$\mathcal{P}$ -Odd Pion Azimuthal Charge Correlations in Heavy Ion Collisions

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We argue that the large instanton induced Pauli form factor in polarized proton-proton scattering may cause, through topological fluctuations, substantial charge-dependent azimuthal correlations for $\pi^\pm$ production in peripheral heavy ion collisions both at RHIC and LHC, thanks to the large induced magnetic field. Our results compare favorably to the measured pion azimuthal correlations by the STAR and ALICE collaborations.

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INTRODUCTION

Large single spin asymmetries in dedicated semi-inclusive deep inelastic scattering were reported by both the CLAS and the HERMES collaborations [1–4]. Similarly large spin asymmetries were reported by the STAR and PHENIX collaborations [5–7] in pion production using a polarized proton beam at collider energies. These large spin asymmetries are due to chirality flip contributions in the scattering amplitude that are not supported by QCD perturbation theory and factorization.

The QCD vacuum supports large instanton-antiinstanton fluctuations that are non-perturbative in nature and a natural source for chirality flip effects. QCD instantons are hedgehog in color-spin, that makes them ideal for triggering large spin asymmetries [8–15]. These large chirality flip contributions are beyond the realm of factorization and provides a QCD based quantitative mechanism for large spin effects in the initial state (Sivers) [16, 17].

In this note we would like to argue that the chirality flips from instanton and anti-instanton in polarized proton-proton collisions may cause, through vacuum topological fluctuations, large pion azimuthal correlations in peripheral heavy ion collisions, thanks to the large induced magnetic field in the prompt phase of the collision. The organization of this note is as follows: we first briefly review the origin of some of the chirality flip effects in the QCD vacuum. We then argue that in peripheral heavy ion collisions, a substantial magnetic field could trigger large polarizations in the protons participating in the collisions. These effects lead to large pion azimuthal correlations that are comparable to those recently reported by the STAR and ALICE collaborations. Our conclusions follow.

P-ODD EFFECTS IN THE INSTANTON VACUUM

It is commonly accepted that in a typical non-central $AuAu$ collision at RHIC as illustrated in Fig. 1, the flying fragments create a large magnetic field that strongly

FIG. 1: 2-pion correlations in $AuAu$ after the collision.

FIG. 2: 2-pion correlations in $AuAu$ at the collision.

polarizes the wounded or participant nucleons in the final state. The magnetic field is typically $eB/m_\pi^2 \approx 1$ at RHIC and $eB/m_\pi^2 \approx 15$ at the LHC and argued to last for about 1-3 fm/c [18]. We recall that in these units $m_\pi^2 \approx 10^{18}$ Gauss which is substantial. As a result, large $\mathcal{P}$ -odd pion azimuthal charge correlations were predicted to take place in peripheral heavy ion collisions [19–22].

In this work, we would like to suggest that a large contribution to the $\mathcal{P}$ -odd pion azimuthal charge correlations may stem from the prompt part of the collision as each of the incoming nucleus polarizes strongly the participating nucleons from its partner nucleus during the col-

lision process as illustrated in Fig. 2. The magnetic field is strong but short lived in the initial state, lasting for about $1/2 \text{ fm}/c$ for a typical heavy ion collision at current collider energies. Polarized proton on proton scattering can exhibit large chirality flip effects through instanton and anti-instanton fluctuations as we now show.

Consider the typical parton-parton scattering amplitude of Fig. 3 with 2-gluon exchanges. In each collision, the colliding “parton” p_i has spin s_i , and thus $u(p_i)\bar{u}(p_i) = \frac{1}{2}\not{p}_i(1 + \gamma_5\not{s}_i)$. The parton p_1 from the A-nucleus encounters an instanton or anti-instanton as depicted by the gluonic form-factor. The latter follows from standard instanton calculus [23]

$$M_\mu^a = t^a [\gamma_\mu - \mathbf{P}_+ \gamma_+ \sigma_{\mu\nu} q^\nu \Psi - \mathbf{P}_- \gamma_- \sigma_{\mu\nu} q^\nu \Psi] \quad (1)$$

with $\gamma_\pm = (1 \pm \gamma_5)/2$ and

$$\Psi = \frac{F_g(\rho_c Q) \pi^4 (n_I \rho_c^4)}{m_q^* g_s^2} \quad (2)$$

and $F_g(x) \equiv 4/x^2 - 2K_2(x)$ with $F_g(0) = 1$. Here $n_I \approx 1/\text{fm}^4$ is the effective instanton density, $\rho_c \approx 1/3 \text{ fm}$ the typical instanton size and $m_q^* \approx 300 \text{ MeV}$ the constitutive quark mass in the instanton vacuum. The momenta of the incoming partons as well as the momentum Q of the transferred gluon are assumed small or $p\rho_c, Q\rho_c \leq 1$. $\mathbf{P}_+ = 1$ stands for an instanton insertion and $\mathbf{P}_- = 1$ for an anti-instanton insertion. In establishing (1), the instanton and anti-instanton zero modes are assumed to be undistorted by the prompt external magnetic field. Specifically, the chromo-magnetic field B_G is much stronger than the electro-magnetic field B , i.e. $|g_s B_G| \gg |eB| \approx \text{or } m_\pi^2 \rho_c^2 \approx 0.004 \ll 1$. The deformation of the instanton zero-modes by a strong magnetic field have been discussed in [24]. They will not be considered here.

