An in-Medium Heavy-Quark Potential from the $Q\bar{Q}$ Free Energy

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Abstract

We investigate the problem of extracting a static potential between a quark and its antiquark in a quark-gluon plasma (QGP) from lattice-QCD computations of the singlet free energy, $F_{Q\bar{Q}}(r)$. We utilize the thermodynamic T -matrix formalism to calculate the free energy from an underlying potential ansatz resummed in ladder approximation. Imaginary parts of both $Q\bar{Q}$ potential-type and single-quark selfenergies are included as estimated from earlier results of the T -matrix approach. We find that the imaginary parts, and in particular their (low-) energy dependence, induce marked deviations of the (real part of the) potential from the calculated free energy. When fitting lattice results of the latter, the extracted potential is characterized by significant long-range contributions from remnants of the confining force. We briefly discuss consequences of this feature for the heavy-quark transport coefficient in the QGP.

Keywords: Heavy-quark potential, Bethe-Salpeter Equation, Quark-Gluon Plasma

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1. Introduction

Heavy-flavor (HF) particles, containing charm or bottom quarks, play a central role in the analysis of strongly interacting matter as produced in ultrarelativistic heavy-ion collisions (URHICs) [1]. The nuclear modification factor and elliptic flow of D and B mesons (as well as their semileptonic decay leptons) are key measures to establish and quantify the strong-coupling nature of the QCD medium [2–4], while the production patterns of charmonia and bottomonia [5–8] are believed to encode information on its deconfinement properties. For both observables, a good understanding of the in-medium interactions of the heavy quarks and quarkonia is mandatory.

Lattice-QCD (lQCD) computations allow to study the properties of heavy-quark (HQ) systems in the quark-gluon plasma (QGP) from first principle. In particular, euclidean correlation functions of quarkonia, HQ susceptibilities and static $Q\bar{Q}$ free energies have been computed with good precision [9, 10]. A non-perturbative T -matrix approach (“Brueckner theory”) for quarkonia and heavy quarks in the QGP has been developed to interpret the lQCD results [11, 12], and the calculated spectral functions and transport coefficients have been applied to both hidden [13] and open HF observables [14] in URHICs. The main theoretical uncertainty in this and other approaches based on the Schrödinger

equation [15–19] remains the definition of the driving kernel or potential. Thus far it has been bracketed by using either the temperature dependent free or internal energies (or combinations thereof), which, roughly speaking, lead to a rather weakly or strongly coupled system, respectively.

In the present paper we address this uncertainty by utilizing the thermodynamic T -matrix to derive an expression for the singlet HQ free energy, $F_{Q\bar{Q}}(r)$. Previous work to extract the HQ potential has been done both nonperturbatively from lQCD using an effective Schrödinger equation and potential for the Wilson loop [20–23], and within weak-coupling schemes [18, 24, 25], where it was found to differ from the free energy [26]. Here, we take a diagrammatic approach by defining the potential within a field-theoretic scattering equation, which leads to a slightly different Schrödinger (or Dyson) equation than in previous work. Starting from the finite-temperature Bethe-Salpeter equation (BSE) for a static quark and antiquark in a mixed coordinate-frequency representation, we resum the ladders into a closed form expression which naturally serves as the definition of the in-medium driving kernel. The pertinent spectral function can then be straightforwardly related to the HQ free energy. A key element in evaluating the resulting expression is the energy dependence of the spectral function, in particular of imag-

inary parts induced by both one- and two-body correlations, i.e., single-quark self-energies and $Q\bar{Q}$ potential-type contributions. The former have been calculated before within the selfconsistent T -matrix formalism [27], while for the latter we take guidance from perturbative calculations [18, 24, 25]. The real part of the potential will be taken from a field-theoretic ansatz for a screened Cornell potential [28], where four parameters characterize its strength and screening properties. This set-up will then be deployed to conduct a fit to IQCD data for the static color-singlet free energy.

