SOFT IMPACTS ON AEROSPACE STRUCTURES

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Abstract

This article provides an overview of the literature dealing with three types of soft impacts of concern for the aerospace applications, namely impacts of rain drops, hailstones and birds against aircraft. It describes the physics of the problem as it has become better understood through experiments, analyses, and numerical simulations. Some emphasis has been placed on the material models and the numerical approaches used in modeling these three types of projectiles.

Keywords: impact, rain, birds, hailstones, aircraft

1- INTRODUCTION

This article deals with the impact of rain drops, hailstones, and birds on aerospace structures. These three types of impacts are usually considered to be soft impacts even if it is difficult to define what constitutes a soft impact as opposed to a hard impact [1]. Soft impacts are sometimes defined as impacts in which the stresses generated far exceed the strength of the projectile but are far below the strength of the target [2] [3]. Often publications dealing with hard impacts such as those occurring when a tool is dropped on a composite structure state that their analytical or numerical models are applicable to bird strikes and hail strikes. The following will show that this is not the case.

Current research emphasis is on the development of efficient approaches for numerical simulation that requires adequate models of the material behavior and efficient numerical methods for handling the large deformations and flow of soft projectiles. In many cases the projectile behaves as an inviscid fluid and sometimes artificial viscosity is introduced to avoid oscillations in the predicted response. Other material models are also used for the projectile while for the target elastic models, viscoelastic models and various plasticity models are used. Modeling a highly deformable projectile is challenging when using Lagrangian or Eulerian descriptions of the deformation. Approaches combining these two continuum descriptions are found to be more efficient and the Smooth Particle Hydrodynamics (SPH) method that considers the material as a series of particles appears as the favorite approach for modeling bird strikes in particular. The article focuses on soft impacts on aircrafts and in particular the impacts due to rain, birds and hailstones.

2- LIQUID IMPACT

Research on this topic started in 1928 with an investigation of the erosion caused by the collapse of cavities on the surface of steam turbine blades [4]. Since then, the scope broadened to include rain erosion on acrylic materials for aircraft transparencies [5] and forward facing airplane components [6], particularly those made out of composite materials. Currently, rain erosion is also a significant issue for the leading edges of wind turbine blades made out of composites with either glass or carbon fiber reinforcement [7] and require special coatings [8]. In addition, to these obvious reasons to study liquid impacts, we will see that at typical impact velocities birds behave like fluid during impact.

Cook [4] discusses the erosion caused by the collapse of cavities on the surface of steam turbine blades. Considering a column of water of impacting a rigid surface with a velocity V , the kinetic energy of a layer of thickness Δh prior to impact is $\frac{1}{2}\rho V^2 A \Delta h$ where A is the cross-sectional area of the column and ρ is the density of the fluid. After impact, the velocity of the fluid is zero and the kinetic energy has been converted into potential energy $\frac{1}{2}p e_v A \Delta h$ where the volumetric strain $e_v = p\beta$. Equating these two energies gives the pressure $p = V\sqrt{\rho/\beta}$. The compressibility β is the inverse of the bulk modulus B and $\sqrt{B/\rho} = C$, the speed of sound in the fluid. This pressure is usually written as

$$p = \rho CV \quad (1)$$

and is commonly called the water hammer pressure. The bulk modulus of water is 2.2 GPa and its density is 1000 kg/m³ so $c = 1483$ m/s. Cook considered impact velocities in excess of 120 m/s. When $V = 120$ m/s, Eq. 1 predicts a water hammer pressure of 178 MPa which is of the same order of magnitude than the yield strength of engineering materials. The wave velocity C is related to the acoustic wave velocity C_a and the particle velocity by $C = C_a + kV$ where k is approximately equal to 2 when $V < 1000$ m/s as indicated in [9]. Therefore, at high impact velocities, the shock wave velocity is significantly higher than the acoustic wave velocity.

Another view of this problem is that the motion of the fluid is governed by the wave equation and that, after impact against the rigid surface, a compressive wave propagates into the fluid with a velocity C and the pressure behind the wave front is the water hammer pressure (Eq. 1). When the water column impacts an elastic half-space, two waves propagate away from the interface: one into the fluid and the other into the solid. The water hammer pressure is

$$p = \frac{z_1 z_2}{z_1 + z_2} V = \frac{\rho_1 c_1 \cdot \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2} V \quad (2)$$

where $z_1 = \rho_1 c_1$ and $z_2 = \rho_2 c_2$ are the mechanical impedances of the liquid and the solid [10]. Eqs. 1, 2 predict the pressures during the initial phase of the impact between a liquid and a solid. As soon as the impact initiates and the shock wave starts to propagate upwards, relief waves initiating at the periphery of a cylindrical liquid projectile start to propagate inwards (Fig. 1). For water jet of radius r impacting a solid surface, the impact phase ends when these release waves reach the central axis at time $t = r/C$. That marks the end of the high pressure phase. After that, the flow can be considered to be incompressible. For incompressible flows, on a stream line, Bernoulli's principle states that $\frac{V^2}{2} + gz + p/\rho = \text{constant}$. When applied to the stream line along the axis of the projectile we found that the stagnation pressure is

$$p_s = \rho V^2 / 2 \quad (3)$$

After that the pressure at the stagnation point falls to $p_s = \frac{1}{2}\rho V^2$. The ratio of the water hammer pressure (Eq. 1) to the stagnation pressure $p/p_s = 2C/V$ is always large which indicates the importance of the initial stage of the liquid-solid impact.

For a spherical water drop impacting a rigid surface [11] [12], the contact radius at the end of the high-pressure stage and the duration of that high-pressure stage are $r = RV/C$ and $\tau = 3RV/2C^2$. The pressure in the compressed region behind the shock wave is not uniform and is highest just behind the contact edge. The pressure at first contact (Eq. 1) increases to approximately three times that value right before jetting [13]. The average pressure during the impact phase is given by $p = \frac{1}{2}\alpha\rho CV$ where the factor $\frac{1}{2}$ is due to the spherical shape of the drop and the factor α tends to one for high impact velocities (Engel [14], 1955).

As the initial high pressure phase of the impact nears its end, the contact radius r_c increases with a velocity equal to the shock velocity [15] (Fig. 2). The contact ends when the angle β reaches the critical value $\beta_c = \sin^{-1}(V/C)$. Prior to this instant, the contact radius expands faster than the radius of the high pressure zone (Fig. 3.-a). Then, the shock wave detaches from the impacted surface and a relief wave propagates towards the inside of the high pressure zone (Fig. 3.b). Finally, a jet develops as the water is free to escape (Fig. 3.c)

In a semi-infinite target, the impact generates three types of waves: a longitudinal compressive wave (P-wave) propagating radially with a velocity c_p , a shear wave (S-wave) propagating with a velocity c_s , and a Rayleigh surface wave propagating along the surface with a velocity c_R (Fig. 4a). In general c_R is very close to c_s . In brittle materials such as glass fractures on the impacted surface are thought to be induced by the surface wave while failures inside the material below the impacted area are attributed to tensile radial stresses following the passage of the shear wave [11]. For impacts on a plate, the compressive wave reflect from the stress-free back face as a tensile wave (Fig. 4.b) which leads to a significant damage called scabbing or spalling [16].

Rain erosion is a function of n , the number of impacts per unit area. Experiments showed that, after an incubation period, the mass loss per unit area m appears to increase linearly with n between two values of n : n_i marking the end of the incubation period and n_f marking the end of

the region in which m increases linearly with n . Springer and Yang [17] present results on rain erosion of coated and uncoated fiber reinforced composite materials.

Studies of the effects of rain drop impacts on optically transparent materials (glass [11] [16], PMMA [10] [13] [18] [19] [20] [21], transparent natural rubber [18] ...) but also on infrared transparent materials such as zinc sulfide (ZnS) [22] [23][24] [25] or zinc-selenide (ZnSe) [23] are available. A finite element simulation of the impact of a 2 mm diameter rain drop on ZnS target [6] at normal and oblique incidence (45°) and high velocity (305 m/s) gives a detailed distribution of radial tensile stresses during the impact. It also shows a strong interaction between the drop and a highly deformable target made out of polyurethane.

