Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated Benders decomposition

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Abstract: Environmental, social and economic concerns motivate the operation of closed-loop supply chain networks (CLSCN) in many industries. We propose a novel profit maximization model for CLSCN design as a mixed-integer linear program in which there is flexibility in covering the proportions of demand satisfied and returns collected based on the firm's policies. Our major contribution is to develop a novel hybrid robust-stochastic programming (HRSP) approach to simultaneously model two different types of uncertainties by including stochastic scenarios for transportation costs and polyhedral uncertainty sets for demands and returns. Transportation cost scenarios are generated using a Latin Hypercube Sampling method and scenario reduction is applied to consolidate them. An accelerated stochastic Benders decomposition algorithm is proposed for solving this model. To speed up the convergence of this algorithm, valid inequalities are introduced to improve the lower bound quality, and also a Pareto-optimal cut generation scheme is used to strengthen the Benders optimality cuts. Numerical studies are performed to verify our mathematical formulation and also demonstrate the benefits of the HRSP approach. The performance improvements achieved by the valid inequalities and Pareto-optimal cuts are demonstrated in randomly generated instances.

Keywords: Robustness and sensitivity analysis, Stochastic programming, Robust optimization, Closed-loop supply chain, Benders decomposition

1. Introduction

The growing need for remanufacturing and recycling due to resource scarcity and environmental concerns requires firms to coordinate the forward and reverse material flows in their supply chains. This motivates the design of a closed-loop supply chain network (CLSCN) to avoid sub-optimality arising from separate design of forward and reverse networks. As pointed out by Klibi et al. (2010), the design of a supply chain network is a crucial strategic decision, the effects of which will persist for many years while the business environment may change. Thus, some important parameters such as demand and costs are significantly uncertain. In addition, because opening or closing a facility is time-consuming and costly, making any change in these

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decisions in response to parameter oscillations is impossible within a short time frame (Pishvaei et al., 2011). Uncertainties are intensified in the reverse supply chain network where the quality and quantity of returned products vary unpredictably and fast. Therefore, the design of CLSCN should be robust to the inherent uncertainty in the network parameters.

Of the few recent relevant papers that consider uncertainty in the CLSCN design problem, most estimate the probability distributions for the parameters and then apply scenario-based stochastic programming (SP; e.g., Salema et al., 2007; Santos et al., 2005). SP is a powerful modeling tool when an accurate probabilistic description of the random variables is known. However, it has three main drawbacks (Bertsimas and Thiele, 2006; Gülpınar et al., 2013). First, in many real-life applications not enough historical data are available to estimate distributions. For instance, predicting demand of a new product is challenging. Secondly, an accurate distribution approximation may require a large number of scenarios. But the more scenarios used for representing uncertainty, the harder it is to solve the problem to optimality. Conversely, if the number of scenarios is limited for computational reasons, the obtained solution may be infeasible for some realizations of uncertain parameters. Even if this occurs with very small probability, it could result in high cost due to the large scale of the CLSCN. Finally, SP models based on expected cost are appropriate when the decision maker worries about the average performance of the system. However, there are situations where the decision maker is concerned with the worst case. We highlight this concern with respect to uncertain demand and return quantities.

To avoid these drawbacks, robust optimization (RO) has emerged as an alternative methodology to cope with uncertainty in the input data. The robust counterpart is a deterministic reformulation of the original problem in which the worst case cost is minimized over all possible values the input parameters may take within predefined uncertainty sets. Two main advantages of RO compared with SP are (Alem and Morabito, 2012): first, independently of the number of uncertain parameters, the robust counterpart can remain computationally tractable, and second, rough historical data and decision makers' experiences can be used to derive the boundaries of uncertainty sets, without the need for precise estimates of probability distributions.

The uncertain parameters we consider in our CLSCN design problem differ qualitatively. Historical data for transportation costs can be used to formulate probabilistic scenarios for them, but no such data for demand and return quantities of a new product exist. Because the purpose of the network is to supply products and collect the returns, we design it for the extreme quantities to ensure that its capacity and configuration will suffice in any event. The need to consider both

types of uncertainty in an integrated network has been emphasized recently by Melo et al. (2009), Klibi and Martel (2012) and Gabrel et al. (2014).

This paper contributes to the CLSCN design literature by developing a novel hybrid robust-stochastic optimization approach and also devising an efficient solution procedure. Specifically, a mixed-integer linear program (MILP) is developed for a multi-period, single-product and capacitated CLSCN. The strategic decisions including locations and capacities of facilities as well as the tactical decisions including inventory levels, production amounts, and shipments among the network entities are determined to maximize the expected worst-case profit. The major contributions can be summarized as follows:

- To integrate both strategic and tactical decisions with flexibility to cover varying proportions of demands and customer returns.
- To simultaneously model two different types of uncertainties including stochastic scenarios for transportation costs and polyhedral uncertainty sets for demand and return quantities, via a hybrid robust-stochastic programming (HRSP) approach.
- To obtain a small but representative set of transportation cost scenarios using Latin Hypercube Sampling (LHS) followed by scenario reduction.
- To strengthen the Benders master problem and improve the quality of the lower bound with two sets of valid inequalities (VI). Pareto-optimal cuts are also used to accelerate the convergence of the solution algorithm.

The remainder of this paper is organized as follows. In the next section, we briefly review the literature on the CLSCN design problem and the relevant solution methods. The problem and its stochastic formulation are defined in Section 3. Then, the HRSP approach is presented in Section 4. In Section 5, the scenario generation and reduction algorithm for transportation costs is presented. The stochastic Benders decomposition (BD) algorithm with some acceleration techniques for improving its convergence is provided in Section 6. Section 7 describes computational experiments and sensitivity analyses that allow us to derive managerial insights about this CLSCN. Finally, Section 8 concludes this paper and offers some suggestions for future research.

2. Literature review

The relevant literature follows two separate but complementary streams. We first review studies of the CLSCN design problem and then discuss solution algorithms. A complete

literature review for the CLSCN design problem based on problem features, supply chain stages, objective, modeling, uncertainty programming, uncertain parameters, decisions, and solution methods is provided in Keyvanshokoo (2015).

2.1. Closed-loop supply chain network design problem

To avoid sub-optimality from modeling and designing forward and reverse networks separately, many researchers have integrated them in the more complex CLSCN (Melo et al., 2009). Many CLSCN models are inspired by facility location theory. In this regard, Melo et al. (2009) and Klibi et al. (2010) presented comprehensive reviews on the facility location models in supply chain planning and on supply chain network design under uncertainty, respectively. Moreover, Pokharel and Mutha (2009) summarized the current developments of reverse supply chains, while Brandenburg et al. (2014) and Dekker et al. (2012) reviewed quantitative models that address environmental and social aspects in the supply chain.

Originally, Fleischmann et al. (2001) considered the integration of forward and reverse flows as a CLSCN using some case studies. They found that this integrated approach could provide a potential for a significant cost savings compared to a segregated approach. The research that followed was primarily carried out with simple facility location models (e.g. Aras et al., 2008). Then, more complex models were proposed especially by considering the real-life characteristics (e.g. Cruz-Rivera and Ertel, 2009). The field has experienced a strong development over the last decade (e.g. Klibi and Martel, 2012; Alumur et al., 2012; Cardoso et al., 2013; Baghalian et al., 2013; Keyvanshokoo et al., 2013; Soleimani and Govindan, 2014; Devika et al., 2014; Gao and Ryan, 2014; Faccio et al., 2014; De Giovanni and Zaccour, 2014; Niknejad and Petrovic, 2014).

Given that all activities in both forward and reverse supply chains are subject to considerable uncertainty, many works addressed the CLSCN design problem where some network parameters such as demand, return and costs are uncertain. In a pioneering step, Salema et al. (2007) extended the model of Fleischmann et al. (2001) to a multi-product and capacitated CLSCN considering uncertainty in demand and return. SP is the most popular tool applied to the configuration of a CLSCN under uncertainty. However, a limited number of studies employed RO (Pishvaei et al., 2011; Vahdani et al., 2012; Hasani et al., 2011). These applied a worst-case robust formulation (Soyster, 1973) which may result in an overly conservative solution. Considering this research gap, we apply a more recent RO approach (Bertsimas and Sim, 2004), which allows a tradeoff between optimality and robustness. To our knowledge, no existing research on CLSCN design combines probabilistic scenarios for some parameters with

uncertainty sets for others. Fanzeres dos Santos et al. (2014) applied a similar hybrid approach in the context of electricity markets.

Minimizing cost has been the primary objective in most CLSCN models. These models typically require that every customer's demand and return has to be satisfied. However, it may not always be optimal to satisfy all demands and returns. Sometimes, there is not much competition in target market, so the cost of losing customers will be very low. Hence, the firm may maximize its profit by losing some customers. On the other hand, sometimes profit is increased with better customer service. This paper includes flexibility to determine what fraction of customers to serve.

2.2. Solution algorithms

Because the CLSCN design problem is an NP-hard combinatorial optimization problem, many solution algorithms including metaheuristic, heuristic, and exact methods have been developed. Most solution methods employ standard commercial packages such as CPLEX to solve mixed-integer programming formulations. However, when the number of discrete variables is large, the resulting models can be solved only by using metaheuristic or heuristic methods to obtain a near optimal solution. But, because CLSCN design involves large investment and greatly influences the operational and tactical costs as well as efficiency of service, developing efficient exact algorithms for solving larger and more realistic cases is worthwhile (de Sá et al., 2013). Among these exact solution approaches, branch-and-bound has been a popular methodology combined with other heuristics or Lagrangian relaxation. There are few papers that develop an exact solution scheme, a shortage highlighted by Klibi and Martel (2012).

As a discrete facility location problem, CLSCN design is an attractive candidate for decomposition. It involves both binary variables related to the strategic configuration, and continuous variables associated with tactical and operational decisions. Keyvanshokoo (2015) found just four papers in which decomposition schemes were applied. Among the decomposition techniques, the BD method (Benders, 1962) is a classical exact algorithm suitable for solving large-scale MILP problems having special structure in the constraint set; i.e., upon fixing the values of the complicating integer variables, the MILP problem reduces to an easy linear program.

However, classical BD and its stochastic version, called the L-shaped method, might not be efficient (Saharidis and Ierapetritou, 2010). The major issues resulting in its slow convergence are (1) solving the relaxed master problem (RMP) which is in fact an integer program or MILP,

and (2) the quality of Benders cuts. To overcome these concerns, different acceleration techniques have been proposed to speed up BD. Magnanti and Wong (1981) defined a cut as Pareto-optimal and achieved significant improvement in convergence by applying such cuts to a problem with degenerate sub-problems. Saharidis et al., (2010) introduced a covering cut bundle strategy by producing a bundle of cuts in each iteration to cover all decision variables of the MP. Some other modifications to this algorithm were presented by McDaniel and Devine (1977), Saharidis et al., (2013), Tang et al., (2013), Sherali and Lunday (2013) and Oliveira et al., (2014) in different applications.

