Optimal control of indoor-air cooling in buildings using a reduced order model

S. Ben Ayed^a, D. Kim^b, J. T. Borggaard^a, E. M. Cliff^a

^a*Interdisciplinary Center for Applied Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA*

^b*School of Mechanical Engineering, Purdue University, West Lafayette, Indiana, USA*

Abstract

In this work, we use Computational Fluid Dynamics (CFD) to generate the distributed dynamic responses of temperature and moisture in a restaurant that correspond to perturbations of input variables. The CFD model is validated with experimental data of the roof top units return temperatures. A Reduced Order Model (ROM) is developed by approximating the responses to these perturbations using a linear time-invariant model. The resulting indoor-air model is coupled to a dynamic envelope model with longer time scales. Assuming the outside temperature distribution and the occupants' loads are known throughout the day, optimal control is applied to our coupled model to minimize the cooling power subject to local and global comfort constraints. The results show that the applied method is more efficient and comfortable than the results given by the experimentally measured response of a conventional set-point control strategy.

Keywords: Optimal control, Building energy, Reduced order model, Berkeley library for optimization modeling, Envelope model, Computational Fluid

Email address: samah09@vt.edu (S. Ben Ayed)

Preprint submitted to Energy

April 24, 2016

10 Dynamics.

11 **1. Introduction**

12 According to The U.S. Energy Information administration, in 2013, 40% of
13 total energy consumption in the U.S. was consumed in residential and commer-
14 cial buildings. This generally consists of heating-ventilation-air-conditioning
15 (*HVAC*), lighting, and plug loads. Thus, even a modest reduction in build-
16 ing energy use represents significant contributions toward the relief of a host
17 of pressing environmental issues. Here we focus on cooling requirements and
18 specifically on the development of models suitable for study of predictive con-
19 trol of the indoor environment.

20
21 Optimal control of the *HVAC* energy in buildings has been the object of
22 various research works during the last decades. Naidu and Rieger wrote an
23 extensive review about advanced control strategies in *HVAC* systems ([1], [2]).
24 Sane et al. [3] presented a survey of problems in controls and optimization re-
25 lated to commercial chilled water cooling plants. They included in multivariable
26 optimal control, dynamic resource allocation and control over networks. Sev-
27 eral works have explicitly reported the savings due to the use of optimal control
28 for *HVAC* systems. House and Smith [4] used optimal control of the supply
29 air temperature to minimize the cost for energy consumption. Comparing this
30 method to the conventional control, they found a cost savings of 11%. Yu and
31 Chan ([5]) used an optimal control strategy to control the cooling towers and

condenser water pumps in a water-cooled chiller system. They concluded that their procedure contributed to annual energy savings of 8.6% and cost savings of 9.9%.

For a good estimate of the optimal energy cost, thermal load modeling is very important. Various models and computational techniques have been adopted. Oldewurtel et al. [6] claims that computer-aided modeling tools like TRNSYS [7] and EnergyPlus [8] are mainly designed to estimate the energy use of building and cannot be readily used for control. They suggested the use of a thermal Resistance-Capacitance (RC) network for control purposes. Many others [9–11] used the same simple RC model to estimate the thermal load. However, Ma et al. [12] used a black box model structure instead of an RC model. They used EnergyPlus to model a commercial building, generated outputs for a nominal configuration, used the input/output data to identify mathematical models for power and temperature, then generated the optimal control. Zlatanovi [13] used experimental data to generate the energy saving estimation model. This was based on monitoring the energy consumption of each HVAC system appliance in a large trade center on a daily basis for one year.