Liquid impacts on composite materials are studied in [19] [26] [27] [28] in connection with rain erosion. Recent experimental studies show that the presence of air between the liquid drop and a solid surface leads to deformation of the bottom surface of the drop and air entrapment during the impact [29] [30]. Contact occurs along a ring enclosing a thin disk of air [31]: a fact that was discussed by Cook [4].

3- BIRD STRIKES

Bird strikes on airplanes and helicopters are regular costly events that can cause loss of aircrafts and the loss of lives. The threat to flying aircraft and the basic understanding of bird-aircraft collisions gained from extensive experimental studies is discussed in subsection 3.1. In recent years, much effort and significant progress has been made to develop material models, numerical approaches, and software to simulate the bird strikes on various components of an airplane. Material models for birds are described in 3.2 and applications to bird strikes on plates, engine components, windshields and other aircraft components are discussed the remainder of this section.

3.1- Introduction to bird strike problems

3.1.1- The threat to flying aircraft

Bird strikes have been a serious threat to aircrafts since the beginning of aviation history. The earliest documented bird strike to a powered aircraft occurred on 7 September 1905 in Dayton, OH during a flight demonstration by Orville Wright. Thorpe [32] shows that from 1912 to 2002,

bird strikes were responsible for at least 55 fatal accidents killing 276 people and destroying 108 civil aircrafts. For transport planes and executive jets bird strikes caused accidents primarily when birds were ingested into the engines (76% of the cases) or when they hit the windshield (7%). For general aviation airplanes, the situation is very much different. Accidents were caused when birds hit the windshield (54%), engines (13%), wings (12%). The difference in the effect of bird strikes between the two different types of aircraft is that: larger aircraft generally are equipped with jet engines and general aviation aircraft are not; (2) stricter requirements are placed on the design of windshield for transport airplanes. These trends tend to explain why a significant portion of the research on bird strikes deals with either the impacts on jet engine blades or on windshields. Statistics indicate that, for helicopters, the areas most likely to be damaged by bird impacts are the windshield (39%), the rotor (11%) and the nose (11%) [33]. Impacts with birds have increased dramatically in recent years due to significantly large increases of large birds (more than 3.6 kg) and to some extent because of increases in air traffic [34].

Bird impacts occur at high speeds and introduce severe loads. Considering an impact velocity of 500 ft/s (152.4 m/s) and a typical density of 56.19 lb/ft³ (900 kg/m³), Eq. 3 predicts that the pressure applied is 1515 psi (10.45 MPa). If, as indicated in [35], this pressure is applied on a 20 in² area, this results in a force in excess of 30,000 lb. At this velocity, the kinetic energy of a 4 lb (1.814 kg) bird is 21.1 kJ. This is a very large amount of energy that has to be absorbed without causing catastrophic failure.

Migratory Canada geese ingested into the engines of US Airways Flight 1549 that crash landed in the Hudson River in New York City on 15 January 2009 [36]. Several non-lethal techniques to reduce bird–aircraft collisions have been proposed. Fernández-Juricic [37] tested the hypothesis that an aircraft color scheme might play a role in bird-strike frequency. It was found that for birds brighter colors would contrast more against the sky than darker colors and, while not conclusive, empirical evidence suggested that brighter aircraft were associated with lower bird-strike rates. Blackwell and Bernhard [38] tested the hypothesis that during daylight, captive birds exposed to an approaching ground-based vehicle exhibiting pulsing 250-W white aircraft landing lights would initiate avoidance behavior more quickly than birds experiencing an oncoming vehicle with nonpulsing (steady) or no lights. Of the five types of birds considered

(captive brown-headed cowbirds, Canada geese, European starlings, herring gulls, and mourning doves), only cowbirds exhibited a response to the landing lights, but not consistently.

3.1.2- Basic understanding from experimental results

Wilbeck [2] presented a detailed account of the literature on soft impact prior to 1978 and a basic understanding of bird impacts on various structures was gained through a several experimental studies. Birds or bird substitutes of different sizes and impact velocities were considered: 60 to 150 g at 350 m/s in [39], 71 to 85 g at 200 m/s in [40], 60 g to 4 kg at 50 to 300 m/s [41], 1.8 and 3.6 kg in [42], 450 g at up to 300 m/s in [43]. Oblique impacts are considered in [41], [43], [44]. It was found that:

- Real birds behave as fluids during impact at velocities larger than 75 m/s [44] or 100 m/s [2].
- Typically, the impact consists of four phases: (1) the shock phase of initial impact in which the pressure reaches the water hammer pressure; (2) the shock pressure decay phase; (3) the steady flow phase during which the pressure along the axis is the stagnation pressure $p = \rho V^2/2$; and (4) the termination phase in which the fluid moves parallel to the surface of the target and the pressure is zero.

In experiments, substitutes are usually preferred to real birds. Gelatin with 10% porosity was found to be an adequate substitute for bird material [41] [44]. The porosity is achieved by the uniform distribution of phenolic microballoons [44]. Other materials have been used to build bird simulants. For example, synthetic ballistic gel already employed as muscle tissue surrogate [45]. Lavoie et al [46] provide a detailed list of ingredient and a procedure for making substitute birds with water, ballistic gelatin, and smaller amounts of other ingredients. In Shupikov et al [47] a bird analog is made of silicon models the bird's muscular tissue and plastic ball fillers and in [48] four parts water to one part of bovine hide gelatine was used to substitute bird..

End on straight cylinders are the best shape to simulate ends on impacts of real birds [41] and a length to diameter ratio of two is recommended [44]. Challita and West [43] found that, when birds strike side-on, no region of steady flow pressure can be detected. The Hugoniot pressure

decay process is still continuing as the trailing edge of the bird reaches the target surface. The density of the bird and its dimensions are usually determined from expressions of statistical analyses of various bird populations. For example, in [49], the bird's density and its torso diameter are given by $\rho = 959 - 63 \log_{10} m$ and $\log_{10} d = -1.095 + 0.335 \log_{10} m$ in terms of its mass m . This information about the geometry, the density and the diameter of the bird is used to design bird substitutes for experiments or to for numerical modeling.

This early work provides a basic understanding of bird impacts normal to a rigid surface. Oblique impacts are more complicated and for impacts on deformable structures the interaction between the deformation of the projectile and that of the target should be accounted for. Section 3.2 describes models for the constitutive behavior of birds during impact, impacts on rigid and deformable plates are discussed in Section 3.2. Overviews of current work on bird impacts on turbofan engine blades, windshields, and leading edges of wings or tail sections are given in Sections 3.3-3.5.

3.2- Constitutive equations

One of the challenges in the simulation of a soft projectile such as a bird is to select a model to represent the behavior of that projectile: the bird in this case. Experimental studies have shown that the bird behaves like a fluid during impact and that the impact has both a shock phase and a flow phase. This suggests that this fluid model should be a compressible fluid.

Using a general approach that can be used for both solids and fluids, the Cauchy stress tensor σ_{ij} is split between a hydrostatic part and a deviatoric part

$$\sigma_{ij} = -p \delta_{ij} + s_{ij} \quad (4)$$

where $p = -\sigma_{kk}/3 = -(\sigma_{11} + \sigma_{22} + \sigma_{33})/3$ is the hydrostatic pressure, and s_{ij} is the deviatoric stress tensor. The pressure is related to the deformation by an equation of state and s_{ij} can be described in various ways depending on the material to be modeled. For an inviscid fluid such as water $s_{ij} = 0$ and many simulations assume that the bird is also inviscid.

3.2.1- Polynomial equation of state

Equations of state describe the pressure-volume behavior. The polynomial equation of state

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 \quad (5)$$

where $\mu = (\rho/\rho_0) - 1$ is the relative change in density and C_0 - C_3 are constants determined from experiments. Usually, $C_0=0$ and often only the second term is retained (e.g. [50] [51] [52]). C_1 is the bulk modulus of the material and is also called K . Additional terms are often added to Eq. 5 (e.g. [52],[53], [54])

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + (C_4 + C_5\mu + C_6\mu^2)E \quad (6)$$

Usually these added terms are not used in modeling birds. However Eq. 6 can be used to model the surrounding air by taking $C_0=C_1=C_2=C_3=C_6=0$ and $C_4 = C_5 = \gamma - 1$ where $\gamma = 1.4$ is the ratio of the specific heats [42] [54].