3. Problem definition and formulation

3.1. Problem definition

As illustrated by Fig. 1, we consider a single product, multi-period, and capacitated CLSCN consisting of manufacturing/remanufacturing, distribution, collection, and disposal centers as well as retailers under demand, return and transportation cost uncertainty. The end-of-use products are collected from retailers, transported to collection centers, and after a quality test, divided into two categories: recoverable products sent to manufacturing/remanufacturing centers and scrapped products shipped to disposal centers. In the forward network, the remanufactured products along with the new ones are supplied to retailers from manufacturing/remanufacturing centers through distribution centers to meet their demand. We also assume a periodic review inventory policy for distribution centers to find inventory levels and include base-stock levels for these facilities as decision variables (Keyvanshokoo et al., 2013). However, it is assumed that the product is perishable and hence the excess amount of product in the retail facilities in one period cannot be used to satisfy the consumer demand of the next period.

Fig. 1. The CLSCN structure

This CLSCN model can apply to companies that are introducing new products to their target market consisting of their previous customers. For example, suppose an exclusive company produces desktop and notebook computers. To improve customer satisfaction, it decides to also provide after-sales service and, to this aim, they want to produce some components. On one hand, due to being an exclusive firm, these spare parts would appeal only to their customers who bought the computers from this company before, so there is not much competition in the target market. Thus, the risk of losing customers will be very low. Then, if a small penalty is considered for not satisfying demand or collecting returns, profit may be maximized by covering just a portion of demand and returns. On the other hand, if the company wants to emphasize satisfaction of customers, a high penalty cost should be considered. Our formulation allows any condition between these two extremes. Most CLSCN design models in the literature aim to satisfy the whole demand and return quantities, or they maximize profit without any attention to how much of the demand and returns they satisfy (Keyvanshokoo et al., 2013; Amin and Zhang, 2013). However, one of our goals is to design the network considering different conditions of the target market that affect the importance of these constraints.

In our model, if the flow from distribution centers to retailers exceeds the retailers' demand, then a surplus cost proportional to the excess is charged. On the other hand, if the retailers' demand is greater than the quantity delivered from distribution centers, then a penalty cost for unsatisfied demand is incurred. In the reverse network, if the flow from retailers to collection centers is greater than the potential returns, which is impossible in practice, then we apply a penalty cost per unit of excess flow. However, if this flow is less than the potential returns, then we impose a scrap cost per unit of uncollected returns. Defining the penalty, surplus, and scrap costs balances the forward flows with the demands and the reverse flows with the return quantities as much as possible while ensuring complete recourse (Birge and Louveaux, 1997) in the stochastic program.

The main concern of this paper is to design the CLSCN in the presence of uncertainty. Two different types of uncertainty are present; one for transportation costs and the other for demand and return quantities. During the last decade, the oscillations in fuel price have dramatically influenced transportation costs and it is quite likely that this uncertainty on fuel price will be sustained (Pishvae et al., 2009). We assume the firm has historical data for transportation cost distributions from its previous sales and so model this uncertainty with probabilistic scenarios.

On the other hand, forecasting the precise distribution of future demands and returns of a new product is very difficult. Demand could be affected by unexpected events such as the appearance of a new competitor and return quantities depend on customer use patterns. Stationarity of probability distributions cannot be guaranteed, especially in our multi-period planning horizon. Even if sufficient data are available to generate credible scenarios, considering many of them for both demands and returns will create computational challenges. Thus, we adopt a RO approach of formulating uncertainty sets instead of probability distributions for these quantities. The CLSCN design problem is to concurrently determine the location of facilities, their capacities, and base-stock levels as the first-stage decisions in light of the recourse production amounts and network flows to meet the worst-case demand and return quantities in each transportation cost scenario.

3.2. Problem formulation

We first formulate a two-stage stochastic program with recourse for the CLSCN design problem assuming full knowledge of probability distributions for uncertain transportation costs. Regarding the uncertain demands and returns, we first consider their nominal values under each scenario for transportation cost. In the following subsection, we explain how uncertainty sets for these parameters are included in the HRSP approach. In the first-stage, strategic decisions such as locations and capacities of facilities as well as distribution base-stock levels are determined as the here-and-now decisions that should be made before realization of any uncertain parameters, and in the second-stage operational decisions such as network flows are made after realization of uncertain parameters. Define the following notation:

Sets:

I	Potential locations of manufacturing/remanufacturing centers, $i \in I$
J	Potential locations available for distribution and collection centers, $j \in J$
K	Fixed locations of retailers, $k \in K$
R	Fixed locations of disposal centers, $r \in R$
S	Set of scenarios for transportation costs, $s \in S$
T	Set of time periods in the planning horizon, $t, p \in T$

Parameters:

$\hat{D}_{ks}^t, \hat{R}_{ks}^t$	Nominal demand (return of used product) of retailer k at time period t in scenario s
F_i^{MC}	Fixed cost for opening manufacturing/remanufacturing center i
F_j^{DC}, F_j^{CC}	Fixed cost for opening distribution (collection) center j

PR_k	Revenue per unit of product sold by retailer k to customers
MC_i, RC_i	Unit manufacturing (remanufacturing) cost in manufacturing/remanufacturing center i
IC_j^t	Inventory cost per unit of product in time period t in distribution center j
CC_j	Unit collection/inspection cost in collection center j
DC_r	Unit disposal cost in disposal center r
PC^D	Penalty cost per unit of non-satisfied demands of retailer
SC^R	Scrap cost per unit of uncollected returns of retailer
SC^D	Surplus cost per unit of excess amounts of flow over demands received by retailers
PC^R	Penalty cost per unit of excess amounts of flow over returns collected from retailers
CLJ_{ijs}^t	Transportation cost per unit of product transported from manufacturing/remanufacturing center i to distribution center j in time period t in scenario s
CJK_{jks}^t	Transportation cost per unit of product transported from distribution center j to retailer k in time period t in scenario s
CKJ_{kjs}^t	Transportation cost per unit of returned product transported from retailer k to collection center j in time period t in scenario s
CJI_{jis}^t	Transportation cost per unit of recoverable product transported from collection center j to manufacturing/remanufacturing center i in time period t in scenario s
CJR_{jrs}^t	Transportation cost per unit of scrapped product transported from collection center j to disposal center r in time period t in scenario s
c_i^{MC}	Capacity cost of manufacturing/remanufacturing center i per unit of products per period
c_j^{DC}, c_j^{CC}	Capacity cost of distribution (collection) center j per unit of products per period
CAP_i^{MC}	Maximum available capacity of manufacturing/remanufacturing center i (units of products per period)
CAP_j^{DC}, CAP_j^{CC}	Maximum available capacity of distribution (collection) center j (units of products per period)
a	Fraction of returned products that can be remanufactured
Pr_s	Probability of transportation cost scenario s

Decision variables:

X_i^{MC}	Binary variable equal to 1 if a manufacturing/remanufacturing center is opened at location i , 0 otherwise
X_j^{DC}, X_j^{CC}	Binary variable equal to 1 if a distribution (collection) center is opened at location j , 0 otherwise
W_i^{MC}	Capacity of manufacturing/remanufacturing center i (units of products per period)
W_j^{DC}, W_j^{CC}	Capacity of distribution (collection) center j (units of products per period)

FIJ_{ijs}^t	Quantity of products transported from manufacturing/remanufacturing center i to distribution center j in time period t in scenario s
FJK_{jks}^t	Quantity of products transported from distribution center j to retailer k in time period t in scenario s
FKJ_{kjs}^t	Quantity of returned products transported from retailer k to collection center j in time period t in scenario s
FIJ_{ijs}^t	Quantity of recoverable products transported from collection center j to manufacturing/remanufacturing center i in time period t in scenario s
FJR_{jrs}^t	Quantity of scrapped products transported from collection center j to disposal center r in time period t in scenario s
PI_{is}^t	Quantity of products produced by manufacturing/remanufacturing center i in time period t in scenario s
BS_j	Base-stock level of product in distribution center j at the beginning of each period

The two-stage stochastic CLSCN design problem can be formulated as follows:

$$\text{Max } Z = Q(X, W, BS) - \sum_{i \in I} [F_i^{MC} X_i^{MC} + C_i^{MC} W_i^{MC}] - \sum_{j \in J} [F_j^{DC} X_j^{DC} + C_j^{DC} W_j^{DC} + F_j^{CC} X_j^{CC} + C_j^{CC} W_j^{CC}] \quad (1)$$

S.t.

$$W_i^{MC} \leq CAP_i^{MC} X_i^{MC} \quad \forall i \in I \quad (2)$$

$$W_j^{DC} \leq CAP_j^{DC} X_j^{DC} \quad \forall j \in J \quad (3)$$

$$W_j^{CC} \leq CAP_j^{CC} X_j^{CC} \quad \forall j \in J \quad (4)$$

$$W_j^{DC} \geq BS_j \quad \forall j \in J \quad (5)$$

$$X_j^{DC} + X_j^{CC} \leq 1 \quad \forall j \in J \quad (6)$$

$$X_i^{MC}, X_j^{DC}, X_j^{CC} \in \{0, 1\}, W_i^{MC}, W_j^{DC}, W_j^{CC}, BS_j \geq 0 \quad \forall i \in I, j \in J \quad (7)$$

where $Q(X, W, BS) = E_{\zeta}[\mathcal{Q}(X, W, BS, \zeta_s)] = \sum_s Pr_s \times \mathcal{Q}(X, W, BS, \zeta_s)$. $E_{\zeta}[\mathcal{Q}(X, W, BS, \zeta_s)]$ denotes the recourse

function. For a given scenario ζ_s , $\mathcal{Q}(X, W, BS, \zeta_s)$ is the optimal objective function value of the second-stage problem (8)-(24):

$$\mathcal{Q}(X, W, BS, \zeta_s) = \text{Max} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} [PR_k FJK_{jks}^t - CJK_{jks}^t FJK_{jks}^t - CKJ_{kjs}^t FKJ_{kjs}^t - CC_j FKJ_{kjs}^t] - \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} [CIJ_{ijs}^t FIJ_{ijs}^t \quad (8)$$

$$+ CJI_{jis}^t FJI_{jis}^t + RC_i FJI_{jis}^t] - \sum_{j \in J} \sum_{r \in R} \sum_{t \in T} [CJR_{jrs}^t FJR_{jrs}^t + DC_r FJR_{jrs}^t] - \sum_{i \in I} \sum_{t \in T} MC_i PI_{is}^t - \sum_{j \in J} \sum_{t \in T} \left[\sum_{i \in I} \sum_{p \leq t} FIJ_{ijs}^p - \sum_{k \in K} \sum_{p \leq t} FJK_{jks}^p \right] IC_j^t$$