Modeling the indoor environment is very important in small to medium commercial buildings that utilize rooftop units (RTUs) to condition large open spaces. Examples include retail stores, restaurants and factories. In fact, for these buildings, the well-mixed assumption is not practical due to non-uniformly distributed diffuser systems, local cooling/heating actions corresponding to each

55 RTU, and non-uniform heat gains. In the present work, we develop a coupled
56 CFD-envelope model having two distinct time scales for a specific restaurant.
57 The CFD model captures important dynamics of the spatial distributions of
58 temperature and moisture inside the building; while the envelope model studies
59 possible inside/outside heat transfer mechanisms, namely convection, conduc-
60 tion and radiation. Various approaches to couple prediction codes for fluid
61 dynamics and solid conduction in a problem of *conjugate heat transfer* have
62 been studied [14, 15].

63 We use optimal control with this coupled model to minimize the cooling energy
64 in a given restaurant. Since the optimization procedure is iterative, a reduced
65 order model that captures the essential dynamics of the problem and has a low
66 computational cost [16, 18, 19] is developed for performance analysis of optimal
67 control for open-spaces by utilizing ROM. For this study, the reduced order
68 model is created from CFD simulations such that the steady-state responses
69 are adequately captured and such that the resulting model easily couples to the
70 envelope model.

71

72 **2. CFD of the restaurant indoor air and Reduced Order Model**

73 Our approach to developing a reduced-order model is to construct a lin-
74 ear time-invariant (LTI) model that approximates the input-output behavior
75 observed in computational fluid dynamic (CFD) simulations. Accordingly, a
76 CFD model for the restaurant was developed using FLUENT. The coupling be-

77 tween the indoor-air and the envelope models is naturally achieved by treating
 78 the bounding surface temperatures as inputs to the CFD and the corresponding
 79 surface heat fluxes as outputs; that is, we use CFD to approximate the Dirichlet-
 80 to-Neumann (dynamic) response. Other CFD inputs include the mass-flow-rate,
 81 temperature, and humidity supplied by each of the roof-top-units (RTU), the
 82 sensible and latent loads provided by occupants, and external loads generated
 83 by the kitchen activity. Additional outputs of interest include air tempera-
 84 ture/humidity at selected zones/locations, and return air properties. The mesh
 85 is formed of 4.70m tetrahedral cells with 624k vertices. We use a refinement of
 86 about 1/3 around the diffuser and returns and approximately 1/7 in the wall
 87 regions where the thermostat temperatures are averaged (around 250 tetrahe-
 88 dral faces). The moisture is treated by integrating the species transport model
 89 in FLUENT. Hence the fluid is a mixture of air and water vapor. This FLU-
 90 ENT feature models the mixing and transport of species by solving conservation
 91 equations describing convection and diffusion for the species. The turbulence
 92 model used in the work is a standard k-Epsilon model with standard wall func-
 93 tions. In fact, it is a simple two-equation model in which the solution of two
 94 separate transport equations allows the turbulent velocity and length scales to
 95 be independently determined. Robustness, economy, and reasonable accuracy
 96 for a wide range of turbulent flows explain this model's popularity in industrial
 97 flow and heat transfer simulations [17].

98 *2.1. Nominal Boundary/Source Conditions*

99 The indoor-air response is decomposed into a nominally steady part and a
100 perturbation. For the steady part we apply fixed boundary and source condi-
101 tions enumerated here:

102 *Bounding Surfaces*; For five of the bounding surfaces we prescribe no-slip
103 flow and fixed temperature (Dirichlet) data

- 104 • North Wall: 27.00°C
- 105 • East Wall: 25.58°C
- 106 • West Wall: 28.43°C
- 107 • Floor: 24.92°C
- 108 • Ceiling: 25.48°C

109 *Kitchen Interface*: The south wall boundary of our computational domain
110 (see Fig. 1) separates the dining zone from the kitchen, which is not modeled.
111 For the solid part of this boundary we impose no-slip flow and adiabatic thermal
112 conditions. For the opening(s) between the kitchen and the dining zones (red)
113 we have prescribed the flow from the kitchen.

- 114 • mass-flow-rate: 0.591 kg.s
- 115 • temperature: 25.00°C
- 116 • water mass-fraction: 16g/kg

117 *Supply Air:* Each supply air vent is a 2 ft by 2 ft ceiling mounted square.
118 The air supply from each RTU is uniformly distributed among the associated
119 vents, and is supplied at 16.60°C. The velocity at each vent is normal (*i.e.*
120 downward), and is spatially distributed using a flattened parabolic profile.