It is often said that the bird is modeled as water mainly because the same equation of state is used and the densities are close. However, it was found that birds can be thought of as a mixture of water and a small percentage of air. Selezneva [55] considered a material model with polynomial EOS. The constants C_1 - C_3 for 0, 10 and 15 % porosity are given in Table 1 and, for these values, the pressure plotted versus the relative density up to 1200 MPa in Fig. 5 shows that a small change in porosity can have a significant effect.

3.2.2- Tait's equation of state

Tait's equation of state proposed in 1888 [56] is widely used to model the behavior of water [57][58] [59] and is usually written as

$$p = B \left[\left(\frac{\rho}{\rho_0} \right)^n - 1 \right] \quad (7)$$

when $\rho > \rho_0$. The constant B is related to the wave velocity by $B = \rho_0 c^2 / n$. For water $n=7.15$, $\rho_0 = 1000 \text{ kg/m}^3$, $B=2.984 \times 10^6 \text{ Pa}$. Tait's equation of state is widely used to model the behavior of birds during impact. Eq. 7 is also called Murnaghan's EOS in many publications in reference to a 1944 article by that author [60].

3.2.3- Equation of state derived from the Rankine-Hugoniot relations

Considering a shock wave moving with a velocity u_s in a fluid initially at rest, the particle velocity $u_1=0$ ahead of the shock wave and $u_2=u_p$ behind the shock wave (Fig. 6-a). In a coordinate system moving with the wave front, the situation is as shown in Fig. 6-b. The equations for conservation of mass and conservation of momentum are

$$\rho_0 u_s = \rho (u_s - u_p) \quad p_1 + \rho_0 u_s^2 = p_2 + \rho (u_s - u_p)^2 \quad (8.a,b)$$

as in [2]. Combining these two equations gives the Hugoniot pressure

$$p_H = p_2 - p_1 = \rho_0 u_s u_p \quad (9)$$

which is the same as Eq. 1 accounting for changes in the shock wave velocity. Eq. 8.a can be rewritten as

$$\frac{u_p}{u_s} = 1 - \frac{\rho_0}{\rho} = \eta \quad (10)$$

where the variables η and μ are defined in terms of the densities before and after the shock by $\eta = 1 - \frac{\rho_0}{\rho} = \mu/(1 + \mu)$. For a linear (Hugoniot) relationship between the shock velocity and the particle velocity

$$u_s = c_0 + k u_p \quad (11)$$

Eq. 10 gives $u_p = u_s \eta$, substituting into Eq. 11 gives $u_s(1 - k\eta) = c_0$ and substituting into Eq. 8.b gives

$$p = \rho_0 c_0^2 \eta / (1 - k\eta)^2 \quad (12)$$

This equation is the simplest of several Mie-Gruneisen equations of state and it is often used to model the behavior of birds. A third order Taylor series expansion of Eq. 11 in terms of μ gives the following expressions for the constants in the polynomial equation of state (Eq. 5)

$$C_1 = \rho_0 c_0^2, \quad C_2 = (2k - 1)C_1 \quad C_3 = (k - 1)(3k - 1)C_1$$

These formulas are used to estimate the constants C_1 - C_3 in the polynomial equation of state (Eq. 4) when only the density and c_0 are known. It is often assumed that the slope of the linear Hugoniot relationship $k=2$ for water.

Using higher order polynomial functions on the right hand side of the Hugoniot relationship (Eq. 12) leads to more complex expressions for the Hugoniot pressure. For example, with the cubic polynomial function $u_s = C + S_1 u_p + S_2 u_p^2 + S_3 u_p^3$ the Hugoniot pressure becomes

$$p_H = \rho_o C^2 \eta / \{1 - S_1 \eta - S_2 \eta^2 u_s - S_3 \eta^3 u_s^2\}^2$$

The three equations of state discussed here (Eqs. 5, 7, 12) are used almost exclusively to model the material behavior of birds during impact. An alternative is to specify the pressure volume relationship in tabular form as a few authors have elected to do.

3.2.4- Viscous fluid models

For Newtonian fluids, the deviatoric stresses s_{ij} are directly proportional to the deviatoric strain rates d'_{ij}

$$s_{ij} = 2 \mu d'_{ij} \quad (13)$$

where μ is the viscosity of the fluid, $d'_{ij} = d_{ij} - \frac{1}{3} d_{kk} \delta_{ij}$, and the rate of deformation tensor is defined in terms of the velocity components v_i as $d_{ij} = (\partial v_i / \partial x_j + \partial v_j / \partial x_i) / 2$. In conjunction with an equation of state such as Eq. 4, for example, this Eq. 13 is used for describing the behavior of birds impacts in several studies [54] [61] [62] [63] [64] [65] [66]. In some cases a small viscosity term like this is introduced to prevent oscillations in numerical simulations.

3.2.5- Elastoplastic models

For some soft impacts, large deformations occur and a consistent formulation is required. To handle large rotations, and Brockman and Held [64] used the Jaumann stress rate tensor $\dot{s}_{ij}^J = \dot{s}_{ij} - s_{ik} \omega_{kj} + s_{jk} \omega_{ki}$ where $\dot{s}_{ij}$ is the material time derivative of the Cauchy stress σ_{ij} . The velocity gradient is split into a symmetric tensor $\mathbf{D}$ called the rate of deformation tensor and an anti-symmetric tensor $\mathbf{W}$ called the spin tensor. In terms of the components of the velocity

vector $\mathbf{v}$, the components of $\mathbf{D}$ and $\mathbf{W}$ are $d_{ij} = (v_{i,j} + v_{j,i})/2$ and $w_{ij} = (v_{i,j} - v_{j,i})/2$. The material remains elastic as long as the equivalent von Mises stress $\sigma_e = \sqrt{\frac{3}{2} s_{ij} s_{ij}}$ is less than the current yield strength

$$\sigma_y = \sigma_y^0 \left[1 + (\dot{\epsilon}/D)^{1/p} \right] + H \epsilon_p \quad (14)$$

which depends on the static yield strength in tension σ_y^0 , the effective strain rate $\dot{\epsilon} = \sqrt{\frac{2}{3} d'_{ij} d'_{ij}}$, the hardening modulus $H = EE'/(E - E')$, ϵ_p is the effective plastic strain, and the constants D and p . E is the elastic modulus and E' is the slope of the stress-strain curve past the yield point (Fig. 7). When strain rate effects are neglected [67], $\dot{\epsilon}$ is assumed to be zero in Eq. 14.

For the elastic part of the deformation, the time derivative of the deviatoric Cauchy stress is related to the deviatoric part of the rate of deformation by $s'_{ij} = 2G d'_{ij}$. Plastic flow is expected to occur when $\sigma_e > \sigma_y$ and in that case the stresses are returned to the yield surface using the radial return correction $s_{ij} \leftarrow s_{ij} \sigma_y / \sigma_e$. In [64], the updated plastic strain is evaluated using $\epsilon_p \leftarrow \epsilon_p + (\sigma_e - \sigma_y) / (3G + H)$ and the pressure p is evaluated using a polynomial equation of state. The effective plastic strain can also be evaluated using $\epsilon_p = \int_0^t \left(\frac{2}{3} \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p \right)^{1/2} dt'$ where the plastic strain rate $\dot{\epsilon}_{ij}^p$ is the difference between the total strain rate $\dot{\epsilon}_{ij}$ and the elastic strain rate $\dot{\epsilon}_{ij}^e$ as in [67]. In [67], the pressure-volume relationship is given in terms of the bulk modulus K and the ratio of the current volume v to the initial volume v_0 . That is, $p = -K \ln(v/v_0)$.

