1 **On the relationship between eruption intensity and volcanic plume height:**
2 **insights from three-dimensional numerical simulations**

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8

9 **Abstract**

10 Height of plumes generated during explosive volcanic eruptions are commonly
11 used to estimate the associated eruption intensity (i.e., mass eruption rate; MER). In
12 order to quantify the relationship between plume height and MER, we performed a
13 parametric study using a three-dimensional (3D) numerical model of volcanic plumes
14 for different vent sizes. The results of five simulations indicate that the flow pattern in
15 the lower region of the plume systematically changes with vent size, and hence, with
16 MER. For MERs $< 4 \times 10^7 \text{ kg s}^{-1}$, the flow in the lower region has a jet-like structure
17 (the jet-like regime). For MERs $> 10^8 \text{ kg s}^{-1}$, the flow shows a fountain-like structure
18 (the fountain-like regime). The flow pattern of plumes with $4 \times 10^7 \text{ kg s}^{-1} < \text{MERs} < 10^8$
19 kg s^{-1} shows transitional features between the two flow regimes. Within each of the
20 two flow regimes, the plume height increases as the MER increases, whereas plume
21 heights remain almost constant or even decrease as MER increases in the transitional
22 regime; as a results, the jet-like and fountain-like regimes show distinct relationships
23 of plume height and MER. The different relationships between the two regimes reflect
24 the fact that the efficiency of entrainment of ambient air in the jet-like regime is
25 substantially lower than that in the fountain-like regime. It is suggested that, in order

26 to estimate eruption intensity from the observed plume heights, it is necessary to take
27 the different flow regimes depending on MER into account.

28

29 **Keywords:** Explosive volcanism; Eruptive plume dynamics; Fluid dynamics models;
30 Three-dimensional simulations, Dusty-gas model

31 **1. Introduction**

32 Explosive eruptions can generate gigantic plumes, called as eruption column or
33 volcanic plume. During such eruptions, a large amount of a mixture of solid
34 pyroclasts (volcanic ash) and volcanic gases are ejected from volcanic vent into the
35 atmosphere. The ejected mixture entrains ambient air and expands because of the
36 heating from the pyroclasts, becoming buoyant and developing an eruption column.
37 As the mixture rises, its density becomes equal to the atmospheric density at the so-
38 called neutral buoyancy level (NBL), because of the atmosphere stratification. Finally,
39 the mixture reaches the top of the column exhausting its momentum and radially
40 spreads around the NBL.

41 Plume heights such as the total height and NBL are primarily determined by the
42 balance between the thermal energy ejected from the vent and the work done during
43 transport of the ejected material and entrained air through the stratified atmosphere
44 (e.g., Morton et al., 1956; Woods, 1988). Therefore, plume heights increases as the
45 eruption intensity (the mass eruption rate; MER) increases (Woods, 1988). This
46 dependency was obtained from dimensional analyses (Morton et al., 1956) and semi-
47 qualitatively studies based on field observations (Wilson and Walker, 1987; Sparks et
48 al., 1997; Mastin et al., 2009), experimental studies (Carazzo et al., 2008), and
49 numerical studies based on one-dimensional (1D) models of volcanic plume (Woods,
50 1988; Costa et al., 2016).

51 The relationships between MER and plume height are commonly derived under
52 the assumptions that the entrainment velocity at the edge of eruption column is
53 approximately proportional to the mean plume velocity along the flow axis at each
54 height and its proportionality coefficient, representing the efficiency of air
55 entrainment, is considered as a constant (e.g., Woods, 1995) or a simple function

56 (Kaminski et al., 2005; Carazzo et al., 2008). On the other hand, a recent study by
57 Suzuki and Koyaguchi (2012) showed that the flow pattern and the efficiency of air
58 entrainment in the lower part of eruption column can fundamentally change
59 depending on the vent size. Their results suggest that these changes affect the
60 dynamics of eruption columns near the vent such as the critical condition for column
61 collapse; however, whether or not these changes near the vent control the plume
62 heights is unclear.