S.t.

$$\sum_{j \in J} FJK_{jks}^t = \hat{D}_{ks}^t \quad \forall k \in K, t \in T \quad (9)$$

$$\sum_{j \in J} FKJ_{kjs}^t = \hat{R}_{ks}^t \quad \forall k \in K, t \in T \quad (10)$$

$$\sum_{k \in K} \sum_{p \leq t} FJK_{jks}^p - \sum_{i \in I} \sum_{p \leq t} FIJ_{ijs}^p \leq 0 \quad \forall j \in J, t \in T \quad (11)$$

$$\sum_{i \in I} \sum_{p \leq t} FIJ_{ijs}^p - \sum_{k \in K} \sum_{p \leq t} FJK_{jks}^p = BS_j \quad \forall j \in J, t \in T \quad (12)$$

$$PI_{is}^t + \sum_{j \in J} FJI_{jis}^t = \sum_{j \in J} FIJ_{ijs}^t \quad \forall i \in I, t \in T \quad (13)$$

$$a \sum_{k \in K} FKJ_{kjs}^t = \sum_{i \in I} FJI_{jis}^t \quad \forall j \in J, t \in T \quad (14)$$

$$(1-a) \sum_{k \in K} FKJ_{kjs}^t = \sum_{r \in R} FJR_{jrs}^t \quad \forall j \in J, t \in T \quad (15)$$

$$PI_{is}^t + \sum_{j \in J} FJI_{jis}^t \leq W_i^{MC} \quad \forall i \in I, t \in T \quad (16)$$

$$\sum_{i \in I} FIJ_{ijs}^t \leq W_j^{DC} \quad \forall j \in J, t \in T \quad (17)$$

$$\sum_{k \in K} FKJ_{kjs}^t \leq W_j^{CC} \quad \forall j \in J, t \in T \quad (18)$$

$$FIJ_{ijs}^t \leq M.X_j^{DC} \quad \forall i \in I, j \in J, t \in T \quad (19)$$

$$FJK_{jks}^t \leq M.X_j^{DC} \quad \forall k \in K, j \in J, t \in T \quad (20)$$

$$FKJ_{kjs}^t \leq M.X_j^{CC} \quad \forall k \in K, j \in J, t \in T \quad (21)$$

$$FJI_{jis}^t \leq M.X_j^{CC} \quad \forall i \in I, j \in J, t \in T \quad (22)$$

$$FJR_{jrs}^t \leq M.X_j^{CC} \quad \forall r \in R, j \in J, t \in T \quad (23)$$

$$FIJ_{ijs}^t, FJK_{jks}^t, FKJ_{kjs}^t, FJI_{jis}^t, FJR_{jrs}^t, PI_{is}^t \geq 0 \quad \forall i \in I, j \in J, r \in R, k \in K, t \in T \quad (24)$$

The objective (1) is to maximize the expected total second-stage profit less the first-stage costs including fixed costs of opening facilities and capacity costs. The second-stage profit (8) includes the revenue from selling new products less transportation costs, inventory costs, manufacturing costs of new products and remanufacturing costs of used products, collection costs of used products, and disposal costs of scrapped product. Constraints (2)-(4) ensure capacity restrictions for manufacturing/remanufacturing, distribution and collection centers, respectively. Constraints (5) guarantee that the capacity of each distribution center is at least equal to its base-stock level. Constraints (6) ensure that at each location j just one of distribution or collection centers is opened. Constraints (9)-(10) assure that the nominal demand of retailers are satisfied by the distribution centers and also the nominal returns of used products from retailers are collected by the collection centers, respectively. In the next section, we will explain how we incorporate uncertainty sets for these parameters. Constraint (11) assures the flow

balance for each distribution centers. Constraints (12) enforce base-stock levels for each distribution center in scenario s and period t . Constraints (13)-(15) ensure the flow balance for manufacturing/remanufacturing and collection centers. Constraints (16)-(18) express the capacity constraints for the manufacturing/remanufacturing, distribution and collection centers, respectively. Constraints (19)-(23) connect the binary variables for facility existence with the corresponding flows, where M is a large number. Finally, constraints (7) and (24) enforce the binary and non-negativity restrictions on decision variables.

4. Hybrid robust-stochastic programming approach

First, we briefly review a RO approach presented by Bertsimas and Sim, 2003, 2004 as a prelude to describing our HRSP formulation. Consider the linear program (LP) where C is an n -vector, A is a $m \times n$ matrix, and b is an m -vector:

$$\text{Min } Cx \text{ s.t. } Ax \leq b, x \geq 0 \quad (25)$$

Assume uncertainty only affects the elements of matrix A . That is, consider a particular row i of A and let J_i represent the set of coefficients in row i of A subject to uncertainty. Each data element $\tilde{a}_{ij}, j \in J_i$ is modeled as a bounded and independent random variable taking value in an interval $[\hat{a}_{ij} - a_{ij}, \hat{a}_{ij} + a_{ij}]$ where $\hat{a}_{ij}$ is the nominal value and a_{ij} is the maximum deviation from this nominal value. With this assumption, LP (25) is reformulated as:

$$\text{Min } Cx \text{ s.t. } \max_{\forall \tilde{a}_{ij} \in J_i} \left(\sum_j \tilde{a}_{ij} x_j \right) \leq b_i \quad \forall i, x \geq 0 \quad (26)$$

Then, we define a scaled deviation $z_{ij} = (\tilde{a}_{ij} - \hat{a}_{ij}) / a_{ij}$ from its nominal value of $\hat{a}_{ij}$ that always belongs to the interval $[-1, 1]$. Note that $\tilde{a}_{ij}$, $\hat{a}_{ij}$ and a_{ij} denote the uncertain value, its nominal value and its maximum deviation from the nominal value, respectively. It is unlikely that all of the uncertain input $\tilde{a}_{ij}, j \in J_i$ will realize their worst-case values simultaneously. Thus, a maximum number of parameters that can deviate from their nominal values for each constraint i is considered as Γ_i , called the budget of uncertainty, where $\Gamma_i \in [0, |J_i|]$. The aggregated scaled deviation of uncertain parameters for constraint i is bounded as $\sum_{j \in J_i} |z_{ij}| \leq \Gamma_i, \forall i$.

The budget of uncertainty plays a crucial role in adjusting the solution's level of conservatism against the robustness. If $\Gamma_i = 0$, it reduces to the nominal formulation where there is no protection against uncertainty. If $\Gamma_i = |J_i|$, the i th constraint is completely protected against

the worst-case realization of uncertain parameters. Finally, if $\Gamma_i \in (0, |J_i|)$ the decision maker considers a trade-off between conservatism and cost of the solution against the level of protection against constraint violation. Based on this definition, the set J_i is defined as $J_i = \{ \tilde{a}_{ij} | \tilde{a}_{ij} = \hat{a}_{ij} + a_{ij}z_{ij}, \forall i, j, z \in \Omega \}$ where $\Omega = \{ z | \sum_{j=1}^n z_{ij} \leq \Gamma_i, |z_{ij}| \leq 1, \forall i \}$. Restating each constraint i as $\sum_j \tilde{a}_{ij}x_j = \sum_j (\hat{a}_{ij} + a_{ij}z_{ij})x_j = \sum_j \hat{a}_{ij}x_j + \sum_j a_{ij}z_{ij}x_j$, LP (26) can be reformulated as:

$$\text{Min } Cx \text{ s.t. } \sum_j \hat{a}_{ij}x_j + \text{Max}_{z_i \in \Omega_i} \sum_j a_{ij}z_{ij}x_j \leq b_i \quad \forall i, x \geq 0 \quad (27)$$

The lower level problem $\text{Max}_{z_i \in \Omega_i} \sum_j a_{ij}z_{ij}x_j$ for a given vector x^* is equivalent to LP (28):

$$\text{Max} \sum_j a_{ij}z_{ij}x_j^* \text{ s.t. } \sum_j z_{ij} \leq \Gamma_i \quad \forall i, 0 \leq z_{ij} \leq 1 \quad \forall j \in J_i \quad (28)$$

Then by introducing the dual variables λ_i and μ_{ij} , the dual of LP (28) is:

$$\text{Min } \Gamma_i \lambda_i + \sum_{j \in J_i} \mu_{ij} \text{ s.t. } \lambda_i + \mu_{ij} \geq a_{ij}x_j^* \quad \forall i, j \in J_i, \mu_{ij} \geq 0 \quad \forall j \in J_i, \lambda_i \geq 0 \quad \forall i \quad (29)$$

The dual (29) is applied to LP (27) to obtain the robust counterpart of LP (25):

$$\text{Min } Cx \text{ s.t. } \hat{a}_i x - \Gamma_i \lambda_i - \sum_{j \in J_i} \mu_{ij} \leq b_i \quad \forall i, \lambda_i + \mu_{ij} \geq a_{ij}x_j \quad \forall i, j \in J_i, \mu_{ij}, \lambda_i \geq 0 \quad \forall i, j \in J_i \quad (30)$$

This RO approach provides an efficient way to determine bounds on the probability of violation of each constraint. Let x_j^* be the robust solution, then the violation probability of the i th constraint is calculated by:

$$\Pr \left(\sum_j \tilde{a}_{ij}x_j^* < b_i \right) \leq 1 - \Phi \left((\Gamma_i - 1) / \sqrt{|J_i|} \right) \quad (31)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. This upper bound provides a way of assigning a proper budget of uncertainty to each constraint when our uncertain parameters are independent and symmetrically distributed random variables in their associated uncertainty set.

In our HRSP approach for CLSCN, within each transportation cost scenario we define polyhedral uncertainty sets for demand and return in each period and for each retailer. Fig. 2 illustrates the arrangement of the polyhedral uncertainty sets for demands of retailers in different periods for different transportation cost scenarios. The uncertain demands and returns are allowed to deviate from a nominal scenario toward a worst-case realization within a constrained polyhedral uncertainty set. For simplicity, Fig. 2 shows uncertainty sets for demand only.

