidate is omitted from the reference set. Such an effect is called the DEA cross reference (DCR). This approach ranks each efficient DMU based on its DCR value. The higher its DCR value, the higher its rank. However, the nature of this approach is the same as the super efficiency, and it still has the multi-platform problem.

Cross-efficiency is another important approach of ranking efficient DMUs. It is first proposed by Doyle & Green (1994) and developed by Liang Wu, Cook, & Zhu (2008), Wu, Liang, & Yang (2009), Wu, Liang, & Chen (2009), Wu, Liang, Yang, & Yan, (2009), Alcaraz, Ramón, & Ruiz (2013), Ruiz (2013) and Wu, Chu, Sun, & Zhu, (2016). This approach allows each DMU to selfishly choose an optimal set of input and output weights to the classical DEA, and defines the average value of a DMU efficiencies based on these optimal weights as its cross efficiency. However, cross-efficiency scores are generally not unique, and depend on which of the alternate optimal solutions to the DEA linear programs is used (Liang, Wu, Cook, & Zhu, 2008).

Common weight DEA is also an approach for ranking DMUs (Cook, Roll, & Kazakov, 1990; Roll, Cook, & Golany, 1991; Kao & Hung, 2005; Wu, Liang, & Yang, 2009; Kao, 2010; Wu, Chu, Zhu, Li, & Liang, 2016). This approach attempts to find a common set of weights to calculate efficiencies for all DMUs, and ranks them based on their efficiencies. Compared to the cross efficiency, this approach evaluates DMUs based on a common platform (a common set of weight), and the obtained efficiencies are comparable. However, it's difficult to find a comprise principle to choose the common set of weights and different principles adopted lead to different common sets of weights chosen as well as ranks among DMUs.

Besides, Salo & Punkka (2011) introduced the efficiency dominance to rank DMUs by considering all possible input and output weights: a DMU is only said to be better than another DMU if its efficiency score is higher for all feasible input and output weights. This approach is fair to rank DMUs but lacks of discrimination for efficient DMUs.

In the current study, we approach the multi-platform problem in efficiency evaluation from the perspective of cooperative game theory. Efficient DMUs are assumed to compete and cooperate with each other in the evaluation. For example, in bank performance evaluation, efficient branches compete with other branches to be the best benchmark because branches and their managers with higher ranks often receive more rewards and support in the future. On the contrary, efficient branches cooperate with each other to construct a common efficient frontier to evaluate inefficient ones, and make the evaluate result comparable.

Combining DEA with game theory is not rare in DEA literature, Nakabayashi & Tone (2006) presented a cooperative game to solve the problem of consensus-making between unit and organization when evaluating performance via DEA models. Li & Liang (2010) used a cooperative game to select indices for DEA. Some researches focused on incorporating game theory with a two-stage DEA model (Zhu, 2004; Liang, Cook, & Zhu, 2008; Zha & Liang, 2010; Du Liang Chen Cook & Zhu, 2011; Li, Chen, Liang, & Xie, 2012; Zhou, Sun, Yang, Liu, & Ma, 2013). Lozano (2012; 2013), Lozano, Mármol, & Hinojosa (2015), Borrero, Hinojosa, & Mármol (2016) and Wu, Zhu, Cook, & Zhu (2016) proposed approaches based on cooperative games in dealing with problems of information and technology sharing among organizations.

The purpose of this paper is to rank efficient DMUs with a common platform. We define a cooperative game with transferable utility (TU-game) to construct the common platform for efficient DMUs. Each efficient DMU is regarded as a player, each possible subset of efficient DMUs is regarded as a coalition. The effect of a coalition on each efficient DMU is defined as its efficiency change proportion (ECP) when the coalition is removed from the reference set. Basing on ECP, we define a characteristic function of the coalition as the sum of the ECPs of all efficient DMUs. We also prove that the characteristic function satisfies super additivity. Then, with the fairness axioms of efficiency and dummy player, symmetry and additivity, a Shapley value is used as a solution to the cooperative game and to rank DMUs. In this paper, we will also extend the proposed approach under the variable returns to scale (VRS) assumption.

The remainder of this paper is organized as follows. Section 2 describes the problem; Section 3 defines ECP to measure the marginal role of a DMU set on an efficient unit; Section 4 presents a common platform approach based TU-game for ranking efficient DMUs; Numerical examples are presented in Section 5; Section 6 extends our common platform approach to the VRS assumption; The conclusion is elaborated in Section 7.

2. Problem description

In this paper, we use the following notations:

Table 1 Notation explanation

N	original DMU set
E^*	the set of DMUs which are on the efficient frontier
S	a subset of E^* , corresponding to a coalition in this paper
j, f	any DMU in original DMU set $N=\{1,2,\dots,n\}$
k, d	an efficient DMU in set E^*
I	index on inputs, $i=1,2,\dots,m$
R	index on outputs, $r=1,2,\dots,h$
x_{ij}	amount of input i consumed by DMU j
y_{rj}	amount of output r produced by DMU j

Notably, E^* defined in **Table 1** is the set of DMUs that are on the efficient frontier. Therefore, their efficiencies are all 1. For the “radial” DEA models in this section, E^* may not only include strongly efficient DMUs, but also include weakly efficient DMUs. Suppose there are n independent DMUs in set N , and each DMU_j ($j \in N = \{1,2,\dots,n\}$) consumes m inputs x_{ij} ($i \in M = \{1,2,\dots,m\}$) to generate h outputs y_{rj} ($r \in H = \{1,2,\dots,h\}$). The efficiency rating for any given DMU_f can be computed by using the standard input-oriented CCR model (Charnes, Cooper, & Rhodes, 1978) as follows:

$$\begin{aligned}
 E_f(N) &= \text{Min } \theta \\
 \text{s.t. } & \sum_{j \in N} \lambda_j x_{ij} \leq \theta x_{if}, i \in M \\
 & \sum_{j \in N} \lambda_j y_{rj} \geq y_{rf}, r \in H \\
 & \lambda_j \geq 0, \forall j, j \in N, \theta: \text{free}
 \end{aligned} \quad , \quad (1)$$

where $E_f(N)$ is the efficiency for DMU_f based on reference set N .