63 In this study, we carried out a set of numerical simulations of volcanic columns
64 using a 3D fluid dynamic model (e.g., Suzuki et al., 2005; Suzuki and Koyaguchi,
65 2009, 2013) for different MERs obtained by varying the vent size. We estimated the
66 plume heights from the simulation results and compared the estimated heights with
67 those predicted by a 1D model based on the Buoyant Plume Theory (BPT) which
68 assumes constant efficiency of air entrainment. From the results of 3D simulations
69 and the comparison with those of the 1D model, we evaluated the dependency of
70 plume heights on MERs (and the vent sizes). The 3D model used here is somewhat
71 simplified compared with recent more sophisticated models such as ASHEE
72 (Cerminara et al., 2016). The model considers the dilute regime described by
73 Elghobashi (1991) and it used the dusty-gas approximation (i.e., Cerminara et al.,
74 2016). It ignores the effects of pressure disequilibrium at the vent (Carcano et al.,
75 2014) and water condensation (Herzog et al., 1998). In addition, the eddy diffusivity
76 due to subgrid turbulence was not explicitly calculated; the large-eddy simulation was
77 not used. Despite these simplifications, however, the model has successfully
78 reproduced the relationship between MER and plume height obtained by the field
79 observations (Suzuki and Koyaguchi, 2009, 2013).

80

81 2. Method and simulation inputs

82 From the results of numerical simulation, we estimated three characteristic
83 heights of the eruptive plume. The first one is the total (or maximum) eruption column
84 height (H_T). This height is defined as the highest level of an assigned concentration
85 threshold (see Suzuki et al., *in review*). Because the total height generally fluctuates
86 with time (e.g., Suzuki and Koyaguchi, 2009), we averaged the results across a
87 specified time window. The second one is the height of the NBL (H_{NBL}), defined as
88 the level where the density difference relative to the atmospheric changes sign from
89 positive to negative. To estimate this height, simulation results were averaged over an
90 appropriate time window and then averaged over the cross-section along the plume
91 axis. The third height is the maximum spreading level H_{MSL} , defined as the height
92 with the maximum radial injection of the erupted material. The H_{MSL} was estimated
93 using horizontally-integrated and temporally averaged profiles of the mass fraction of
94 the erupted material.

95 We carried out five simulations of eruption plumes with variable vent size, i.e.,
96 variable MER. The vent radius, R_0 , ranged from 60 m to 267 m; the MER, $\dot{m}_0$, ranged
97 from $2.0 \times 10^7 \text{ kg s}^{-1}$ to $4.0 \times 10^8 \text{ kg s}^{-1}$. The other parameters were kept fixed among
98 the simulations. Magmatic temperature and water content were assumed to be 1000 K
99 and 2.84 wt %, respectively, similar to the conditions set for Group H in Suzuki and
100 Koyaguchi (2012). The pressure at the vent is assumed to be in equilibrium with the
101 atmospheric condition. According to the equation of state, the initial density of the
102 ejected material ρ_0 is estimate to be 7.72 kg m^{-3} . The exit velocity w_0 is 230 m s^{-1}
103 corresponding to the Mach number of 2.0, and it is assumed to be constant during
104 eruption. The MER and vent radius is related by an equation of $\dot{m}_0 = \rho_0 \pi R_0^2 w_0$. The
105 common parameters used are listed in Tables 1 and the values considered in the

different Runs are reported in Table 2. The details of the 3D and 1D models used in this study are described below.

2.1. 3D time-dependent model

In order to investigate how variation of MER due to changes of vent size influences the efficiency of air entrainment and the plume dynamics and heights, we used the pseudo-gas model (Marble, 1970; Suzuki et al., 2005). In the framework of this model, the disequilibrium between the solid pyroclasts and the gas phases is neglected (Cerminara et al., 2016); the mixture of the ejected material (solid pyroclasts and water vapour) and the entrained air is treated as a single fluid whose density is calculated from the mixing ratio of the ejected material and entrained air. This model solves a set of partial differential equations describing the conservation of mass, momentum, and energy, the equation of state, and the equation describing the thermodynamic state of the mixture of pyroclasts, volcanic gas, and air. Details of the numerical procedures used in this study are described in Suzuki et al. (2005) and Suzuki and Koyaguchi (2009).