Fig. 2. Uncertainty characterization of the HRSP approach

To develop the uncertainty sets, first we define the positive and negative deviation percentages from the nominal scenario for demands and returns, respectively, as follows:

$$\eta D_{ks}^{t+} = \frac{D_{ks}^t - \hat{D}_{ks}^t}{\Delta D_{ks}^{t+}} \quad \text{if } D_{ks}^t > \hat{D}_{ks}^t, \quad \eta D_{ks}^{t-} = \frac{\hat{D}_{ks}^t - D_{ks}^t}{\Delta D_{ks}^{t-}} \quad \text{if } D_{ks}^t < \hat{D}_{ks}^t \quad (32)$$

$$\eta R_{ks}^{t+} = \frac{R_{ks}^t - \hat{R}_{ks}^t}{\Delta R_{ks}^{t+}} \quad \text{if } R_{ks}^t > \hat{R}_{ks}^t, \quad \eta R_{ks}^{t-} = \frac{\hat{R}_{ks}^t - R_{ks}^t}{\Delta R_{ks}^{t-}} \quad \text{if } R_{ks}^t < \hat{R}_{ks}^t \quad (33)$$

Then, the uncertainty sets of demand and return for each scenario of transportation costs are:

$$J_s^D = \left\{ D_{ks}^t \mid D_{ks}^t = \hat{D}_{ks}^t + \Delta D_{ks}^{t+} \times \eta D_{ks}^{t+} - \Delta D_{ks}^{t-} \times \eta D_{ks}^{t-}, \forall k, t, \forall \eta D_{ks}^{t+}, \eta D_{ks}^{t-} \in K^D \right\} \quad (34)$$

where

$$K^D = \left\{ \eta D_{ks}^{t+}, \eta D_{ks}^{t-} \mid 0 \leq \eta D_{ks}^{t+} \leq 1, 0 \leq \eta D_{ks}^{t-} \leq 1, \sum_{k \in K} \sum_{t \in T} (\eta D_{ks}^{t+} + \eta D_{ks}^{t-}) \leq \Gamma_s^D \right\}.$$

Similarly,

$$J_s^R = \left\{ R_{ks}^t \mid R_{ks}^t = \hat{R}_{ks}^t + \Delta R_{ks}^{t+} \times \eta R_{ks}^{t+} - \Delta R_{ks}^{t-} \times \eta R_{ks}^{t-}, \forall k, t, \forall \eta R_{ks}^{t+}, \eta R_{ks}^{t-} \in K^R \right\} \quad (35)$$

where

$$K^R = \left\{ \eta R_{ks}^{t+}, \eta R_{ks}^{t-} \mid 0 \leq \eta R_{ks}^{t+} \leq 1, 0 \leq \eta R_{ks}^{t-} \leq 1, \sum_{k \in K} \sum_{t \in T} (\eta R_{ks}^{t+} + \eta R_{ks}^{t-}) \leq \Gamma_s^R \right\}$$

The dimension of these sets is $|K| \times |T|$ for each transportation cost scenario. The parameter Γ_s^D in (34) is the budget of uncertainty for demand in scenario s via which we can constrain the number of periods in which the demand may deviate from its nominal value. Similar definitions apply to the polyhedral uncertainty sets for returns in (35).

Allowing for this uncertainty implies that constraints (9) and (10) may not be satisfied. In the HRSP, we relax these constraints and penalize their violation in the objective function. By doing so, we provide the decision maker with flexibility to allow violations in either direction. For example, if the company is in a very competitive market setting where it does not want to lose any customer, then we can increase the penalty for unsatisfied demand. On the other hand, if the manufacturing and remanufacturing resources of company are restricted or the cost price of the product is high, and also if there is not much competition in the target market, then it can increase the surplus cost per unit of excess flow to retailers. In Section 7.1.2, we investigate the effects of these parameter values on the CLSCN design solution.

Our purpose is to minimize the worst-case costs associated with violations of (9)-(10). To incorporate the uncertainty sets (34)-(35) in the stochastic formulation (1)-(8),(11)-(24), we isolate the objective function terms containing random demand and return parameters for scenario s as the following nonlinear expression:

$$PC_s(FJK, FKJ) = \max_{D_{ks}^t \in J_s^D} \left[\sum_{k \in K} \sum_{t \in T} \left(D_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D, \sum_{k \in K} \sum_{t \in T} \left(\sum_{j \in J} FJK_{jks}^t - D_{ks}^t \right) \times SC^D \right] \\ + \max_{R_{ks}^t \in J_s^R} \left[\sum_{k \in K} \sum_{t \in T} \left(R_{ks}^t - \sum_{j \in J} FKJ_{kjs}^t \right) \times SC^R, \sum_{k \in K} \sum_{t \in T} \left(\sum_{j \in J} FKJ_{kjs}^t - R_{ks}^t \right) \times PC^R \right] \quad (36)$$

For each transportation cost scenario, this term represents the worst-case value for penalty, scrap and surplus costs. We reformulate this nonlinear optimization problem (36) as the following LP for each scenario s by defining auxiliary variables $Z1_s$ and $Z2_s$:

$$\min_{Z1_s, Z2_s} PC_s(FJK, FKJ) = Z1_s + Z2_s \quad (37)$$

S.t.

$$\sum_{k \in K} \sum_{t \in T} \left(D_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D \leq Z1_s, \forall D_{ks}^t \in J_s^D \quad (38)$$

$$\sum_{k \in K} \sum_{t \in T} \left(\sum_{j \in J} FJK_{jks}^t - D_{ks}^t \right) \times SC^D \leq Z1_s, \forall D_{ks}^t \in J_s^D \quad (39)$$

$$\sum_{k \in K} \sum_{t \in T} \left(R_{ks}^t - \sum_{j \in J} FKJ_{kjs}^t \right) \times SC^R \leq Z2_s, \forall R_{ks}^t \in J_s^R \quad (40)$$

$$\sum_{k \in K} \sum_{t \in T} \left(\sum_{j \in J} FKJ_{kjs}^t - R_{ks}^t \right) \times PC^R \leq Z2_s, \forall R_{ks}^t \in J_s^R \quad (41)$$

$$Z1_s, Z2_s \geq 0 \quad (42)$$

The constraints (38)-(42) should be satisfied for all realizations of the uncertain demands and returns in their polyhedral uncertainty sets. We find their robust counterparts, explained in detail for constraint (38). From the set definition (34), we can rewrite the constraints (38) as:

$$\text{Max}_{D_{ks}^t \in J_s^D} \left[\sum_{k \in K} \sum_{t \in T} \left(D_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D \right] \leq Z1_s \quad (43)$$

which can be transformed into:

$$\sum_{k \in K} \sum_{t \in T} \left(\hat{D}_{ks}^t - \sum_{j \in J} FJK_{jks}^t \right) \times PC^D + \text{Max}_{\eta D_{ks}^{t+}, \eta D_{ks}^{t-} \in K^D} \left[\sum_{k \in K} \sum_{t \in T} \left(\Delta D_{ks}^{t+} \times \eta D_{ks}^{t+} - \Delta D_{ks}^{t-} \times \eta D_{ks}^{t-} \right) \times PC^D \right] \leq Z1_s \quad (44)$$

In this constraint, we optimize over the positive and negative deviation percentages from nominal scenario for uncertain demand. We expand the maximization problem in (44) considering constraints from polyhedral uncertainty sets as follows:

$$\begin{aligned} & - \text{Min}_{\eta D_{ks}^{t+}, \eta D_{ks}^{t-} \in K^D} \left[\sum_{k \in K} \sum_{t \in T} \left(-\Delta D_{ks}^{t+} \times \eta D_{ks}^{t+} + \Delta D_{ks}^{t-} \times \eta D_{ks}^{t-} \right) \times PC^D \right] \\ & S.t. \\ & -\eta D_{ks}^{t+} \geq -1, \forall t \in T, k \in K \quad : \alpha 1_{ks}^t \\ & -\eta D_{ks}^{t-} \geq -1, \forall t \in T, k \in K \quad : \alpha 2_{ks}^t \\ & \sum_{k \in K} \sum_{t \in T} \left(-\eta D_{ks}^{t+} - \eta D_{ks}^{t-} \right) \geq -\Gamma_s^D \quad : \beta 1^D \\ & \eta D_{ks}^{t+}, \eta D_{ks}^{t-} \geq 0 \end{aligned} \quad (45)$$

Then we take the dual as:

$$\begin{aligned} & \text{Max}_{\alpha 1_{ks}^t, \alpha 2_{ks}^t, \beta 1^D} \left[-\Gamma_s^D \times \beta 1^D - \sum_{k \in K} \sum_{t \in T} \left(\alpha 1_{ks}^t + \alpha 2_{ks}^t \right) \right] \\ & S.t. \\ & -\alpha 1_{ks}^t - \beta 1^D \leq -\Delta D_{ks}^{t+}, \quad \forall t \in T, k \in K \\ & -\alpha 2_{ks}^t - \beta 1^D \leq \Delta D_{ks}^{t-}, \quad \forall t \in T, k \in K \\ & \beta 1^D, \alpha 1_{ks}^t, \alpha 2_{ks}^t \geq 0, \forall t \in T, k \in K \end{aligned} \quad (46)$$

In this LP (46), since the second constraint is actually redundant, we can remove $\alpha 2_{ks}^t$.

According to strong duality theory, we then replace the objective function (46) without $\alpha 2_{ks}^t$ in the constraint (44) and, hence, the robust counterpart of constraint (38) is equivalent to the system of inequalities:

$$\left(\sum_{k \in K} \sum_{t \in T} \left(\hat{D}_{ks}^t + \alpha 1_{ks}^t \right) + \beta 1^D \times \Gamma_s^D - \sum_{k \in K} \sum_{t \in T} \sum_{j \in J} FJK_{jks}^t \right) \times PC^D \leq Z1_s \quad (47)$$

$$\alpha 1_{ks}^t + \beta 1^D \geq \Delta D_{ks}^+, \quad \forall t \in T, k \in K$$

$$\alpha 1_{ks}^t, \beta 1^D \geq 0, \quad \forall t \in T, k \in K$$

The robust counterpart of the other constraints is found by the same procedure. Finally, our hybrid robust-stochastic formulation of this CLSCN design problem is:

$$\begin{aligned} \text{Max } Z = & \sum_{s \in S} \text{Pr}_s \cdot \left(\sum_{j \in J} \sum_{k \in K} \sum_{t \in T} [PR_k FJK_{jks}^t - CJK_{jks}^t FJK_{jks}^t - CKJ_{kjs}^t FKJ_{kjs}^t - CC_j FKJ_{kjs}^t] - \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} [CIJ_{ijs}^t FIJ_{ijs}^t \right. \\ & + CJI_{jis}^t FJI_{jis}^t + RC_i FJI_{jis}^t] - \sum_{j \in J} \sum_{r \in R} \sum_{t \in T} [CJR_{jrs}^t FJR_{jrs}^t + DC_r FJR_{jrs}^t] - \sum_{j \in J} \sum_{t \in T} \left(\sum_{i \in I} \sum_{p \leq t} FIJ_{ijs}^p - \sum_{k \in K} \sum_{p \leq t} FJK_{jks}^p \right) IC_j^t \\ & \left. - \sum_{i \in I} \sum_{t \in T} MC_i PI_{is}^t - Z1_s - Z2_s \right) - \sum_{i \in I} [F_i^{MC} X_i^{MC} + C_i^{MC} W_i^{MC}] - \sum_{j \in J} [F_j^{DC} X_j^{DC} + C_j^{DC} W_j^{DC} + F_j^{CC} X_j^{CC} + C_j^{CC} W_j^{CC}] \end{aligned} \quad (48)$$

S.t.