- 121 • RTU#1- mass-flow-rate: 2.9320 kg/s; water mass-fraction 7 g/kg; dis-
122 tributed over 19 supply vents (blue)
- 123 • RTU#2- mass-flow-rate: 0.5320 kg/s; water mass-fraction 9 g/kg; dis-
124 tributed over 04 supply vents (red)
- 125 • RTU#3- mass-flow-rate: 0.3990 kg/s; water mass-fraction 9 g/kg; dis-
126 tributed over 03 supply vents (purple)
- 127 • RTU#4- mass-flow-rate: 0.5320 kg/s; water mass-fraction 9 g/kg; dis-
128 tributed over 04 supply vents (green)

129 Note that RTU#3 supplies four vents but only three of these are in the compu-
130 tational domain.

131 *Return Air:* Each of the return vents is ceiling mounted. These are specified
132 as *pressure outlet* with the flow normal to the boundary, and the pressure at
133 40 Pa below that on the floor. The pressure difference approximates the static
134 head loss under vertical equilibrium (*i.e.* $\Delta P = -\rho g \Delta h$) at a ceiling height of
135 3.4 m.

136 *Dining Area Source:* The dining area is the roughly 69 m³ volume shown in
137 yellow in Fig. 1. The intent is to simulate the sensible/latent loads produced

138 by the occupants. In this volume we uniformly distribute an energy source of
139 approximately 8,750 W, and a moisture source of approximately 3.08 g/s.

140 *Wine Bar Area Source:* The winebar area is the roughly 23 m³ volume that
141 surrounds the winebar in the north end of the domain. The intent is to sim-
142 ulate the sensible/latent loads produced by the occupants. In this volume we
143 uniformly distribute an energy source of approximately 3,310 W, and a moisture
144 source of approximately 1.10 g/s.

145 2.2. Steady state response

146 The system was subjected to fixed boundary and source conditions chosen
147 to represent a mean condition in the space (see § 2.1). FLUENT was run in both
148 steady and unsteady modes; the latter consisted of seven hours of simulated
149 time with external (boundary & source) conditions fixed. Some responses for
150 the last hour are shown in Fig. 2. The black horizontal lines are averages over
151 the one-hour window.

152 The meaning of these various temperatures can be understood in terms of
153 Fig. 1. In particular

- 154 • **dining zone** is the yellow rectangular volume in the center of the figure.
- 155 • **wine_bar zone** is the yellow volume in the shape of a reflected **D** in the
156 left of the figure.
- 157 • **fireplace** is the surface of the red box in the left foreground (to the left
158 of the entrance *bump-in*).

159 Note that the *room average* is the least oscillatory: it is the mass-averaged
160 temperature over the entire room. The *dining* and *wine_bar* values are mass-
161 averaged over the cited volumes. The *fireplace* value is averaged over the external
162 surfaces of the designated red-box, and exhibits the largest oscillations. Also,
163 note that the values at the *right* end ($t = 0$) do not generally match the average,
164 and this is particularly noticeable in the *fireplace* data. One might expect that
165 the oscillations would decay and the internal flow in the room would become
166 steady. In fact there is no guarantee that a steady flow exists, moreover the
167 computed results depend on the grid which is likely somewhat too coarse.

168 2.3. FLUENT model validation

169 In the early morning of 21 August 2013, a test to validate the FLUENT
170 results was conducted at the Harvest Grill site. At approximately 03:30 the
171 supply air temperature from RTU1 was reduced from about 16.5°C to about
172 9.5°C as shown in Figure 3. A FLUENT run was made wherein a piecewise linear
173 approximation of this temperature profile was prescribed for the RTU1 supply
174 air while (most) other source/boundary conditions were maintained at nominal
175 values. The only exception was that the temperature of the (supply) kitchen air
176 was changed from the nominal 25°C to 21°C. The change was made because at
177 the time of the experiment there was no cooking activity in the kitchen. The
178 chosen value (21°C) was the ambient temperature in the RTN3 return air which
179 is closely coupled to the kitchen air. Initial data for the simulation was a result
180 of previous *steady state* runs.