The simulations were designed to describe the injection of a mixture of pyroclasts and volcanic gas from a circular vent located at 0 km above sea level (a.s.l.) in a windless atmosphere. Mid-latitude atmospheric conditions were taken into account and the profiles of the temperature gradients considered are indicated in Table 1. The atmospheric density and pressure were calculated from the hydrostatic relationship. Free-slip conditions were assumed at the ground boundary, whereas free outflow/inflow conditions were applied at the upper and side boundaries of the computational domain.

130 Numerical calculations were performed on a generalized coordinate system. The
131 vent diameter consists of 16 grid points. The grid size increases with distance from the
132 vent at a constant rate (by a factor of 1.02) up to the length scale of vent radius. The
133 partial differential equations are solved by the Roe scheme (Roe, 1981) with MUSCL
134 (Monotone Upstream-centred Scheme for Conservation Laws) interpolation (van
135 Leer, 1977) for spatial integration and the time splitting method for time integration.

136 To obtain a statistically stationary configuration of eruption columns, the 3D
137 simulations were carried out for sufficiently long time. The choice of the critical
138 threshold and time window for averaging were decided *a posteriori* based on the
139 sensitivity of results (the procedure of the choice is described in Suzuki et al., *in*
140 *review*). The simulated time, and the time window and output interval for averaging
141 are also listed in Table 2. The plume heights obtained from the 3D simulations are
142 denoted by H_T^{3D} , H_{NBL}^{3D} , and H_{MSL}^{3D} .

143

144 2.2. 1D steady model

145 Several sophisticated 1D models of eruption column have been proposed and
146 compared among them by different research groups (Costa et al., 2016). Here we used
147 a simple model with uniform entrainment coefficients proposed by Woods (1988).
148 This model is designed to describe the physical quantities in a steady eruption column
149 as a function of the height above the vent. In this model, the fluxes of mass,
150 momentum, and specific enthalpy along the plume are calculated (Cerminara, 2015).
151 To compare the 1D calculation results with those obtained from the 3D pseudo-gas
152 model, we ignore the disequilibrium effects between solid pyroclasts and gas phases
153 in the 1D model.

From the calculations using the 1D model, we can obtain the total height of eruptive plume (H_T^{1D}) and the height of the NBL (H_{NBL}^{1D}). In the 1D model, the maximum spreading level is practically considered to be same as H_{NBL}^{1D} . We compared H_T^{3D} with H_T^{1D} , and then H_{NBL}^{3D} and H_{MSL}^{3D} with H_{NBL}^{1D} .

3. Simulation results

As we mentioned above, we carried out five Runs with different vent sizes, and hence different MERs. In all 3D simulations describing the time evolution of the eruption plumes, an eruption column and umbrella cloud were developed. That is, the radial length scale of umbrella cloud became comparable to or longer than the total height of eruption column. In the following, we describe the representative results for Run 1 (characterized by the lowest MER) and Run 5 (characterized by the highest MER), and the quantitative features of plume heights.

3.1 Run 1

Figure 1 shows the time-averaged, vertical cross-sections of different physical quantities obtained for the small vent size case: $R_0 = 60$ m and $\dot{m}_0 = 2.0 \times 10^7$ kg s⁻¹.

The eruption column develops over 20 km. The edge of umbrella cloud reaches 25–28 km from the central axis of the flow. In the lower column, the eruption column has an inner core with a high concentration of the erupted material (Fig. 1a), density larger than the ambient air (Fig. 1b), and the large upward velocity (Fig. 1c). The inner core is surrounded by an outer shear layer having a lower particle concentration and density lower than the ambient air. The inner core is progressively eroded by the outer shear layer with height and disappears at about 5 km. We refer to the region near the vent where the flow is primarily controlled by the initial momentum as the momentum region. Above the momentum region, the cloud density becomes lower

179 than the ambient air across the eruption column (i.e., the plume region, defined as the
 180 buoyancy-driven flow). In this region, the concentration of the erupted material
 181 decreases continuously (Fig. 1a) and the upward velocity increases (Fig. 1c). At $z >$
 182 14 km, the cloud density becomes higher than the ambient air (Fig. 1b) and the cloud
 183 collapses outward and downward (Figs. 1d and 1e, respectively). This region is
 184 referred to as the overshooting top, which is also characterized by a temperature lower
 185 than the ambient air (figures not shown). From the overshooting top, the cloud spreads
 186 radially around $z \sim 15$ km as a gravity current (e.g., Costa et al., 2013). The edge of the
 187 umbrella cloud reaches $x \sim 7$ km at 400 s and $x \sim 25$ km at 2400 s.