Constraints (2)–(7), (11)–(24)

$$\left(\sum_{k \in K} \sum_{t \in T} (\hat{D}_{ks}^t + \alpha 1_{ks}^t) + \beta 1^D \Gamma_s^D - \sum_{k \in K} \sum_{t \in T} \sum_{j \in J} FJK_{jks}^t \right) PC^D \leq Z1_s, \forall s \in S \quad (49)$$

$$\left(\sum_{k \in K} \sum_{t \in T} (\hat{D}_{ks}^t - \alpha 2_{ks}^t) - \beta 2^D \Gamma_s^D - \sum_{k \in K} \sum_{t \in T} \sum_{j \in J} FJK_{jks}^t \right) SC^D \geq -Z1_s, \forall s \in S \quad (50)$$

$$\left(\sum_{k \in K} \sum_{t \in T} (\hat{R}_{ks}^t + \gamma 1_{ks}^t) + \lambda 1^R \Gamma_s^R - \sum_{k \in K} \sum_{t \in T} \sum_{j \in J} FKJ_{kjs}^t \right) SC^R \leq Z2_s, \forall s \in S \quad (51)$$

$$\left(\sum_{k \in K} \sum_{t \in T} (\hat{R}_{ks}^t - \gamma 2_{ks}^t) - \lambda 2^R \Gamma_s^R - \sum_{k \in K} \sum_{t \in T} \sum_{j \in J} FKJ_{kjs}^t \right) PC^R \geq -Z2_s, \forall s \in S \quad (52)$$

$$\alpha 1_{ks}^t + \beta 1^D \geq \Delta D_{ks}^+, \quad \forall t \in T, k \in K, s \in S \quad (53)$$

$$\alpha 2_{ks}^t + \beta 2^D \geq \Delta D_{ks}^-, \quad \forall t \in T, k \in K, s \in S \quad (54)$$

$$\gamma 1_{ks}^t + \lambda 1^R \geq \Delta R_{ks}^+, \quad \forall t \in T, k \in K, s \in S \quad (55)$$

$$\gamma 2_{ks}^t + \lambda 2^R \geq \Delta R_{ks}^-, \quad \forall t \in T, k \in K, s \in S \quad (56)$$

$$Z1_s, Z2_s, \alpha 1_{ks}^t, \beta 1^D, \alpha 2_{ks}^t, \beta 2^D, \gamma 1_{ks}^t, \lambda 1^R, \gamma 2_{ks}^t, \lambda 2^R \geq 0, \quad \forall t \in T, k \in K, s \in S \quad (57)$$

In this formulation, the parameters Γ_s^D and Γ_s^R control the trade-off between the robustness and the level of conservatism of the obtained solution at each scenario for transportation cost by restricting the number of times that demands and returns deviate from the nominal scenario in their associated uncertainty sets. As a result, higher values for the parameters Γ_s^D and Γ_s^R increase the level of robustness at the expense of a lower expected profit.

5. Scenario generation and reduction algorithm for transportation costs

To obtain transportation cost scenarios over multiple periods, we combine forecast errors into a tree. As in Schütz et al. (2009) we use a deterministic P th order autoregressive process as the

forecasting method, and add a realization of error term ε_{t+1}^s , to the predicted transportation cost at time $t+1$ to obtain the transportation cost in scenario s denoted by c_{t+1}^s :

$$\hat{c}_{t+1}^s = \alpha + \sum_{i=1}^P \beta_i c_{t+1-i} + \varepsilon_{t+1}^s \quad (58)$$

where α is a constant parameter, β_i is an autoregressive parameter, and c_{t+1-i} is the historical transportation cost at period $(t+1-i)$. Then, a transportation cost scenario is generated as:

$$\hat{c}_{t+j}^s = \begin{cases} \alpha + \sum_{i=1}^P \beta_i c_{t+j-i} + \varepsilon_{t+j}^s & j=1 \\ \alpha + \sum_{i=1}^{j-1} \beta_i \hat{c}_{t+j-i}^s + \sum_{i=j}^P \beta_i c_{t+j-i} & 1 < j \leq P \\ \alpha + \sum_{i=1}^P \beta_i \hat{c}_{t+j-i}^s, & j > P \end{cases} \quad (59)$$

The error terms are assumed to be independent and normally distributed with mean zero and variance σ_ε^2 . Fig. 3 illustrates the tree of NS scenarios for prediction errors on costs associated with all arcs between a given pair of facility sets (e.g., from I to J), where NF is the number of such arcs.

Fig. 3. Scenario tree for the prediction error term for each time period t

Based on this scenario tree for the prediction error terms, the procedure to combine forecasting and scenario generation for a given arc $f=(i,j)$ is illustrated by Fig 4.

Fig. 4. Forecasting and scenario generation scheme for transportation costs

To construct different scenarios for the error terms, most previous studies used Monte Carlo simulation (MCS). Instead, we apply Latin hypercube sampling (LHS) introduced by Olsson et al. (2003). MCS often requires a large sample size to approximate an input distribution, but LHS is designed to accurately approximate the input distribution through sampling in fewer iterations compared with MCS. Moreover, this method covers more of the domain of the random variables than MCS with the same sample size (Fattahi et al., 2014; Shi et al., 2013). To generate the scenario tree for $|T|$ periods, suppose that in each period the error terms are generated using LHS. Since the error terms are period-independent, using this procedure results in an exponentially increasing number of scenarios which makes CLSCN model hard to solve. To efficiently reduce the number of scenarios, a backward reduction technique (Dupačová et al., 2003) is used. The goal is to reduce a scenario tree from m to n scenarios, where $n \ll m$. We find the scenario whose removal will require the least probability mass to be redistributed, remove it, redistribute its probability, and then repeat this procedure until we have only n scenarios left. The pseudo-code for our scenario generation and reduction algorithm is provided in Keyvanshokoo (2015).

6. Solution methodology

The L-shaped method introduced by Van Slyke and Wets (1969) is a BD method applied to two-stage recourse SP problems. The MILP is decomposed into a master problem (MP) that involves the first-stage decision variables, and Benders sub-problems (BSP) to optimize the second-stage decision variables. The BSPs are scenario-specific and connected by the first-stage variables. The BSP of this CLSCN can be formulated by fixing the first-stage variables to the

given values $\{X_i^{MC} = \tilde{X}_{i,it}^{MC}, X_j^{DC} = \tilde{X}_{j,it}^{DC}, X_j^{CC} = \tilde{X}_{j,it}^{CC}, W_i^{MC} = \tilde{W}_{i,it}^{MC}, W_j^{DC} = \tilde{W}_{j,it}^{DC}, W_j^{CC} = \tilde{W}_{j,it}^{CC}, BS_j = \tilde{BS}_{j,it}\}$ at iteration it . The BSP includes the objective function (8) with the added term $-Z1_s - Z2_s$ subject to the constraints (11)-(24) and (49)-(57) in which the first-stage variables have been fixed to these given values. Because our formulation possesses complete recourse, the BSP is feasible for the given values of first-stage variables, and an optimality cut (OC) may be deduced from an optimal solution to the dual of the sub-problem (DSP). Thus, if vectors y and h represent the dual variables of the constraints (11)-(23) and (49)-(56) respectively, then the DSP which obtains a lower bound for the objective function of the original CLSND problem at each iteration it is formulated as follows:

$$\begin{aligned}
DSP: \text{Min } Z_{s,it}^{DSP} = & \sum_{j \in J} \sum_{t \in T} \left(\tilde{BS}_{j,it} y 2_{js}^t + \tilde{W}_{j,it}^{DC} y 7_{js}^t + \tilde{W}_{j,it}^{CC} y 8_{js}^t \right) + \sum_{i \in I} \sum_{t \in T} \tilde{W}_{i,it}^{MC} y 6_{is}^t + \sum_{r \in R} \sum_{j \in J} \sum_{t \in T} M \cdot \tilde{X}_{j,it}^{CC} y 13_{jrs}^t \\
& \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(M \cdot \tilde{X}_{j,it}^{DC} y 9_{ijs}^t + M \cdot \tilde{X}_{j,it}^{CC} y 12_{jis}^t \right) + \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} \left(M \cdot \tilde{X}_{j,it}^{DC} y 10_{jks}^t + M \cdot \tilde{X}_{j,it}^{CC} y 11_{kjs}^t \right) \\
& - \sum_{k \in K} \sum_{t \in T} \left(\Delta D_{ks}^{t+} h 5_{ks}^t + \Delta D_{ks}^{t-} h 6_{ks}^t + \Delta R_{ks}^{t+} h 7_{ks}^t + \Delta R_{ks}^{t-} h 8_{ks}^t \right)
\end{aligned} \tag{60}$$

Subject to:

$$y 1_{js}^t - y 2_{js}^t + y 10_{kjs}^t - PC^D h 1_s + SC^D h 2_s \geq PR_k - CJK_{jks}^t + IC_j^t, \quad \forall j \in J, k \in K, t \in T \tag{61}$$

$$-y 1_{js}^t + y 2_{js}^t - y 3_{is}^t + y 7_{js}^t + y 9_{ijs}^t \geq -CIJ_{ijs}^t - IC_j^t, \quad \forall j \in J, i \in I, t \in T \tag{62}$$

$$y 3_{is}^t - y 4_{js}^t + y 6_{is}^t + y 12_{jis}^t \geq -CIJ_{jis}^t - RC_i, \quad \forall j \in J, i \in I, t \in T \tag{63}$$

$$a y 4_{js}^t + (1-a) y 5_{js}^t + y 8_{js}^t + y 11_{kjs}^t - SC^R h 3_s + PC^R h 4_s \geq -CKJ_{kjs}^t - CC_j^t, \quad \forall j \in J, k \in K, t \in T \tag{64}$$

$$-y 5_{js}^t + y 13_{rjs}^t \geq -CJR_{jrs}^t - DC_r, \quad \forall j \in J, r \in R, t \in T \tag{65}$$

$$y 3_{is}^t + y 6_{is}^t \geq -MC_i, \quad \forall i \in I, t \in T \tag{66}$$

$$PC^D \Gamma_s^D h 1_s - h 5_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{67}$$

$$SC^D \Gamma_s^D h 2_s - h 6_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{68}$$

$$SC^R \Gamma_s^R h 3_s - h 7_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{69}$$

$$PC^R \Gamma_s^R h 4_s - h 8_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{70}$$

$$PC^D h 1_s - h 5_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{71}$$

$$SC^D h 2_s - h 6_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{72}$$

$$SC^R h 3_s - h 7_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{73}$$

$$PC^R h 4_s - h 8_{ks}^t \geq 0, \quad \forall k \in K, t \in T \tag{74}$$

$$h 1_s + h 2_s \leq 1 \tag{75}$$

$$h 3_s + h 4_s \leq 1 \tag{76}$$

$$y1_{js}^t, y6_{is}^t, y7_{js}^t, y8_{js}^t, y9_{ijs}^t, y10_{jks}^t, y11_{kjs}^t, y12_{jis}^t, y13_{jrs}^t, h1_s, h2_s, h3_s, h4_s, h5_{ks}^t, h6_{ks}^t, h7_{ks}^t, h8_{ks}^t \geq 0 \quad (77)$$

$$\forall j \in J, i \in I, k \in K, t \in T$$

Then, based on the DSP's solution, the general MP which produces an upper bound for the objective function of original CLSN design model at each iteration can be written as:

$$\text{MP: Max } Z^{\text{MP}} = \sum_s \text{Pr}_s \times \theta_s - \sum_{i \in I} F_i^{\text{MC}} X_i^{\text{MC}} - \sum_{i \in I} C_i^{\text{MC}} W_i^{\text{MC}} - \sum_{j \in J} F_j^{\text{DC}} X_j^{\text{DC}} - \sum_{j \in J} C_j^{\text{DC}} W_j^{\text{DC}} - \sum_{j \in J} F_j^{\text{CC}} X_j^{\text{CC}} - \sum_{j \in J} C_j^{\text{CC}} W_j^{\text{CC}} \quad (78)$$

