**Distinguishing styles of explosive eruptions at Erebus, Redoubt and Taupo volcanoes using
multivariate analysis of ash morphometrics**

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Abstract

The style and dynamics of volcanic eruptions control the level and type of hazards posed for local populations and can have a temporary long-range impact on climate if eruptions are extremely energetic. The purpose of this study is to provide a statistical approach to ash morphometrics in order to provide a means by which to evaluate diverse eruption styles and mechanisms of fragmentation. The methodology presented can be applied to tephra deposits worldwide and may aid volcanic hazard mitigation by better defining a volcano's history of explosive behavior.

Ash-sized grains were collected from tephra deposits on Mount Erebus, Antarctica (< 10 ka, phonolitic unit SC4), Mount Redoubt, Alaska (2009, andesitic events 2-4 & 9-18), and the Taupo volcano, New Zealand (1.8 ka, rhyolitic unit 3D). Coarse ash from each deposit was carefully hand-sieved to ~1 mm diameter and display diverse morphologies that vary from grains that are moderately vesicular and more rectangular (blocky) to highly vesiculated (spongy) grains that vary from angular to sub-rounded. A total of 264 grains were imaged by scanning electron microscopy. Morphometric properties were determined using image processing software and then evaluated by several statistical methods.

Discriminant analysis of all parameters was found to be the best at differentiating the tephra deposits and allowing for interpretation of eruptive styles in conjunction with field observations. A linear array of data forming a positive slope in factor space, which explains > 99% of the total data variance, is interpreted to represent a continuum between fragmentations involving water-magma interaction ("wet") to grains that were formed predominately by magmatic ("dry") fragmentation mechanisms. The Taupo Hatepe ash, which was deposited from a phreatoplinian eruption column, has the highest factor values in the array, which signifies, in

part, more rectangular/blocky morphologies with smooth grain edges. Factor values for the 2009 Redoubt eruption (events 2-4) are nearly as high as Hatepe ash and based on this we suggest that it was produced, in part, by phreatomagmatic fragmentation. This is supported by field observations that document melting and eruption through glacial ice during the early phases of the 2009 activity. Redoubt ash grains from later stages of the same eruption (events 9-18) show a significant shift to lower values in factor space (more irregular/vesiculated grains) and are interpreted to be a consequence of 'drier' conditions. Coarse ash data from Mount Erebus are completely separated from Taupo and Redoubt grains in factor space due primarily to the difference in mean gray value, which is a proxy for vesicle density and size. The vesicle characteristics (larger and deeper) are consistent with documented strombolian-style activity and the scatter in grain shape data support fragmentation by a mixture of wet and dry processes as has previously been proposed based on deposit characteristics and resemblance to tephra produced by current activity.

Keywords: ash morphometry, phreatomagmatic fragmentation, discriminant analysis, factor analysis

1. Introduction

The eruption mechanism of a volcano can play a tremendous role in the hazards that it can produce. Large explosive eruptions can have significant effects not only on the local environment but on a global scale when the eruption column penetrates the stratosphere. Furthermore, the size and shape of these particles are important for assessing hazards related to

the dispersion and sedimentation of the particles, the reaction of these particles with the gases in the eruption plume, and the threats that these particles pose to aviation and public health (Bagheri et al., 2015).

Volcanic ash is defined as the pyroclastic ejecta that is less than 2 mm in size and formed from the explosive fragmentation of magma during its ascent and eruption at the Earth's surface (Fisher, 1961). The energy for fragmentation is derived from the thermal energy of the magma. Ash becomes finer as the fragmentation energy increases. It has been shown that the production of fine ash ($< 64 \mu\text{m}$) requires fragmentation energy that is 50-1000 times greater than the production of coarse ash ($64\text{-}2000 \mu\text{m}$) (Zimanowski et al., 2003). For many years volcanologists have used parameters such as size and angularity as well as surface morphology to describe the energy and type of fragmentation mechanism responsible for pyroclastic deposits. In the case of angularity, the mechanism of fragmentation has been determined semi-quantitatively through the use of visual charts divided into categories such as angular, sub-angular, sub-rounded, rounded, etc. There are also other visual references for ash textures that correspond to specific fragmentation mechanisms (e.g., Heiken and Wholetz, 1985). Over the years, there have been many attempts to categorize ash morphologies quantitatively using a variety of measured parameters. These morphological parameters include form factor, circularity, roundness, sphericity, etc. However, the conclusion in most of these studies is that no singular parameter can distinguish between magmatic and phreatomagmatic mechanisms. For this reason multiple parameters must be examined together in order to assess the mechanism of fragmentation (Dellino and La Volpe, 1996; Ersoy et al., 2006; Ersoy et al., 2007; Jordan et al., 2014; Leibrandt and Le Pennec, 2015)

The aim of this study is to refine and produce a methodology to determine the fragmentation mechanism from specific ash morphologies of known origin, such as blocky, smooth edged shapes that typify phreatomagmatic produced grains and highly irregular shapes that typify magmatically fragmented grains, using SEM and image processing techniques. It is shown that using multiple morphological parameters with the aid of multivariate analysis can be successfully used to interpret fragmentation mechanisms and provide insight on the eruptive style producing coarse ash. The development of a quantitative method of determining eruption style from ash morphology may be used as a prediction tool to manage risk in volcanically active areas.

2. Background

2.1. Mechanisms for the formation of ash

There are two fundamental mechanisms that cause explosive eruptions and the formation of ash (Heiken, 1972; Sheridan and Wohletz, 1983); 1) exsolution and expansion of primary gases within magma (magmatic or “dry” eruptions) and 2) interaction of magma with external water (phreatomagmatic or “wet” eruptions). In many cases both magmatic and hydromagmatic fragmentation mechanisms occur together during the course of an eruption (Wohletz, 1983; Dellino et al., 2012; Wohletz et al., 2013). Magmatic fragmentation is the result of a complex relationships between driving mechanisms for eruption and the rheological behavior of magma (Cashman and Scheu, 2015) but principally it is the end result of rapid depressurization of magma, which liberates volatiles such as CO₂, SO₂, and H₂O to form bubbles that grow and coalesce (i.e. vesiculation). When bubble over-pressure exceeds the strength of the surrounding magma or when the volume of the rising and expanding magma package exceeds the critical

strain rate, the gas phase separates from liquid $\pm$ solid phases. Phreatomagmatic fragmentation is the result of a complex interplay of contact dynamics between magma and external water that lead to thermohydraulic explosion (Zimanowski et al. 2015). The chemical composition of magma influences melt rheology and primary volatile content and is intimately linked with numerous other inter-related physical properties of magma (e.g., viscosity, bubble size, shape & density, crystallinity, melt porosity & permeability) that, along with external factors (e.g., conduit architecture), determine eruption dynamics.

Explosive eruptions produce ash particles that exhibit a wide range of size, angularity and surface morphologies. The vesicularity of particles can vary from ‘simple’ ash, defined as ash particles that have very restricted size distribution of bubble diameters (Genareau et al., 2013), to ‘complex’ ash, which preserve a diversity of vesicle sizes on a surface, as well as complete and non-fragmented vesicles within the interior of the particle. In this study we focus on complex ash particles as the relatively large size of our particles allows for multiple surface vesicle observations.

2.2. *Tephra deposits*

Tephra deposited by four separate eruptions documented at three different volcanoes were selected for this study in order to compare morphometric parameters and to evaluate which of those parameters, or what combination of those, are indicative of eruption style. Compositionally the deposits are evolved and range from intermediate to felsic (53 to 74 wt.% SiO_2), representative of alkaline (Erebus) and calc-alkaline (Redoubt and Taupo) magma series.