188 Figure 2 illustrates the time evolution of the mass fraction of the erupted
 189 material (ξ) and the density difference relative to the atmospheric density along the
 190 central axis estimated from the spatial average within 8 grid points from the central
 191 axis. The plume height increases at the first 300 s. After this time, the maximum
 192 height of eruption cloud oscillates at the period of 200–300 s. The total heights
 193 defined by the threshold of $\xi = 0.01$, and 0.001 are almost identical. In some periods
 194 (for example, $t = 1800$ – 2000 s), the position of $\xi = 0.001$ is 2 km higher than that of ξ
 195 $= 0.01$, but that deviation is 10% of the total height. According to these results, we
 196 chosen the mass fraction of $\xi = 0.01$ – 0.001 to define the maximum points of the
 197 plume and the time window of [400s:2400s] for time averaging for Run 1. The total
 198 height H_T^{3D} was estimate to be 21.1–23.2 km.

199 The time-averaged vertical profile of ξ integrated in the region where $10 \text{ km} < x$
 200 $< 30 \text{ km}$ and $\xi > 0.001$ is provided in Fig. 3. This profile can be approximately by a
 201 Gaussian function $\xi = \xi_c \exp[-(z - z_c)^2 / (2\sigma_z^2)]$ with mean value $z_c = 14.7$ km and
 202 standard deviation $\sigma_z = 1.7$ km. The standard deviation based on the chi-square
 203 function, representing the goodness of fit, is 0.0026, whereas $\xi_c = 0.0154$. From this

204 result, $H_{\text{MSL}}^{3\text{D}}$ was estimate to be 14.7 ± 1.7 km. The vertical profile of the density
 205 difference (Fig. 4b) indicated that the density difference has negative value below
 206 13.4 km, whereas it has positive values above that level. Therefore, $H_{\text{NBL}}^{3\text{D}}$ is 13.4 km.
 207 This is within the range of $H_{\text{MSL}}^{3\text{D}}$ and lower than its mean value z_c . In general, the
 208 results of this analysis depends on the time window and the spatial range for
 209 averaging if the umbrella cloud does not sufficiently develop. We confirmed that
 210 $H_{\text{MSL}}^{3\text{D}}$ does not change largely even when the different time window and spatial range
 211 are applied.

212 **3.2 Run 5**

213 Run 5 is characterized by a large vent size: $R_0 = 267$ m and $\dot{m}_0 = 4.0 \times 10^8$ kg
 214 s^{-1} . The global features of the eruption plume are similar to those of Run 1 but with
 215 larger eruption column, umbrella cloud, and overshooting top (Fig. 5).

216 The high concentration of the erupted material reaches about 30 km and the
 217 dilute part rises over 40 km (Fig. 5a). The radial length scale of umbrella cloud is 80–
 218 100 km. Because the eruption is more intensive than Run 1, the overshooting top is
 219 more remarkable (Fig. 5b), the radial velocity of umbrella cloud is larger (Fig. 5d),
 220 and the upward velocity in the buoyant region is larger than those for Run 1 (Fig. 5c).

221 The flow pattern in the lower column for Run 5 is significantly different from
 222 that for Run 1. The lower column has a fountain-like structure below 5 km in Run 5,
 223 whereas it has a jet-like structure in Run 1. This fountain-like structure is referred to
 224 as a radially suspended flow (Neri and Dobran, 1994). The radially suspended flow is
 225 characterized by a high concentration of the erupted material and density larger than
 226 that of the surrounding ambient. The erupted material is mixed with the ambient air by
 227 the large-scale eddy around the top of the radially suspended flow. The sign of density
 228 difference rapidly changes from negative to positive at this height.