The super efficiency model (Peterson and Anderson, 1993) for evaluating efficient units can be formulated as follows:

$$\begin{aligned}
 E_f(N) &= \text{Min } \theta \\
 \text{s.t. } & \sum_{\substack{j \in N \\ j \neq f}} \lambda_j x_{ij} \leq \theta x_{if}, i \in M \\
 & \sum_{\substack{j \in N \\ j \neq f}} \lambda_j y_{rj} \geq y_{rf}, r \in H \\
 & \lambda_j \geq 0, \forall j, j \in N, j \neq f, \theta: \text{free}
 \end{aligned} \quad . \quad (2)$$

If DMU f is an efficient DMU, then its efficiency computed by the super efficiency model is not less than 1. While if DMU f is an inefficient DMU, then its super efficiency score equals to its CCR score.

Figure 1 Efficient frontiers formed by different efficient DMUs

We use the example described in Figure 1 to explain the multi-platform problem in detail. Figure 1 plots four DMUs: A , B , C , and D . Each DMU has two inputs X_1 and X_2 with a single output of unity. Three efficient DMUs A , B , and C and one inefficient DMU D are also shown. If the super efficiency model is used to evaluate A , the efficient frontier is A_1BC and the super efficiency score of A is calculated as OA/OA_1 . In a similar manner, B 's super efficiency score is OB/OB_1 and the efficient frontier is AB_1C . The reference set of C is ABC_1 when C is under evaluation and its super efficiency score is OC/OC_1 . Notably, the efficient frontiers based on the super efficiency model are changing when different efficient DMUs are evaluated. Three different efficient frontiers are used when evaluating super efficiencies of A , B , and C , respectively. These three efficient frontiers imply that A , B , and C are not evaluated with the same evaluation standards. Hence, super efficiencies of A , B , and C are not comparable.

Meanwhile, if we evaluate all efficient DMUs based on one single frontier (the same reference set), then resulted efficiencies may not be fair. For example, denote the set of removed efficient DMUs as S and the set of all DMUs as N . When S is removed, the new reference set is denoted as N/S . The efficiency of DMU k can be computed by model (3) as follows:

$$\begin{aligned}
 E_k(N/S) = \text{Min } & \theta \\
 \text{s.t. } & \sum_{j \in N/S} \lambda_j x_{ij} \leq \theta x_{ik}, i \in M \\
 & \sum_{j \in N/S} \lambda_j y_{rj} \geq y_{rk}, r \in H \\
 & \lambda_j \geq 0, \forall k \in E^*, j \in N/S
 \end{aligned} \quad (3)$$

Using Figure 1 as an example, if $S=\{A\}$, then the rest of efficient DMUs set $N\setminus\{A\}$ forms an efficient frontier A_1BC . Based on this frontier, DMU A 's score calculated by model (3) is higher than 1, whereas DMU B 's score equals to 1. The scores are comparable for the DMUs based on one common efficient frontier A_1BC , but unfair for DMU B for only considering the subset $S=\{A\}$. On the contrary, the scores would be comparable but the situation would be unfair for DMU A if we only consider the subset $S=\{B\}$. The reason is that when we set $S=\{B\}$, the resulted frontier AB_1C would be of advantage to DMU B 's super efficiency, whereas be of disadvantage to DMU A 's super efficiency.

Therefore, a conflict arises, that is, if we use the super efficiency model, efficient DMUs can be discriminated while super efficiencies resulted may be inequitable or incomparable. If we use CCR model, the resulted efficiencies are equitable and comparable, while efficient DMUs can't be discriminated.

3. Marginal effect of an efficient DMU set on an efficient unit

In this section, we define a new measure, which quantifies the marginal impact of a particular DMU set on an efficient unit in efficiency evaluation. This particular DMU set should be a subset of E^* .

Definition 1: The effect of an efficient DMU set S on an efficient DMU_d is defined as $ECP_d(S)$:

$$ECP_d(S) = \frac{E_d(N/S)}{E_d(N)} - 1 = E_d(N/S) - 1, \forall d \in E^*, S \subseteq E^*, \quad (4)$$

where $E_d(N/S)$ is the efficiency score of DMU_d computed by model (3) with the DMU set N/S and $E_d(N)=1$ is computed by model (1). Obviously, $\forall d \in E^*, S \subseteq E^*$, if $d \notin S$, then $ECP_d(S)=0$; if $d \in S$, then $ECP_d(S) \geq 0$; if $S=\emptyset$, then $ECP_d(S)=0$.

When we remove an efficient DMU set S from the reference set, efficient DMUs outside S are still on the frontier, therefore, if $d \notin S$, then $ECP_d(S)=0$. While if $S=\emptyset$, which means every efficient DMU is outside S , then we also have $ECP_d(S)=0$.