181 Figure 4 displays the comparisons of the FLUENT and measured data for the

182 return air temperatures in RTN1 and RTN2. Note that these start at rather
 183 different values; that is, the initial condition in the FLUENT simulation does
 184 not well-represent the overnight conditions of the test. To compare the trends
 185 we shifted the FLUENT predictions so that the initial values match. Figure 5
 186 displays the shifted comparisons for all four return air locations. In the time
 187 interval [03:30, 05:00] the FLUENT predictions are very close to the data. After
 188 05:00 the supply air temperatures for the other RTU's began to change; a feature
 189 not captured in the FLUENT simulation.

190 *2.4. Input Variables*

191 Twenty-two of the boundary conditions discussed in § 2.1 were identified as
 192 useful for injecting control or disturbance to the system. These are enumerated
 193 in Table 1.

194 For each input we ran a FLUENT simulation beginning from the final state of
 195 the *steady* simulation and proceeding for 7200 s. In each run, one of the inputs
 196 in Table 1 was subjected to a ramp input beginning at $t = 8$ s, ramping to the
 197 appropriate perturbation value over a 60 s span, then remaining constant at the
 198 perturbed value.

199 *2.5. LTI Model*

200 Our approach is to construct an LTI model that approximates the perturbed
 201 CFD responses generated by FLUENT simulations. Whereas there are a number
 202 of approaches to extract reduced-order LTI models from response data (for ex-
 203 ample [18, 19]), we are motivated to find a low order model with the appropriate

204 asymptotic behavior in a computationally efficient way.

205 *Nominally Steady Outputs* In order to describe perturbed responses in a
206 meaningful way, we first deal with defining *steady* values of the output variables.
207 The output variables are listed in Table 2. Outputs 1 to 5 represent the average
208 thermal flux through the walls which will be coupled to a building envelope
209 model as shown in Fig. 10. Outputs 6 to 9, 10 to 13 and 14 to 17 represent the
210 return air conditions, i.e. the average of the mass flow rates, temperatures and
211 humidities respectively. These are used to calculate HVAC power consumption.
212 Outputs 18 to 21 indicates local temperatures at thermostat locations. In order
213 to measure comfort, air conditions at some local area of interests and zonal
214 volume averaged temperatures and humidities were chosen as outputs. Outputs
215 22 to 33 are the conditions at the local points while the rests are the zonal
216 conditions. The final two outputs 42 and 43 are left for the average temperature
217 and moisture of the whole space.

218 It is clear from Fig. 2 that notable oscillations persist in the output variables.
219 There are several potential explanations for this observed behavior:

- 220 1. there is no guarantee that the Reynolds-Averaged Navier-Stokes PDE
221 model has a steady solution;
- 222 2. the grid may not be sufficiently resolved to represent the PDE behavior;
223 and/or,
- 224 3. the iterative procedure for the nonlinear algebraic system may not be
225 sufficiently converged.

226 Whatever the cause(s) we simply define steady values for the outputs by aver-

227 aging over a time-window. In particular, steady output values are computed by
 228 averaging the observed outputs over the final 60 minutes of the nominally steady
 229 solution. Steady values for the four output variables in Fig. 2 are indicated by
 230 solid lines. Steady values for all of the output variables are listed in Table 2.