2. Static $Q\bar{Q}$ Free Energy in Ladder Approximation

Our objective in this section is to relate the static correlation function of an infinitely massive quark and antiquark to the HQ free energy, $F_{Q\bar{Q}}(r)$ [18, 29–31], and evaluate it in ladder approximation (BSE or T -matrix scheme). Toward this end we first carry out the ladder-diagram resummation of the BSE in the imaginary-time (Matsubara) formalism, and then reduce the BSE to the T -matrix (potential approximation).

The one-particle propagator in the infinite-mass limit is given by [18]

$$\begin{aligned} G^{(1)}(v_n, \mathbf{r}') &= \int \frac{d^3\mathbf{p}'}{(2\pi)^3} e^{i\mathbf{p}'\cdot\mathbf{r}'} \frac{1}{iv_n - \varepsilon_{\mathbf{p}'} - \Sigma^{(1)}(v_n, \mathbf{p}')} \\ &\approx \int \frac{d^3\mathbf{p}'}{(2\pi)^3} e^{i\mathbf{p}'\cdot\mathbf{r}'} \frac{1}{iv_n - M - \Sigma^{(1)}(v_n)} \equiv \delta(\mathbf{r}') \tilde{G}^{(1)}(v_n), \end{aligned} \quad (1)$$

where the δ -function implies that the particle never moves; $\Sigma^{(1)}$ denotes the single-particle selfenergy and M is its (heavy) mass; $v_n = (2n+1)\pi/\beta$ is a discrete fermionic Matsubara frequency (n integer, $T = 1/\beta$: temperature), adopting the conventions of Ref. [32] with $\hbar = 1$. The two-particle Green function inherits the δ -function structure [18]¹:

$$\begin{aligned} \tilde{G}^>(-i\tau, \mathbf{r}_1, \mathbf{r}_2 | \mathbf{r}'_1, \mathbf{r}'_2) \\ \equiv \delta(\mathbf{r}_1 - \mathbf{r}'_1) \delta(\mathbf{r}_2 - \mathbf{r}'_2) \tilde{G}^>(-i\tau, r), \end{aligned} \quad (2)$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Here and in the following the “tilde” notation is used to denote the reduced Green function without the spatial δ -functions. The static $Q\bar{Q}$ free energy, $F_{Q\bar{Q}}$, can be defined in terms of the $Q\bar{Q}$ Green function as [18]

$$F_{Q\bar{Q}}(r) = -\frac{1}{\beta} \ln \left(\tilde{G}^>(-i\beta, r) \right). \quad (3)$$

¹Color and spin indices are suppressed.

The task in the following is to calculate $\tilde{G}^>(-i\tau, r)$ in Eq. (2) within a BSE (or potential model) in the Matsubara formalism. In a “mixed” frequency-coordinate representation it is given by

$$\begin{aligned} \tilde{T}(z_\lambda, v_n, v_m | r) &= K(z_\lambda, v_n - v_m, r) \\ &- \frac{1}{\beta} \sum_k K(z_\lambda, v_n - v_k, r) G_0^{(2)}(z_\lambda - v_k, v_k | r) \tilde{T}(z_\lambda, v_k, v_m | r) \end{aligned} \quad (4)$$

where $K(z_\lambda, v_n - v_m, r)$ is the interaction kernel,

$$G_0^{(2)}(z_\lambda - v_n, v_n | r) \equiv \tilde{G}^{(1)}(z_\lambda - v_n) \tilde{G}^{(1)}(v_n) \quad (5)$$

the uncorrelated two-particle propagator, and v_m, v_n and v_k are the single-quark frequencies for incoming, outgoing and intermediate states, respectively. The total external frequency z_λ is conserved because of imaginary-time translational invariance. The full two-particle (or meson) Green function takes the form [33, 34]

$$\begin{aligned} \tilde{G}(z_\lambda, r) &= \tilde{G}_0^{(2)}(z_\lambda) \\ &+ \frac{1}{\beta^2} \sum_{k,l} \tilde{G}_0^{(2)}(z_\lambda - v_k, v_k) \tilde{T}(z_\lambda, v_k, v_l | r) \tilde{G}_0^{(2)}(z_\lambda - v_l, v_l), \end{aligned} \quad (6)$$