2.2.1. Taupo - 1.8 ka, Hatepe Ash unit 3 subunit D (HA3-D)

The Taupo volcano in New Zealand has experienced two rhyolitic caldera-forming eruptions that occurred at 27 and 1.8 ka (Houghton et al., 2010). The younger event produced seven tephra deposits erupted from at least three vents on a NE-SW aligned fissure along the eastern portion of Lake Taupo. In total this eruptive episode ejected $\sim 22 \text{ km}^3$ dense-rock equivalent (DRE) of chemically uniform rhyolite. The third and fourth phases of this eruption produced a total of 1.75 and 0.8 km^3 (DRE) phreatoplinian fall deposits, respectively (Houghton et al., 2003; Houghton et al., 2010). Phreatoplinian-style activity refers to large scale explosive eruptions that emit sustained water-rich plumes (i.e. gas, solid, melt) that produce wide-spread fall deposits (Houghton et al., 2015). Unit 3 is spread over an area of $> 15,000 \text{ km}^2$ with subunits 3A, 3B, and 3D having relatively symmetrical isopach patterns (Houghton et al., 2003). Unit 3, known as the Hatepe Ash, consists of fine ash that is dominated by highly vesicular pumice (65-90%) with few lithics ($< 8 \text{ wt } \%$). Subunit 3D, the deposit evaluated in this study, contains aggregate-bearing ash falls separated by thin laterally-continuous partings of coarse ash and pumice lapilli (Houghton et al., 2003).

Tephra deposited during the third phase of the 1.8 ka eruption show transitions between wetter, less intense phases (subunits 3B and 3D) and a drier, more intense phases (subunit 3C) (Houghton et al., 2003). Overall Unit 3 shows a decrease in all grain-size classes $> 1 \text{ mm}$ outwards from the source (Walker, 1981). Houghton et al. (2010) suggest that the shift to smaller grain sizes is related to a change from lower to higher magma/water ratios. The fragmentation was initiated by exsolution of magmatic volatiles and that interaction between magma particles and external water occurred at a shallow level within the conduit above the fragmentation surface (Houghton et al., 2003; 2010).

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152 *2.2.2. Redoubt – 2009, fall deposit (AT-1617 & 1876)*

153 The Redoubt Volcano, Alaska is an andesitic, ice-covered stratovolcano that stands at
 154 3108 m and is located on the west side of Cook Inlet within Lake Clark National Park (Wallace
 155 et al., 2013). This volcano has erupted three times historically prior to 2009; 1902, 1966-68 and
 156 1989-90. The eruption in 2009 took place between March 15th and April 4th. During this time, the
 157 volcano generated 20 tephra-producing explosions (events) lasting between < 1 and 31 minutes.
 158 It has been estimated that over 80,000 km² of the surrounding area received at least minor ash
 159 fall (0.8-0.2 mm in thickness) and that plume heights were between 4.6 and 19 km above sea
 160 level with the majority over 12 km. The 2009 Redoubt eruption fits the general classification of
 161 plinian-style activity which is characterized by large scale explosive eruptions that produce
 162 sustained plumes that reach heights in the tens of kilometers (Cioni et al., 2015).

163 The first explosive eruptive phase started on March 22 and continued for six hours,
 164 generating a total of five distinct plumes known as events 1-5 (Wallace et al., 2013). Deposits
 165 from each event are generally indistinguishable except in locations where snowfall separated
 166 events 2-4 from events 1 and 5. The samples in this study were collected from events 2-4 (AT-
 167 1876) and are fine-grained and contain vesicular material (22-54 wt %) as well as dense juvenile
 168 material (38-72 wt %). Event 5 contains a significant amount of lithic clasts from wall rock
 169 incorporation by vent widening and ice clasts from the destruction of the glacier over the vent
 170 area (Schaefer, 2012). In late March, between the 26th and 28th, ten explosive events (9-18)
 171 created a composite tephra layer to the north and east of the volcano. The samples in this study
 172 from events 9-18 (AT-1617) are generally fine grained, mostly ash less than a few millimeters in
 173 diameter, and consists of sub-equal amounts of vesicular and dense juvenile material. All tephra

deposits produced by the 2009 eruption contain accretionary lapilli, indicating that external water, sourced from melting of the summit glacier, was abundant and incorporated within volcanic plumes (Schaefer, 2012; Van Eaton et al., 2015).

Apart from a small phreatic explosion on March 15, all tephra producing events are considered to be caused by magmatically-driven explosions (Wallace et al., 2013; Schaefer et al., 2012). Wallace et al. (2013), however, also postulates that the greater proportion of fine ash in events 9-18 was sourced from either pyroclastic flows (i.e. the elutriation of fine ash into plumes) or from explosive magma-water interaction (i.e. higher fragmentation efficiency; Zimanowski et al., 2015; Liu et al., 2015).

2.2.3. Erebus - < 10 ka, fall deposit SC4 (BW03-015)

Mount Erebus is a large volume (~1670 km³) phonolitic stratovolcano that rises to an elevation of 3794 m above sea level on Ross Island located in the southern Ross Sea of Antarctica. It is the world's southernmost active volcano and contains a long-lived convecting anorthoclase-phonolite lava lake in its main summit crater that regularly produces strombolian-style eruptions (Kyle et al., 1992; Oppenheimer et al., 2011). Strombolian-type activity is characterized by weak, discrete and short duration explosions that eject pyroclasts to heights of only a few hundred meters if they are large (e.g., up to a meter) and less than a kilometer for ash-size material. (Taddeucci et al., 2015).

The SC4 deposit is classified as a pyroclastic tuff breccia (*cf.* White and Houghton, 2006) and drapes over a small portion of the active summit cone of the main crater and is better exposed along the southeastern rim of the inactive side crater (Panter and Winter, 2008). The

maximum thickness of this tephra is ≥ 10 m and consists of crudely bedded material with sub-horizontal accumulations of flattened lava bombs (> 1 meter in length) surrounded by lapilli- to ash-sized clasts. The age of the SC4 deposit is unknown. Harpel et al. (2004) and Kelly et al. (2008) used the $^{40}\text{Ar}/^{39}\text{Ar}$ method to date young lavas and tephra on and around Mount Erebus. The SC4 is grouped with other post-caldera lavas and tephra deposited on the summit plateau that range in age from 17 ka to <1 ka. A sequence of englacial tephra layers on the upper flanks of Mount Erebus have a bracketed age between 71 and 15 ka (Harpel et al., 2004). Nineteen undated layers are stratigraphically younger than this sequence. Panter and Winter (2008) speculated that the SC4 deposit may have been produced by an eruption that correlates to one of these younger layers. In a more recent study, Parmelee et al. (2015) revised the age of summit plateau lavas using cosmogenic (^3He and ^{36}Cl) exposure dating and found them to be < 10 ka.

The inspection of the ash-sized clasts in the SC4 deposit by Panter and Winter (2008), revealed two distinct morphological types; spongy (i.e. vesicular) and blocky. They attribute the spongy morphology to be indicative of eruptions of vesiculating magmas (magmatic); whereas, the blocky, less vesicular morphologies are attributed to being a direct result of the explosive interaction of magma with an external water source (phreatomagmatic). Therefore, Panter and Winter's (2008) overall interpretation of this deposit is that it is a proximal pyroclastic fall deposit that was produced by a combination of magmatic and phreatomagmatic activity during a single eruptive phase.