3.3- Numerical models for soft projectiles

An important consideration for numerical simulations is the choice of the way the deformation is described. In a Lagrangian description, nodes in a computational mesh correspond to specific material particles. The mesh deforms with the material and that has some advantages in tracking free surfaces and interfaces between materials but there are difficulties with large deformations that result in severe mesh distortion. The Eulerian description uses a fixed grid and the material moves relative to it. It requires a grid large enough to cover the region where the material is initially but also where it might be in the future. There are difficulties in tracking the location of

free surfaces and interfaces. The Arbitrary Lagrangian Eulerian (ALE) description the grid may be moved and/or deformed in a prescribed way. This approach allows to handle cases with more distortion than the Lagrangian description and with more resolution than with an Eulerian approach since the region to be meshed will be much smaller [68]. In addition to these mesh-based methods, meshless methods have been developed including the SPH method [69]. The main difference is that the formulation is focused on the motion of particles instead of the deformation of meshes.

Bird impact problems are solved using a Lagrangian approach in [61] [62] [65] [70] [71] [72] [73] [74], the ALE approach in [52] [53] [54] [75] [76] [77] [78] [79] [80] [81] [82] [83], and a variant of the ALE approach called coupled Eulerian Lagrangian (CEL) is used in [72] [84] [85] [86] [87] [88] [89]. The Eulerian approach is rarely used in bird impact problems [90] because a large grid is needed which leads to high computational costs. A discussion of these three methods and examples are given in [91] [92] [93] [94].

The SPH method is used in bird impact simulations [33] [45] [49] [54] [71] [79] [82] [95] [96] [97] [98] [99] [100] [101] [102] [103] [104] [105] [106] [107] [108] [109] [110] [111] [112] [113]. To obtain an accurate description of the pressure applied on the surface of the target the SPH model requires a large number of particles: 1856 particles in [97], 13,871 in [79], 35,000 in [103] [114], or 65,000 in [71]. Other meshless methods are available but besides the SPH method, it appears that only the discrete element method is used for bird impact problems in [115]

3.4- Bird impact on plates

Often tests are conducted using rigid plates as a target in order to better understand the mechanics of soft impacts and generate data to validate numerical simulations. Many numerical analyses use the impact against a rigid surface for validation purposes (e.g. [46] [65] [89] [103] [116] [117][118]). Impacts on deformable plates are also considered to study the interaction between the deformation of the soft body and that of the structure.

3.4.1- Bird impact on rigid plates

The maximum pressure is generated during the first phase of the impact and can be determined using the expression for the Hugoniot pressure (Eq. 9). During the flow phase the pressure on the axis of the projectile during a normal impact is the stagnation pressure $p = \rho V^2/2$,

It is known that “If the natural period of the impacted structure is long compared with the duration of the impact, the impact may be considered to be an impulsive event” so that the impulse is the only parameter required to analyze the response [41]. This is the case for many bird-windshield collisions. Considering the conservation of momentum during normal impact, the impulse applied on a rigid surface $I = \int F dt = mV$. Ref. [41] assumes that the bird behaves like a fluid, the impact begins when the leading edge of the bird first touches the target and ends when the trailing edge reaches the target, and that the bird does not decelerate during impact. Then the impact duration is equal to the length of the projectile divided by the impact velocity ($\Delta T = L/V$). The average force applied on the target given by $F = I / \Delta T$ is $F = \rho AV^2$.

The pressure distribution due to the normal impact of a water jet can be estimated by the expression given by Banks and Chandraskhara

$$p = \frac{1}{2} \rho V^2 \exp\{-\zeta_1 (r/a)^2\} \quad (15)$$

or by that of Leach and Walker

$$p = \frac{1}{2} \rho V^2 \left\{ 1 - 3 \left(\frac{r}{\zeta_2 a} \right)^2 + 2 \left(\frac{r}{\zeta_2 a} \right)^3 \right\} \quad (16)$$

where $\zeta_1 = 0.5$ and $\zeta_2 = 2.58$, r is the radius and a is the radius of the cylinder [2]. Fig. 7 shows how the pressure spreads beyond the initial radius of the cylinder, the difference between the predictions of Eq. (15,16) is small. This distribution is similar to that obtained in the numerical analysis of Airolidi and Cacchione [119].

For oblique impacts on a rigid surface (Fig. 9.a), the back of the cylinder is assumed to keep moving with the same velocity V through the effective length $L_{\text{eff}} = L + D / \tan \theta$. The contact duration is $\Delta t = L_{\text{eff}} / V$ and experiments [41] verified that the normal component of the momentum transferred to the plate $\int F dt = mV \sin \theta$. The impulse momentum relation can be written as $mV \sin \theta = F_{\text{avg}} \Delta t$. The average force is estimated by $F_{\text{avg}} = mV^2 \sin \theta / L_{\text{eff}}$. Barber

et al [41] observed that the impact force varies in a triangular fashion (Fig. 9.b) with a maximum force $F_{\max}$ occurring at $\Delta t/5$ so $F_{\max} = 2 F_{\text{avg}}$. These are general trends followed by experimental results.

3.4.2- Bird impact on deformable plates

Uncoupled loads models assume only small to moderate deformations of the structure prior to catastrophic failure. This way the loads applied to the structure are not affected by its deflections and the analysis is greatly simplified. Following [41], such an analysis is based on four assumptions: (1) the target is completely rigid; (2) The bird behaves as a fluid during impact; (3) the component of bird momentum normal to the target is completely transferred to the target; (4) there is no deceleration of the bird during impact. Uncoupled loads models can successfully predict the response in the case of normal impacts but, for oblique impacts, changes in load location during the impact are significant and are not accounted for in these models [120]. Current state of the art simulations consider bird impacts as a coupled problem and, with the availability of software, the main issue in the development of an adequate model is the proper description of the material behaviors and the discretization of the bird.

Rosenblatt et al [121] simulated the short term response to the impact of a 3 oz (85 g) sphere of water (radius 1.07 in or 2.73 cm) with a velocity of 675 ft/s (206 m/s) impact either a rigid surface or a thin plate at normal or oblique incidence. Results for the normal impact on a 0.136 in (3.45 mm) thick stainless steel plate show a significant deflection of the plate that significantly reduces the applied force. The maximum deflection 152 μs after the start of the impact is larger than 10 mm and permanent deformations remain after the interaction with the water ceases.

Experiments showed that oblique impacts on deformable metallic plates produce two types of deformations depending on the type of response induced: (1) locally rigid target usually deform quasi-statically, the entire target participates in the deformation; (2) locally deforming targets refers to cases in which the bending deformation is limited to a relatively small region surrounding the impact and has not reached the boundaries [41]. The first case is called a boundary controlled impact which occurs when the projectile is much heavier than the target while the second case is a wave controlled impact and it occurs when the projectile is significantly lighter than the target [122].

Bird impacts often induce permanent deformation for materials that deform plastically and the yield strength is strain rate depend. A few expressions used to describe this strain rate dependence are used extensively and they are discussed in the following. Airolidi and Cacchione [119] study bird impact on circular polycarbonate plates. This material is assumed to be elastoplastic and the yield stress for that material is assumed to vary with strain rate as

$$\sigma_y / \sigma_{y0} = 1 + (\dot{\epsilon}/D)^{1/p} \quad (17)$$

where σ_y is the dynamic yield strength, σ_{y0} is the static yield strength, and p and D are constants. Strain rate effects on the yield strength of metals including strain rate effects is also described by

$$\sigma_y = [a + b(\epsilon_p)^n] [1 + (\dot{\epsilon}/D)^{1/p}] \quad (18)$$

where ϵ_p is the effective plastic stress, $\dot{\epsilon}$ is the strain rate, and a,b, n, D, and p are material constants. The second bracket is the Cowper –Symmonds factor accounting for strain rate. Eq. 18 is used for 7075 aluminum alloy [123], steel and aluminum alloy [95], 2024-T3 aluminum facesheets in a sandwich plate [124]

The behavior of metals subjected to plastic deformations with high strain rate and temperature effects is often described by the empirical Johnson-Cook relation

$$\sigma_y = [a + b(\epsilon_p)^n] [1 + c \ln(\dot{\epsilon}/\dot{\epsilon}_0)] (1 - T^{*m})$$

where σ is the stress, ϵ_p is the effective plastic strain, $\dot{\epsilon}$ is the strain rate, $\dot{\epsilon}_0$ is a reference strain rate ($\dot{\epsilon}_0 = 1 \text{ s}^{-1}$ in [101]), $T^{*m} = (T - T_r)/(T_{rm} - T_r)$ where T_r is the room temperature and T_m is the melting point of the material. The First bracket gives the shape of the curve, the second bracket accounts for strain rate effects, and the last term accounts for the effect of temperature. Constants a, b, c, m, n are determined for each material. The Johnson-Cook model was used for aluminum alloys LY-12 and 2024-T3 in [101]

Simulations of bird strikes on composite plates can be found in [76] [87] [89] [106][107]. With cohesive elements, Heimbs and Bergmann were able to predict the size of delaminations and examined the effect of preloads on damage induced by bird strikes [89].