where $\tilde{G}_0^{(2)}(z_\lambda) \equiv -\frac{1}{\beta} \sum_k \tilde{G}_0^{(2)}(z_\lambda - v_k, v_k)$ denotes the energy loop summation (closing the two in- and outgoing lines), and likewise for $\tilde{G}(z_\lambda, r)$. The discrete energy dependencies and summations render Eq. (4) of matrix type, $T_{nm} = K_{nm} + \sum_k [K G_0^{(2)}]_{nk} T_{km}$, which is amenable to a formal solution via matrix inversion, $T_{nm} = \sum_k [1 - K G_0^{(2)}]_{nk}^{-1} K_{km}$, which, however, is not very practical.

To proceed, we adopt a potential approximation, i.e., neglect energy transfers in the interaction kernel so that $K(z_\lambda, v_n - v_k, r) \rightarrow V(z_\lambda, r)$, keeping a possible dependence on the external energy parameter z_λ representing, e.g., effects due to “dynamical screening” (in analogy to the description of electromagnetic plasmas [35]). With this approximation, the Matsubara summations in Eq. (4) decouple and one can resum the geometric series to find

$$\begin{aligned} \tilde{T}(z_\lambda, v_n, v_m | r) &= \\ V(z_\lambda, r) - \frac{1}{\beta} \sum_k V(z_\lambda, r) \tilde{G}_0^{(2)}(z_\lambda - v_k, v_k) \tilde{T}(z_\lambda, v_k, v_m | r) \\ &= \frac{V(z_\lambda, r)}{1 - V(z_\lambda, r) \tilde{G}_0^{(2)}(z_\lambda)} \equiv \tilde{T}(z_\lambda | r) \end{aligned} \quad (7)$$

The corresponding full two-particle Green function can also be written in compact form as

$$\tilde{G}(z_\lambda, r) = \frac{1}{[\tilde{G}_0^{(2)}(z_\lambda)]^{-1} - V(z_\lambda, r)}. \quad (8)$$

The above expression is a key formula of our derivation, as it can serve as the definition of the interaction kernel in the T -matrix formalism. This definition slightly differs from what has been adopted in previous approaches [18, 20, 23, 25], as can be seen from inspecting the resulting Dyson equation for the meson spectral function, see Eq. (13) below.

Next we use the analyticity of the Green function to obtain the correlation function in imaginary-time via a spectral representation,

$$\tilde{G}^>(-i\tau, r) = \int_{-\infty}^{\infty} dE' \sigma_V(E', r) \frac{e^{E'(\beta-\tau)}}{e^{\beta E'} - 1}, \quad (9)$$

with the spectral function corresponding to Eq. (8),

$$\sigma_V(E, r) = -\frac{1}{\pi} \tilde{G}_I(E + i\epsilon|r). \quad (10)$$

We use the subscript “ V ” to indicate that the spectral function is calculated in potential approximation. Here and in the following, the subscripts “ R ” and “ I ” are short-hand notations for the real and imaginary part of a given quantity. Since we work in the static limit ($M \rightarrow \infty$), we redefine the spectral function relative to the $Q\bar{Q}$ threshold by a shift $E = E' - 2M$, i.e., $\tilde{\sigma}_V(E, r) \equiv \sigma_V(E + 2M, r)$, which further simplifies Eq. (9) to

$$\tilde{G}^>(-i\tau, r) \equiv \frac{\tilde{G}^>(-i\tau, r)}{e^{-\tau(2M)}} = \int_{-\infty}^{\infty} dE \tilde{\sigma}_V(E, r) e^{-\tau E} \quad (11)$$

Combining Eqs. (3), (10) and (11), we arrive at the final expression for $F_{Q\bar{Q}}(r)$ within the T -matrix approach,

$$F_{Q\bar{Q}} = -\frac{1}{\beta} \ln \left[\int_{-\infty}^{\infty} dE \tilde{\sigma}_V(E, r) e^{-\beta E} \right] \quad (12)$$

In the following section, in the spirit of a variational approach, we will fit a physically motivated ansatz for the potential to IQCD data of the in-medium free energy.