3. Methodology

The samples used in this study were selected from three well documented eruptions of evolved magmas that represent a diversity of eruption styles and likely a variety of ash producing

fragmentation mechanisms. Bruce Houghton provided a sample from the Taupo 1.8 ka Unit 3D deposit (HA3-D) in December 2014. The sample was collected 15 km downwind (east) of the source and arrived at the lab sealed in a plastic bag and wet. A portion of the sample was subsequently oven dried on the lowest setting (50°C) for roughly one hour before gently hand sieving. All Redoubt tephra samples were collected by Wallace et al. (2013) at field stations between March 18 and July 11, 2009 (see Fig. 2 in Avery, 2015). The first sample, USGS Tephra Lab ID AT-1617 (field# K LW007) was collected on March 28th 2009 at NAD 83 latitude of 60.64138° and longitude of -151.35099°, which is ~79 km from the vent. The sample is described as being a bulk sample originating from events 9-18 with a thickness of 0.1 cm and a mass per unit area of 5.79 g/m² collected from a 0.16 m² area. This ash is described as being coarse with well sorted light gray grains. The second sample, USGS Tephra Lab ID AT-1876 (field# K LW105F) was collected on May 3rd 2009 at NAD 83 latitude of 61.05196° and longitude of -152.45895°, which is ~43 km from the vent. The sample is described as being a bulk sample originating from events 2-4 with a thickness of 0.3 cm. The sample was from the lowest tephra, number three from the top, and represents a contiguous ash layer with moderate to coarse grains and no finer ash. The sample from Mount Erebus (BW03-015) was collected from a bomb-supporting matrix within the tephra layer (SC4) in 2003 (Panter and Winter, 2008). This sample contains a high quantity of ash (33 wt. %) with the highest proportion (15.1 wt. %) being in the medium to coarse size fraction (0.25-1mm diameter).

3.1. SEM analysis

Samples were first sieved to grain sizes between 1.18 mm and 850 µm [U.S. standard mesh #16 (-0.25 φ) and #20 (0.25 φ) respectively)], creating a subset of grains approximately 1

mm in diameter for all deposits. The Redoubt sample AT-1876 was also sieved for a second time to extract grains between 850 and 600 μm in diameter [(U.S. standard mesh #20 and #30 (0.75 ϕ)] to test for grain size bias in image analysis. The smaller size fraction from AT-1876 is designated AT-1876s. Individual ash grains were then mounted in rows on graphite tape adhered to an SEM stub, using a dissecting microscope. A piece of copper tape was placed at the side of the first grain on the stud to mark the beginning in some cases. All mounted specimens were subsequently coated with a thin layer of gold-palladium to enhance conductivity of the material.

Two different instruments for scanning electron microscopy were used in this study. All Erebus BW03-015 sample grains were imaged using a LEO 435VP SEM at Old Dominion University, Virginia. The accelerating voltage was set to 15 kV and all images were taken at a magnification of 50 x in depth mode for working distances over 25 mm. Samples from Redoubt and Taupo were imaged in low magnification mode at the University of Toledo, Ohio with a Hitachi S-4800 II SEM. The images were taken at the same magnification of 50 x for consistency and the accelerating voltage was set to 10 kV with a beam current of 10 μA . There are two image resolutions (1024 x 768 pixels and 1280 x 960 pixels) as a result of these slightly different SEM instruments. To correct for this difference all calculations were made according to pixel size and scale of the image.

3.2. Image analysis

The ash grains were separated from the background using threshold subtraction techniques that removed smooth continuous backgrounds from all images using Adobe Photoshop®. The cropped images were then loaded into ImageJ (<http://imagej.nih.gov/ij/>), an

open source software for image analysis, and analyzed for parameters including: area, mean gray value, standard deviation, centroid coordinates, particle perimeter, bounding rectangle dimensions, ellipse of best fit, circularity, aspect ratio, roundness, elongation, solidity and various Feret calculations. The screening of parameters for assumptions and relationships among the explanatory variables used different exploratory data analysis (EDA) techniques. A total of five uncorrelated morphological parameters were used in the analysis which is described in (Table 1). The mean gray value is defined as the average gray value within the particle selection. The range of gray scale values goes from 0 (white) to 255 (black). The mean gray value is the sum of these non-negative gray values divided by the number of pixels in the selection. Thus it follows that the mean gray value increases as pixels get darker. The standard deviation calculation refers to the standard deviation of the gray values used to generate the mean gray value. Other parameters such as rectangularity and compactness were calculated using Image Particle Analysis (IPA) formulas (Table 1) for the comparison of ash particles developed by Dellino and La Volpe (1996). The final parameter, angularity, was calculated using a formula found in Manga et al., 2011 and Russ, 2011 (Table 1). The combination of parameters Aspect Ratio, Rectangularity, Compactness, and Angularity will be hereafter abbreviated as ARCA.

3.3. Statistical analysis

In this study several statistical techniques are employed in order to discriminate ash populations and to interpret whether or not they were produced by different eruptive fragmentation mechanisms. These techniques include *k*-means cluster analysis and R-mode factor analysis in R software (<http://www.r-project.org/>), two-sample *t*-testing on log transformed data, and discriminant analysis using XLSTAT®, a statistical software suite by

Addinsoft. Using the morphological parameters generated from SEM images and subsequent ImageJ software analysis, we provide new multidimensional data on all four deposits and evaluate the constraints (grain size, and proximal versus distal comparisons) and application of this method of categorization.

The purpose of k -means analysis, which is an unsupervised learning algorithm, is to cluster data into partitions based on parameter similarities, where the number of clusters is assigned by the user. This method uses the centroids of clusters, determined by minimizing the sum of the squared errors, in order to characterize the data (Ding and He, 2004). In this method, scatterplots that show visual representation of the data are generated that is later used for understanding similarities and dissimilarities of groups and subsequent assumptions associated with the groups. The equation for this method is determined by minimizing the sum of squared method:

$$J_K = \sum_{k=1}^K \sum_{i \in C_K} (x_i - m_k)^2 \quad \text{Eq. (1)}$$

where, J_k is the objective function, $(x_1, \dots, x_n) = X$ is the data matrix, and $m_k = \sum_{i \in C_K} x_i / n_k$ is the centroid of the data cluster C_k and n_k is the number of points in C_k . The objective of the k -means clustering algorithm is to characterize the data using the centroids of the clusters (Ding and He, 2004; Wagstaff et al., 2001). The algorithm begins with an initial guess for the cluster centroids based on the number of clusters specified. Then the distances between the observations and the nearest cluster are calculated and the mean value is determined from the new clusters. This process is repeated until there is no movement of the centroid values or based on other termination conditions such as assigned number of iterations or a threshold value that falls below the objective function.

Factor analysis and discriminant analysis are additional multivariate analysis techniques that are used to distinguish distinct sets of observations. Factor analysis allows for the reduction of multidimensional data into a few factors that are linear combinations of the original parameters (*cf.* Dellino and La Volpe, 1996). The extraction of eigenvalues and eigenvectors are the first step in this process as they describe the proportion of variance explained by each component and the weights for computing and interpreting the components. Examining these proportions can lead to a decision for using the smallest number of factors which account for most of the variance of the observed parameters. Principal Component Analysis (PCA) was used in the selection of the number of factors by direct examination of the eigenvalues and by implementing scree plots to visually assesses the percentage of variance explained in the data. R-mode factor analysis is used to interpret observed relationships between variables in terms of the samples (Voudouris et al., 1997). This method of analysis allows for the preservation of variance through the linear combination of variables (Dellino and La Volpe, 1996). Discriminant function analysis is a technique that predicts group membership based on a linear combination of the interval variables as well as understanding of relationship between group membership and the variables that are used to discriminate the most between the groups. Outputs of this analysis include the percentage of samples correctly classified by the analysis. This includes a confusion matrix that shows the proportions of the subjects in actual groups that are reassigned to predicted groups during analysis.

4. Results

4.1. Visual description of ash

4.1.1. Taupo

Pumice from the Hatepe HA3-D tephra deposit, Taupo volcano, is highly to extremely vesicular (58-90%) with bubbles that range from small spherical types to larger stretched long-tube types and is crystal-poor (2-3.5 wt.%), containing plagioclase and quartz (Houghton et al., 2003; 2010). The gray-white coarse ash examined in this study is angular to sub-angular, spherical to sub-prismoidal (Fig. 1A) and contains two populations based on vesicle size distribution. Visually, one populations shows a uniform microvesicular texture (vesicles < 20 μm) while a second populations display a greater range in vesicle size with a significant proportion of vesicles in the range of 20 to 80 μm , a few large vesicles 100-200 μm in diameter and long-tube fabrics (Fig. 1A).