3.5- Bird impacts on engine blades

It is often said that bird impacts are the major cause of blade damage for aircraft engines. Many types of birds can cause significant engine damage and their size varies greatly: doves with a mass of 120 g [125], american kestrels with a mass of 120 g [125], snowy owls [125], Eastern Canada geese with a mass of 7 kg [126], starlings typically weighting 3 oz (85 g) and flocking in large numbers [126]. Ref [126] mentions several incidents between a commercial aircraft and a flock of starlings or gulls that left hundreds of dead birds on the runway. On 26 July 2005, at least one turkey vulture was struck by the external fuel tank on the Space Shuttle Discovery during launch at the Kennedy Space Center in Florida [125]. The first major accident with fatalities in which bird-ingestion was a contributing factor occurred in October, 1960 when a Lockheed Electra ingested a large number of starlings and sustained partial power loss on three of its four engines. The aircraft crashed into Boston Harbor. This event led to the introduction of certification requirements for engines demonstrating flocking-bird ingestion capability [127].

Shown here as an idealized right cylinder, the bird moves towards the fan with a velocity $\bar{V}_a$ which is the velocity of the airplane while the fan rotates at high speeds (Fig. 10). The velocity of the bird relative to the rotating blades is $\bar{V}_{relative} = \bar{V}_a - \bar{V}_t$ where $\bar{V}_t$ is the linear velocity of the blade. The figure shows how two consecutive blades slice the bird into three pieces along planes parallel to the direction of the relative velocity. The idealized slice in the middle impacts the second blade shown in Fig. 10 and slides along its surface all the way to the trailing edge. Similarly, the top piece will impact the first blade and the bottom piece will hit a third blade not shown in the figure. Several situations arise depending on the size of the bird, the spacing of the blades and the velocities but several blades can be impacted by a single bird (This is clearly an oblique impact on a curved surface that should be analyzed as a three-dimensional problem [3]. The impact is most severe on the leading edge (or slicing edge) of the blade which may have to be reinforced by metal in the case of composite fan blades.

Analytical approaches [128] can simulate bird impacts on a single rotating turbofan blade but in general the problem is more complex and requires a numerical approach to model a full fan and the casing around it. As indicated in Fig. 10, soft projectiles are split into several impactors that come in contact with three or four blades [53] [112] [129]. Because of the damage induced by the impact, blade tips impact and slide against the casing which can cause failure of the case

[129]. A progressive damage simulation of the entire fan stage predicted the detachment of a large number of blades resulting from a bird strike, in accordance with experimental results [114].

Birds are generally modeled as inviscid fluids with an equation of state relating the pressure to the relative change in volume or density. Usually, the behavior of the bird is described either by polynomial EOS [53] [62] [65][103] or the Tait EOS [99] [108]. The simple EOS $p = C_1 \mu$ is used to model the behavior of the bird in several simulations of impact against fan blades [53] [82] [114] [129]

With rotating blades, centrifugal effects lead to significant deformation and initial stresses prior to impact. Generally a preliminary nonlinear analysis is performed to initialize stresses and strains in the blade prior to the impact simulation [53] [82] [83] [112][114] [129].

Most investigations on bird impacts with turbofan engines focus on impacts against rotating blades but two other engine components have also been studied. A spinner is an aerodynamic cone, a fitted over a propeller hub or at the centre of a turbofan engine [112]. Results for bird impacts at 100 m/s at a 60° angle on a spinner rotating at 7800 rpm show that significant damage can be induced and important differences were observed depending on whether the spinner was rotating or stationary. Designs were developed for spinners to deflect the bird upon impact instead of having it trapped into the pocket formed during the deformation of spinner [111]. Anghileri et al [102] [109][110] examine bird impact damage onto turbofan-engine intakes and show examples of experimental and numerical results. The displacements from that pre-analysis are applied as initial conditions for the impact analysis

3.6- Bird impacts on windshields

Bird strikes on transparencies are a major threat for both commercial and military aircrafts. Several publications present photographic evidence of severe damage to aircraft canopies by bird impact including complete perforation [130], [131]. The canopies of some military aircrafts are made out of polycarbonate [64] [73] [130] [132]. Often it consists of a single layer but it can also be made of several layers. For example, McCarty [132] studies canopies with 4 layers of polycarbonate joined by three silicone interlayers. The stress-strain behavior of that material is

nonlinear and tests at 25, 40, 80, 100°C indicate that this behavior is strongly affected by temperature [73]. Canopies are also made of polymethyl methacrylate (PMMA). Historically, PMMA has often been called acrylic glass and many studies of birdstrikes on transparencies made with that material are available [67] [74] [80][132] [133] [134].

In the most basic construction, laminated glass for aircraft windshields consists of two glass panels joined by a polyvinyl-butylal (PVB) interlayer (e.g. [104]). More complex constructions can be used. For example, Refs. [104] [135] consider five layer laminates with three glass layers and two polyvinyl-butylal (PVB) interlayers.

3.6.1- Constitutive models for transparencies

In the basic windshield configuration with two glass panels joined by a polyvinyl-butylal (PVB) interlayer (e.g. [104], [136]), the glass layers are assumed to be elastic and failure is determined using the von Mises yield criterion while the yield strength is assumed to vary with strain rate according to the Cowper-Symmonds law (Eq. 17). In [33] [104] [136] the instantaneous shear modulus is taken to vary as

$$G(t) = G_{\infty} + (G_0 - G_{\infty}) e^{-\beta t} \quad (19)$$

From [136], typical values of the material properties taken are $G_0 = 330$ MPa, $G_{\infty} = 0.69$ MPa, and $\beta = 12.6 \text{ s}^{-1}$. With this value of β , the shear modulus does not vary much during a typical impact which is why, in some analyses, PVB is assumed to be elastic. The deviatoric stresses s_{ij} and the deviatoric strains e_{ij} are related by

$$s_{ij} = 2 \int_0^t G(t - \tau) \dot{e}_{ij}(\tau) d\tau \quad (20)$$

and the volumetric behavior of PVB is assumed to be linear ($p = -K \epsilon_{kk}$) in [136] or to follow an EOS given in tabular form [104].

Tests by Hooper et al [137] showed that the behavior of PVB can be represented by the generalized Maxwell model (Fig. 11) in which the shear relaxation modulus is given as a function of time t by the Prony series

$$G(t) = G_{\infty} + \sum_{i=1}^n G_i e^{-t/\tau_i} \quad (21)$$

G_∞ is the long term modulus, G_i and τ_i are the shear moduli and relaxation times for each of the Maxwell elements in parallel. At time $t=0$, the instantaneous shear modulus is

$$G_o = G_\infty + \sum_{i=1}^n G_i \quad (22)$$

Numerical values of G_i/G_o and τ_i for $i=1-6$ in [137] show that G_i is small and the relaxation times are much larger than the duration of typical impacts when i is larger than two. The same model (Eq. 21) is used in [138] [139]. The same expression can be used for the bulk relaxation modulus $K(t)$ with a long term modulus K_∞ , K_i , and a different set of relaxation times τ_i [139]. Experimental results for PVB [137] [140] show that the behavior is linear for strains below 20%. The constitutive equations for this linear generalized Maxwell model are

$$\sigma_{ij}(t) = \int_0^t 2G(t-\tau) \frac{de_{ij}}{d\tau} d\tau + \delta_{ij} \int_0^t K(t-\tau) \frac{d\varepsilon_{kk}}{d\tau} d\tau \quad (23)$$

where e_{ij} is the deviatoric strain and ε_{kk} is the volumetric strain [141]. Eq. 19 is obtained when $n=1$ in Eq. 21 and Eq. 20 is the first term in Eq. 23 dealing with deviatoric stresses. Nonlinear constitutive models for describing the large deformations can be developed considering the interlayer material to be hyperelastic following some of the well-known models from rubber elasticity (e.g. [140][142]). This approach gives the nonlinear first term in Eq. 21. To account for rate effects, the other terms in the Prony series are added [142].