231 *CFD Perturbations* As noted above, in each run, one of the inputs in Table
 232 1 was subjected to a ramp input beginning at $t = 8$ s, ramping linearly to
 233 the corresponding perturbation value over 60 s, then remaining constant at the
 234 perturbed value. In each case the output variables (see Table 2) were recorded
 235 at 4 s intervals (1800 samples). For each input, the perturbed output values
 236 were computed as:

$$y_{\text{pert}}(t_j) = Y_{\text{data}}(t_j) - Y_{\text{nom}} \quad j = 1, \dots, 1800 .$$

237 *Best Fit* There are several elegant approaches to extract a model from time-
 238 response data ([19, 20]). Our approach decomposes the problem into sub-
 239 problems by treating each input response separately. That is, we hypothesize
 240 a structure for a low-order model and formulate an optimization problem to
 241 find appropriate parameter values. Specifically, for each input we propose a
 242 first-order model of the form

$$\dot{x}(t) = -\frac{1}{\tau}x(t) + b u(t) \tag{1}$$

$$y(t) = C x(t) + D u(t) , \tag{2}$$

243 where τ is a time constant, b is a scalar, and $C, D \in \mathbb{R}^{43}$. We take $b = 0.01$
 244 (scaling). Most of the 22×43 input/output pairs are represented as strictly-
 245 proper transfer functions ($D = 0$). However, for the first five inputs (surface

temperatures) the heat flux output for that surface has a single non-zero entry in D , since the surface temperature is directly coupled to the heat flux on that surface. Thus, we expect that for each surface heat flux output (the first five) we have $D^j = h\delta_{i,j}$, whereas for all other outputs we have $D = 0$.

The time-scales of the building envelope are typically much longer than those of the indoor-air. Therefore, during most of the response the indoor-air system will be near equilibrium. For this reason we require that the ROM accurately represent the indoor-air dynamics for responses near equilibrium. This requirement is not naturally imposed in other model reduction methods and leads to our *ad-hoc* approach. Since the input perturbation function u goes to a constant value (say, u_{ss}), then with $\tau > 0$ the dynamics (1) will go to the stable equilibrium

$$x_{ss} = \tau b u_{ss} \Rightarrow y_{ss} = (C \tau b + D) u_{ss} .$$

Solving for C we find

$$C(\tau, D) = \frac{y_{ss}/u_{ss} - D}{\tau b} , \quad (3)$$

where the vector $D \in \mathbb{R}^{43}$ has (at most) a single non-zero entry. To implement (3) we must specify a method for computing a steady-state value of the perturbed output, *i.e.* $y_{ss} \in \mathbb{R}^{43}$. Because the output data includes residual oscillations it is prudent to use some form of averaging. For the results presented here we averaged $y(t_i)$ over the last 120 samples (out of 1800 total), that is $i \in \{1681, 1682, \dots, 1800\}$. This amounts to an 8 minute wide sampling window.

265 With these preliminaries we define a scalar-valued error function

$$J(\tau, D) = \sum_{i=1}^{1800} [y_{\text{pert}}(t_i) - y(t_i; \tau, D)]^2, \quad (4)$$

266 where $y(t_i; \tau, D)$ is computed from (1)-(2) with the state-to-output map given
 267 by (3), and the input (u) is the step-like input described in § 2.5. For each
 268 input we compute optimal values of τ and (possibly) D by minimizing the error
 269 functional J in (4). These calculations were done using MATLAB's `fminsearch`
 270 procedure.

271 The five non-zero values for the D arrays are;

- 272 1. q_north: $2.912 \text{ W/m}^2/^{\circ}\text{C}$
- 273 2. q_east: $2.038 \text{ W/m}^2/^{\circ}\text{C}$
- 274 3. q_west: $3.514 \text{ W/m}^2/^{\circ}\text{C}$
- 275 4. q_floor: $4.281 \text{ W/m}^2/^{\circ}\text{C}$
- 276 5. q_ceiling: $1.423 \text{ W/m}^2/^{\circ}\text{C}$

277 Note that these can be interpreted as (time and spatial) averages of the film
 278 cooling coefficient on the respective surfaces.