4.1.2. Redoubt

The coarse ash examined from tephra that was deposited by events 2-4 (AT-1876) and 9-18 (AT-1617) during the 2009 Mount Redoubt eruption consist of light gray, angular and moderate to highly vesicular grains (Fig. 1B & 1C). The vesicles have an overall spherical to ovoid shape but also show complex forms produced by coalesced bubbles. Vesicular grains from sample AT-1617 have vesicles that range in size from 30 to 60 μm with a few of larger dimension (~100 μm). Vesicular grains from sample AT-1876 have smaller vesicles that range in size from 10 to 30 μm but, relative to AT-1617, have a higher proportion of larger vesicles 60-110 μm (Fig. 1C). The crystal content of tephra in the later events (9-18) is higher (15.5 wt.%) relative to the earlier events (1.2 wt.%) and consist of plagioclase, amphibole and pyroxene (Wallace et al., 2013). The crystals observed in the sieved size fraction used in this study are

similar in size (~ 1 mm) and mostly free from the vesiculated grains. However, crystals ~100 μm in size are found within a few grains.

4.1.3. Erebus

The inspection of coarse ash from the SC4 tephra deposit on Mount Erebus reveal two principal morphologic types; spongy and blocky (Panter and Winter, 2007). The blocky ash is dark-brown to greenish-black, moderately vesicular (*cf.* Houghton and Wilson, 1989) with smooth curvilinear surfaces that form tabular or prismatic grains. The vesicles are folia-shaped and separated by thick bubble walls (Fig. 1D). The spongy ash is light-brown and highly vesicular with spherical to ovoid-shaped vesicles and has overall grain shapes that are highly irregular (Fig. 1D). Both ash morphologies are lithic-free and poorly crystalline, containing sparse phenocrysts (2-3 vol.%, 50 to 500 μm long dimension) of anorthoclase, clinopyroxene, magnetite and apatite. Vesicle size ranges from <10 to ≥ 500 μm in diameter in both ash types.

4.2. Exploratory analysis

Several statistical methods are used to evaluate similarities and dissimilarities in ash morphology between the four deposits from the three volcanoes. We ran *k*-means for both 2 and 3 clusters for all ash samples and compared them to the standard deposit and sample plots as shown in Fig. 2. Cluster analysis in both cases was run using the multiplication of the ARCA parameters. The RectComp on the *x*-axis in the figure is an abbreviation for the multiplication of the unitless values of Rectangularity and Compactness while the AnguAR on the *y*-axis is the abbreviation of the multiplication of the unitless values Angularity and Aspect Ratio. As

RectComp values increase, morphology is more rectangular and regular. As AnguAR values increase, morphology is more irregular and less elongate. The first analysis, *k*-means 2 clusters, Erebus ash grains clustered mostly with the Taupo grains (79% and 64% respectively); whereas, the Redoubt grains clustered separately (64%), with the exception of sample AT-1876s (Fig. 2A). The position of the cluster center for Erebus and Taupo (i.e., Class 2) grains have values of 0.99 for RectComp and 0.40 for AnguAR while the position of the cluster center for Redoubt (i.e., Class 1) grains have values of 0.73 for RectComp and 0.62 for AnguAR. This finding suggests that the Erebus grains were more similar in morphology to Taupo grains and were therefore grouped together (Fig. 2B). Furthermore, the position of this cluster, having lower RectComp (0.73 vs 0.99) and higher AnguAr (0.62 vs 0.40) values, suggests this grouping exhibits morphologies that are more rectangular, regular, and elongate.

For the second analysis, *k*-means 3 clusters, there was some overlap between all deposits (Fig. 2C). As in the previous analysis, Erebus (48%) and Taupo (65%) dominated the first cluster where the cluster center was characterized by lower value of 0.83 for the RectComp and higher value of 0.50 for the AnguAr (Fig. 2D). The second cluster was dominated by Erebus (46%) and Taupo (27%) with cluster center values of 0.69 for both RectComp and AnguAr. Finally, the third cluster was dominated by Redoubt grains (47%) and had cluster center values of 1.11 and 0.34 for RectComp and AnguAr. RectComp and AnguAr values across these clusters indicate a transition from regular rectangular and elongate morphologies (Erebus and Taupo) to irregular and less elongate morphologies (Redoubt). Furthermore, Redoubt deposit AT-1876 (events 2-4, collected 43 km from vent) is mostly split between groups 1 and 2 (52%); whereas, deposit AT-1617 (events 9-8, collected 79 km from vent) is concentrated mostly within cluster group 3

(74%). This indicates that not only is there a transition in morphologies between different volcanic deposits, but also within separate deposits of the same eruption.

Univariate statistics was generated for each parameter and compared by volcano of origin (Erebus, Redoubt, Taupo) and deposit sample (BW03-015, AT-1617, AT-1876, AT-1876s, and HA3-D). In order to understand grain differences and parameter relationships the data set was explored by different plotting techniques. The box plots in Fig. 3 illustrate the distribution of the rectangularity and the angularity parameters grouped by volcano of origin. For example, the rectangularity for Erebus and Taupo is associated with lower median values of 1.13 and 1.18, respectively, than the Redoubt deposit which has a median value of 1.30 (Fig. 3). In addition, for the rectangularity parameter the Redoubt has higher standard deviation than Erebus and Taupo (0.231, 0.154, and 0.121 respectively). The coefficients of variation for Erebus and Taupo are lower (13.6 %, 10.2 %) than for Redoubt (17.8 %), which imply that the shape of the particles are more regular for Erebus and Taupo. The aspect ratio parameter shows that Erebus and Taupo are associated with higher median values of 1.35 and 1.28 than the Redoubt deposit which has a median value of 1.25. The aspect ratio values suggest that the particles from Redoubt are less slim but have sharper edges and are more irregular than Erebus and Taupo. The angularity is higher for Erebus (0.43) and Taupo (0.41) than for Redoubt (0.36), this confirms the rectangularity parameter and that the shape of the Redoubt particles are irregular and less rectangular than the shape of ash from the other deposits. In addition, the shape of both the Taupo and Erebus grains are more elongate and more regular, with Erebus being the most elongate as shown in Fig. 1D.

Factor analysis (R mode) was then conducted on the calculated ARCA parameters for all tephra samples in this study (Fig. 4). The scatterplot shows that values of the two most

important factors account for a total of 80.6 % of the amount of variation. In the plot, Factor 1 explains a total of 48.2 % while Factor 2 a total of 32.4% of the total data variance. The loadings results from the factor analysis suggest that Factor 1 is most likely associated with rectangularity and angularity while Factor 2 is most likely associated with aspect ratio and compactness. The plot shows Erebus ash more concentrated on the right-hand side of the factor analysis chart (circles), while Redoubt ash is primarily concentrated in the middle (crosses) and the Taupo ash extends more to the left (triangles) with higher negative Factor 1 values (Fig. 4). This indicates that the Erebus ash has a higher aspect ratio value and the Redoubt ash has higher rectangularity value.

4.3. Grain size effect

Understanding the effect of size distribution on image analysis is important in order to make meaningful intra and inter-sample comparisons. To evaluate this effect we processed two different size fractions from Redoubt sample AT-1876. The sample was gently hand sieved and separated into 1.18 - 850 μm and 850 - 600 μm diameter fractions. The smaller size fraction is designated AT-1876s. Approximately 28 grains of each of size fraction was analyzed using identical methodologies. Two-sample *t*-tests were then used to determine whether or not morphological differences are a consequence of different sized grains (Table 3).

