On the origin of plasma density blobs

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Abstract

By keeping nonlinear Boltzmann factor in electron density dependence on electrostatic potential it is demonstrated that large plasma density blobs, often seen in experiment inside separatrix, can exist within the framework of drift wave dynamics. The estimates show that plasma density in a blob can be ~ 3 times higher than average plasma density, but hardly exceeds this limit, which in a ball park is in agreement with experimental observations.

Key words: plasma; drift waves; blobs

It is known that intermittent blobby transport plays a very important role in the transport of edge plasma in magnetic fusion devices (e.g. see Ref. 1-3). Although the mechanism, propelling high plasma density filaments (blobs) on the outer side of a tokamak in the Scrape-Off-Layer (SOL) is rather well understood [1-4], the physics responsible for the formation of the blobs is still under the questions. Meanwhile there is compelling experimental evidence that high-density blobs exist already inside the separatrix (e.g. see Ref. 5-7) where they move mainly in poloidal direction and once in a while cross the separatrix and appear in the SOL. In what follows we demonstrate that it is plausible that the formation of these high plasma density blobs is inherent for the drift wave plasma turbulence. To do this we start with the reconsideration of the Hasegawa-Mima equation [8] and will not pursue a standard assumption about smallness of electrostatic fluctuations.

Following [9] we consider Boltzmann electrons

$$n_e(\vec{r}, t) = \hat{n}(\vec{r}_\perp) \exp\{\phi(\vec{r}, t)\}, \quad (1)$$

(where $\phi = e\varphi / T_e$, e is the electron charge, φ is the electrostatic potential, T_e is the electron temperature which we assumed to be constant, and $\hat{n}(\vec{r}_\perp)$ describes electron density in the absence of electrostatic potential) and ion continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0, \quad (2)$$

where we assume cold ion approximation for ion velocity

$$\vec{V}_i = \vec{V}_0 - \rho_s^2 \frac{d}{dt} \nabla \phi \quad \text{and} \quad \vec{V}_0 = -D_B (\nabla \phi \times \vec{b}), \quad \frac{d(\dots)}{dt} = \frac{\partial(\dots)}{\partial t} + (\vec{V}_0 \cdot \nabla)(\dots), \quad (3)$$

where $\rho_s^2 = T_e M / (c / eB)^2$, $D_B = c T_e / eB$, M is the ion mass and B is the strength of the magnetic field. After we take $\hat{n}(\vec{r}_\perp) = n_0 \exp(-\Lambda x)$ and, using the quasi-neutrality condition, substitute the expressions (1) and (3) into Eq. (2) we find

$$\frac{d e \phi}{dt} - \rho_s^2 \nabla \cdot \left(e \phi \frac{d}{dt} \nabla \phi \right) - \Lambda e \phi \vec{e}_x \cdot \left(\vec{V}_0 - \rho_s^2 \frac{d}{dt} \nabla \phi \right) = 0, \quad (4)$$

which we'll call drift-wave with non-linear electrons (NLE) equation. Assuming that $|\phi| \ll 1$ and neglecting the drift caused by inertia in the last term of Eq. (4) we obtain the Hasegawa-Mima equation (HM):

$$\frac{d}{dt}(\phi - \rho_s^2 \nabla^2 \phi) - \Lambda \vec{e}_x \cdot \vec{V}_0 = 0. \quad (5)$$

Let us consider these equations for the case where $|\partial(\dots)/\partial y| \gg |\partial(\dots)/\partial x|$, that implies that we are looking for a streamer-like structures. For NLE we find

$$\frac{\partial e^\phi}{\partial t} - \rho_s^2 \frac{\partial}{\partial y} \left(e^\phi \frac{\partial^2 \phi}{\partial y \partial t} \right) + U_{DW} \frac{\partial e^\phi}{\partial y} = 0, \quad (6)$$

while for HM

$$\frac{\partial}{\partial t} \left(\phi - \rho_s^2 \frac{\partial^2 \phi}{\partial y^2} \right) + U_{DW} \frac{\partial \phi}{\partial y} = 0, \quad (7)$$

where $U_{DW} \equiv \Lambda D_B$ is the phase velocity of linear drift waves in a long wavelength approximation.

From Eq. (6,7) we see that the NLE conserves $\langle \exp(\phi) \rangle_y = \text{const.}$ while the HM conserves $\langle \phi \rangle_y = \text{const.}$, here $\langle \dots \rangle_y$ is just averaging over coordinate y . This implies that both NLE and HM maintain a constant averaged density along y . We also notice that Eq. (6) obtained from NLE is nonlinear, while Eq. (7) following from the HM model becomes linear.