Laminated glass for windshield applications also uses transparent polymers such as polymethyl methacrylate (PMMA), polycarbonate (PC), or polyurethane (PU). Nonlinear viscoelastic material models can be used for these materials (e.g. [134])

3.6.2- Constitutive models for birds

In the analysis of bird strikes on windshields, the bird is usually modeled as an inviscid fluid ($s_{ij}=0$ in Eq. 4). Many studies use a polynomial equation of state while others use the Mie-Gruneisen type EOS. In [49] [104] [135], the pressure is given in tabular form as a function of the volumetric strain ε_v which is the natural log of the relative volume. A one term Taylor series expansion gives $\varepsilon_v = -\mu$.

Another approach is to model the bird as a viscoelastic fluid. The volumetric behavior is represented by a linear relationship (Eq. 5 where C_1 is the only non-zero term) with a bulk modulus $C_1=K=60$ MPa and the deviatoric deformations by a viscoelastic shear model (Eq. 19) with $G_0=6.2$ MPa, $G_\infty = 0.3$ MPa, and $\beta=0.5$ s⁻¹ as in [143].

3.6.3- Impact response

Refs. [104] [135] consider five layer laminates with three glass layers and two polyvinyl-butylal (PVB) interlayers. All layers are 4 mm thick for a total thickness of 20 mm. The 1.8 kg (4 lb) bird in that study has a density 950 kg/m³ and is idealized as a right circular cylinder with a recommended length to diameter ratio $L/D=2$ and a 106 mm diameter. This study pointed out the need to accurately model the supporting frame and its interaction with laminated glass.

Some investigations consider more than the bird and the canopy during impact. For example, in [35] a two degree of freedom model (Fig. 12) representing the first mode of transparency and the first mode of the supporting frame that is attached to the airframe is subjected to sine pulse duration of 0.5 ms. That pulse represents a typical load induced by a bird strike. The idea behind this simplified model is to examine the role that the frame can play in absorbing energy which can hardly be accomplished by the canopy alone.

Belytschko [144] studied bird strikes on canopies of military aircrafts. A simulation of the complete scenario of a bird-strike event is presented including the bird-canopy impact, the canopy-pilot impact, and the resulting response of the pilot. It includes a “simplified model” of the upper body including the spine and the head that consists of 50 masses connected by springs with axial, bending and torsional stiffnesses.

3.7- Bird impact on aircraft leading edges

Bird strikes on aircrafts occur on wings, radome shields [52] and vertical and horizontal stabilizers in addition to impacts on engine blades and windshields. Photographs show significant damage induced by bird impacts on leading edges of wings (Doubrava [131] , [145]), on Pitot tubes (Doubrava [131]) and on the fuselage of airplanes and helicopters [36]. Regulations (FAR 25.571) require that airplanes must be capable of successfully completing a flight after an impact with a 1.81 kg (4 lb) bird at the expected cruising velocity.

Simulations of bird strikes on the leading edges of aircraft wings are presented in several publications in which the leading edge is made out of: (1) laminated composite materials [87] [118] [146] [147]; (2) fiber metal laminates (FML) [97] [148] [149] [150], (3) a tensor skin laminate [151] [152]; (4) a sandwich construction with a FML outside skin, an aluminum honeycomb core, and an aluminum inside skin [49] [105] or other sandwich structures [52].

Composite materials are used extensively in both commercial and military aircrafts. In these materials, impacts are known to induce damage that will significantly affect the structure's load carrying capability. The challenge in the simulation is to predict the damage induced by the bird strike and in particular to predict the size and location of the debonding between plies. In [118] a 1.8 kg bird is modeled as an inviscid fluid and only the C_1 term is retained in the polynomial equation of state so that the pressure is equal to the bulk modulus times μ , the relative change in volume. The motion of the bird substitute is described using the SPH method. The target is a wing leading edge profile made of glass fabric/epoxy composite with a $[0/45]_{2S}$ layup modeled with eight stacked shell elements to predict the size delaminations. Bayandor et al [147] addressed the same problem modeling the bird using the SPH method and the Tait or Murnaghan equation of state. Delamination is modeled using cohesive elements with a bilinear cohesive law between plies, the laminated being modeled using a stacked shell approach [118] [146] [147]. The onset of delamination is predicted using the mixed mode fracture criterion $(G_I/G_{Ic})^n + (G_{II}/G_{IIc})^n = 1$. G_I and G_{II} are mode I and mode II the strain energy release rates and the subscript c indicates their critical values. Often, the exponent n is taken to be one by some authors or two by others. Once delamination is initiated, its growth is controlled by the cohesive law. In these three references delamination is assumed to be controlled by the transverse normal stress at the interface.

Fibre metal laminates (FML) are of laminated composite materials consisting of alternating plies and fiber reinforced epoxy. The concept of FMLs originated at Fokker in the Netherlands in the 1950s [148]. Glass-based FMLs under the name commercial name GLARE can be found on the Boeing 777, the Bombardier Learjet 125 and the upper fuselage of the A380 [148]. The constitutive model for the composite plies includes two damage parameters to account for loss of rigidity under transverse and shear loading and also accounts for strain rate effects [148] [149]. The laminate is modeled using a stacked shell finite element approach with tied contact connections to model delaminations. The behavior of the bird is modeled using Tait's EOS and the SPH method is used for its discretization [97] [150].

The tensor skin concept was initially to increase the survivability of helicopter crashes on water surfaces [153] and is now considered for use in aircraft wing leading edges to resist bird strikes [151] [152]. The typical configuration has three types of plies: the 'carrying', 'tensor' and 'cover' plies (Fig. 13). During normal operations, load carrying plies transfer aerodynamic loads to the ribs and provide the required stiffness of the wing leading edge. Tensor plies, consist of folded loops that unfold when high loads are applied as in the case of bird impacts. Tensor plies are usually composite plies with high molecular weight polyethylene (Dyneema) fabric reinforcement. The cover ply just provides a smooth external surface for aerodynamic purposes. Under large impacts the tensor skin unfolds and acts as a membrane to stop the projectile.

Numerical analyses of bird impacts on the trailing edge of aircraft wings are presented in several references [72] [84] [85] [86] [113].

Bird impacts on the leading edges of vertical stabilizers were analyzed in [50] [154]. The bird is modeled using solid brick elements with a linear equation of state $p = K\mu$ where the bulk modulus $K=2200$ MPa. The density, mass, and initial velocity of the bird are 950 kg/m^3 , 3.62 kg (8 lb), and 129 m/s (250 knots) respectively. The impact energy in this case is 30 kJ. Bird impacts on horizontal tail plane leading edges were simulated in [155]

4- HAIL IMPACT

The risk of severe damage caused by the impact of hailstones on airplanes has long been recognized. Souter and Emerson [156] present an extensive review of 131 case histories from

1938 to 1952. Their findings are: (1) The leading edges of the wing and tail are the areas more susceptible to damage; (2) damage to the fuselage is usually confined to the nose and cockpit areas and windshields or canopies can be broken or cracked; (3) engine cowlings can be damaged like leading edges.