Subject to:

Constraints (2)-(7), $\theta_s \geq 0, \forall s \in S$

$$\theta_s \leq \sum_{j \in J} \sum_{t \in T} \left(BS_{j,it} \tilde{y}2_{js}^t + W_{j,it}^{\text{DC}} \tilde{y}7_{js}^t + W_{j,it}^{\text{CC}} \tilde{y}8_{js}^t \right) + \sum_{i \in I} \sum_{t \in T} W_{i,it}^{\text{MC}} \tilde{y}6_{is}^t + \sum_{r \in R} \sum_{j \in J} \sum_{t \in T} M.X_{j,it}^{\text{CC}} \tilde{y}13_{jrs}^t + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left(M.X_{j,it}^{\text{DC}} \tilde{y}9_{ijs}^t \right. \\ \left. + M.X_{j,it}^{\text{CC}} \tilde{y}12_{jis}^t \right) + \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} \left(M.X_{j,it}^{\text{DC}} \tilde{y}10_{jks}^t + M.X_{j,it}^{\text{CC}} \tilde{y}11_{kjs}^t \right) - \sum_{k \in K} \sum_{t \in T} \left(\Delta D_{ks}^{t+} \tilde{h}5_{ks}^t + \Delta D_{ks}^{t-} \tilde{h}6_{ks}^t + \Delta R_{ks}^{t+} \tilde{h}7_{ks}^t + \Delta R_{ks}^{t-} \tilde{h}8_{ks}^t \right), \forall s \in S \quad (79)$$

In this MP, the constraint (79) represents the optimality cut where $(\tilde{y}, \tilde{h})$ indicates the extreme point of the dual polyhedron that results from solving the DSP. This RMP provides an upper bound to the optimal objective value of the MP. At a given iteration of BD, the RMP is first solved to obtain the values of first-stage decisions. Then, these values are used to solve DSP to obtain an extreme point and a new optimality cut (79) is included in the RMP. But this algorithm may require a large number of iterations to converge. To improve the slow convergence of BD, acceleration techniques such as generation of valid inequalities, disaggregation of Benders cuts (Dogan and Goetschalckx, 1999), Pareto-optimal cut generation scheme (Magnanti and Wong, 1981; Papadakos, 2008), covering cut bundle strategy (Saharidis et al., 2010), local branching (Rei et al., 2009), generation of maximal non-dominated cuts (Sherali and Lunday, 2013), and dynamically updated near-maximal Benders cuts (Oliveira et al., 2014) have been proposed. Of these, we employ the following acceleration strategies.

6.1. Valid inequalities

One of the critical reasons for slow convergence of BD is the low quality of the RMP solutions at the primary iterations (Saharidis and Ierapetritou, 2010). To avoid this inefficiency, a series of valid inequalities may be derived to be included into RMP to restrict the feasible region and produce higher quality solutions. Consequently, the gap between the lower and upper bounds will be decreased and the algorithm will converge to an optimal solution faster. The following two types of VI are developed:

- (1) Force the capacity of established facilities to at least equal the sum of maximal downstream requirements:

$$\sum_{i \in I} W_i^{MC} \geq \sum_{j \in J} BS_j \quad (80)$$

$$\sum_{j \in J} W_j^{DC} \geq \max_{s \in S, t \in T} \sum_{k \in K} \hat{D}_{ks}^t \quad (81)$$

$$\sum_{j \in J} W_j^{CC} \geq \max_{s \in S, t \in T} \sum_{k \in K} \hat{R}_{ks}^t \quad (82)$$

Constraints (80)-(82) apply this idea to manufacturing/remanufacturing, distribution, and collection centers, respectively.

(2) Force the opening of at least one facility of each type:

$$\sum_{i \in I} X_i^{MC} \geq 1 \quad (83)$$

$$\sum_{j \in J} X_j^{DC} \geq 1 \quad (84)$$

$$\sum_{j \in J} X_j^{CC} \geq 1 \quad (85)$$

It is easy to verify that they preserve complete recourse. Moreover, at the beginning of the L-shaped algorithm, we need an initial feasible solution of first-stage variables. We can obtain a good initial feasible solution by solving the optimization problem with the objective (78) without its first term subject to constraints (2)-(7) and the VIs (80)-(85).

6.2. Pareto-optimal cuts generation scheme

Magnanti and Wong (1981) proposed a procedure for generating Pareto-optimal cuts to strengthen the optimality cuts. A cut is called Pareto-optimal if no other cut makes it redundant and similarly, the optimal dual solution corresponding to that cut is called Pareto-optimal. If a DSP has multiple optimal solutions, as typically occurs in BSPs with network structure such as ours, the Pareto-optimal cut is the strongest among all the alternative cuts that could be generated. To generate a Pareto-optimal cut, consider our CLSCND problem as the MILP problem $\text{Max } c_1^T x + c_2^T y \text{ s.t. } Ax + By \leq b, x \geq 0, y \in \{0, 1\}$. Fixing integer variables $y = \tilde{y}$, the general form of the SP is $\text{Max } c_1^T x \text{ s.t. } Ax \leq b - B\tilde{y}, x \geq 0$ and then its DSP can be written as $\text{Min } (b - B\tilde{y})^T u \text{ s.t. } A^T u \geq c_1^T, u \geq 0$. Let u^* be the optimal solution of the DSP and y^c be a core point of the solution space of RMP. A Pareto-optimal cut can be obtained by solving the following problem, which is also called Magnanti-Wong problem:

$$\text{Min } (b - B\tilde{y}^c)^T u \text{ s.t. } A^T u \geq c_1^T, (b - B\tilde{y})^T u \geq (b - B\tilde{y}^c)^T u^*, u \geq 0 \quad (86)$$

The challenge at each iteration is to identify and update a core point, which is required to lie inside the relative interior of the convex hull of the sub-region defined in terms of the MP variables. To deal with this problem, Papadakos (2008) proved that it is not necessary to use a core point y^c to obtain a Pareto-optimal cut. Instead, a convex combination of the current MP solution and the previously used core point suffices for obtaining a new core point at each iteration as $y_{it}^c \leftarrow \lambda y_{it-1}^c + (1-\lambda)y_{it}^{MP}$, $0 < \lambda < 1$. For the first iteration, y_0^c is set to equal the solution of MP. Fig. 6 depicts the pseudo-code for the accelerated multi-cut L-shaped algorithm with the Pareto-optimal cut scheme. In step *iii*, the corresponding Magnanti-Wong problem (86) is solved to obtain a Pareto-optimal cut.

Fig. 6. The pseudo-code of the accelerated L-shaped algorithm

7. Computational results

In Section 7.1, the mathematical formulation is verified by performing sensitivity analysis on some important parameters in small instances. Then in Section 7.2, stability analysis is done to

verify that our developed scenario generation and reduction algorithm generates appropriate scenario trees. Finally in Section 7.3, we describe computational experiments using the proposed stochastic BD algorithm for solving large-scale CLSCN design problems.

7.1. Sensitivity analysis of the hybrid robust-stochastic CLSCN design formulation

To assess the model performance, two test instances described in Table 1 are considered. We generate scenarios for uncertain transportation costs. Then, for each scenario uncertainty sets of demand and return are developed. To do so, we sample nominal demands from a uniform distribution specified in Table 1. Then, we determine maximum positive and negative deviations from the nominal scenario such that the deviation interval of uncertain demand is a subset of the interval defined in Table 1. The same procedure is used to obtain uncertainty sets for returns.

Table 1. Characteristics and transportation costs scenarios in the test instances 1 and 2.

Instance Size $ I * J * K * R * T $	Scenarios (s)	Scenario Probability	Transportation costs	Nominal Demands	Nominal Returns
3*8*10*2*2	1	0.5	Unif(5,10)	Unif(2100,2850)	Unif(450,1050)
	2	0.2	Unif(10,15)	Unif(2350,2950)	Unif(580,1200)
	3	0.3	Unif(15,20)	Unif(2150,2650)	Unif(460,1150)
8*12*20*5*6	1	0.1	Unif(5,9)	Unif(1500,2000)	Unif(350,850)
	2	0.3	Unif(7,12)	Unif(1900,2450)	Unif(450,1050)
	3	0.1	Unif(6,11)	Unif(2500,3100)	Unif(850,1350)
	4	0.3	Unif(5,10)	Unif(2100,2850)	Unif(450,1050)
	5	0.2	Unif(10,15)	Unif(2350,2950)	Unif(680,1200)

Other parameters are generated randomly according to the uniform distributions specified in Table 2. The instances are solved by GAMS 23.5 using ILOG-CPLEX 11.0. To explore the effects of the main parameters on solutions, sensitivity analysis is performed on operational costs, penalty costs analysis, selling price and uncertainty budgets.

Table 2. The distributions from which the parameters used in the test instances are generated.