Consider the effect of removing a DMU in S . When S contains only one DMU, our definition that characterizes the effect of deleting an efficient DMU does exactly what the super efficiency model does. In the super efficiency model, each efficient DMU is measured by a new efficient frontier composed by the efficient DMU set without the targeted one. Based on the super efficiency model, the efficiency score of each efficient DMU may be higher than 1. For example, we suppose an

efficient DMU d has a super efficiency score of 1.2 contrary to its CCR score of 1. Such a difference between the two scores is attributed to the two different efficient frontiers used in the different models. The super efficiency model forms a new efficient frontier by removing the DMU under consideration from the DMU set, whereas the CCR model forms an efficient frontier composed of all efficient DMUs including the DMU under consideration. When S has more than one DMU, if $d \in S$, then the efficient DMU d is evaluated by a new efficient frontier composed of the DMU set when the DMUs in S have been removed. The score $ECP_d(S)$ is higher than or equivalent to 0. In this case, our definition also characterizes such an effect of removing an efficient DMU set.

The definition of $ECP_d(S)$ captures the idea that the proportion of the efficiency score of DMU d will change when coalition S is removed from reference set. In other words, the value $ECP_d(S)$ illustrates the extent of the effect of coalition S on DMU d . Based on **Definition 1**, if we obtain $ECP_d(S)$, then the efficiency of DMU d against the new efficient frontier enveloped by the DMU set N/S , namely, $E_d(N/S)$, can be easily computed. Previously, to ensure comparability and equitability in the evaluation, we need to consider all possible efficient frontiers comprised of different efficient DMU sets.

Figure 2 ECP of each DMU

We use Figure 2 to illustrate how to calculate $ECP_d(S)$ and clarify the concept of ECP. The figure shows five DMUs: A , B , C , D , and M . Each DMU consumes two inputs, X_1 and X_2 to generate a unit of output. The figure also shows that A , B , C , and D are efficient DMUs in the CCR model while M is an inefficient DMU. If $S = \{A, D\}$ and D is the targeted DMU, then

$$ECP_D(S) = \frac{ECP_D(N/S)}{ECP_D(N)} - 1 = ECP_D(N/S) - 1 = \frac{OD'/OD}{OD/OD} - 1 = \frac{DD'}{OD}. \quad ECP_D(S) \text{ means } D\text{'s ECP}$$

when DMU set S is removed from the reference set. Similarly,

$$ECP_A(S) = \frac{ECP_A(N/S)}{ECP_A(N)} - 1 = ECP_A(N/S) - 1 = \frac{OA'/OA}{OA/OA} - 1 = \frac{AA'}{OA}.$$

4. Cooperative game in determining the rank of each efficient DMU

In this section, we assume that efficient DMUs cooperate with each other to maintain a common efficient frontier. Evaluation results based on the common efficient frontier are comparable and equitable. We use a cooperative game to construct a common platform and consider the competitive and cooperative relationships among efficient DMUs in E^* . Each efficient DMU is regarded as a player, E^* is regarded as a grand coalition.

Definition 2: A common platform cooperative game with transferable utility (TU-game) can be denoted as $\langle E^*, C \rangle$, which consists of

- 1) a finite set E^* , the set of players (efficient DMUs)
- 2) a function C assigned to every subset (coalition) S of E^* , that is, $\forall S \subseteq E^*$, a real number $C(S)$ exists. $C: 2^{E^*} \rightarrow R$ is denoted as the characteristic function, and $C(S)$ is the total payoff of coalition S . The coalition E^* can be called as grand coalition.

Definition 3: For a given coalition S , the value of corresponding payoff $C(S)$ is defined as

$$C(S) = \sum_{d \in S} ECP_d(S), \forall S \subseteq E^*, \quad (5)$$

where S is a subset of efficient unit set E^* . $C(S)$ is available for all possible divisions among the players of S , and $C(\emptyset) = 0$. The definition means that the higher the effect of a coalition S on the efficient DMUs, the higher the payoff of S .

Given that $ECP_d(S)$ measures the effect of coalition S on DMU d , $C(S)$ is the effect of coalition S on the original efficient frontier. In other words, $C(S)$ captures the concept of how much the efficient frontier will change when the coalition S does not participate in the evaluation. In this study, $C(S)$ is also defined as the overall payoff of coalition S in the TU-game. Therefore, $C(S)$ should be divided among players of S .

We evaluate the performance of bank branches as an example. In the first step, the traditional DEA models are used to classify efficient branches and non-efficient branches. Then, in the second

step, a cooperative game is used for evaluating efficient branches. In this study, rewards are regarded as the payoff, and we assume that players derive a same utility from the same amount of rewards. Efficient branches cooperate with each other to obtain a higher total rewards (payoff, $C(S)$). The total rewards $C(S)$ can also be allocated to each efficient branch with fairness in the TU-game.

Theorem 1: The characteristic function C is super-additive. Specifically, for any $S_1 \subseteq E^*$ and $S_2 \subseteq E^*$ with $S_1 \cap S_2 = \emptyset$, we obtain

$$C(S_1 \cup S_2) \geq C(S_1) + C(S_2).$$

Proof. See Appendix.

We seek a fair mechanism to allocate the expected overall gains among efficient DMUs in the coalition. First, following Osborne & Rubinstein (1994), we introduce three axioms to formalize notions of fairness as follows.