229 The time evolution of the vertical distributions of the main variables (Fig. 6)
 230 indicate a strong oscillation of plume heights. The top of the plume oscillates around
 231 the height of 25–40 km at a period of about 300 s. The position of the threshold of $\xi =$
 232 0.05 almost coincides with the positions of $\xi = 0.01, 0.001$ before 1100 s. After this
 233 time, the dilute parts develop above the column and the position of $\xi = 0.05$ is 5–10
 234 km lower than the position of $\xi = 0.001$. Considering the threshold of $\xi = 0.05$ –0.001
 235 and the time window of [1000 s: 4800 s] for time averaging, H_T^{3D} is estimated to be
 236 31.3–41.7 km. This range is wider than that for the small vent case (Run 1).

237 The time-averaged vertical profile of ξ integrated in the region where $30 \text{ km} < x$
 238 $< 130 \text{ km}$ and $\xi > 0.001$ is shown in Fig. 7. This profile can be approximate by a
 239 Gaussian function with $z_c = 18.8 \text{ km}$ and $\sigma_z = 4.1 \text{ km}$, yielding $H_{MSL}^{3D} = 18.8 \pm 4.1$
 240 km. The standard deviation based on the chi-square function is 0.0049, whereas $\xi_c =$
 241 0.0154.

242 Fig. 8 shows that the density difference along the central axis changes from
 243 negative to positive at 12.6 km, giving $H_{NBL}^{3D} = 12.6 \text{ km}$, which is substantially lower
 244 than H_{MSL}^{3D} .

245 3.3 Flow Regimes and Plume Heights

246 Table 3 describes whether the flow belongs to the jet-like or fountain-like
 247 regimes for five different MERs or vent radii. In addition, the table summarizes the
 248 characteristic plume heights, i.e., H_T^{3D} , H_{MSL}^{3D} , and H_{NBL}^{3D} . When the MER is equal to or
 249 larger than 10^8 kg s^{-1} , the flow is clearly in the fountain-like regime. On the other
 250 hand, the case with $R_0 = 60 \text{ m}$, $\dot{m}_0 = 2.0 \times 10^7 \text{ kg s}^{-1}$ is characterised by jet-like
 251 structures. The case with $R_0 = 84 \text{ m}$, $\dot{m}_0 = 4.0 \times 10^7 \text{ kg s}^{-1}$ (Run 2) belongs to the
 252 transitional regime, i.e., the flow is between the jet-like and fountain-like regime.

253 Suzuki and Koyaguchi (2012) have suggested that the transition between
 254 the jet-like and fountain-like regimes is determined by whether the unmixed
 255 core is eroded by the annular mixing layer or not in the momentum region. From
 256 the comparison between the height where the unmixed core disappears and the
 257 height where the initial momentum is exhausted, they have shown that the
 258 condition separating the jet-like and fountain-like regimes is determined by a
 259 certain critical value of Richardson number (Ri_{crit}) as:

$$Ri_{crit} = \frac{1}{2(a_1 M + a_2)}$$

260 where the Richardson number is calculated from the exit velocity (w_0), the
 261 densities of ejected material (ρ_0) and ambient air (ρ_a), and the vent radius (R_0)
 262 as

$$Ri \equiv \frac{(\rho_0 - \rho_a)gR_0}{\rho_a w_0^2}.$$

263 Here, M is the Mach number of the exit velocity and a_1 and a_2 are empirical
 264 constants; according to the results by Suzuki and Koyaguchi (2012), $a_1 = 1.3 -$
 265 3.4 and $a_2 = 0.16 - 4.7$. When the velocity, pressure, and density at the exit are
 266 considered to be fixed, the transition condition separating the jet-like and
 267 fountain-like regimes is given by a critical value of MER. In the present case (the
 268 exit velocity is Mach 2, the exit pressure is 1 atm and the exit density is 7.72 kg
 269 m^{-3}), we can estimate the critical MER as $\dot{m}_0 > 1.2 - 21.0 \times 10^7 \text{ kg s}^{-1}$, which is
 270 consistent with the results in Table 3.