3. Potential Extraction from the Free Energy

The two-particle Green function in medium receives contributions from the medium-modified interaction

kernel, V , and from the single-particle selfenergies, $\Sigma^{(1)}$, arising from individual interactions of each particle with the heat bath which figure into $\tilde{G}_0^{(2)}$. Both are, in principle, complex quantities and are contained in our T -matrix expression for the full two-particle Green function, Eq. (8); we write the pertinent spectral function as

$$\tilde{\sigma}_V(E, r) = -\frac{1}{\pi} \tilde{G}_I(E + 2M + i\epsilon|r) = \frac{1}{\pi} \frac{-[V_I(E, r) + \Sigma_I^{(2)}(E)]}{[E - V_R(E, r) - \Sigma_R^{(2)}(E)]^2 + [V_I(E, r) + \Sigma_I^{(2)}(E)]^2}, \quad (13)$$

where we use $\Sigma^{(2)}$ to denote the two-particle selfenergy that arises from the uncorrelated single-particle selfenergies [27, 33]. We now make concrete ansätze for the potential and selfenergies in terms of a field-theoretic model for the in-medium Cornell potential [28] and previous results from the T -matrix formalism [27]. The former leads to the following real parts of potential and uncorrelated selfenergy,

$$V_R(E, r) = -\frac{4}{3} \alpha_s \frac{e^{-m_D r}}{r} - \sigma \frac{e^{-m_s r}}{m_s} \quad (14)$$

$$\Sigma_R^{(2)}(E) = -\frac{4}{3} \alpha_s m_D + \sigma \frac{1}{m_s}, \quad (15)$$

corresponding to the r -dependent and r -independent terms of the screened Cornell potential, respectively (additional contributions from the uncorrelated 2-particle propagator are small [27] and therefore neglected in the present exploratory calculation). Their strengths are characterized by the strong coupling, $\alpha_s = g^2/(4\pi)$, and string tension σ , along with corresponding screening masses m_D and m_s , respectively. As in our previous work [12], we utilize the latter as independent fit parameters, where, for simplicity, we choose linear ansätze, $m_{D,s} = c_{D,s} g T$ with two constant coefficients, $c_{D,s}$ (which turn out to be close of order one). The imaginary part of the uncorrelated 2-particle selfenergy, $\Sigma_I^{(2)}$, follows from the folding integral of the single-particle propagators which contain the respective in-medium selfenergies. In principle, this quantity follows automatically from a selfconsistent calculation, once the potential is specified, as was carried out in Ref. [27]. For our exploratory study in the present work, we do not enforce selfconsistency but instead employ the energy dependence of $\Sigma_I^{(1)}$ as found in Ref. [27], see Fig. 3 in there. Its features are quite similar to the perturbative calculations of Ref. [31] (cf. Fig. 5 in there), but with smeared thresholds due to the nonperturbative heavy-light interaction (T -matrix). It turns out that the

resulting uncorrelated 2-particle selfenergy can be well represented by a Gaussian form,

$$\Sigma_I^{(2)}(E) = D \exp\left(-\frac{(E - \frac{\Sigma_R^{(2)}}{2})^2}{2(CT)^2}\right) \frac{4}{3} \alpha_s T. \quad (16)$$

With $C \simeq 1.6$ we can reproduce the energy dependence of the single-particle width of Ref. [27], being very similar for both internal and free energies as underlying potential. The strength parameter D , however, which we define relative to the perturbative value, $\frac{4}{3}\alpha_s T$, markedly depends on the underlying potential, varying by a factor of 2-3 between free and internal energies; as mentioned above, we here use it as a fit parameter.