- 1), **Efficiency and Dummy Player:** All the gains generated by the grand coalition should be allocated. If a DMU i does not make any contribution to any of the coalitions it joins, that is, $ECP_i(S) = 0$ for all S such that $i \in S$, then the allocated value for DMU i is zero..
- 2), **Symmetry:** We set an interchange function $\pi: E^* \rightarrow E^*$, which changes the index of DMU from i to $\pi(i)$. Then, we have that the allocation for i equals to $\pi(i)$.
- 3), **Additivity:** For any two games C_1 and C_2 , we have that the allocation for $C_1 + C_2$ is equal to the sum of allocation for C_1 and C_2 .

Notably, k_1 and k_2 are interchangeable in axiom **Symmetry** and are defined as:

$\sum_{d \in S \cup \{k_1\}} ECP_d(S \cup \{k_1\}) = \sum_{d \in S \cup \{k_2\}} ECP_d(S \cup \{k_2\})$ for any S that contains neither k_1 nor k_2 . In this axiom, if k_1 and k_2 are interchangeable, then they have equal contribution for any possible coalitions in E^* . Therefore, they should also be allocated with an equal value from the coalition. For the super efficiency common platform TU-game, if DMU k_1 and DMU k_2 have same inputs and outputs, then they should also have a same ranking.

As discussed in Shapley (1953) and Myerson (1991), Shapley value is the unique allocation that satisfies the above axioms, and can be computed by the following Formula (6):

$$\varphi_k(C) = \sum_{k \in S, S \subseteq E^*} \frac{(p-1)!(q-p)!}{q!} [C(S) - C(S/\{k\})], \quad (6)$$

where p is the number of units in DMU set S , i.e. $p = |S|$, and q is the number of units in E^* i.e. $q = |E^*|$. S can be any possible subset of efficient DMU set E^* .

Definition 3 shows that S cannot be equivalent to E^* when no inefficient unit exists in N . Since the Shapley value method needs to compute $C(S)$, when all efficient DMUs are omitted. If at least one inefficient DMU exists, then the Shapley value for each efficient DMU can be calculated; if no inefficient DMU exists in the DMU set, then the efficient frontier does not exist when we omit all efficient units. In this case, a new DMU needs to be formed. In our approach, we create a new DMU as $X_i^- = \max_{k \in E^*} \{X_{ik}\}$, $i = 1, \dots, m$ and $Y_r^- = \max_{k \in E^*} \{Y_{rk}\}$, $r = 1, \dots, s$. Such a new unit is denoted as E^- . After that, we can calculate the Shapley value for each efficient DMU based on Formula (6).

5. Numerical examples

In this section, we illustrate the proposed approach using two data sets. One is an illustrative data, and the other is a real-world data of 14 Chinese city commercial banks (CCCBs) in 2012. In the two examples, we compute the Shapley value and rank of each efficient DMU and compare the evaluation results with the super efficiency model.

5.1 Illustrative data example

The illustrative data set contains eight DMUs, as shown in Table 2. Each DMU consumes two inputs: X_1 and X_2 , to generate two outputs: Y_1 and Y_2 . Our approach and the super efficiency model are applied to this data set. The evaluation results are shown in Table 3.

Table 2 Illustrative data set

	Input X_1	Input X_2	Output Y_1	Output Y_2
A	2	7	3	1
B	2	12	4	1
C	5	5	2	1
D	10	4	2	1
M	3	6	1	1
F	10	6	1	1

<i>G</i>	4	12	2.5	1
<i>H</i>	5	11	3.5	1

Based on the CCR model, DMUs *A*, *B*, *C*, *D*, and *M* are efficient, whereas DMUs *F*, *G*, and *H* are inefficient. Therefore, the main assignment of the evaluation is to compare the five efficient DMUs. Table 3 shows the efficiency and rank of each efficient unit based on the super efficiency model and our common platform approach. Comparing the 2nd and 4th columns, we can find that the ranks of unit *C* and unit *D* are exchanged.

Table 3 Results of CCR model

Unit	Super efficiency	Rank (SE)	Shapley value	Rank	Rank change
<i>A</i>	1.3582	1	2.1517	1	0
<i>B</i>	1.3333	2	1.5705	2	0
<i>C</i>	1.1000	4	0.7827	3	+1
<i>D</i>	1.2500	3	0.6753	4	-1
<i>M</i>	1.0417	5	0.3881	5	0

5.2 Real-world example

Our proposal is also applied to the data set of 14 CCCBs in 2012. The data is from each bank's annual report. Each bank has three inputs and two outputs, as shown in Table 4. The inputs include Fixed assets, Employee's pay and General expenses. The outputs include Profit and Loans. The inputs and outputs are measured in thousand Yuan.

Table 4 Data set of Chinese commercial banks of 2012

Bank	Fixed assets	Employees' pay	General expenses	Profit	Loans
Jiujiang Bank <i>A</i>	775,310	283,893	644,280	2,193,085	26,392,924
Chengdu Bank	1,120,795	679,087	1,674,877	3,262,998	91,799,733
Baotou Bank	5,086,018	420,404	2,867,703	2,845,104	57,320,923
Hangzhou Bank	1,114,333	1,239,207	3,187,168	4,474,572	148,341,428
Huishang Bank <i>C</i>	1,178,797	1,155,569	2,433,518	5,680,038	159,941,475
Harbin Bank	5,810,688	332,044	2,466,586	3,789,997	85,298,079
Jiangsu Bank	3,203,353	2,864,359	6,617,403	9,625,877	342,827,271
ZHHR Bank <i>F</i>	150,570	207,802	877,996	636,046	21,152,061