Let us consider traveling wave, $\partial(\dots)/\partial t = -U_{HM/NLE} \partial(\dots)/\partial y$, solutions of Eq. (6,7) satisfying conditions $\langle \phi \rangle_y = 0$ for HM and it's analog $\langle \exp(\phi) \rangle_y = 1$ for NLE.

For HM we find just simple sinusoidal oscillation of ϕ with amplitude ϕ_0 , which formally should be small

$$\phi = \phi_0 \sin(\kappa_{HM} y), \quad (8)$$

where κ_{HM} is the wave number satisfying the following inequality

$$\kappa_{HM}^2 = \rho_s^{-2} (U_{DW} / U_{HM} - 1) > 0, \quad (9)$$

which requires $U_{HM} < U_{DW}$. We note that Eq. (9) can be recast as $U_{HM} = U_{DW} / (1 + \kappa_{HM}^2 \rho_s^2)$, which, taking into account Eq. (8), implies that the speed U_{HM} equals to the phase velocity of linear drift wave with the wavenumber κ_{HM} , which is not surprising since Eq. (7) is simply a linear equation for the drift wave potential.

For traveling wave solution for the NLE equation we arrive to the following equation

$$\kappa_{\text{NLE}}^2 \frac{d e^\phi}{dy} + \frac{d}{dy} \left(e^\phi \frac{d^2 \phi}{dy^2} \right) = 0, \quad (10)$$

where $\kappa_{\text{NLE}}^2 = \rho_s^{-2} (U_{\text{DW}} / U_{\text{NLE}} - 1)$, which at this moment may be both positive and negative.

From Eq. (10) we find

$$\frac{d^2 \phi}{dy^2} = -\kappa_{\text{NLE}}^2 + C e^{-\phi} \equiv -\frac{\partial}{\partial \phi} W(\phi), \quad (11)$$

where C is the integration constant and

$$W(\phi) = \kappa_{\text{HME}}^2 \phi + C(e^{-\phi} - 1), \quad (12)$$

is the effective potential (we notice that addition of any constant, C_W , to $W(\phi)$ does not change Eq. (11), therefore we choose such C_W that $W(0) = 0$, which makes our further analysis more convenient). From Eq. (11) we find the first integral

$$\frac{1}{2} \left(\frac{d\phi}{dy} \right)^2 + W(\phi) = E, \quad (13)$$

where E is the effective energy. Therefore, as always, it is convenient to analyze Eq. (11), as the equation of motion of quasi-particle considering ϕ it's the effective coordinate and y as the effective time.

Since we have to satisfy $\langle \exp(\phi) \rangle_y = 1$, our solution, which we assume to be periodic, should be bounded by some “turning points” (where $d\phi/dy = 0$) at positive ϕ_{max} and at negative ϕ_{min} (if both ϕ_{max} and ϕ_{min} have the same sign $\langle \exp(\phi) \rangle_y = 1$ cannot be held) satisfying the conditions

$$W(\phi_{\text{max}}) = W(\phi_{\text{min}}) = E. \quad (14)$$

Moreover, since the existence of the solution (13) requires $W(\phi_{\text{min}} < \phi < \phi_{\text{max}}) < E$, ensuring that our quasi-particle moves across the “potential well”, from Eq. (12) one sees that it is only possible when both κ_{NLE}^2 and C are positive, which implies $U_{\text{NLE}} < U_{\text{DW}}$. Finally, using Eq.

(13), it is easy to show that we can express the condition $\langle \exp(\phi) \rangle_y = 1$ only in terms of ϕ . For this purpose it is convenient to introduce $\hat{E} = E / \kappa_{\text{NLE}}^2$, $\hat{C} = C / \kappa_{\text{NLE}}^2$, and

$$\hat{W}(\phi) = \phi + \hat{C}(e^{-\phi} - 1), \quad (15)$$

as a result we arrive to the following equation

$$\int_{\phi_{\min}}^{\phi_{\max}} \frac{e^{\phi} d\phi}{\sqrt{\hat{E} - \hat{W}(\phi)}} = \int_{\phi_{\min}}^{\phi_{\max}} \frac{d\phi}{\sqrt{\hat{E} - \hat{W}(\phi)}}. \quad (16)$$

Thus, to find the solution of Eq. (11) which corresponds to our constrain $\langle \exp(\phi) \rangle_y = 1$, we need to find the solution of Eq. (16) and then use the “turning” points $\phi_{\max}$ (or $\phi_{\min}$) as the boundary conditions in Eq. (11). We will consider $\hat{C}$ as the input parameter and will try to find $\hat{E}(\hat{C})$ as the solution of Eq. (16).