271 The three different plume regimes, i.e. jet-like, transitional, and fountain-like
 272 regimes, can be clearly visualized in Fig. 9 where the plume characteristic heights,
 273 H_T^{3D} , H_{MSL}^{3D} , and H_{NBL}^{3D} , are shown. All these quantities show a similar trend (for the

sake of comparison the results from Suzuki and Koyaguchi (2015) with $2.5 \times 10^6 \text{ kg s}^{-1}$ are also plotted). For $\dot{m}_0 < 4.0 \times 10^7 \text{ kg s}^{-1}$ (the jet-like regime) and for $\dot{m}_0 > 10^8 \text{ kg s}^{-1}$ (the fountain-like regime), and plume heights increase as MERs increase. For $4.0 \times 10^7 \text{ kg s}^{-1} < \dot{m}_0 < 10^8 \text{ kg s}^{-1}$ (the transitional regime), however, plume heights remains almost constant or even decrease as MERs increase.

These variations in plume heights suggest that the efficiency of entrainment substantially change as the flow pattern changes from the jet-like regime to the fountain-like regime. In Fig. 9, H_T^{1D} and H_{NBL}^{1D} are also illustrated for variable entrainment coefficients. From the comparison of H_T^{3D} with H_T^{1D} , we can estimate the effective value of the entrainment efficiency for 1D BPT models, k_T . The value of k_T is 0.03–0.04 in the jet-like regime, whereas it is 0.05–0.07 fountain-like regime. The effective value of the entrainment efficiency for 1D models can be also estimated comparing H_{MSL}^{3D} with H_{NBL}^{1D} . The latter method indicates an effective entrainment coefficient k_{MSL} of about 0.04 for the jet-like regime, and a value of 0.10 fountain-like regime. If we consider that the value of H_{NBL}^{3D} instead of H_{MSL}^{3D} represents the NBL in the 1D model, the effective value of the entrainment coefficient, k_{NBL} , would be 0.05–0.10 for the jet-like regime and substantially larger than 0.10 for the fountain-like regime with $\dot{m}_0 > 10^8 \text{ kg s}^{-1}$.

The enhancement of entrainment in the fountain-like regime is also supported by the velocity distribution around the lower part of the eruption column. In Figs. 1e and 5e we can see the flow field of the ambient atmosphere largely modified by the presence of the fountain; the large eddy of the fountain strengthen the inward flow in the ambient atmosphere just above the fountain in Run 5 (see Fig. 5e). Because of this inward flow, the mass fraction of erupted material drastically decreases just above the fountain in Run 5 (Fig. 8a), whereas it gradually decreases with height Run 1 without

the fountain (Fig. 4a), suggesting that the entrainment is enhanced by the large eddy of the fountain.

4. Volcanological implications

Although further systematic studies are required in order to fully understand the problem of the interaction between column dynamics and flow field in the ambient atmosphere, the results in Fig. 9 are sufficient to show that the dynamics and efficiency of air entrainment are different between the jet-like and fountain-like regimes, and that this difference affects the relationship between plume height and MER with significant volcanological implications.

In common practice, MERs necessary to initialize tephra dispersal codes are estimated from heights of plume directly observed using BPT models (e.g., Folch et al., 2012) or inferred from field studies for past eruptions (e.g., Carey and Sparks, 1986; Bursik et al., 1992; Koyaguchi and Ohno, 2001; Bonadonna and Costa, 2013). A few empirical relationships relating these two quantities were recently proposed (Sparks et al. 1997; Mastin et al., 2009; Degruyter and Bonadonna, 2012; Woodhouse et al., 2013; Carazzo et al., 2014). Uncertainties characterizing these estimations are typically very large affecting tephra hazard assessment (Bonadonna et al., 2015 and references therein).

Most of the methods for estimating MERs are based on the validity of the similarity assumption of BPT (Costa et al., 2016). On the other hand, our results indicate that such a similarity assumption of BPT starts to lose its validity in the transitional regime because of the role of fountain-like structures. Not taking the different plume behaviour in the different regimes into account may introduce strong bias in MERs estimations and misinterpret the actual plume dynamics.