Tianjin Bank D	1,267,597	296,940	1,820,934	3,298,603	118,767,291
Chongqing Bank E	1,015,688	142,024	1,577,106	2,518,447	75,256,873
Dalian Bank	1,154,635	619,940	2,327,875	2,253,330	98,880,747
Hankou Bank B	1,088,335	95,372	1,221,129	2,414,775	57,757,208
Nanjing Bank	2,172,992	635,475	2,721,211	4,980,404	121,962,186
Ningbo Bank	2,397,820	624,031	3,529,395	5,098,041	142,564,629

Among the 14 banks, 6 ones are efficient based on the CCR model. The efficient unit set E^* contains Jiujiang Bank (A), Hankou Bank (B), Huishang Bank (C), Tianjin Bank (D), Chongqing Bank (E), and ZHHR Bank (F). According to Definition 2 and 3, we can compute $C(S)$ and $C(S/\{k\})$ of these banks, respectively. Table 5 shows the results of Jiujiang Bank (A).

Table 5 Procedures for computing the Shapley value of Jiujiang Bank

S	$C(S)$	$C(S/\{k\})$	S	$C(S)$	$C(S/\{k\})$
$\{A\}$	0.4778	0	$\{A,B,C,D\}$	1.9254	1.1347
$\{A,B\}$	0.9230	0.4279	$\{A,B,C,E\}$	3.0088	2.2178
$\{A,C\}$	1.1502	0.3851	$\{A,B,C,F\}$	1.6593	0.8682
$\{A,D\}$	0.7522	0.2743	$\{A,B,D,E\}$	4.0401	3.5356
$\{A,E\}$	0.5750	0.0972	$\{A,B,D,F\}$	1.2392	0.7387
$\{A,F\}$	0.5132	0.0354	$\{A,B,E,F\}$	2.3632	1.8681
$\{A,B,C\}$	1.6040	0.8130	$\{A,C,D,E\}$	1.8988	1.1341
$\{A,B,D\}$	1.2039	0.7034	$\{A,C,D,F\}$	1.5269	0.7622
$\{A,B,E\}$	2.3278	1.8327	$\{A,C,E,F\}$	1.3027	0.5375
$\{A,B,F\}$	0.9583	0.4632	$\{A,D,E,F\}$	1.1989	0.7211
$\{A,C,D\}$	1.4716	0.7069	$\{A,B,C,D,E\}$	4.7118	3.9212
$\{A,C,E\}$	1.2474	0.4823	$\{A,B,C,D,F\}$	1.9806	1.1900
$\{A,C,F\}$	1.2055	0.4404	$\{A,B,C,E,F\}$	3.0641	2.2731
$\{A,D,E\}$	1.1635	0.6857	$\{A,B,D,E,F\}$	4.0754	3.5710
$\{A,D,F\}$	0.7875	0.3097	$\{A,C,D,E,F\}$	1.9541	1.1893
$\{A,E,F\}$	0.6104	0.1325	$\{A,B,C,D,E,F\}$	4.7670	3.9764

Table 6 shows the Shapley value of each DMU based on Formula (6). Since there are six DMUs in efficient set E^* , based on the number of DMUs in the coalition S , the Shapley value can be decomposed into six parts. In Table 6, $p=1$ represents scores obtained when S contains only one DMU, $p=6$ represents scores obtained when S contains all the six efficient DMUs.”

The six efficient DMUs in Table 6 have different Shapley values. Therefore, our approach can discriminate efficient DMUs which can't be done by traditional CCR model. In addition, the

efficiency of each DMU is obtained based on a common platform, which has considered all possible evaluation standards. Table 7 shows the comparison of our approach with the super efficiency model.

Table 6 Shapley values of six banks

NO	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	Shapley value
<i>A</i>	0.0796	0.0898	0.1002	0.1108	0.1214	0.1318	0.6335
<i>B</i>	0.0713	0.1155	0.1785	0.2592	0.3564	0.4688	1.4497
<i>C</i>	0.0642	0.0760	0.0882	0.0997	0.1092	0.1153	0.5525
<i>D</i>	0.0457	0.0578	0.0888	0.1376	0.2030	0.2838	0.8168
<i>E</i>	0.0162	0.0703	0.1431	0.2339	0.3414	0.4644	1.2692
<i>F</i>	0.0059	0.0066	0.0072	0.0079	0.0085	0.0092	0.0453

Table 7 also shows the rank of each DMU based on the super efficiency model and our approach, respectively. The last bank, Chongqing Bank (*E*) has the same rank via the two approaches, while the first five banks have different ranks.

Table 7 Evaluation results of the super efficiency model and our approach

NO.	SE	Rank (SE)	Shapley value	Rank	Rank change
<i>A</i>	1.4778	1	0.6335	4	-3
<i>B</i>	1.4279	2	1.4497	1	+1
<i>C</i>	1.3851	3	0.5525	5	-2
<i>D</i>	1.2743	4	0.8168	3	+1
<i>E</i>	1.0972	5	1.2692	2	+3
<i>F</i>	1.0354	6	0.0453	6	0

We use Jiujiang Bank (*A*) and Chongqing Bank (*E*) as examples to explain the difference. The efficiency computed by super efficiency model of Jiujiang Bank (*A*) is 1.4778. This score is the highest among the scores of the six banks. However, the Shapley value of Jiujiang Bank (*A*) in our model is 0.6335, which is just the fourth higher among the six efficient DMUs. The super efficiency of Chongqing Bank (*E*) is 1.097 while its Shapley value is 1.269. The rank of this bank in the super efficiency model is the fifth. Its rank increases to the second based on our approach.