In terms of (1), the contribution of Fig.3 to the differential cross section is

$$d\sigma \sim \frac{g_s^4}{|p_1 - k|^4} \text{tr} [M_\mu^a \not{p}_1 (1 + \gamma_5 \not{s}_1) \gamma_0 (M_\nu^b)^\dagger \gamma_0 \not{k}] \times \text{tr} [\gamma^\mu t_a \not{p}_2 (1 + \gamma_5 \not{s}_2) \gamma^\nu t_b \not{k}'] \quad (3)$$

which can be decomposed into $d\sigma \approx d\sigma^{(0)} + d\sigma^{(1)}$ in the dilute instanton liquid. The zeroth order contribution is

$$d^{(0)}\sigma \sim 64g_s^4 \frac{2(k \cdot p_2)(p_1 \cdot p_2) + (k \cdot p_1)(p_1 \cdot p_2 - k \cdot p_2)}{|p_1 - k|^4} \quad (4)$$

where we used $k' = p_1 + p_2 - k$. The first order contribution is

$$d^{(1)}\sigma \sim \frac{64g_s^4}{|p_1 - k|^4} [(p_1 \cdot p_2)^2 + (k \cdot p_2)(p_1 \cdot p_2)] \times (k \cdot s_1) (\mathbf{P}_+ - \mathbf{P}_-) \Psi \quad (5)$$

after using $p_1 \cdot s_1 = 0$ and $p_1^2 = k^2 = 0$. Converting to standard parton kinematics with $p_1 \rightarrow x_1 P_1$, $p_2 \rightarrow x_2 P_2$ and $k \rightarrow K/z$, we obtain for the ratio of the $\mathcal{P}$ -odd to $\mathcal{P}$ -even contributions in the differential cross section

$$\frac{d^{(1)}\sigma}{d^{(0)}\sigma} = \frac{x_1(P_1 \cdot P_2)^2 + \frac{1}{z}(K \cdot P_2)(P_1 \cdot P_2)}{2(K \cdot P_2)(P_1 \cdot P_2) + (K \cdot P_1)(\frac{x_1}{x_2}P_1 \cdot P_2 - \frac{K \cdot P_2}{zx_2})} \times (K \cdot s_1) (\mathbf{P}_+ - \mathbf{P}_-) \Psi \quad (6)$$

Now consider the kinematics appropriate for the collision set up in Fig. 2,

$$\begin{aligned} P_{1/2} &= \frac{\sqrt{s}}{2}(1, 0, 0, \pm 1) \\ K &= (E, K_\perp \cos \Delta\phi, K_\perp \sin \Delta\phi, \frac{\sqrt{s}}{2}x_F) \\ s_1 &= (0, 0, s_1^\perp, 0) \end{aligned} \quad (7)$$

where $K_\perp$ and $E^2 = K_\perp^2 + sx_F^2/4 + m_\pi^2$ are the transverse momentum and total squared energy of the outgoing pion respectively. x_F is the pion longitudinal momentum fraction. Thus

$$\begin{aligned} \lim_{s \rightarrow \infty} \frac{d^{(1)}\sigma}{d^{(0)}\sigma} &= (\sin \Delta\phi) s_1^\perp \frac{x_F + x_1 z}{x_F z} \frac{K_\perp}{m_q^*} \frac{\pi^3 (n_I \rho_c^4)}{8\alpha_s} \\ &\times F_g \left(\rho \sqrt{\frac{x_1}{x_F z} (K_\perp^2 + m_\pi^2)} \right) (\mathbf{P}_- - \mathbf{P}_+) \end{aligned} \quad (8)$$

We note that Eq. 8 vanishes after averaging over the instanton liquid background which is $\mathcal{P}$ -even

$$\left\langle \frac{d^{(1)}\sigma}{d^{(0)}\sigma} \right\rangle = 0 \quad (9)$$

since on average $\langle \mathbf{Q} \rangle = \langle \mathbf{P}_+ - \mathbf{P}_- \rangle = 0$.