Finally, we have to specify the imaginary part of the potential; its magnitude and r dependence have been determined previously in the weak coupling limit and for large bound-state size, $T \gg 1/r$, as $V_I^{\text{pert}} = \frac{4}{3}\alpha_s T \Phi(m_D r)$ [18, 24], where $\Phi(x)$ is a smoothly varying function which vanishes for $x=0$ and approaches 1 for large x (see Ref. [25] for different distance regimes). However, in our definition of the potential, we have identified the non-zero part at infinite distance with the uncorrelated selfenergy. Upon subtracting this part and again allowing for a nonperturbative enhancement with the same strength parameter D and energy dependence as in Eq. (16) we arrive at the form

$$V_I(E, r) = D \exp\left(-\frac{(E - \frac{\Sigma_R^{(2)}}{2})^2}{2(CT)^2}\right) \frac{4}{3} \alpha_s T [\Phi(Bm_D r) - 1]. \quad (17)$$

To accommodate the possibility of an r dependence different from what is given by Φ (e.g., due to string interactions), while keeping the same asymptotic limits, we rescaled its argument by a constant B (which turns out to be of order one). We note that nontrivial energy dependencies in both imaginary parts, V_I and $\Sigma_I^{(2)}$, are crucial to ensure convergence of the integration in Eq. (12). In this sense, the potential is required to go beyond the static approximation.

As an example of our method we perform a fit to a set of free energies computed in lattice QCD with $N_f = 2 + 1$ dynamical flavors [36], for temperatures $T=1.2, 1.7, 2.2 T_c$, with $T_c=196\text{MeV}$ [36]. The following parameter values were used: The Coulomb Debye mass is $m_D = 1.22gT$, close to the expected HTL value, while the string interaction is screened less with $m_s = 0.8gT$; for the string tension itself a constant value of $\sigma = 1.58\text{GeV/fm}$ turns out to be sufficient, which is larger than in vacuum but significantly smaller than in previous fits to the in-medium free energy [12, 28]. For

the strong coupling we utilize a weak temperature running, $\alpha_s = 0.288 - 0.05(\frac{T}{1.2T_c} - 1)$, decreasing from 0.288 at $1.2T_c$ to 0.238 at $2.4T_c$. For the dimensionless coefficients we find $B = 0.73$, which delays the approach of Φ to its asymptotic value of one, $C=2.07$ in Eq. (16), which produces slightly longer tails in the energy dependence of the HQ selfenergies than in the microscopic calculations of Ref. [27], and $D = 1 + \frac{3}{(T/1.2T_c)^2}$, which characterizes a nonperturbative enhancement by up to a factor of 4 close to T_c , decreasing at higher temperatures [27].

The resulting fit to the IQCD free energies is displayed in Fig. 1, showing good agreement. Reproducing the temperature dependence essentially ensures that the internal energies, $U = F + TS$ with $S = -dF/dT$, also agree with the lattice data. The underlying two-body potential turns out to lie significantly above $F_{Q\bar{Q}}$, but below $U_{Q\bar{Q}}$, at intermediate and long distances, not unlike what was found in Ref. [22]. However, the potential differs from both free and internal energy in that its derivative, characterizing the inter-quark force, is larger than for both $F_{Q\bar{Q}}$ and $U_{Q\bar{Q}}$, in a region around $r \simeq 1\text{ fm}$. This is a remnant of the confining string term in the hot medium, caused by the relatively small screening mass, m_s . Even though this force is not so large, it has a rather long range, encompassing a relatively large volume. In the heavy-light quark sector, this enables heavy quarks to effectively couple to more particles in the heat bath, relative to a short-range force. Our preliminary calculations of the HQ diffusion coefficient indeed indicate values comparable to earlier calculations using the internal energy [12], which has a stronger force but over a shorter range. In this sense, the potential obtained in the present study may still imply a strongly coupled QGP.