Grain size was found to have an effect on only three parameters: rectangularity, circularity, and angularity. For rectangularity, the smaller size fraction had a mean of 1.15; whereas, the larger size fraction had a mean of 1.31 (Table 2). This indicates that the smaller size fraction is less irregular and more rectangular. This relationship was found to be significant with

a p -value of 0.0017 (95% confidence interval, Table 3). Finally, the angularity, or R value of the grains for the smaller fraction were higher (0.41) than the larger fraction (0.34), indicating that the larger grain size fractions are overall more angular (p -value: 0.0236) (Tables 2 and 3). In summary, the smaller size fraction (AT-1876s) is less irregular, being less angular and more rectangular and irregular relative to the coarser size fraction.

4.4. Proximal vs distal effect (Redoubt)

Here we evaluate whether or not coarse ash (~1 mm diameter) from the 2009 Redoubt eruption deposited at different distances for the source vent can be distinguished morphologically. Tephra sample AT-1617 was deposited ~79 km to the east-northeast of the vent and sample AT-1876 was deposited ~43 km to the north of the vent. It is important to keep in mind that these two deposits were produced from multiple plumes representing separate events. For analysis, approximately 28 grains each from samples AT-1617 (events 9-18) and AT-1876 (events 2-4) are compared for all parameters using two-sampled t -test.

In total, five parameters were determined significant: mean gray value, standard deviation of the mean gray value, solidity, rectangularity, and compactness (Table 3). For the mean gray value of the particle, the more proximal deposit (AT-1876) had a value of 70.35; whereas, the more distal deposit (AT-1617) had a lower value of 67.49 (p -value: 0.0135). The solidity value of the proximal sample was 0.7921; however, the more distal sample had a value of 0.6756. The solidity values indicate that grains from the more proximal deposit have a smoother surface than grains from the distal deposit (p -value: 0.0098). In the case of rectangularity, grains from the more proximal deposit (AT-1876) had a mean value of 1.31; whereas, grains from the more

distal deposit (AT-1617) displayed a higher mean value of 1.43 (Table 2). This indicates that the distal deposit is less rectangular (p -value: 0.0301). Furthermore, the compactness value of the more proximal deposit (0.71) was significantly lower than the more distal deposit (0.77), indicating once again that the proximal grains were slightly more rectangular (p -value: 0.0047) (Table 2).

4.5. Discriminant analysis

Discriminant analysis was utilized to test both the ARCA parameters as well as all other measured parameters. The first discriminant analysis was run only with ARCA parameters, where the within-class covariance matrices were found to be significantly different, by the Box's test (chi-squared and Fisher's F) and Kullback's test. This indicates that the discriminant analysis is a quadratic function and each class (sample) must have a separate covariance matrix. Furthermore, the eigenvalues for Factors 1 and 2 explain 94.57 % of the total variance in the data set. Rectangularity, compactness, and angularity were predominantly correlated with Factor 1; whereas, aspect ratio was primarily correlated with Factor 2 (Fig. 5). Analysis of prior and posterior classification were run in which several grains were re-classified based on similarity with another group. From the confusion matrix (Table 4), it is shown that only 44.32 % of the grains were correctly classified, with the Erebus BW03-015 sample having the highest percent correct followed by the Redoubt sample AT-1617 and Taupo HA3-D.

The second discriminant analysis was run on the morphological parameters of mean gray value, standard deviation of the mean gray value, ARCA parameters, and solidity (Fig. 6A). Similarly, the within-class covariance matrices were found to be significantly different. The first

two factors explained a total of 99.35 % of the variance. The mean gray value, standard deviation of the mean gray value, and aspect ratio were mostly correlated with Factor 1; whereas, all other parameters were highly correlated with Factor 2. Analysis of posterior classification that take into account prior probabilities were similarly run, in which several grains were once again re-classified based on similarity with another deposit. From the respective confusion matrices (Table 4), it is shown that 78.03 % of the grains were correctly classified. In this case, 99% of the Erebus grains were classified correctly followed by Taupo (86%) and Redoubt sample AT-1617 (70%). Most of the Redoubt sample AT-1876 grains still classified incorrectly with the majority classifying as being from the Taupo HA3-D.

All previous discriminant analyses found that the most confusion in re-classifying grains was with the Redoubt AT-1876 sample, and specifically, the smaller size fraction (AT-1876s) in the first discriminant analysis. For this reason a final discriminant analysis (Fig. 6B) was run where all parameters are evaluated but the smaller ash size fraction (600 – 850 μm) sieved from Redoubt tephra sample AT-1876 (AT-1876s) was excluded. As before, the within-class covariance matrices were found to be significantly different. Additionally, the eigenvalues for Factors 1 and 2 explain 99.84% of the total data variance. The mean gray value and standard deviation of the mean gray value were again highly correlated with Factor 1, with 98% and 90% of the variances explained. All other parameters were highly correlated with Factor 2. Analysis of predicted observation values were again similarly run, in which individual grains are classified into groups based on the highest probability of belonging to a particular group. The confusion matrix is used to summarize the percentage of correctly classified grains as the ratio of the total number of grains and to calculate accuracy of the classification. The confusion matrix (Table 4) shows that 82% of the grains were now correctly classified. In this case 99% of the Erebus grains

were classified correctly followed by Taupo (86%) and Redoubt sample AT-1617 (70%). Approximately 79% of the grains from Redoubt sample AT-1876 still classified incorrectly, with the majority falling in the Taupo HA3-D class.

5. Discussion

Ash morphologies differ greatly based on the physical mechanisms of magma fragmentation and eruption style. Although the chemical composition of the magma has a fundamental influence on the fragmentation process, composition alone does not reliably predict eruption style or resultant particle shape and texture. In this study we use a combination of simple particle morphometrics, in conjunction with multivariate analysis techniques to evaluate whether or not specific fragmentation processes are distinguishable. In addition, using the methodology presented, specific eruption styles may be inferred in discriminant function space.

5.1. Distance versus eruption dynamics

For the two deposits from the 2009 Redoubt eruption; sample AT-1617 (~79 km from vent, events 9-18) and AT-1876 (~43 km from vent, events 2-4), five parameters were found to be significantly distinct between the two deposits (Table 3). The mean gray value was greater in the more proximal deposit (AT-1876), indicating a darker average shade of gray. The main control of mean gray value is vesicle size and depth on the surface of a grain. Ash in both samples contain two distinct size populations of vesicles (i.e. complex ash, *cf.* Genareau et al., 2013). Small vesicle diameters dominate in both samples, however ash from sample AT-1876 has a greater proportion of larger diameter vesicles relative to sample AT-1617. It follows that

the significantly lower mean gray value of ash in sample AT-1617 relative to AT-1876, is due to the fact that AT-1617 has less of the larger (and deeper) vesicles.

A bimodal distribution in vesicle size indicates two bubble nucleation events prior to eruption. The larger vesicles represent earlier bubble nucleation and their growth lower within the conduit system, whereas, the smaller vesicles reflect explosive decompression during eruption that stimulate higher rates of bubble nucleation and greater bubble densities (Genareau et al., 2012; Genareau et al., 2013). The lower proportion of the larger vesicles in the more distal ash from sample AT-1617 (the more distal deposit) agree with a higher energy eruption scenario, which is also supported by the overall higher plume heights during this phase of the eruption (Schaefer et al., 2012; Wallace et al., 2013). Unlike the Taupo and Redoubt deposits, ash from Mount Erebus tephra was collected within a few hundred meters from the vent source. The Erebus SC4 ash has an average mean gray value of 146.6, which is significantly higher than Redoubt ash (67.5 and 70.3 for AT-1617 and AT-1876, respectively) due to the occurrence of much larger and deeper vesicles (Table 2 and Fig. 1D). Although the difference in distance from vent between Erebus and Redoubt samples is substantial, we interpret the difference in vesicle size and distribution to be controlled by eruption dynamics, not distance, which will be discussed in more detail below.