Parameter	Range	Parameter	Range	Parameter	Range
F_i^{MC}	Unif(2100000, 3100000)	MC_i	Unif(120,160)	CAP_i^{MC}	Unif(12000, 22000)
F_j^{DC}	Unif(831500, 1000000)	RC_i	Unif(20,40)	CAP_j^{DC}	Unif(12000, 20000)
F_j^{CC}	Unif(831500, 1000000)	IC_j^t	Unif(5,10)	CAP_j^{CC}	Unif(4800, 8100)
PC^D, PC^R	Unif(150,600)	CC_j	Unif(60,80)	C_i^{MC}	Unif(50, 100)
SC^D, SC^R	Unif(50,150)	DC_r	Unif(1,5)	C_j^{DC}	Unif(30, 50)
a	Unif(0.7,1)	PR_k	Unif(160,230)	C_j^{CC}	Unif(30, 50)

7.1.1. Operational costs

First, we examine solution sensitivity to some essential costs, such as remanufacturing, collection, and manufacturing costs. Changing these operational costs affects the amount of demands satisfied and the amount of returns collected. To investigate these effects, one cost at a time is multiplied by some constant coefficients. Then we examine the sensitivity of expected coverage of demand and returns, as well as profit, over the scenarios.

Table 3. Expected coverage of return and profit for different remanufacturing costs.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of return	Change coefficient	Profit	Expected coverage of return
0.5	3095177	96.88%	0.5	607672	93.90%
1	2934364	96.88%	1	570964	93.90%
2	2621310	88.76%	2	-1277780	91.60%
10	276568	78.66%	10	-3351230	85.23%
30	-5072070	0.00%	50	-56133500	33.70%

Table 4. Expected coverage of return and profit for different collection costs.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of return	Change coefficient	Profit	Expected coverage of return
0	4264758	99.98%	0	751722	99.87%
0.6	3454197	98.93%	0.2	715681	95.78%
0.8	3187979	96.75%	0.6	607672	93.90%
0.9	3061048	96.23%	0.8	383767	93.90%
1	2934364	89.88%	1	570964	93.90%
2	1754915	75.76%	2	-2487887	87.65%
8	-5072070	54.34%	8	-4468432	66.42%

The fluctuation of the optimal expected profit and the expected coverage of return and demand over scenarios are demonstrated in Tables 3 and 4 for different values of remanufacturing and collection costs, respectively. Increasing these costs results in decreasing the expected coverage of return as well as the profit. In fact, with extreme increase in these operational costs, the expected coverage of return decreases to zero because collecting and acquiring end-of-use products is no longer economical. However, changing the collection and remanufacturing costs has no effect on the expected coverage of demand.

Table 5. Expected coverage of demand and return and profit for different manufacturing costs.

Test instance 1	Test instance 2
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Change coefficient	Profit	Expected coverage of demand	Expected coverage of return	Change coefficient	Profit	Expected coverage of demand	Expected coverage of return
0.5	5955323	99.38%	88.76%	0.5	9659102	97.60%	93.90%
0.8	4129472	97.53%	88.76%	0.8	5630294	96.40%	93.90%
1	2934364	97.53%	96.88%	1	570964	95.90%	93.90%
2	2383885	85.44%	96.88%	2	-10001500	95.30%	93.90%
3	-6554420	76.19%	99.01%	3	-21934000	83.40%	99.60%
4	-8825490	59.30%	99.01%	4	-30067200	67.10%	99.60%
5	-8872220	24.94%	99.21%	5	-34444100	34.30%	99.60%

Table 5 shows that when the manufacturing cost increases, the expected coverage of demand and profit will decrease, but the expected coverage of returns will increase. When the manufacturing cost increases in the forward network, the system tries to satisfy demand by remanufacturing used products collected from retailers. Thus, with increase in manufacturing cost, the expected coverage of returns increases and, since the remanufactured products are not sufficient to meet the demand, the expected coverage of demand decreases.

7.1.2. Penalty and other costs related to retailers

Next we investigate the impact of penalty costs for unsatisfied demand and scrap costs for uncollected returns on the expected coverage of demand and return, respectively, followed by the relation between surplus cost and the expected coverage of demand and also the relation between penalty cost and the expected coverage of return. There is an inverse relation between the surplus and penalty costs in the forward network and also between the scrap and penalty costs in the reverse network. In the forward network, the CLSCN seeks a trade-off between the penalty and surplus costs such that their total is minimized. Likewise, in the reverse network, the optimization achieves a trade-off between the penalty and scrap costs such that their total is also minimized. These costs serve to balance the forward flows with the demand and the reverse flows with the return quantities as much as possible.

Table 6. Expected coverage of demand and profit for different penalty costs for unsatisfied demands.

Test instance 1			Test instance 2		
Penalty cost of unsatisfied demand	Profit	Expected coverage of demand	Penalty cost of unsatisfied demand	Profit	Expected coverage of demand
0	6881686	78.23%	0	8011126	74.56%
155	5756598	85.43%	50	5458363	83.22%
300	5696535	89.68%	100	4747939	93.34%

800	5535406	92.56%	350	3558672	96.81%
1200	5427877	99.48%	450	3264315	98.57%
2400	5134039	99.96%	750	2973672	99.78%

Table 7. Expected coverage of return and profit for different scrap costs for uncollected returns.

Test instance 1			Test instance 2		
Scrap costs of uncollected return	Profit	Expected coverage of return	Scrap costs of uncollected return	Profit	Expected coverage of return
0	6881686	0	0	8011126	0
100	5089580	65.43%	50	6407165	58.22%
300	4660283	88.76%	150	5937110	81.78%
500	4304763	98.74%	450	5413307	96.57%
800	4302200	99.84%	750	4315328	99.78%

As the results in Table 6 show, increasing the penalty cost for unsatisfied demands results in higher expected coverage of demand and lower total profit. A similar sensitivity of the expected coverage of return to its corresponding scrap cost of uncollected returns is also seen in Table 7.

Table 8. Expected coverage of demand and profit for different surplus costs of excess amount of flows over demand.

Test instance 1			Test instance 2		
Surplus cost for demand	Profit	Expected coverage of demand	Surplus cost for demand	Profit	Expected coverage of demand
0	3084707	99.98%	0	1028633	98.75%
60	2976581	98.59%	75	462961	98.42%
100	2915489	97.69%	100	360710	97.24%
800	2313456	95.52%	900	-653970	96.41%
1200	2236937	85.48%	1200	-773171	95.27%
2000	1931361	75.66%	5000	-1138770	86.78%

Table 9. Expected coverage of return and profit for different penalty costs of excess amount of flows over return.

Test instance 1			Test instance 2		
Penalty cost of return	Profit	Expected coverage of return	Penalty cost of return	Profit	Expected coverage of return
0	3082146	99.14%	0	1028633	99.71%
20	3044299	98.39%	25	838454	96.52%
50	2993617	97.66%	55	635474	94.21%
100	2985514	88.76%	100	407039	93.83%
1000	2941578	86.28%	1000	-1350870	89.22%
2000	2853703	85.36%	5000	-2085200	87.78%

Furthermore, as Table 8 illustrates, increasing the unit surplus cost for excess amount of flows over demands in retailers lowers the expected coverage of demand and the total profit. A similar result for penalty costs on the excess amount of flows over returns is shown in Table 9.

7.1.3. Selling price

The selling price has an important influence on the total profit and the expected coverage of demand. The sensitivity is explored by multiplying the price by a constant coefficient.

Table 10. Expected coverage of return and profit for different selling prices.

Test instance 1			Test instance 2		
Change coefficient	Profit	Expected coverage of demand	Change coefficient	Profit	Expected coverage of demand
0.2	-15235300	21.92%	0.2	-41582100	83.40%
0.5	-10527400	59.31%	0.4	-31680300	92.30%
0.7	-5258650	85.44%	0.7	-15665600	94.80%
1	2934364	97.53%	1	570964	95.90%
1.5	12190600	98.43%	2.5	84338690	98.70%
2.5	49782290	99.48%	5	225508800	99.80%
5	128181300	99.86	10	508778800	100.00%

From Table 10, increasing the selling price causes the expected coverage of demand and profit to increase. The reason for this system behavior is that the cost prices of the product for different customers are different and so the system tries to satisfy the demand of customers whose cost price is less than the selling price. Increasing the selling price raises the number of such customers and so the expected coverage of demands will increase.

7.1.4. Budget of uncertainty

The effect of uncertainty is studied by changing the budget of uncertainty parameters for uncertain demands and returns. We define ρ as the level of variability of the uncertain parameter respect to its nominal value and consider the values $\rho = 5\%, 10\%, 20\%$ and 30% . With the help of this parameter we can change the radius of the polyhedral uncertainty sets. In test instance 1, for each level of variability of uncertain demand, we vary Γ_s^D in each scenario s from 0 (the nominal formulation) to its maximum value $|K| \times |T| = 20$ (the worst-case formulation) by 1, while maintaining $\Gamma_s^R = 0$ to investigate just the uncertainty in demand. In addition, a bound on the probability of constraint violation is computed according to equation (31) under the assumption of symmetric distributions for independent demand and return quantities. The percent decrease in

the optimal profit value and the constraint violation probability bound for each scenario s independently are plotted in Figures 7 and 8, respectively, as functions of Γ_s^D and Γ_s^R . Here, the relative decrease in optimal profit is calculated as $(z^N - z^R)/z^N$ where z^N and z^R are the nominal and robust optimal profits, respectively.

Fig. 7. Optimal profit decrease and probability of robust constraint violation as a function of Γ_s^D

Fig. 8. Optimal profit decrease and probability of robust constraint violation as a function of Γ_s^R

As expected, we observe from Figures 7 and 8 that for each level of variability, the magnitude of reduction in profit increases while the constraint violation probability decreases with as the uncertainty budget increases. However, the bound on the probability of constraint violation computed according to equation (31) applies only to a single robust constraint. To our knowledge, no bounds have been developed for probability of violating multiple robust constraints together. To investigate this, we compute an empirical frequency of constraint violation in a simulation. Test instance 1 is solved for different values of Γ_s^D and Γ_s^R as they are increased from 0 to 20 in increments of one. Then, for each solved test instance 1 with different values of Γ_s^D and Γ_s^R , we generate random values a thousand times for the collection of uncertain demands and returns uniformly within their associated polyhedral uncertainty sets. Next, based on the optimal values for the decision variables and these sampled values of demand and return quantities, we check to see whether the robust constraints are feasible or not, and so obtain the violation frequency of our hybrid robust-stochastic problem. To compare with each other for test instance 1, these frequencies as well as the constraint violation probability bound for each scenario s , which is calculated based on equation (31), are plotted together in Figure 9 as functions of Γ_s^D and Γ_s^R .

Fig. 9. Violation frequency and optimal profit value decrease as a function of Γ_s^D and Γ_s^R

In Figure 9, the red curve with diamond markers shows the deviation from optimal profit for different values of Γ_s^D and Γ_s^R . Also, the green plot with star markers shows the violation probability of a single constraint computed using equation (31). However, according to the approximate violation frequencies illustrated by the blue plot with triangle markers, if we choose $\Gamma_s^D, \Gamma_s^R \leq 4$, then the overall violation probability is approximately 1, which means that when the highest objective value is obtained it is not robust with respect to changing the values of our uncertain parameters. By increasing the values of Γ_s^D and Γ_s^R from 4 to 14 constraint violation frequency decreases sharply. Finally, if we choose our budget of uncertainty parameters from $\Gamma_s^D, \Gamma_s^R > 14$, then there is not much difference in violation frequency and in fact it is close to 0, but the objective value may be unacceptably high. This suggests that moderate values of the uncertainty budgets, such as 10, may provide a good tradeoff between cost and robustness.