The above example of a fit is not unique, mostly due to uncertainties in the imaginary parts, which in the future will have to be calculated in a selfconsistent scheme as laid out in Ref. [27].² However, several qualitative, model-independent observations can be made. First, if the imaginary parts, $(V + \Sigma^{(2)})_I(E)$, are taken to zero, the spectral function σ_V becomes a δ function in energy and the potential in the denominator is forced to become the free energy, thus recovering the weak-coupling analysis of Ref. [18]. This underlines the key role of the imaginary parts of both potential and single-quark selfenergy in raising the potential above the free energy, i.e., rendering a stronger force, as controlled by the strength pa-

²The imaginary parts implicit in our fit (with $D \simeq 4$ close to T_c) are comparable to what was found before with the internal energy as potential, which indicates that they are in the expected range of the selfconsistent solution.

Figure 1: Our fit (blue lines) to the color-singlet $Q\bar{Q}$ free energies computed in lattice QCD (black dots) [36] at 3 different temperatures, $T=1.2, 1.7, 2.2 T_c$ in the left, middle and right panel, respectively. The red lines are the real parts of the underlying potential, while the green lines represent the corresponding internal energies.

parameter D in Eqs. (16) and (17). The E -dependence of $(V + \Sigma^{(2)})_I(E)$ (the “width of the width”), encoded in the width parameter C , also plays an important role, which reiterates the need for microscopic calculations, such as the T -matrix approach. The dependence on the real part is more robust and “model independent” in the sense that it can be largely determined by the small distance behavior of free energy where $\Phi(x)$ forces the imaginary part to become small. Let us also comment on the magnitude of the string tension exceeding its vacuum value by about 60%. This appears to be a rather generic feature within the screened Cornell potential ansatz. In previous works utilizing this ansatz [12, 28], where the strength of the string term was additionally allowed to develop a temperature dependence, fits to the free energy preferred string tensions of up to a factor of 3-4 times the vacuum one, albeit with a much stronger screening than in our present potential fit. As a result, the in-medium free energy never exceeds the vacuum potential (as in our present work). Similar features are also found in applications of the Dyson-Schwinger approach to quarkonia at finite T [37]. A possible microscopic mechanism for an enhanced string tension could be the release of magnetic monopole in the near- T_c region, as suggested, e.g., in Ref. [38].

4. Conclusions and Outlook

Utilizing a thermodynamic T -matrix approach, we have derived an expression for the static heavy-quark free energy that directly relates to the underlying interaction kernel, resummed in ladder approximation. The such defined kernel serves as our definition of an in-medium potential, $V_{Q\bar{Q}}(r)$, in the context of a diagrammatic many-body approach. Key elements of this framework are a non-trivial energy dependence of the pertinent $Q\bar{Q}$ spectral function, induced by the imaginary parts of both one- and two-body type, i.e., in the

single-quark selfenergies and the potential. We have applied this set-up to fit lattice-QCD “data” for the color-singlet free energy in the spirit of a variational scheme for the underlying potential. To facilitate this, we made an ansatz for the real part of the potential in terms of a screened Cornell potential, augmented with an imaginary part motivated by perturbative studies but with variable magnitude. We additionally accounted for complex heavy-quark self-energies with an energy dependence motivated by earlier selfconsistent implementations of the many-body theory. In the weak-coupling limit, with small imaginary parts, our formalism yields a potential close to the free energy, in agreement with existing literature. However, we have found a solution where the underlying potential significantly deviates from the weak coupling limit, exhibiting a rather long-range remnant of the confining force surviving well above T_c , and with large associated imaginary parts. This scenario is not inconsistent with what was obtained earlier using the internal energy as a potential approximation. First estimates indicate that the pertinent HQ diffusion coefficient is around $D_s(2\pi T) \simeq 3-5$, suggestive for a strong coupling regime.

Several further investigations are in order to scrutinize our findings. First, the calculation needs to be carried out selfconsistently, by computing the heavy-quark selfenergies from the underlying T -matrices and reinserting the former into the latter [27]. Second, the free energies should be updated with the most recent lattice-QCD results. Third, quarkonium correlators, HQ susceptibilities and transport coefficients should be computed and systematically compared to lattice data. The role of possible three-body correlations may need to be addressed. If the theoretical framework passes these tests, applications to heavy-quark and quarkonium phenomenology are warranted.

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