The evaluation results of Jiujiang Bank (*A*) and Chongqing Bank (*E*) in the two approaches are vastly different. The difference is due to the existence of multi-standard problems in the super efficiency model. The second column in Table 6 that is, $p=1$, is computed by the super efficiency

score and the weight of $1/q$. Thus, the super efficiency score can be deemed as one step ahead of our model because q is determined. The highest efficiency score is obtained by Jiujiang Bank (A). Therefore, the rank by the second column is consistent with the result by the super efficiency model. However, when the number of subset increases to two, Jiujiang Bank (A) does not obtain the highest score. On the contrary, Chongqing Bank (E) has more effect on the coalition when $p > 2$ (The S has more than two DMUs). As a result, Chongqing Bank (E) has more effect to the efficient frontier than Jiujiang Bank (A). Thus, the Shapley value of Chongqing Bank (E) is higher than Jiujiang Bank (A).

Figure 3 Rank of banks in super efficiency model and our model

6. Extension to VRS

This section extends the proposed approach from the constant return to scale (CRS) assumption to the VRS assumption. Due to the infeasibility of the super efficiency model based the VRS assumption (Baker & Talluri, 1997; Seiford & Zhu, 1999), here we use two extensions of the slack-based measure model to replace the traditional super efficiency model in the proposed approach. One is the super slack-based measure (S-SBM) model developed by Tone (2002) and the other is the joint SBM (J-SBM) model. The two models are always feasible (Tone, 2002; Chen, 2013).

We first introduce the initial SBM model developed by Tone (2001) as follows.

$$\begin{aligned}
E_f^{SBM}(N) &= \text{Min} \frac{1 - \frac{1}{m} \sum_{i \in M} s_{if}^- / x_{if}}{1 + \frac{1}{s} \sum_{r \in H} s_{rf}^+ / y_{rf}} \\
s.t. \quad &\sum_{j \in N} \lambda_j x_{ij} = x_{if} - s_{if}^-, \forall i \in M \\
&\sum_{j \in N} \lambda_j y_{rj} = y_{rd} + s_{rf}^-, \forall r \in H \\
&\sum_{j \in N} \lambda_j = 1 \\
&\lambda_j \geq 0, s_{if}^- \geq 0, s_{rf}^- \geq 0, \forall i, r, j
\end{aligned} \tag{7}$$

Model (7) is a fractional programming, and can be transformed to the following linear programming via Charnes–Cooper transformation (Charnes, Cooper, & Rhodes, 1978; Tone, 2001).

$$\begin{aligned}
E_f^{SBM}(N) &= \text{Min} \quad t - \frac{1}{m} \sum_{i \in M} s_{if}^- / x_{if} \\
s.t. \quad &t + \frac{1}{s} \sum_{r \in H} s_{rf}^+ / y_{rf} = 1 \\
&\sum_{j \in N} \lambda_j x_{ij} = tx_{if} - s_{if}^-, \forall i \in M \\
&\sum_{j \in N} \lambda_j y_{rj} = ty_{rd} + s_{rf}^-, \forall r \in H \\
&\sum_{j \in N} \lambda_j = 1 \\
&\lambda_j \geq 0, s_{if}^- \geq 0, s_{rf}^- \geq 0, \forall i, r, j
\end{aligned} \tag{8}$$

Contrary to radial projection models, weakly efficient situations do not exist in SBM models. Therefore, the TU-game players set $E^* = \{f : E_f^{SBM}(N) = 1\}$ in this section only include the strong efficient DMUs. Then, similar to the model (3) in Section 2, when a subset S ($S \subseteq E^*$) is removed from the reference set, the super efficiencies $E_d^{SSBM}(N/S), \forall d \in E^*$ can be calculated by the following S-SBM model (9).

$$\begin{aligned}
E_d^{SSBM}(N/S) &= \text{Min} \frac{1 + \frac{1}{m} \sum_{i \in M} s_{id}^- / x_{id}}{1 - \frac{1}{s} \sum_{r \in H} s_{rd}^+ / y_{rd}} \\
s.t. \quad &\sum_{j \in N/S} \lambda_j x_{ij} = x_{id} + s_{id}^-, \forall i \in M \\
&\sum_{j \in N/S} \lambda_j y_{rj} = y_{rd} - s_{rd}^-, \forall r \in H \\
&\sum_{j \in N/S} \lambda_j = 1 \\
&\lambda_j \geq 0, s_{id}^- \geq 0, s_{rd}^- \geq 0, \forall i, r, j
\end{aligned} \tag{9}$$

Fractional programming (9) can be transformed to the following linear programming (10).

$$\begin{aligned}
E_d^{SSBM}(N/S) &= \text{Min} \quad t + \frac{1}{m} \sum_{i \in M} s_{id}^- / x_{id} \\
s.t. \quad &t - \frac{1}{s} \sum_{r \in H} s_{rd}^+ / y_{rd} = 1 \\
&\sum_{j \in N/S} \lambda_j x_{ij} = tx_{id} + s_{id}^-, \forall i \in M \\
&\sum_{j \in N/S} \lambda_j y_{rj} = ty_{rd} - s_{rd}^-, \forall r \in H \\
&\sum_{j \in N/S} \lambda_j = 1 \\
&\lambda_j \geq 0, s_{id}^- \geq 0, s_{rd}^- \geq 0, \forall i, r, j
\end{aligned} \tag{10}$$

Definition 4: Same as CRS models, we also define an ECP in this section, and we can also obtain the following equation.

$$ECP_d^{SSBM}(S) = E_d^{SSBM}(N/S) - 1, \forall d \in E^*, S \subseteq E^* \tag{11}$$

Following Tone (2002) and Chen (2013), we can easily prove: if $d \in S$, $ECP_d^{SSBM}(S) \geq 0$; if $d \in E^* / S$, $ECP_d^{SSBM}(S) = 0$; if $S = \emptyset$, which means no DMUs have been removed from the reference set, then $ECP_d^{SSBM}(S) = 0$.