The lower solidity values for grains in the more distal Redoubt sample AT-1617 would indicate that the morphology is rougher than grains in the more proximal sample AT-1876 (Table 2); however, solidity is not a straight forward parameter. Solidity is the surface area of the particle divided by the convex area. For a circle or an ellipse the area and convex areas would be presumed equal, yielding a ratio of 1 for solidity. Likewise, a smooth rectangle or other non-curved shape would also yield a solidity value of 1. The difference in rectangularity between the

two samples are significantly different with AT-1876 having a more rectangular shape than AT-1617. Consequently, ash from the more proximal deposit has a more rectangular/blocky shape than ash from the more distal deposit. This may be due to effects of secondary abrasion on the rectangularity parameter (Jordan et al., 2014). The abrasion of ash particles would create more rounded grains and this will occur in large eruption columns (e.g., plinian) as a result of particles colliding with each other during transport (Textor et al., 2006). Therefore, particles from sample AT-1876 would be expected to have a higher solidity and rectangularity values given their closer proximity to the vent at Redoubt. However, another possible explanation for the more rectangular/blocky shape of the more proximal ash (AT-1876, events 2-4) relative to more distal ash (AT-1617, events 9-18) is that fragmentation mechanisms may have been influenced by magma-water interaction in the early stages of the eruption. In the months preceding the 2009 Redoubt eruption melting of the Drift glacier, then occupying the summit crater, were reported. One such observation was the opening of an ice pit that by the end of February, expanded greatly and exposed a sub-glacial waterfall (Schaefer et al., 2012). On March 15 there was a small phreatic explosion, deemed event 0, which produced a small amount of fine grained tephra before the first “magmatically-driven” eruptions (events 1-6) occurred seven days later (Wallace et al., 2013). Ice clasts within deposits from event 5 and the presence of accretionary lapilli in all pyroclastic fall deposits from this eruption (Wallace et al., 2013; Schaefer et al., 2012) indicates that external water (i.e. melting of glacial ice) was readily available to influence magma fragmentation processes.

5.2. Insight from factor analysis

The Dellino and La Volpe (1996) conducted a multivariate R-mode factor analysis on rectangularity, compactness, circularity, and elongation parameters calculated from fine ash samples that is used as a standard methodology for several subsequent studies (e.g., Lautze et al., 2013; Iverson et al., 2014; Leibrandt and Le Pennec, 2015). Dellino and La Volpe (1996) developed a two factor diagram with a qualitative line drawn dividing grains with morphologies that are indicative of magmatic fragmentation from those that were produced by phreatomagmatic fragmentation. They concluded that this factor diagram of ash from Monte Pilato-Rocche eruptions on the island of Lipari could be applied generally and that the addition of new fine ash deposits from other volcanic areas could confirm or adjust the arbitrary boundary between these two eruption styles. Ersoy et al. (2006) conducted a study in which they applied the Dellino and La Volpe (1996) parameters in conjunction with factor analysis in an attempt to distinguish base surge and phreatoplinian ash generated from eruptions at Mount Nemrut in Turkey. They could not distinguish ‘wet’ and ‘dry’ fragmentation mechanisms based only on particle outline parameters.

To test the applicability of the Dellino and La Volpe (1996) method in determining fragmentation mechanism, R-mode factor analysis was conducted on the calculated ARCA parameters (where angularity and aspect ratio are almost analogous to circularity and elongation) for all tephra samples (Table 5). Factor analysis of ash in this study did not produce a clear separation of morphologies (Fig. 4) and therefore, for our data, there is no clear delimitation to base interpretations for the causes of fragmentation. There is, however, a general clustering of data for each deposit. Erebus ash is more concentrated on the right-hand side of the factor analysis chart (circles), while Redoubt ash is primarily concentrated in the middle (crosses) and the Taupo ash to the left (triangles) with higher negative Factor 1 values (Fig. 4). High and

positive Factor 1 values indicate relatively low rectangularity and higher angularity values. Low rectangularity values (closer to 1) indicate a more rectangular and regular shape; whereas, higher angularity values (closer to 1) indicate a more regular shape. Based on this reasoning and t-testing of the means for rectangularity and angularity between all deposits, Erebus ash is significantly more rectangular and regular on average relative to the other deposits (Tables 2 and 3).

The shape of Erebus grains may be a consequence of the deposit's close proximity to the vent relative to Redoubt and Taupo deposits, however, it may also reflect the difference in eruption style. Based on deposit characteristics Panter and Winter (2007) conclude that the Erebus SC4 tephra was produced by strombolian-style activity, while both Redoubt and Taupo deposits were from plinian and phreatoplinian activity, respectively (Wallace et al., 2013; Schaefer et al., 2012; Houghton et al., 2003). Therefore it was important for us to compare Erebus ash to ash examined by Lautze et al. (2013) from the 'type' locality for this style of activity, which is the Stromboli volcano, Italy. Striking similarities are found when comparing images of the ash from Stromboli with Erebus ash. Ash from both locations have large spherical vesicles with smooth walls. Furthermore, density curves for compactness and elongation were compared (Avery, 2015). For both parameters the distribution of the curve and the mean of Stromboli ash match the Erebus ash, where all grains trend towards less elongate and less irregular forms.

5.3. Insight from Discriminant Analysis

To evaluate the characterization of the data from R-mode factor analysis and in an attempt to refine interpretations of fragmentation mechanisms and eruption styles, ARCA and all other parameters determined for coarse ash between 1180 and 850 μm in diameter from each deposit were analyzed using discriminant function analysis. We consider the marked separation of Erebus ash data from Redoubt and Taupo ash data, and the well-defined positive linear array of Redoubt and Taupo ash data in factor space (Fig. 6) to be interpretable with respect to eruption dynamics. The linear array of Taupo and Redoubt ash data is considered to be a result of morphologies formed by gradation between wetter and drier fragmentation mechanisms. More rectangular/blocky ash with smoother grain edges (i.e. higher solidarity), which plots towards the positive Factor 2 end of the array, would thereby represent those produced by phreatomagmatic fragmentation and the more irregular (highly vesiculated or ‘spongy’) ash, which plots towards the negative Factor 2 end of the array, would represent those produced by magmatic fragmentation. For Taupo, this agrees with the Houghton et al. (2003) interpretation that the fragmentation during the 1.8 ka eruption transitioned between wetter and drier conditions. For Redoubt, the earlier phase (events 2-4), which produced tephra sample AT-1876, may have involved some magma-water interaction as the eruption broke through and melted back the glacier covering the vent, while the later phase (events 9-18), which produced sample AT-1617, occurred under drier conditions dominated by magmatically-driven fragmentation.

The Erebus ash has significantly higher, positive mean gray values (responsible for most of the variance in Factor 1) that separates it from Redoubt and Taupo ash data (Fig. 6). Recall that the high mean gray values can be explained by the fact that the Erebus ash processes much larger and deeper vesicles (Fig. 1D). Vesicle density and size in pyroclasts can be used to understand eruption dynamics at the time of formation (Moitra et al., 2013). A decrease in

confining pressure during magma ascent allows dissolved volatiles to form bubbles of supercritical fluid by decreasing pressure-dependent solubility. If bubble growth is limited by viscosity the ambient pressure may decrease faster than the gas pressure inside the bubble, thus pressurizing the bubble with respect to the surrounding melt. Plinian style eruptions are driven by the rapid expansion or fragmentation of pressurized bubbles. Strombolian-style activity at Erebus, however, is controlled by irregularities in the near-summit portion of its conduit system that influence magma degassing. Specifically, the shallowest (< 1.5 km) of the conduit system have significantly non-vertical, non-planar geometries that have a controlling role in the growth, storage, and eruption of the large gas slugs (Ilanko et al., 2015; Oppenheimer et al., 2011). Recall that the SC4 deposit is considered to have formed by strombolian-style activity that is similar in nature to the current activity (Panter and Winter, 2007).