P-ODD CORRELATIONS IN AA COLLISIONS THROUGH INSTANTONS

Now consider hard pp collisions in peripheral AA collisions as illustrated in Fig. 2. The Magnetic field is strong enough at the collision to partially polarize the colliding protons. Say $c\%$ of the wounded protons from a given nucleus get polarized by the partner colliding nucleus. For simplicity, we set $s_\perp u(x, Q^2) = c\% \Delta_s u(x, Q^2)$ and $s_\perp d(x, Q^2) = c\% \Delta_s d(x, Q^2)$, with $\Delta_s u(x, Q^2)$ and

FIG. 3: Gluon Exchange. The blob is an instanton or anti-instanton insertion. See text.

$\Delta_s d(x, Q^2)$ as the spin polarized distribution functions of the valence up-quarks and valence down-quarks in the proton respectively. We also assume that the outgoing u quark turns to π^+ and that the outgoing d quark turns to π^- . With this in mind, we may rewrite the ratio of differential contributions in (8) following [25–28] as

$$\frac{d\mathbf{N}}{d\phi_\alpha} \sim 1 - 2a_\alpha \sin(\phi - \Psi_{RP}) \quad (10)$$

with $\alpha = \pm$ or

$$a_+ = \frac{\Delta_s u(x, Q^2)}{u(x, Q^2)} \Upsilon \mathbf{Q} \quad a_- = \frac{\Delta_s d(x, Q^2)}{d(x, Q^2)} \Upsilon \mathbf{Q} \quad (11)$$

and

$$\Upsilon \equiv \frac{x_F + xz}{x_F z} \frac{K_\perp}{m_q^*} \frac{\pi^3 (n_I \rho_c^4)}{16\alpha_s} F_g \left(\rho \sqrt{\frac{x}{x_F z}} (K_\perp^2 + m_\pi^2) \right) \quad (12)$$

While on average $\langle a_\alpha \rangle = 0$ since $\langle \mathbf{Q} \rangle_V = 0$, in general $\langle a_\alpha a_\beta \rangle \neq 0$ for the 2-particle correlations. Explicitly

$$\begin{aligned} -\langle a_{\pi^+} a_{\pi^-} \rangle &= -\left(\frac{\Delta_s u(x, Q^2)}{u(x, Q^2)} \frac{\Delta_s d(x, Q^2)}{d(x, Q^2)} \right) \Upsilon^2 \langle \mathbf{Q}^2 \rangle_V \\ -\langle a_{\pi^+} a_{\pi^+} \rangle &= -\left(\frac{\Delta_s u(x, Q^2)}{u(x, Q^2)} \right)^2 \Upsilon^2 \langle \mathbf{Q}^2 \rangle_V \\ -\langle a_{\pi^-} a_{\pi^-} \rangle &= -\left(\frac{\Delta_s d(x, Q^2)}{d(x, Q^2)} \right)^2 \Upsilon^2 \langle \mathbf{Q}^2 \rangle_V \end{aligned} \quad (13)$$

According to [29], $\Delta_s u(x, Q^2)/u(x, Q^2) = 0.959 - 0.588(1 - x^{1.048})$ and $\Delta_s d(x, Q^2)/d(x, Q^2) = -0.773 + 0.478(1 - x^{1.243})$. For reasonable values of $\langle x \rangle$, $\langle a_{\pi^+} a_{\pi^+} \rangle \sim \langle a_{\pi^-} a_{\pi^-} \rangle \sim -\langle a_{\pi^+} a_{\pi^-} \rangle$ as expected [25–28].

A more quantitative comparison to the reported data in [25, 28] can be carried out by estimating the fluctuations of the topological charge $\mathbf{Q}$ in the prompt collision 4-volume $V \approx (\tau^2/2)\Delta\eta V_\perp(b)$. In the latter, $\tau \approx 1/2$ fm/c is the prompt proper time over which the induced magnetic field is active, $\Delta\eta$ is the interval in pseudo-rapidity and $V_\perp(b)$ the transverse collision area for fixed impact parameter b . Through simple geometry

$$V_\perp(b) = 2R^2 \left(\arccos\left(\frac{b}{2R}\right) - \frac{b}{2R} \sqrt{1 - \left(\frac{b}{2R}\right)^2} \right) \quad (14)$$

where R is the radius of two identically colliding nuclei. $\mathbf{Q}^2$ involves a pair $\mathbf{P}, \mathbf{P}'$ of instanton-antiinstanton. Specifically,

$$\langle \mathbf{Q}^2 \rangle_V = \langle (\mathbf{P}_+ - \mathbf{P}_-)(\mathbf{P}'_+ - \mathbf{P}'_-) \rangle_V \quad (15)$$

If we denote by $N_\pm$ the number of instantons and antiinstantons in V , with $N = N_+ + N_-$ their total number, then in the instanton vacuum the pair correlation follows from

$$\langle \mathbf{Q}^2 \rangle_V \equiv \left\langle \left(\frac{N_+ - N_-}{N_+ + N_-} \right)^2 \right\rangle_V \approx \frac{\langle (N_+ - N_-)^2 \rangle_V}{\langle (N_+ + N_-)^2 \rangle_V} \quad (16)$$