To get some insights let us start with analysis of the potential $\hat{W}(\phi)$. First we note that it has only one extremum (minimum) at $\phi = \phi_{\text{ext}} = \ln(\hat{C})$. Depending on the magnitude of $\hat{C}$, ϕ_{ext} can be both positive ($\hat{C} > 1$) and negative ($\hat{C} < 1$), but $\hat{W}(\phi_{\text{ext}}) = \ln(\hat{C}) - \hat{C} + 1$ is always negative except the case $\hat{C} = 1$, where $\hat{W}(\phi_{\text{ext}}) = 0$. Schematically $\hat{W}(\phi)$ for different $\hat{C}$ is shown in Fig.1.

From Fig. 1 one can see that for $\hat{C} \gg 1$ quasi-particle spends most of the time at large $\phi > 0$. Therefore, taking into account that in this case $\exp(\phi) \gg 1$, we conclude that the solution of Eq. (16) exist only for $\hat{C} < \hat{C}_{\text{crit}} \sim 1$. An exact value of $\hat{C}_{\text{crit}}$ can be found numerically, but this goes beyond the scope of this paper.

For $\hat{C} = 1$ and small ϕ we have $\hat{W}(\phi) = \phi^2 / 2$. Expanding $\exp(\phi) \approx 1 + \phi$ in the left hand side of Eq. (16) we find the equality holds for any $\hat{E}$. However, in practice, taking into account both “anharmonic” features of $\hat{W}(\phi)$ and only approximate validity of expansion $\exp(\phi) \approx 1 + \phi$, some “quantization” of $\hat{E}$ will appear which will determine the value of $\hat{E}$. Nonetheless, since for $\hat{C} \sim 1$ the effective potential $\hat{W}(\phi)$ is close to be symmetric and, therefore, one could expect that the solutions will be close to the sinusoidal oscillations of $\phi(y)$ from Eq. (8).

For $\hat{C} \ll 1$ $\hat{W}(\phi)$ is strongly squeezed toward negative ϕ where quasi-particle spends most of its time. As a result, due to presence of $\exp(\phi)$ in the left hand side of Eq. (16), the regions of major contributions to the left and right hand sides of Eq. (16) are separated. While the major contribution to the left hand side gives the region of positive ϕ , the magnitude of the integral on the right hand side is mainly determined by negative ϕ . Then for $\hat{C} \ll 1$ we find $\phi_{\max} \approx \hat{E}$ and

$$\int_{\phi_{\min}}^{\phi_{\max}} \frac{e^{\phi} d\phi}{\sqrt{\hat{E} - \hat{W}(\phi)}} \approx \sqrt{\pi} \exp(\hat{E}). \quad (17)$$

To estimate the integral on the right hand side of Eq. (16) we take into account fast growth of exponent in the expression for $\hat{W}(\phi)$ we can take $\phi_{\min} \approx \alpha \phi_{\text{ext}} = \alpha \ell \ln(\hat{C})$, where $\alpha \gtrsim 1$ is some numerical factor. Since $|\phi_{\min}| \gg 1$ the fast growing exponential part in the expression for $\hat{W}(\phi)$ contributes only within a small range of ϕ ($\Delta\phi \sim 1$) and, therefore, can be neglected. So on the right hand side of Eq. (16) we can take $\hat{W}(\phi) \approx \phi$. As a result we have

$$\int_{\phi_{\min}}^{\phi_{\max}} \frac{d\phi}{\sqrt{\hat{E} - \hat{W}(\phi)}} \approx 2\sqrt{\hat{E} - \alpha \ell \ln(\hat{C})}. \quad (18)$$

Then from the expressions (17, 18) we have the following approximate solution

$$\exp(\hat{E}) \approx 2\sqrt{-(\alpha/\pi)\ell \ln(\hat{C})}. \quad (19)$$

Recalling that $\phi_{\max} \approx \hat{E}$ and $n_e \propto \exp(\phi)$ we conclude that this solution corresponds to a strong local increase of plasma density, density blob, in comparison to the average one, up to

$$n_{\text{blob}} / \langle n_e \rangle \approx 2\sqrt{-(\alpha/\pi)\ell \ln(\hat{C})}. \quad (20)$$

These plasma density blobs are surrounded by plasma with depleted density. Since $\phi_{\min} \approx \ell \ln(\hat{C})$ we have virtually plasma hole, where plasma density, n_{hole} , drops up to

$$n_{\text{hole}} / \langle n_e \rangle \approx \hat{C} \ll 1. \quad (21)$$

Using Eq. (13) and our estimate (18) we find the distance between two consecutive blobs,

$$L_2 = \int_{\phi_{\min}}^{\phi_{\max}} d\phi / \sqrt{(E - W(\phi))/2},$$

$$L_2 \approx 2^{3/2} \sqrt{-\alpha \ell n(\hat{C})} / \kappa_{\text{NLE}}. \quad (22)$$