Hail impacts can occur either when the airplane is on the ground or in flight. For ground hail impacts the size of the hailstones can be estimated from historical records and their terminal velocities can also be predicted. Studies of the frequency distribution of maximum hailstone diameters started in 1899 and the data indicates that the probability of hailstones having a diameter larger than 25.4 mm (1 in) is just 0.07 [157]. In vertical fall, hailstones reach a terminal velocity when its weight is balanced by the air resistance. That is when $mg = \frac{1}{2} \rho_{\text{air}} C V^2 A$ where g is the gravitational acceleration, ρ_{air} is the density of the air, C is the drag coefficient. The mass of a spherical ice projectile $m = \pi \rho_i d^3 / 6$ and its projected area is $A = \pi d^2 / 4$. The terminal velocity can be written as

$$V = \left(\frac{4 \rho_i g}{3 \rho_{\text{air}} C} d \right)^{1/2} = K \sqrt{d} \quad (24)$$

Typically, the density of hailstones is 900 kg/m^3 [157], the density of air of 1.24 kg/m^3 , $C=0.45$ and $g=9.81 \text{ m/s}^2$. Eq. 24 indicates that the kinetic energy of hailstones increases with d^4 which is why many authors consider that size is the major factor affecting damage induced by hailstone impacts on crops and structures. Typically, hailstones have a mass of 63 grams and a velocity of 37.3 m/s which gives a kinetic energy of 43 J (Alves [145]). Inflight hail impacts occur when airplanes fly through a storm and in that case the impact velocity is essentially the velocity of the aircraft. For example, in Ref. [158] simulated the impact on hailstones as spherical masses of 99.6 g and a diameter of 60 mm, yield stress of 17 MPa and an ultimate tensile strain of 0.05. For ground hail impacts, impact energies were either 90 or 136 J. With the same hailstones in-flight impacts at 225 m/s (812 km/h) resulted in kinetic energy 2543 J.

The following discusses models for the constitutive behavior of ice and the damage induced by the impact of hailstones on both metallic and composite structures.

4.1- Material model for ice impacts

Selecting a model to describe the behavior of the projectile or the target during impact should be based on the physics of the problem and what is to be predicted. The compressive stress-strain behavior of ice is known to exhibit two types of behaviors: (1) ductile behavior for low strain rates and (2) brittle behavior at high strain rates [159]. During the ductile phase, strain hardening occurs and the maximum strength increases with strain rates, collapse occurs when the strain reaches 10% or more. The strain-rate hardening exponent $m=0.3$ ($\sigma \propto \dot{\epsilon}^m$). When the ice becomes brittle it collapses when shortened by about 0.1% and the strength decreases with strain rate. The transition between the two regimes corresponds to the highest compressive strain. The same behavior is observed in tension [160]. With low velocity impacts, hailstones remain in one piece even if many cracks develop [161]. In that case the projectile can be thought to behave as a solid and “the microstructure (equiaxed or monocrystalline) does not seem to have a significant influence on the residual deformation of the plate. The ice clearly behaves as a solid during the impact because the plastic deformations of the plate under the impact spot produced by the impact are localized” [162]. On the other hand, “at a velocity significantly greater than that required to produce fracture in the ice, the mechanical properties become relatively insignificant, and the impact forces are governed by the shape and mass of the projectile” [163], suggesting that the ice behaves as a fluid.

4.1.1- Ice as a brittle solid

Schulson gives an extensive review dealing with the brittle failure of ice [164]. In several publications, ice is modeled as a brittle solid in which damage occurs when tensile strains develop [165]. This model is adapted from a previous model developed for concrete. The equivalent tensile strain is defined in terms of the principal strains e_i as

$$\bar{\epsilon} = \sqrt{\sum_i (H_i e_i)^2}$$

where Heaviside function H_i is zero if $e_i < 0$ and $H_i = 1$ if $e_i > 0$. Damage develops when $\bar{\epsilon}$ reaches a threshold value K that depends on a single scalar damage variable D . That can be written in the form of a failure criterion $f = \bar{\epsilon} - K(D) = 0$. A procedure for updating D is described that also include a damage delay effect [165]. The elastic properties of damaged elements in the model are reduced according to $\{\sigma_i\} = D [C_{ij}]\{\epsilon_i\}$. A slightly different version is given in Chuzel et al [166].

4.1.2 – Viscoelastic models

Studies of the mechanical behavior of ice show that the stress-strain behavior is affected by strain rate which suggests the use of a viscoelastic models. Khoo et al [167] [168] describe a viscoelastic model that can be represented by a spring-mass model in one dimension. It consists of a Maxwell element in series with a Kelvin-Voigt element (Fig. 14) and produces three types of strain: the elastic strain ϵ_e , the viscous strain ϵ_v , and the delayed strain ϵ_d . This model is generalized to two dimensions [167] [168] and three dimensions [169].

Ice was also modeled as a viscoelastic material by Zamankhan to examine the impact of an ice ball with a stationary ice block [170] using the generalized Maxwell model described in Subsection 3.6.1 with a single term in the Prony series (Eq. 21). The simulation accurately predicted the coefficients of restitution obtained from experiments. The point was made that this model is inadequate for this problem because an unrealistic value of the relaxation time had to be used to achieve good agreement with experiments. Instead, it was shown that an elastoplastic model with strain-rate dependent yield strength provides a good fit to the experimental data while correctly modeling the physics of the problem.

4.1.3- Elastoplastic models for ice impacts

In some simulations, hail is modeled as a solid deforming plastically. Several issues must be addressed in developing such models including: (1) is ice isotropic or anisotropic?; (2) what is the yield or failure criterion?; (3) how is the post-yield behavior modeled? The behavior of ice has been studied extensively for many years and is known that there are many types of ice and that their internal structure is quite complex and affect their material properties. Many studies assume that on a macroscopic scale ice is isotropic and successfully analyze problems dealing

with glaciers or with iceberg-ship interactions. In general, failure strengths in tension and compression are different and that should be accounted for in failure criteria for multiaxial loading.

To study the movement of glaciers, Nye [171][172] assumes that ice is an isotropic homogeneous solid and yielding does not depend on the hydrostatic stress and follows the Mises or the Tresca yield criterion. The von Mises yield criterion is expressed in terms of $J_2 = s_{ij}s_{ij} / 2$, the second invariant of the deviatoric stress tensor, and the yield strength in tension σ_y as

$$f = J_2 - (\sigma_y^2/3) = 0 \quad (25)$$

or, in terms of the equivalent (or von Mises) stress $\sigma_e = \sqrt{3J_2} = \sqrt{\frac{3}{2}s_{ij}s_{ij}}$, Eq. 25 can be written as $\sigma_e - \sigma_y = 0$. The strain energy density of an isotropic elastic material can be written in terms of only two invariants: J_2 and $I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$, the first invariant of the stress tensor. Experiments showed that for metals, yielding is independent of I_1 (or equivalently of the hydrostatic stress) and von Mises criterion is applicable. However, for many materials yielding does not depend on J_2 alone and a many other criteria have been introduced. Ref. [173] shows that most criteria can be written in terms of J_2 and I_1 and can be classified into four categories. Yield criteria for anisotropic materials are also discussed in [173].

Pernas-Sánchez et al [174] use the 1952 Drucker- Prager yield criterion

$$f = \sqrt{J_2} - A - BI_1 \quad (26)$$

where $J_2 = s_{ij}s_{ij} / 2$ is the second invariant of the deviatoric tensor, $I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$ is the first invariant of the stress tensor and the constants A and B depend on σ_t and σ_c the yield strengths in tension and compression: $A = \frac{2}{\sqrt{3}} \frac{\sigma_c\sigma_t}{\sigma_c - \sigma_t}$, $B = \frac{1}{\sqrt{3}} \frac{\sigma_t - \sigma_c}{\sigma_c + \sigma_t}$. Eq. 26 can be seen as the von Mises criterion in terms of the equivalent stress with an added I_1 term.

Based on experimental results from previous investigators, the yield strength of ice in compression was assumed to vary with strain rate but the yield strength in tension is expected to remain constant [174]. A non-associated plastic flow rule is developed to describe the inelastic

behavior of ice. Other failure criteria are used to study iceberg-ship collisions but have not been used for hail impact simulations.

In some simulations, the deformations of the projectile may remain small and it is not necessary to distinguish between the different types of stress. To describe the deformation of soft impactors the approach described by Brockman et al (Section 3.2.5) is often used. Carney et al [175] extended this approach accounting for the fact that the strength of the ice is different in tension and compression and that the effect of strain rate are different in tension and compression.