Definition 5 Similarly, the characteristic function $C_d^{SSBM}(S)$ for common platform TU-game based on S-SBM model is defined as

$$C^{SSBM}(S) = \sum_{d \in S} ECP_d^{SSBM}(S). \quad (12)$$

Obviously, if $S = \emptyset$, then $C_d^{SSBM}(S) = 0$.

Theorem 2: The characteristic function $C_d^{SSBM}(S)$ is super-additive.

Proof. See Appendix.

From **Definition 4**, **Definition 5**, and **Theorem 2**, we learn that the TU-game defined in Section 4 can also be applied here. After calculating the results by S-SBM model (10), a Shapley value can be computed as the unique solution for the fairness axioms presented in Section 4.

We use the data set in Table 2 to illustrate the VRS case of the proposed approach. Results are presented in Table 8. Similarly, five DMUs are VRS-efficient, while the last three DMUs are inefficient. The 2nd and 3rd columns show the super efficiency and ranks obtained by S-SBM model, the 4th and 5th columns are the Shapley value and ranks based on the common platform approach, and the last column shows rank changes between the two approaches. It shows DMU A and D are ranked differently.

Table 8 Results of SBM model based on VRS assumption

Unit	Super efficiency	Rank (SE)	Shapley value	Rank	Rank change
A	1.2976	4	0.7635	3	-1
B	2.0000	2	1.2252	2	0
C	1.0667	5	0.3252	5	0
D	1.6667	3	0.7221	4	+1
M	2.1667	1	1.3734	1	0

Notably, the common platform S-SBM method can also be applied to the CRS assumption. Here we use the S-SBM model as an example to extend the proposed approach in this paper. Similarly, we can use the J-SBM (Chen, 2013) to extend it as well.

7. Conclusion

In this paper, we propose an approach to evaluate DMUs using a common platform to compute the efficiencies of DMUs in a comparable and equitable way. The set of all efficient DMUs is regarded as a grand coalition and each efficient unit is a player. Based on the axioms of fairness, Shapley values

are obtained for the cooperative game. Simultaneously, we extend our method to a VRS case. As a real-world example, our approach is applied to analyze the efficiencies of 14 CCCBs. Based on our model, the Shapley values of these banks and a full ranking are given.

A limitation is that the proposed approach only calculates the efficiencies of efficient units. Inefficient DMUs, which can be evaluated in the CCR model, cannot be directly compared with the evaluations of efficient DMUs computed by our approach. In addition, when many efficient DMUs exist, the algorithm becomes complicated and requires much time to run. Another problem is that some inefficient units probably have an effect on efficient DMUs. This effect is an issue because the cooperative game may not be super-additive when considering all DMUs in the DMU set as players. For future research, the proposed approach may be extended to compute the efficiency of inefficient unit in a similar way. Regarding all units in the DMU set as a coalition is also recommended.

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Appendix

Lemma 1: $E_d(N/\{k\}) \geq E_d(N), \forall k, d \in E^*$

Proof: If $k = d$, then $E_d(N/\{k\})$ denotes the super efficiency of d and $E_d(N/\{k\}) \geq E_d(N), \forall k \in E^*$ (Lee, Chu, & Zhu, 2011). If $k \neq d$, then $E_d(N/\{k\}) = E_d(N) = 1$ as k and d are both efficient DMUs.

Lemma 2: $ECP_d(S) \geq 0, \forall d \in E^*, S \subseteq E^*$.

Proof. $E_d(N) = 1$, If $d \in S$ and $E_d(N/S) \geq 1$, then based on Lemma 1, we have

$$ECP_d(S) = \frac{E_d(N/S)}{E_d(N)} - 1 = E_d(N/S) - 1 \geq 0. \quad \text{If } d \notin S \text{ and } E_d(N/S) = 1, \text{ then}$$

$$ECP_d(S) = \frac{E_d(N/S)}{E_d(N)} - 1 = E_d(N/S) - 1 = 0.$$

Lemma 3: $E_d(N/S) \geq E_d(N), \forall d \in E^*, S \subseteq E^*$

Proof. Based on Lemma 2, we have $ECP_d(S) \geq 0, \forall d \in E^*, S \subseteq E^*, ECP_d(S) = \frac{E_d(N/S)}{E_d(N)} - 1 \geq 0$,

$$\frac{E_d(N/S)}{E_d(N)} \geq 1. \text{ Therefore, } E_d(N/S) \geq E_d(N).$$

Lemma 4: $ECP_d(S_1 \cup S_2) \geq ECP_d(S_1) + ECP_d(S_2), \forall d \in E^*, S_1 \subseteq E^*, S_2 \subseteq E^*, S_1 \cap S_2 = \emptyset$.