7.2. Stability analysis of scenario generation and reduction algorithm

One of the main criteria which should be satisfied by a scenario generation and reduction method is stability. If several scenario trees with the same input data are generated and we solve our problem with these scenario trees, then we should obtain (approximately) the same optimal value of the objective function. That is, if we generate $|L|$ scenario trees $\xi_l, l=1, \dots, L$ using our scenario generation and reduction algorithm, solve the CLSCN design problem with each one of these scenario trees, and obtain the optimal solution $x_l^*, l=1, \dots, L$, then stability means that we should have $f(\hat{x}_l^*, \xi_l) \approx f(\hat{x}_u^*, \xi_u), \forall u, l \in 1, \dots, L$ where $f(\hat{x}_l^*, \xi_l)$ is the optimal objective function value with respect to the scenario tree l . This type of stability means that the real performance of the optimal solution x_l^* is stable; i.e., it is not dependent upon which scenario tree we choose (Kaut and Wallace, 2007). To carry out this stability analysis, we generate 8 scenario trees for test instances 1 and 2, solve the hybrid robust-stochastic CLSCN design problem with the other input data held constant and then the optimal objective function values are reported in Table 11. The lack of any substantial difference between the optimal objective values indicates stability.

Table 11. Stability analysis of the scenario generation and reduction algorithm

Test instance 1		Test instance 2	
Scenario Tree	Optimal objective function value	Scenario Tree	Optimal objective function value
1	2934364	1	570964

2	2945120	2	572356
3	2975218	3	570852
4	3012312	4	575123
5	3024521	5	569978
6	2898959	6	572123
7	2955412	7	570768
8	3003252	8	572325

7.3. Computational efficiency of the accelerated L-shaped algorithm

We measure the computational efficiencies achieved by adding VIs and employing Pareto-optimal cuts in terms of computing times, number of Benders cycles and the quality of lower bounds. The characteristics of test instances are described in Table 12. The transportation cost scenarios are based on an $AR(I)$ process with $\alpha = 30$, $\beta_1 = 0.15$, and error terms normally distributed with a mean 0. The accelerated BD algorithm is coded in MATLAB and tested on a computer with CPU Intel Core i7, 2.5 GHz and 8 GB RAM. CPLEX 11.0 is used to solve MP and BSPs. We solve each instance both with and without VIs and Pareto-optimal cuts.

Table 12. Characteristics and size of the generated test instances.

Instance	$ I $	$ J $	$ K $	$ R $	$ T $	Number of scenarios	Number of variables	Number of constraints
1	8	12	20	5	3	5	8.75×10^3	9.76×10^3
2	10	14	20	8	6	10	6.26×10^4	6.83×10^4
3	12	16	25	10	8	15	1.75×10^5	1.88×10^5
4	15	18	30	12	9	20	3.55×10^5	3.77×10^5
5	18	20	35	16	12	24	7.48×10^5	7.78×10^5
6	20	24	40	18	12	28	1.17×10^6	1.23×10^6
7	24	26	43	20	14	32	1.89×10^6	1.96×10^6
8	26	30	45	25	16	35	2.74×10^6	2.85×10^6

7.3.1. Effectiveness of the valid inequalities

Table 13 compares the effects of different combinations of VIs on the lower bounds, optimality gap, the number of BD iterations (Iters) and computational times. Here, BD denotes the BD algorithm without any VI, and BDVI1, BDVI2, and BDVI12 denote that VIs (80)-(82), (83)-(86), and (80)-(86), respectively, are added to the MP. The stopping criteria are (1) optimality gap below a threshold value 0.009 or (2) a maximum of 70 Benders iterations is reached. In Table 13, we see that VI2 itself does not make any significant impact on either the number of Benders iterations or the optimality gap. However, BDVI12 consistently improves the lower bound as compared with the classical BD algorithm, and so increases the convergence rate.

Moreover, when the maximum number of iterations is reached, for example in instances 3 and 8, the gaps provided including both VIs are better than those provided with either VI alone. Of the two sets of VIs alone, the first (VI1) improves the lower bound more and so is more efficient.

7.3.2. Effectiveness of Pareto-optimal cuts

In Table 13, the lower bound, the optimality gap and the number of Benders iterations are also displayed for the BD variants with Pareto-optimal cuts and also with a hybrid strategy that combines the VIs with the Pareto-optimal cuts. Note that in our computational experiments, a core point approximation $\bar{y}_0^c$ is initialized with a feasible solution to our RMP and then we update the approximation at each successive iteration by setting $\bar{y}_i^c \leftarrow \lambda \bar{y}_{i-1}^c + (1-\lambda) \hat{y}_i^{MP}$. We set $\lambda=0.5$ according to empirical observations of Papadakos (2008) and Oliveira et al., (2014). The BDVI12 variant has lower optimality gaps than the BD variant with Pareto-optimal cuts. The hybrid strategy achieves even better results in terms of optimality gap and the iteration count compared with other variants of BD algorithm. Table 13 also displays the computational times for solving each test instance by each BD variant and also by directly solving the extensive form by CPLEX. The CPLEX computational times are smaller for the test instances with few scenarios. As the number of scenarios increases, the BD algorithm, especially with the hybrid strategy, outperforms CPLEX. Moreover, we see that when we apply the BD algorithm with Pareto-optimal cut, the number of iterations and optimality gap are decreased for most test instances compared with BD, BDV1, BDV2, and BDV12. But, this algorithm has the highest computational time compared with all BD algorithms. The smaller numbers of Benders iterations when using Pareto-optimal cuts do not necessarily mean smaller computational times in fact, the computational times are increased as a result of the time spent to solve the Magnanti-Wong problem to obtain the Pareto-optimal cuts. Here, each iteration is more effective than each iteration in other BD algorithms and that is why we have less the number of iterations and optimality gap compared with BD, BDV1, BDV2, and BDV12, but each iteration takes longer. However, when we add both VI1 and VI2 to the BD algorithm with Pareto-optimal cut as the BD algorithm with hybrid strategy, it gives us the best performance in terms of both computational time and also optimality gaps and number of Benders cycles for large-scale instances such as instance 5, 6, 7, and 8. Therefore, the Pareto-optimal cuts generation scheme plus adding both valid inequalities demonstrates the best performance in general.

Table 13. Lower bounds, optimality gap, number of Benders iterations and computational times (in seconds) for different BD algorithms and CPLEX

Test NO.	BD				BDV1				BDV2				CPLEX
	Z_{lb}	Gap (%)	Iters	Time	Z_{lb}	Gap (%)	Iters	Time	Z_{lb}	Gap (%)	Iters	Time	Time
1	20489040	0.315	6	25.19	20497213	0.275	6	19.08	20489040	0.315	6	21.57	11.06
2	76750546	0.532	15	88.08	76750546	0.532	13	82.21	76750546	0.532	14	85.12	35.48
3	140005839	1.243	70	121.56	140317678	1.018	70	119.32	140014137	1.237	70	121.34	58.32
4	177735807	0.843	25	162.78	177825740	0.792	24	141.11	177825740	0.792	25	155.62	159.12
5	164097472	0.767	18	287.54	164097472	0.767	17	271.37	164097472	0.767	18	285.09	332.34
6	156060139	0.567	10	399.91	156094286	0.545	8	375.76	156060139	0.567	10	388.23	465.12
7	180275935	0.821	21	684.23	180281299	0.818	21	655.34	180275935	0.821	21	675.54	- *
8	173109952	1.353	70	1018.45	173118493	1.348	70	899.21	173109952	1.353	70	986.24	- *
Test NO.	BDV12				BD with Pareto-optimal cut				BD with hybrid strategy				
	Z_{lb}	Gap (%)	Iters	Time	Z_{lb}	Gap (%)	Iters	Time	Z_{lb}	Gap (%)	Iters	Time	
1	20503960	0.242	5	19.08	20504983	0.237	5	27.08	20519053	0.167	3	24.01	
2	76814729	0.448	11	79.88	76814729	0.448	11	92.43	76814528	0.448	11	90.34	
3	140345464	0.998	70	119.32	140317678	1.018	70	145.21	140695119	0.747	15	122.67	
4	177908700	0.745	23	141.11	177938726	0.728	22	189.34	177966995	0.712	19	157.17	
5	164136565	0.743	17	271.37	164149600	0.735	15	298.32	164149600	0.735	15	268.54	
6	156105154	0.538	8	375.76	156094500	0.545	7	467.78	156130215	0.522	5	372.62	
7	180281299	0.818	20	655.34	180292029	0.812	19	699.12	180297394	0.809	16	646.46	
8	173243278	1.275	70	899.21	173109952	1.335	70	1146.13	173109952	1.156	70	888.32	

* The dashes means that we were not able to solve these test instances.

Conclusion

In this paper, a mixed-integer linear programming model for a multi-period, single-product and capacitated CLSCN design problem is formulated to maximize the expected profit. As the major contribution, a hybrid robust-stochastic programming approach is developed to model qualitatively different uncertainties. We assume historical data exist for transportation costs and use them to generate probabilistic scenarios by a scenario generation and reduction algorithm. Then, in each scenario for transportation costs, polyhedral uncertainty sets are proposed for demand and return quantities in the absence of historical data for a new product. Some numerical instances are created to analyze and validate formulation. To solve this combinatorial problem, an accelerated stochastic BD algorithm is proposed. Two groups of valid inequalities are added to the master problem to efficiently improve the lower bound, and Pareto-optimal cuts are also applied to further accelerate convergence. The computational results demonstrate that the combination of all valid inequalities is most effective for improving the lower bound. Also, the Pareto-optimal cut generation scheme results in significant improvement for some instances where the number of Benders iterations is large. Overall, the combination of VIs and Pareto-optimal cuts demonstrates the best average performance.

As this paper introduces a novel combination of robust and stochastic optimization in the context of CLSCN design, there are some opportunities for future research such as applying other robust optimization approaches and even other uncertainty sets such as ellipsoidal ones, as well as investigating the management of disruption risk in the CLSCN design problem. Moreover, to solve this large-scale problem, other versions of the BD approach such as a Benders-based branch-and-cut approach, where a single branch-and-cut tree is constructed and then the Benders cuts are added during the exploration of this tree, can be applied for performance comparisons.

Acknowledgements: The authors thank the associate editor and the anonymous referees for their valuable and constructive comments on this research, which improved the content and presentation substantially. This material is based upon work supported by the National Science Foundation under Grant No. 1130900.

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