It is important to re-emphasize here that we do not regard the chemical composition of magma to be a primary control on the ash morphology in this study. Recall that the Erebus ash, which is phonolitic, is morphologically similar to ash from the Stromboli volcano, Italy, which is basaltic (Lautze et al., 2013). Other studies have found that significant compositional changes do not coincide with variations in ash morphology (Cioni et al., 2014) and, conversely, that ash produced by eruption of compositionally homogeneous magmas can have quite variable morphologies (D’Orsano et al., 2011). Therefore, we consider the first-order control for the displacement of Erebus ash towards higher Factor 1 values relative to ash from Taupo and Redoubt (Fig. 6) to be due to differences in eruption style and fragmentation process, namely strombolian versus plinian styles of eruption.

The vertical spread in Erebus ash grains above and below the Factor 2 zero-line in Fig. 6 is a result of a mixture of rectangular/blocky (positive Factor 2 values) grains with more

irregular/highly vesiculated grains (negative Factor 2 values) that formed by rapid cycling between ‘dry’ strombolian and phreatomagmatic explosions during a single eruptive phase as was originally proposed by Panter and Winter (2008). This style of alternating eruptive style/fragmentation mechanism is also documented by a late Pleistocene, andesitic cone complex in the Ohakune Volcanic Complex, New Zealand (Kòsik et al., 2016).

6. Conclusions

Processes that fragment magmas clearly dictate the resultant morphology of ash particles observed in tephra deposits. The resulting shapes are complex due to the many factors involved in fragmentation. However, by evaluating several of the most appropriate shape parameters using multivariate statistical methods it is possible to correctly identify a deposit’s fragmentation mechanism with some certainty. In this study the application of ARCA parameters alone were insufficient to make this distinction. Specifically these parameters were not sufficient to account for the relative amount and size of vesicles in a grain. Furthermore, the particle perimeter alone cannot account for the majority of the vesicularity in any one grain since it does not account for ‘non-breaching’ vesicles, or those vesicles that do not intersect the perimeter. However, parameters such as mean gray value and solidity can account for some of the vesicularity-related differences in grains. The combination of all parameters produce a linear data array in discriminant factor space (Fig. 6) that is interpreted to reflect variations in the processes controlling magma fragmentation. Specifically the involvement of external water during the 2009 eruption of the Redoubt volcano (magmatic with some hydromagmatic influence) and for the 1.8 ka Taupo eruption that produced the Hatepe (Unit 3D) ash (magmatic → phreatomagmatic). The clear separation of the Erebus ash from Redoubt and Taupo ash in the

same analysis is most likely due to the difference in eruption style (strombolian versus plinian, respectively) that, in turn, ultimately relate to magma composition and system dynamics.

The method developed in this study can be used as a tool to help determine what fragmentation process is responsible for the formation of volcanic ash. Furthermore, this method could be advanced with the addition of samples created by other eruption styles and compositions. New samples could potentially constrain the existing factor space regions of eruption style morphology and identify new regions within factor space. For example, the significant gap between ash data for the Erebus and ash from Taupo and Redoubt deposits in Fig. 6 may be populated by grain morphologies that are representative of activity that are intermediate in their explosivity and eruption dynamics (e.g., ‘violent’ strombolian, subplinian, vulcanian). Overall, the cost effective and simple design of this method of classification can be used to evaluate eruption styles for both modern and ancient deposits.

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854 **Table Captions**

855 Table 1. Parameter formulas and sources. Note A is the area of the particle, b and w are the base
856 and the width of the smallest bounding rectangle, the major and minor axes are the primary and
857 secondary axes of the best fitting ellipse, p is the perimeter of the particle, and c is the perimeter
858 of a circle with area A .

859
860 Table 2. Mean parameter values for samples. Parameters are ordered: mean gray value, standard
861 deviation of the mean gray value, aspect ratio, solidity, rectangularity, compactness, and
862 angularity. Samples AT-1617 and AT-1876 are samples from the Redoubt volcano where AT-
863 1876s is the smaller sieved size fraction (600-800 μm) of this sample. Sample HA3-D originates
864 from the Taupo volcano and BW03-015 originates from the Erebus volcano.

865
866 Table 3. Significance testing on volcanic and sample relationships. Results of significance testing
867 on all parameters using two-sample t -tests with assumed unequal variances on log-transformed
868 data. Note those marked with an asterisk (*) are statistically insignificant having p -values > 0.05
869 (95% confidence interval).

870
871 Table 4. Confusion matrices. Samples categories include Redoubt AT-1617, Redoubt sample
872 AT-1876 (1.18mm-850 μm size), Redoubt AT-1876s (850-600 μm size), Taupo sample HA3-D,
873 and Erebus sample BW03-015. Categories of “from” represent the number of samples from that
874 particular sample that were identified “to” the corresponding sample by the model. The DA

category corresponds to the discriminant analysis trial number. The percent correct refers to the amount of grains from each sample that were correctly identified as their actual sample.

Table 5. Summary of R-mode factor analysis run on ARCA parameters for all deposits. The eigenvalue shows the amount of variation explained by each factor. It is common to drop all factors less than 1.0 as they do not contribute as much to the variance in the data set. The percentage explained is the portion of the entire data set variance explained by each factor. The cumulative percentage explained is the combined variance for factor and the previous factors. For example, Factor 2 explains approximately 32.4 % of the total data variance; furthermore, Factors 1 and 2 combined explain approximately 80.6 % of the total variance within the data set. The square of the R-loadings represent the amount of variance explained by that factor for each parameter.

Figure Captions

Figure 1. Representative grains for studied samples. (A) corresponds to grains from the Taupo HA3-D sample. (B) represents grains from the Redoubt AT-1876 sample. (C) represents grains from the Redoubt AT-1617 sample. Finally, (D) represents grain morphologies from the Erebus BW03-015 sample.

Figure 2. Scatterplots of ARCA parameters where *a*) shows the result from *k*-means cluster analysis using a total of two clusters, *b*) represents the visualization of the ARCA parameters grouped by volcanic origin, *c*) shows the result from *k*-means cluster analysis using a total of three clusters, and *d*) represents the visualization of the ARCA parameters grouped by deposit sample.

Figure 3. Distribution of data values associated with Rectangularity, Aspect Ratio, and Angularity parameters. The box plots in the figure are grouped by volcanic origin and summarize each of parameters based on five number summary statistics including: the median of the data, the lower and upper quartiles (25% and 75%), and the minimum and maximum values. Figure 4. ARCA parameter R-mode factor analysis with representative grains. Black circles indicate Erebus grains, red triangles represent Redoubt grains, and green crosses represent Taupo grains. Inset shows vector labels for each parameter and the total variance represented by each factor is given as a percentage. Characteristic samples and their corresponding ARCA measurements are given for each quadrant. **(A)** Aspect Ratio (1.51), Rectangularity (1.07), Compactness (0.63), Angularity (0.43). **(B)** Aspect Ratio (2.35), Rectangularity (1.00), Compactness (0.58), Angularity (0.44). **(C)** Aspect Ratio (1.88), Rectangularity (1.07), Compactness (0.60), Angularity (0.42). **(D)** Aspect Ratio (1.34), Rectangularity (1.79), Compactness (0.58), Angularity (0.15). **(E)** Aspect Ratio (1.51), Rectangularity (1.45), Compactness (0.70), Angularity (0.26). **(F)** Aspect Ratio (1.19), Rectangularity (1.70), Compactness (0.85), Angularity (0.23). **(G)** Aspect Ratio (1.17), Rectangularity (1.54), Compactness (0.82), Angularity (0.27).

