Comments on: “On the sound attenuation in fluid due
to the thermal diffusion and viscous dissipation” [Phys.
Lett. A 379 (2015) 1799–1801]

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Abstract

We point out and correct the erroneous expression for what Hu et al. term the ‘sound attenuation coefficient’ (i.e., a_s) that appears in their recent Letter [Phys. Lett. A 379 (2015) 1799–1801].

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The aims of the present Letter are as follows: (i) show that the ‘improved’ expression for a_s , the ‘sound attenuation coefficient’ associated with harmonic plane waves in thermoviscous fluids, that appears in Ref. [1, Eq. (9)] is incorrect; (ii) provide the needed correction; and (iii), re-establish that the classical expression for a_s is the correct one.

To this end we observe that the expression for k^2 given in Ref. [1, Eq. (7)] can be recast in the more explicit form

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$$k^2 = \frac{\omega}{2\alpha} \left\{ \frac{\rho v_s^2 + j\omega(\alpha\rho\gamma + \varphi)}{\omega\gamma\varphi - j\rho v_s^2} - \sqrt{\frac{\omega^2(\alpha\rho\gamma - \varphi)^2 - \rho^2 v_s^4 - 2j\omega\rho v_s^2[\alpha\rho(\gamma - 2) + \varphi]}{(\rho v_s^2 + j\omega\gamma\varphi)^2}} \right\}, \quad (1)$$

a result which is equivalent to the corresponding expression given by Morse and Ingard [2, Eq. (6.4.26)₁], and which we have independently verified as correct. Here, as in Ref. [1], k is the complex-valued wave number corresponding to the pressure field; $\omega(> 0)$ denotes the angular frequency; $\rho(> 0)$ is the equilibrium state value of the fluid's mass density; $\gamma(> 1)$ denotes the ratio of specific heats; v_s represents the sound speed in the undisturbed fluid; $\alpha(> 0)$ is the thermal diffusivity; φ , which Hayes [3, p. 38] has termed the longitudinal coefficient of viscosity, is defined as $\varphi = 4\mu/3 + \eta$, where $\mu(> 0)$ and $\eta(\geq 0)$ are the shear and bulk viscosity coefficients, respectively; and $j = \sqrt{-1}$.

The first step in determining a_s is to expand Eq. (1) about $\omega = 0$. Omitting the details, the application of the binomial theorem yields, after simplifying,

$$k^2 \sim -\frac{\omega^2}{v_s^2} \left\{ 1 - j \left[\frac{\alpha(\gamma - 1) + \varphi/\rho}{v_s^2} \right] \omega - \left[\frac{\alpha^2(\gamma - 1)(\gamma - 2) + 3\alpha(\gamma - 1)\varphi/\rho + \varphi^2/\rho^2}{v_s^4} \right] \omega^2 \right\} \quad (\omega \rightarrow 0); \quad (2)$$

see also Lamb [4, Eq. (32)], whose analysis is restricted¹ to the special case $\eta = 0$.

We pause now to note that it is here, at this stage, i.e., the derivation of the small- ω expression for k^2 , that an error occurs in Ref. [1]. Specifically, the first approximate expression for k^2 given on Ref. [1, p. 1800] is incorrect, as use of the software package MATHEMATICA and comparisons with Eq. (2) (above) and Ref. [2, Eq. (6.4.23)₁] all reveal. The problem arises because, rather than expand their square-root² about $\omega = 0$, Hu et al. opted to do so under the assumption $4|(ad - bc)(a + d)^{-2}| \ll 1$, i.e.,

$$4\omega\alpha\rho \left| \frac{j\rho v_s^2 - \gamma\varphi\omega}{[\rho v_s^2 + j\omega(\gamma\alpha\rho + \varphi)]^2} \right| \ll 1. \quad (3)$$

While the present authors are unable to cite physical justification for its use, the authors of Ref. [1] appear to have adopted this assumption in an effort to obtain a more accurate (but undoubtedly ad hoc) expression for a_s .