4.2- Hail damage

Depending on the impact velocity, hailstone can experience large deformations and fragmentation. Several approaches have been used in the simulation of hailstones [176]: (1) the Lagrangian finite element formulation; (2) the Arbitrary Lagrangian Eulerian (ALE) method; and (3) the Smooth Particle Hydrodynamic (SPH) method. Using these three approaches on several hail impact problems it was concluded that the Lagrangian formulation is adequate only in the initial stages of the impact when the deformation of the ice does not cause severe element distortion. Difficulties were encountered when using the ALE method for the ice and a Lagrangian description of the structure. The SPH method is deemed to be the most efficient for this type of problem [176]. References [53], [146], [165], [166], [174], [177], [178], [179],[180], use it to model hailstones.

4.2.1- Dent depth for hail impacts on metallic plates

McNaughtan and Chisman [181] studied the effect of impact velocity, impact angle, plate material properties and thickness on the damage caused by one inch diameter hail at speeds up to 2500 ft/s (762 m/s). Under normal impact it was found that the dent depth varies linearly with impact velocity. For oblique impacts the dent depth varies with the normal component of the velocity. Hail penetration speed is proportional to the square root of the plate thickness. In [156], a model is developed to predict the dent depth produced by a mass m impacting a plate with a velocity V . It is assumed that:

1. a region with a radius r is deformed plastically into a conical shape and that w , the depth of the dent, is small compared to r
2. the deformation is resisted entirely by membrane forces and the deformation is purely plastic and irreversible.
3. All the kinetic energy of the ice sphere is expended by straining of the skin.

With these assumptions, the dent depth is given by

$$w = V \sqrt{m} / \sqrt{2\pi\sigma_y t} \quad (27)$$

where V is the initial velocity of the sphere, m is its mass, σ_y and t are the yield stress and the thickness of the plate. Experimental results can be fitted by Eq. 27 when $1/\sqrt{2\pi}$ is replaced by a constant k determined by curve fitting [156]. Recalling the relationship between V and the diameter of the sphere (Eq. 24), Eq. 27 indicates that, for ground hail, w is proportional to the square of the diameter. Another analytical model for this problem was presented showing that the initial stage of permanent deformations dominated by bending and is followed by another stage controlled by membrane deformations [182]. Experiments show that in that second stage the dent depth increases linearly with the velocity of the projectile.

Fiber metal laminates (FMLs) are considered for use in primary aircraft structures. Experimental results show that permanent indentation of FML plates increases with the impact energy [183][184] [185]. Starikov [186] fitted experimental data for plates made out of aluminum or fiber metal laminates by $w = A T^B / (\sigma_y t)$ where T is the initial kinetic energy of the projectile and A and B are determined by curve fitting.

4.2.2-Hail impact on composite structures

Foreign object impacts on composite structures generate several types of damage: fiber failures, matrix cracks, and delaminations [122,187,188]. During a hailstorm, a given part can be subjected to multiple impacts and the cumulative effect can be significant [189]. The effect of hail impact damage on the fatigue life of composites [190] and on the residual strength of sandwich panels [191] were shown to be significant.

Fig. 7 in [192] shows photographs taken during the impact of a 42.7 mm diameter ice ball against a hard surface at 73.5 m/s at four different times after initial contact: 0 μ s, 91 μ s, 182 μ s, and 455 μ s. The deformation of the sphere suggest that it behaves like a fluid. In addition, assuming that the back of the sphere moves at a constant 73.5 m/s, the impact duration is estimated by $t=D/V$ is 585 μ s, a value consistent with the results in [192]. In addition, experimental results indicate that the maximum impact force F varies linearly with the initial kinetic energy of the ball [192]. Dimensional analysis suggests that the force should be proportional to the kinetic energy and inversely proportional to the diameter ($F \sim \frac{1}{2}mV^2 / D$). Similar photographs from Asp and Juntikka [193] for a 25 mm diameter sphere at 96 m/s show what the authors call “local crushing and the subsequent fluid-like flow”. Similar behavior is shown in [194], a study that focuses on impact-induced delaminations to laminated composites and in [161] where both experiments and simulations show the propagation of longitudinal cracks oriented along the direction of travel.

5- CONCLUSION

This article presents a review of the literature dealing with three types of soft impact problems of interest to the aerospace industry. It provides a list of 194 references that is far from exhaustive but gives a comprehensive view of the current state of knowledge for this type of soft impacts. The impact of rain drops lead to erosion or fracture. Bird strikes are severe threats due to the significant kinetic energy of the bird as it hits critical components of the airplane which should be able to complete its flight safely. Hailstones can cause significant damage since they also have significant kinetic energy levels before impact.

In each case, a basic understanding of the physics of the problem has been achieved through experimental and simple analytical approaches. This body of work makes it clear that such soft impact are significantly different than hard impacts like those due to tool drops on aircraft structures. The analysis should account for the fluid-like behavior of the projectiles and the kinetic energy to be dissipated is much larger in the case of bird strikes or hail impacts. In recent years, an increasingly large number of publications report on numerical simulations of this type of soft impacts. This is made possible by the availability of software capable of modeling the large deformations of the projectile using the SPH method or other approaches. That type of

software also allows to experiment with different models for the constitutive behavior of the materials for the projectile and for the target. It was shown that plausible results can be obtained with inappropriate constitutive models in the case of ice impacts.

Numerical simulations of soft impacts are usually performed as a coupled problem: one numerical model for the projectile coupled with a numerical model for the target. This is sometimes referred to as a two-way coupling so that the projectile generates a load on the structure and in turn the deformation of the structure affects the motion of the projectile. Accurate analyses require a detailed discretization of both domains as discussed earlier. A more efficient approach might be what is called one-way coupling or an uncoupled approach. Essentially, it assumes that the loading generated by the impact can be determined assuming that the target is rigid so that the pressure distribution over its surface is calculated considering the deformation of the projectile only. Then, this load history is used to calculate the response of the structure. This approach is applicable when the deformation of the structure is small enough that it does not affect the deformation of the projectile or when the impact and the deformation of the structure occur on two different time scales. For the normal impact of a right cylinder on a rigid surface it is possible to actually predict the pressure distribution on that surface during the impact. Numerical simulations indicate that the situation more complex for oblique impacts and for the impact of spherical projectiles or projectiles of other shapes. A better understanding of these cases and an investigation of the possible use of uncoupled analysis could lead to more efficient simulations. When two-way coupling is required, it should be possible to obtain non-dimensional results to predict spatial and temporal pressure distributions without having to perform a full analysis.

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Table 1: Coefficients in polynomial equation of state for birds different porosity levels Selezneva [55]

	0% porosity	10% porosity	15% porosity
C_1	2060	28	6.9
C_2	6160	-85	-3180
C_3	10300	35000	31000

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Figure 1: Interaction of a soft impactor with a rigid surface (Wilbeck [2]): (a) cylindrical impactor prior to impact; (b) shock wave propagating upwards and relief waves propagating inwards; (c) relief waves reached the axis of the cylinder; (d) end of the high pressure phase and beginning of flow phase of the impact

Figure 2: Spherical water drop at the end of the impact phase [15]

Figure 3: Impact of a spherical drop against a rigid surface: (a) contact radius expands faster than the speed of sound in water; (b) compressive wave front detaches from the rigid surface and relief wave propagates towards the inside of the compressed zone ; (c) outwards flow [15]

Figure 4: Stress waves propagating in the target following an impact

Figure 5: Effect of porosity on the pressure density behavior of birds

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Figure 12: Two degree of freedom model for canopy frame impact analysis [35]: M_w and k_w are the modal mass and stiffness of the first vibration mode of the canopy, m , k and C are the equivalent mass, stiffness and stiffness coefficients of a supporting frame.

Figure 13: Tensor skin laminates

Figure 14: Viscoelastic model in which an applied stress produces an elastic strain ϵ_e , a viscous strain ϵ_v , and a delayed strain ϵ_d .

Figure(s)

Figure 1

Figure 2

Figure 3

(a)

(b)

Figure 4

Figure 5

Figure 6

Figure 7:

Figure 8

(a)

(b)

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14