324 Individuating the three regimes on the basis of the value of MERs can also
325 help to better classify the eruptive plumes and improve parameterizations aimed to
326 estimate the associated MER. Our results, obtained in windless conditions, confirm
327 that the difference between weak and strong plume is based on the different physics
328 (Costa et al., 2016) rather than on the effects of winds alone. In fact, independently of
329 wind intensity, a weak plume is characterized by a jet-like behaviour in the lower
330 region having a lower entrainment efficiency with respect a strong plume having a
331 fountain-like behaviour and a higher entrainment efficiency. The transitional regime
332 separating the jet-like from the fountain-like regimes roughly lays on the boundary
333 between small-moderate and sub-plinian eruptions classified by Bonadonna and Costa
334 (2013). This implies that the characterization of flow regimes is necessary for a better
335 estimation of MER inferred from plume heights particularly for these eruption styles.

336
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453

454 **TABLE CAPTIONS**

455

456 Table 1. Common input parameters and constants for simulations.

457

458 Table 2. Input parameters for simulations.

459

460 Table 3. Plume heights and flow regime obtained from the 3D simulations.

461

462

463 **FIGURE CAPTIONS**

464 Figure 1. Simulation results of Run 1. Time-averaged, vertical cross-sections of (a)
465 the mass fraction of the erupted material, (b) the density difference relative to
466 the atmospheric density at the same vertical position $(\rho - \rho_a) / \rho_a$, (c) the upward
467 velocity, (d) the radial velocity, and (e) the flow field near the vent. The time
468 window for averaging is [400 s: 2400 s]. The positions corresponding to $\xi = 10^{-2}$,
469 10^{-3} , and 10^{-4} are shown by solid curves in the figures.

470

471 Figure 2. Timewise distributions of (a) the mass fraction of the erupted material and
472 (b) the density difference relative to the atmospheric density at the same vertical
473 position along the central axis for Run 1. The positions corresponding to $\xi = 10^{-2}$
474 and 10^{-3} are also shown by the solid curves in the figures.

475

476 Figure 3. Time-averaged vertical profile of ξ within the umbrella cloud for Run 1,
477 horizontally integrated at each height for $10 \text{ km} < x < 30 \text{ km}$. The time window
478 for averaging is [400 s: 2400 s]. The dashed curve is the fitted Gaussian
479 function: $\xi = 0.0154 \exp[-(z - 14.7)^2 / (2 \times 1.7^2)]$, where the standard
480 deviation based on the chi-square function is 0.0026.

481

482 Figure 4. Time-averaged vertical profiles of physical quantities for Run 1. (a) The
483 mass fraction of the erupted material ξ and (b) the density difference relative to
484 the atmospheric density at the same vertical position. Solid curves represent the
485 values horizontally integrated using the method proposed by Cerminara (2015),
486 whereas dashed curves represent the spatial average within 8 grid points from
487 the central axis. The time window for averaging is [400 s: 2400 s]. The dotted

488 curve in Fig. 4a represents the vertical profile of ξ calculated by the 1D BPT
489 model with $k = 0.1$. The arrow in Fig. 4b indicated $H_{\text{NBL}}^{3\text{D}}$ where the cloud
490 density equals the atmospheric density.

491

492 Figure 5. Simulation results of Run 5. Time-averaged, vertical cross-sections of (a)
493 the mass fraction of the erupted material, (b) the density difference relative to
494 the atmospheric density at the same vertical position $(\rho - \rho_a) / \rho_a$, (c) the upward
495 velocity, (d) the radial velocity, and (e) the flow field near the vent. The time
496 window for averaging is [1000 s: 4800 s]. The positions corresponding to $\xi =$
497 0.05 , 10^{-2} , and 10^{-3} are shown by solid curves in the figures.

498

499 Figure 6. Timewise distributions of (a) the mass fraction of the erupted material and
500 (b) the density difference relative to the atmospheric density at the same vertical
501 position along the central axis for Run 5. The positions corresponding to $\xi =$
502 0.05 , 10^{-2} , and 10^{-3} are also shown by the solid curves in the figures.

503

504 Figure 7. Time-averaged vertical profile of ξ within the umbrella cloud for Run 5,
505 horizontally integrated at each height for $30 \text{ km} < x < 130 \text{ km}$. The time
506 window for averaging is [1000 s: 4800 s]. The dashed curve is the fitted
507 Gaussian function: $\xi = 0.0517 \exp[-(z - 18.8)^2 / (2 \times 4.1^2)]$, where the
508 standard deviation based on the chi-square function is 0.0049.