Figure 5. Discriminant analysis of all samples for ARCA parameters only. Two factor chart from the first discriminant analysis run on the four ARCA parameters. All grains from each sample were analyzed including Redoubt AT-1617, Redoubt AT-1876 (corresponds to the 1 mm size fraction), Redoubt AT-1876s (corresponds to the 850 – 600 μ m size fraction), Taupo HA3 D, and Erebus BW03-015. Factor 1 represents 85.23% of data variance and Factor 2 represents 9.34% data variance, for an overall variance of 94.57% represented by both factors. Inset map shows component vectors.

Figure 6. Second and third discriminant analyses. (A) Discriminant analysis of all parameters. All grains from each sample were analyzed including Redoubt AT-1617, Redoubt AT-1876 (corresponds to the 1 mm size fraction), Redoubt AT-1876s (corresponds to the 850 – 600 μ m size fraction), Taupo HA3 D, and Erebus BW03-015. Factor 1 represents 97.75% of data variance and Factor 2 represents 1.59% data variance, for an overall variance of 99.35% represented by both factors. Inset map shows component vectors. (B) Discriminant analysis of all parameters but excluding sample AT-1876s. Two factor chart produced by the third discriminant analysis run on all parameters excluding the Redoubt sample AT-1876s (corresponding to the 850 - 600 μ m size fraction). Analyzed samples include Redoubt AT-1617, Redoubt AT-1876 (corresponds to the 1 mm size fraction), Taupo HA3-D, and Erebus BW03-015. Factors 1 and 2 represent 97.99% and 1.85% of total data variance respectively, for a combined representative variance of 99.84% by both factors. The first inset map shows the spread of the left-hand side of the graph along with the centroids for the deposits. The second inset map shows vectors for all parameter measurements.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Table 1

<i>Parameter</i>	<i>Formula</i>	<i>Explanation</i>	<i>Reference</i>
<i>Angularity</i>	$\frac{4\pi \times A}{p^2}$	Angularity is a measure that describes the sharpness of a grain's corners and edges, regardless of shape. A perfect rounded grain would have angularity value of 1 while grains with extremely sharp edges would have angularity value of 0.	Manga et al. (2011) Russ (2011)
<i>Aspect Ratio</i>	$\frac{\text{major axis}}{\text{minor axis}}$	The degree of aspect ratio or so called "elongation" is expressed as ratio of two different grain parameters including major and minor axis of best fitting ellipse. The degree of elongation is expressed by an index that shows grain's long and slender properties where large values are associated with slim and thin grain shape.	Russ (2011)
<i>Solidity</i>	$\frac{\text{Area}}{\text{Convex Area}}$	Solidity represents the ratio of area of the two-dimensional (2-D) projection of the grain to the convex area that is defined as the area enclosed by an imaginary "string" wrapped around the grain called convex hull. A high solidity value of 1 represents a smooth grain edge while low solidity value of 0 represents a rough grain edge.	ImageJ
<i>Compactness</i>	$\frac{A}{b \times w}$	Compactness represents the degree to which a shape is compact that is defined as the ratio of the grain area to the area of the bounding rectangle. Values close to 1 indicate the particle shape is close to a rectangle.	Lautze et al. (2013) Dellino and La Volpe (1996)
<i>Rectangularity</i>	$\frac{p}{2b + 2w}$	Rectangularity is the degree to which a grain is rectangular defined as the perimeter of the grain to the perimeter of the bounding rectangle of the grain. Rectangularity values close to 1 indicate a more regular and rectangular shape	Jordan et al. (2014) Dellino and La Volpe (1996)

Table 2

	<i>BW03-015</i>	<i>AT-1617</i>	<i>AT-1876</i>	<i>AT-1876s</i>	<i>HA3-D</i>
<i>Mean Gray Scale</i>	146.55136	67.48745	70.34546	71.20555	74.58175
<i>StdDev</i>	68.72199	34.08194	31.58832	30.09351	26.13471
<i>Aspect Ratio</i>	1.3823531	1.253433	1.320852	1.247122	1.347812
<i>Solidity</i>	0.7453627	0.67563	0.792097	0.865633	0.886077
<i>Rectangularity</i>	1.1312123	1.433122	1.314048	1.147643	1.177633
<i>Compactness</i>	0.676674	0.766088	0.709044	0.668239	0.703805
<i>Angularity</i>	0.4291944	0.306284	0.344965	0.4127	0.405971
<i>Number of clasts</i>	100	27	28	28	81

Table 3

	<i>Erebus x Redoubt</i>	<i>Erebus x Taupo</i>	<i>Redoubt x Taupo</i>	<i>AT-1617 x AT-1876</i>	<i>AT-1876 x AT-1876s</i>
<i>Mean</i>	3.8029E-131	5.358E-115	1.97364E-08	0.01346208	0.52672164*
<i>StdDev</i>	6.33519E-25	1.60563E-32	0.003876333	0.00673981	0.29161057*
<i>Aspect Ratio</i>	0.000174399	0.26347175*	0.016760245	0.1448191*	0.12853074*
<i>Solidity</i>	0.28481519*	1.28415E-15	7.78084E-07	0.00977061	0.11955477*
<i>Rectangularity</i>	3.79338E-08	0.018046835	9.43665E-05	0.03008574	0.001697036
<i>Compactness</i>	0.001269517	0.001970384	0.46943672*	0.00472483	0.07300785*
<i>Angularity</i>	1.86614E-05	0.27069530*	0.000156998	0.2656070*	0.023552018

Table 4

<i>from \ to</i>	<i>DA</i>	<i>AT-1617</i>	<i>AT-1876</i>	<i>AT-1876s</i>	<i>BW03-015</i>	<i>HA3-D</i>	<i>Total</i>	<i>% correct</i>
<i>AT-1617</i>	1	16	0	0	1	10	27	59.26%
	2	19	1	0	0	7	27	70.37%
	3	19	1	-	0	7	27	70.37%
<i>AT-1876</i>	1	6	1	0	8	13	28	3.57%
	2	7	4	3	0	14	28	14.29%
	3	7	6	-	0	15	28	21.43%
<i>AT-1876s</i>	1	0	1	0	19	8	28	0.00%
	2	2	3	14	0	9	28	50.00%
	3	-	-	-	-	-	-	-
<i>BW03-015</i>	1	3	0	2	69	26	100	69.00%
	2	0	0	0	99	1	100	99.00%
	3	0	0	-	99	1	100	99.00%
<i>HA3-D</i>	1	5	0	3	42	31	81	38.27%
	2	5	4	2	0	70	81	86.42%
	3	5	6	-	0	70	81	86.42%
<i>Total</i>	1	30	2	5	139	88	264	44.32%
	2	33	12	19	99	101	264	78.03%
	3	31	13	-	99	93	236	82.20%

Table 5

	<i>Factor 1</i>	<i>Factor 2</i>	<i>Factor 3</i>	<i>Factor 4</i>
<i>Eigenvalue</i>	1.92702806	1.29770441	0.72514304	0.05012449
<i>Percent Explained</i>	48.175701	32.44261	18.128576	1.253112
<i>Cumulative Percent Explained</i>	48.1757	80.61831	98.74689	100
<i>R-loadings:</i>				
<i>Aspect Ratio</i>	0.08181818	0.78557879	0.6132313	0.01091532
<i>Rectangularity</i>	-0.98729949	-0.01286463	0.0182749	0.1572903
<i>Compactness</i>	-0.22069508	-0.78166751	0.5817435	-0.04317458
<i>Angularity</i>	0.94703075	-0.26344027	0.1016411	0.15297407