Recalling that $\kappa_{\text{NLE}}^2 = \rho_s^{-2} (U_{\text{DW}} / U_{\text{NLE}} - 1)$ we see that, even though we can rewrite this expression as $U_{\text{NLE}} = U_{\text{DW}} / (1 + \kappa_{\text{NLE}}^2 \rho_s^2)$, but from Eq. (22) it follows that κ_{NLE} is not equal to $2\pi / L_2$, which might be considered as an effective wavenumber of a nonlinear structure. Moreover, since for $\hat{C} \ll 1$ we have $2\pi / (\kappa_{\text{NLE}} L_2) < 1$ we find that U_{NLE} is smaller than the phase velocity of linear drift wave with the wavenumber $2\pi / L_2$.

Defining the effective size of the blob, δ_b , as the distance where $\phi > 0$, from Eq. (13, 19) we find

$$\delta_b \approx \left\{ 2^{3/2} \ell n \left(2 \sqrt{-(\alpha / \pi) \ell n(\hat{C})} \right) \right\}^{1/2} / \kappa_{\text{NLE}}. \quad (23)$$

Comparing the expressions (22, 23) we conclude that for $\hat{C} \ll 1$ $\delta_b \ll L_2$ so that the blob indeed looks like a solitary structure rather similar to experimental observations made with the Gas Puff Imaging (GPI) technique in Ref. 5, 6. In addition, we notice that the ratio $n_{\text{blob}} / \langle n_e \rangle$ from Eq. (20) has a very weak dependence on the parameter $\hat{C}$ and taking, as an example, $\hat{C} \sim 0.1$ and $\alpha \sim 3$ we find $n_{\text{blob}} / \langle n_e \rangle \sim 3$, while for $\hat{C} \sim 10^{-2}$ and $\alpha \sim 3$ we have $n_{\text{blob}} / \langle n_e \rangle \sim 4$ which, again, seems to agree with the GPI observations.

Thus, by keeping nonlinear Boltzmann factor in electron density dependence on electrostatic potential we demonstrated analytically the existence of large plasma density structures within the simplest framework of drift wave dynamics. Our estimates show that these structures propagate in poloidal direction with the speed smaller than the phase velocity of linear drift waves and plasma density in the streamers can be ~ 3 times higher than average plasma density, but hardly exceeds this limit. In a ballpark these findings are consistent with experimental observations [5, 6] of plasma density blobs inside the separatrix.

However, in our analytical analysis two major simplifications were made: i) we assumed that the variation of electrostatic potential in radial direction is much slower than that in poloidal one, and ii) we neglected the effects of curvature of the magnetic field lines and the spatial variation of the magnitude of the magnetic field strength.

As a result of the first simplification our nonlinear structures on poloidal-radial projection look like streamers, while GPI diagnostic show more round-like blobs inside the separatrix (e.g. see Ref. 5, 6). Similar round-like blobs observed in the SOL. But, we should remember that the GPI diagnostic is based on the light emission of neutral gas puffed from the main chamber wall. Therefore, due to neutral ionization, neutral density and radiation in the puff are decreasing toward the core tokamak plasma. As a result, radial extend of the radiation image can be smaller than that of real plasma density in the blobs. We also notice that the blobs moving through separatrics into SOL can be affected by the sheared electric field, which can alter their shape [1, 3]. Finally, it is also plausible that two-dimensional (2D) Eq. (4) allows nonlinear solutions of a more round form resembling drift wave vortices (e.g. see ref. 10 and the references therein).

Our second simplification excludes plasma polarization, which, finally, propels blobs in radial direction [4]. Although this effect is inversely proportional to a tokamak major radius, R , and, therefore, is smaller than $\vec{E} \times \vec{B}$ drifts in Eq. (4), which are inversely proportional to $\delta_b \ll R$, in a long run it should be taken into account. However, simultaneous consideration of 2D and polarization effects can only be done numerically, while the main goal of this paper was the demonstration of the existence of large plasma density structures/blobs within the simplest framework of drift wave dynamics and of course more work still needed to learn the formation and dynamics of these structures.

Acknowledgements. The author thanks Prof. A. I. Smolyakov for fruitful discussions. This material is based upon the work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under Award No. DE-FG02-04ER54739 at UCSD

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Figure captures

Fig. 1. Schematic view of the function $\hat{W}(\phi)$ for different $\hat{C}$.