Proof. To prove Lemma 4, we need to prove

$$\frac{E_d(N/(S_1 \cup S_2))}{E_d(N)} + 1 \geq \frac{E_d(N/S_1)}{E_d(N)} + \frac{E_d(N/S_2)}{E_d(N)},$$

which means $E_d(N/(S_1 \cup S_2)) + E_d(N) \geq E_d(N/S_1) + E_d(N/S_2)$.

Given that $E_d(N) = 1$, then we have $E_d(N/(S_1 \cup S_2)) + 1 \geq E_d(N/S_1) + E_d(N/S_2)$.

If $d \in S_1$, then $E_d(N) = E_d(N/S_2) = 1$; and based on Lemma 3, we have $E_d(N/(S_1 \cup S_2)) \geq E_d(N/S_1)$.

If $d \in S_2$, then $E_d(N) = E_d(N/S_1) = 1$; and based on Lemma 3, we have $E_d(N/(S_1 \cup S_2)) \geq E_d(N/S_2)$.

If $d \notin S_1$ and $d \notin S_2$, then $E_d(N) = E_d(N/S_1) = E_d(N/S_2) = E_d(N/(S_1 \cup S_2)) = 1$.

Therefore, $ECP_d(S_1 \cup S_2) \geq ECP_d(S_1) + ECP_d(S_2), \forall d \in E^*, S_1 \subseteq E^*, S_2 \subseteq E^*, S_1 \cap S_2 = \emptyset$.

Theorem 1: The characteristic function C is super-additive. In particular, for any $S_1 \subseteq E^*$ and $S_2 \subseteq E^*$ with $S_1 \cap S_2 = \emptyset$, we obtain

$$C(S_1 \cup S_2) \geq C(S_1) + C(S_2).$$

Proof. Based on Lemma 4, we obtain

$$ECP_d(S_1 \cup S_2) \geq ECP_d(S_1) + ECP_d(S_2), \forall d \in E^*, S_1 \subseteq E^*, S_2 \subseteq E^*, S_1 \cap S_2 = \emptyset \text{ and}$$

$$\sum_{d \in S_1 \cup S_2} ECP_d(S_1 \cup S_2) \geq \sum_{d \in S_1 \cup S_2} ECP_d(S_1) + \sum_{d \in S_1 \cup S_2} ECP_d(S_2).$$

Thus, we can say that the characteristic function C is super-additive.

Theorem 2: The characteristic function $C_d^{SSBM}(S)$ is super-additive.

Proof.

To prove that $C_d^{SSBM}(S)$ is super-additive is equivalent to proving that, for any two sets $S_1 \subseteq E^*$, $S_2 \subseteq E^*$ and $S_1 \cap S_2 = \emptyset$, there is

$$C^{SSBM}(S_1 \cup S_2) \geq C^{SSBM}(S_1) + C^{SSBM}(S_2) \quad (\text{A.1})$$

As defined in **Definition 4** and **Definition 5**, we obtain

$$C^{SSBM}(S_1 \cup S_2) = \sum_{d \in S_1 \cup S_2} ECP_d^{SSBM}(S_1 \cup S_2) = \sum_{d \in S_1 \cup S_2} E_d^{SSBM}(N/(S_1 \cup S_2)) - p_3, \quad (\text{A.2})$$

$$C^{SSBM}(S_1) = \sum_{d \in S_1} ECP_d^{SSBM}(S_1) = \sum_{d \in S_1} E_d^{SSBM}(N/S_1) - p_1, \quad (\text{A.3})$$

$$C^{SSBM}(S_2) = \sum_{d \in S_2} ECP_d^{SSBM}(S_2) = \sum_{d \in S_2} E_d^{SSBM}(N/S_2) - p_2, \quad (\text{A.4})$$

where $p_3 = p_1 + p_2$.

The formula $\sum_{d \in S_1 \cup S_2} E_d^{SSBM}(N/(S_1 \cup S_2))$ can be decomposed as

$$\sum_{d \in S_1} E_d^{SSBM}(N/(S_1 \cup S_2)) + \sum_{d \in S_2} E_d^{SSBM}(N/(S_1 \cup S_2)).$$

Comparing $E_d^{SSBM}(N/(S_1 \cup S_2))$ with $E_d^{SSBM}(N/S_1)$ for arbitrary $d \in S_1$, we learn that the reference set $N/(S_1 \cup S_2) \subseteq N/S_1$, which means moving the frontier of $N/(S_1 \cup S_2)$ to N/S_1 , would increase inputs or decrease outputs. Then, we learn that the optimal slacks of model (11) satisfy the following inequalities: $s_{id}^-(N/(S_1 \cup S_2)) \leq s_{id}^-(N/S_1)$ and $s_{rd}^+(N/(S_1 \cup S_2)) \leq s_{rd}^+(N/S_1)$ for arbitrary $r \in H$ and $i \in M$.

Then, we obtain $E_d^{SSBM}(N/(S_1 \cup S_2)) \geq E_d^{SSBM}(N/S_1), \forall d \in S_1$.

Similarly, we can obtain $E_d^{SSBM}(N/(S_1 \cup S_2)) \geq E_d^{SSBM}(N/S_2), \forall d \in S_2$.

Finally, we obtain $\sum_{d \in S_1 \cup S_2} E_d^{SSBM}(N/(S_1 \cup S_2)) \geq \sum_{d \in S_1} E_d^{SSBM}(N/S_1) + \sum_{d \in S_2} E_d^{SSBM}(N/S_2)$.