Assuming $N_\pm$ to be large in V it follows that [30]

$$\langle \mathbf{Q}^2 \rangle_V \approx \frac{\langle N \rangle_V}{\langle N \rangle_V (\langle N \rangle_V + 4/\mathbf{b})} \quad (17)$$

The deviation from the Poissonian distribution in the variance of the number average reflects on the QCD trace anomaly in the instanton vacuum or $\langle N^2 \rangle_V - \langle N \rangle_V^2 = 4/\mathbf{b} \langle N \rangle_V$ and vanishes in the large N_c limit [30]. Here $\mathbf{b} = 11N_c/3$ is the coefficient of the 1-loop beta function $\beta(\rho_c) \approx \mathbf{b}/\ln(\Lambda\rho_c)$ (quenched). Thus

$$\langle \mathbf{Q}^2 \rangle_V \approx \frac{1}{n_I(\tau^2 \Delta\eta V_\perp(b)/2) + 4/\mathbf{b}} \quad (18)$$

where we have used that the mean $\langle N \rangle_V = n_I V$ in the volume V . The topological fluctuations are suppressed by the collision 4-volume. Note that we have ignored the role of the temperature on the the topological fluctuations in peripheral collisions. Temperature will cause these topological fluctuations to deplete and vanish at the chiral transition point following the instanton-antiinstanton pairing [31]. So our results will be considered as upper-bounds.

For simplicity, we will set $\langle x \rangle \approx 0.01$ for each parton and $\langle z \rangle \approx 0.5$. The measured multiplicity spectra in [32] at different centralities suggest $m_\pi < \langle K_\perp \rangle < 3m_\pi$. We will set $\langle K_\perp \rangle = 2m_\pi$ in our analysis. We will assume a moderate polarization or $c\% = 15\%$ in the collision volume for a general discussion. We fix $\tau = 1/2$ fm to be the maximum duration of the magnetic field polarization, and set the pseudo-rapidity interval approximately $(-1, 1)$ for both STAR [25] and ALICE [28]. The radius of the colliding nuclei will be set to $R = 1\text{fm} \times \sqrt[3]{A}$ where A is the atomic number.

FIG. 4: Pion azimuthal charge correlations versus the data [25] from STAR at $\sqrt{s} = 200\text{GeV}$. See text.

FIG. 5: Pion azimuthal charge correlations versus the data [25] from STAR at $\sqrt{s} = 200\text{GeV}$. See text.

The centrality is approximated as $n\% = b^2/(2R)^2$ [33]. Our results are displayed in Fig. 4 for $AuAu$ and Fig. 5 for $CuCu$ collisions at $\sqrt{s} = 200\text{GeV}$ (STAR), and in Fig. 6 for $PbPb$ collisions at $\sqrt{s} = 2.76\text{TeV}$ (ALICE). We recall that [34]

$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle \equiv -\langle a_\alpha a_\beta \rangle \quad (19)$$

For the like-charges the results compare favorably with the data. For the unlike charges they overshoot the data especially for the heavier ion. Our results show a difference between $\pi^+\pi^+$ and $\pi^-\pi^-$ as we only retained the protons in our analysis. The inclusion of the neutrons would result into the same charge correlations for $\pi^+\pi^+$ and $\pi^-\pi^-$ by isospin symmetry.

CONCLUSIONS

Large chirality flips from instanton and anti-instanton contributions as assessed in polarized pp experiments may contribute substantially to $\mathcal{P}$ -odd azimuthal correlations in *unpolarized* AA collisions at RHIC and LHC,

FIG. 6: Pion azimuthal charge correlations versus the data from ALICE [28] at $\sqrt{s} = 2.76\text{TeV}$. See text.

thanks to the topological fluctuations in the QCD vacuum and a large induced magnetic field in the prompt part of the collision. The effect is stronger in peripheral collisions and subside in central collisions for a moderate prompt polarization. Simple estimates based on the collision geometry and the magnetic field profile, compare favorably to the currently reported pion azimuthal charge correlations by the STAR and ALICE collaborations.

Since most of our effects involve peripheral pp collisions and the fluctuations of the topological charge in a finite volume, we expect it to be less sensitive to final state interactions. The bulk charge multiplicities in most peripheral events are overall consistent with pp multiplicities.

While the details of our results depend on the fraction of the polarized would-be wounded nucleons as well as the duration of the prompt collision, the overall trend does not. Our arguments involve only promptly polarized protons in the presence of topological fluctuations in the confined vacuum, and therefore complement the chiral magnetic effect suggested in the deconfined vacuum [19–22].

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