509

510 Figure 8. Time-averaged vertical profiles of physical quantities for Run 5. (a) The
511 mass fraction of the erupted material ξ and (b) the density difference relative to
512 the atmospheric density at the same vertical position. Solid curves represent the

513 values horizontally integrated using the method proposed by Cerminara (2015),
 514 whereas dashed curves represent the spatial average within 8 grid points from
 515 the central axis. The time window for averaging is [1000 s: 4800 s]. The dotted
 516 curve in Fig. 8a represents the vertical profile of ξ calculated by the 1D BPT
 517 model. The arrow in Fig. 8b indicated $H_{\text{NBL}}^{3\text{D}}$ where the cloud density equals the
 518 atmospheric density.

519

520 Figure 9. Characteristic heights of eruption column as a function of the MER $\dot{m}_0$ for
 521 the simulation results of Runs 1–5. Circles represent $H_{\text{T}}^{3\text{D}}$, squares represent
 522 $H_{\text{MSL}}^{3\text{D}}$, and crosses represent $H_{\text{NBL}}^{3\text{D}}$. The results from Suzuki and Koyaguchi
 523 (2015) are also plotted. Note that the Mach number of the exit velocity is 2.0 in
 524 the present study, whereas it is 1.0 in Suzuki and Koyaguchi (2015). Solid
 525 curves are $H_{\text{T}}^{1\text{D}}$, and dashed curves are $H_{\text{NBL}}^{1\text{D}}$ with the entrainment coefficient k
 526 = 0.03, 0.05, 0.07, and 0.10.

527

528

529 Table 1. Common input parameters and constants for simulations.

Variable	Value
Vent elevation	0 m
Exit velocity	230 m s ⁻¹
Exit temperature	1000 K
Exit water fraction	0.0284
Exit density	7.72 kg m ⁻³
Gravity body force	9.81 m s ⁻²
Gas constant of volcanic gas	462 J kg ⁻¹ K ⁻¹
Gas constant of atmospheric air	287 J kg ⁻¹ K ⁻¹
Specific heat of solid pyroclasts	1100 J kg ⁻¹ K ⁻¹
Specific heat of volcanic gas	1348 J kg ⁻¹ K ⁻¹
at constant volume	
Specific heat of air	717 J kg ⁻¹ K ⁻¹
at constant volume	
Atmospheric density at 0 km	1.29 kg m ⁻³
Atmospheric temperature at 0 km	273 K
Atmospheric temperature gradient	
0–11 km	–6.5 K km ⁻¹
11–20 km	0.0 K km ⁻¹
20–32.2 km	1.0 K km ⁻¹
32.2–47.4 km	2.8 K km ⁻¹

530

531

532 Table 2. Input parameters for simulations.

Run	MER $\dot{m}_0$ (kg s ⁻¹)	Vent radius R_0 (m)	Time window [t_1 : t_2] (s)	Time interval (s)	Grid number $x \times y \times z$	Number of cores	Execution time (hour)
1	2.0×10^7	60	[400: 2400]	1.0	1520×1520×510	512	1188
2	4.0×10^7	84	[400: 2000]	1.0	1040×1040×510	512	2736
3	1.0×10^8	134	[400: 2000]	1.0	776×784×510	384	720
4	2.0×10^8	189	[1000: 3600]	2.0	1024×1024×256	512	432
5	4.0×10^8	267	[1000: 4800]	2.0	1024×1024×256	512	480

533

534 Table 3. Plume heights and flow regime obtained from the 3D simulations.

Run	H_T^{3D} (km)	H_{NBL}^{3D} (km)	H_{MSL}^{3D} (km)	Regime
Run 1	21.1–23.2	13.4	14.7±1.7	Jet
Run 2	22.4–25.9	13.0	16.6±2.8	Jet/Fountain
Run 3	22.3–24.5	11.4	14.6±2.6	Fountain
Run 4	25.1–29.8	11.4	15.9±3.0	Fountain
Run 5	31.3–41.7	12.6	18.8±4.1	Fountain

535

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9