¹A limitation easily remedied by replacing '4 $\nu/3$ ' with ' φ/ρ ' throughout Ref. [4, Art. 360].

²See the second line of the *last* exact expression for k^2 given on Ref. [1, p. 1800].

Continuing, we take the square-root of Eq. (2) and let $k_{1,2}$ correspond to the ‘ $\pm$ ’ cases of the extraction. On expanding the resulting expression about $\omega = 0$, it is a tedious, but straightforward, task to establish that

$$k_2 \sim - \left[\frac{\varphi/\rho + \alpha(\gamma - 1)}{2v_s^3} \right] \omega^2 - j \left(\frac{\omega}{v_s} \right) \quad (\omega \rightarrow 0), \quad (4)$$

where, assuming propagation to the right, we have taken the ‘ $-$ ’ case to ensure that our plane wave solution is physically admissible. Since, in the present context, a_s is defined as the absolute value of the leading-order term in the low-frequency expansion of $\text{Re}(k_2)$, it follows from Eq. (4) that

$$a_s := \left[\frac{\varphi/\rho + \alpha(\gamma - 1)}{2v_s^3} \right] \omega^2, \quad (5)$$

which we note was derived *directly*³ from Ref. [1, Eq. (7)], and which we observe is *exactly* the classical expression for the attenuation coefficient (see, e.g., Refs. [5, 6]). In this regard see also Lighthill [7, §3.1], who points out that a_s can also be expressed in terms of the quantity δ , which he termed the ‘diffusivity of sound’, and which is given by

$$\delta = \nu [\varphi/\mu + (\gamma - 1)/\text{Pr}], \quad (6)$$

where $\nu = \mu/\rho$ denotes the kinematic viscosity and we now introduce the Prandtl number $\text{Pr} = \nu/\alpha$.

Now for $\eta = 0$, which is true for monatomic gases [6, p. 550], Eq. (5) reduces to what Lighthill [7, §3.4] calls the ‘Stokes–Kirchhoff formula’, namely,

$$a_{S-K} = \left[\frac{\alpha(\gamma - 1) + 4\nu/3}{2v_s^3} \right] \omega^2, \quad (7)$$

which we observe can also be extracted from Lamb’s [4, Eq. (32)]. Eq. (7) is, as might be expected, in good agreement with experimental measurements carried out in monatomic gases; see, e.g., Ref. [6, p. 519] and those cited therein. We also observe here that Hu et al. *incorrectly*⁴ attach the names ‘Stokes–Kirchhoff’ to the expression stated in Ref. [1, Eq. (11)], which, it should be noted, is actually equivalent to Eq. (5) of the present Letter.

³We stress this fact because Hu et al. appear to suggest that such is not possible; see the penultimate paragraph on Ref. [1, p. 1800].

⁴In the sense that *both* Stokes [8] and Kirchhoff [9] assumed $\eta = 0$; recall Lighthill [7, §3.4].

In this Letter we have shown that the expression for a_s stated in Ref. [1, Eq. (9)] is, contrary to the claim of Hu et al., not an ‘improved’ version of the classical result but, rather, simply incorrect. Through our analysis, we traced the origin of the problem to Hu et al.’s approach to expanding the square-root term in their expression for k^2 . Unfortunately, the smallness assumption they selected (see Eq. (3)) introduced an error that then propagated throughout the remainder of Ref. [1], and in so doing impacted Ref. [1, Eq. (9)]. Our findings leave no doubt that, in the context of classical acoustics, the expression for a_s given in Eq. (5) is, in fact, the correct one.

Lastly, while Hu et al. are correct in pointing out that a_s generally *underestimates* experimental measurements of acoustic absorption in fluids⁵, the correct (i.e., rigorous) approach to removing such discrepancies cannot be based on ad hoc, mathematically-driven assumptions. Rather, as pointed out by Pierce [6, p. 519], it involves extending the classical theory to include contributions to a_s from other phenomena, in particular, those due to relaxation processes; see, e.g., Ref. [6, §10-7] and those cited therein.

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⁵The exception, as noted above, being monatomic gases.

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