279 2.6. ROM-FLUENT Comparisons

280 Fig. 6 provides comparisons of the FLUENT and the reduced-order-model
 281 (ROM) responses to a change in the east wall temperature. The input pertur-
 282 bation ramps to $+5^{\circ}\text{C}$ over 60 s and the corresponding ramp in the East wall
 283 heat flux is seen in the upper graph. In the lower graph the ceiling heat flux
 284 is seen to decrease by 0.6 W/m^2 , presumably the result of an increase in the

285 air temperature engendered by the increased wall temperature. Note that the
286 graphs display only the first 15 minutes of the response; the data fits are based
287 on two hours of data.

288 Similarly, Fig. 7 provides comparisons of the FLUENT and the reduced-order-
289 model (ROM) responses to a 5°C decrease in the RTU#1 supply air tempera-
290 ture. The upper graph displays the change in return 1 air temperature, whereas
291 the lower graph characterizes the temperature in the dining zone.

292 Fig. 8 provides also comparisons of the FLUENT and the reduced-order-model
293 (ROM) responses to a 3.2g/kg increase in the kitchen inflow moisture. The
294 upper graph displays the change in return 1 water vapor mass fraction, whereas
295 the lower graph shows the response of the dining zone water vapor mass fraction.

296 **3. Development of a thermal building envelope model**

297 In order to connect a reduced-order indoor air model to a building enve-
298 lope model, a general formula derived in [21] is adopted. The majority of the
299 derivations in this section is the same as [21], but it is slightly modified for the
300 coupling.

301 *3.1. Conduction through walls*

302 A finite volume formulation is used to describe the heat conduction through
303 walls and is depicted in Fig. 9.

$$\begin{aligned}
\vec{q}_j &\equiv \begin{bmatrix} q_{gen}|_j^1 \\ q_{gen}|_j^2 \\ \dots \\ q_{gen}|_j^{Nw} \end{bmatrix}, \vec{T}_j \equiv \begin{bmatrix} T_j^1 \\ T_j^2 \\ \dots \\ T_j^{Nw} \end{bmatrix}, \tilde{H}_j^{L/R} \equiv \begin{bmatrix} h_{cd}^{L/R}|_j^1 & & 0 \\ & h_{cd}^{L/R}|_j^2 & \\ & & \dots \\ 0 & & & h_{cd}^{L/R}|_j^{Nw} \end{bmatrix}, \\
\text{and } \tilde{C}_j &\equiv \begin{bmatrix} \rho C w|_j^1 & & 0 \\ & \rho C w|_j^2 & \\ & & \dots \\ 0 & & & \rho C w|_j^{Nw} \end{bmatrix},
\end{aligned}$$

equation ?? can be expressed in the following matrix form

$$\tilde{C}_j \frac{d\vec{T}_j}{dt} = \tilde{H}_{cd,j}^L \vec{T}_{j-1} - (\tilde{H}_{cd,j}^L + \tilde{H}_{cd,j}^R) \vec{T}_j + \tilde{H}_{cd,j}^R \vec{T}_{j+1} + \vec{q}_j. \quad (5)$$

Note that $\vec{T}_j$ is a group of all of the wall temperature nodes belonging to an individual zone. More precisely, $\vec{T}_j = \begin{bmatrix} T_j^1 & T_j^2 & \dots & T_j^{Nw} \end{bmatrix}^T$ where N_w is the number of walls in a zone and the superscript T represents the transpose.

3.2. Heat balance at the exterior surfaces

For any i^{th} outside wall (connected to the external ambient) belonging to an individual zone, the heat balance equation at the surface is

$$\rho_1^i C_1^i w_1^i \frac{dT_1^i}{dt} = h_{cv,ex}^i (T_a - T_1^i) + h_{cd|1}^R (T_2^i - T_1^i) + \alpha_1^i q_{SWR}^i + q_{LWR}^i, \quad (6)$$

where T_1^i represents the wall temperature of the first node which is set to be an outside surface of the wall. With the assumptions that the outside surface is gray and diffuse and the air is a non-participating radiation media, net long wavelength interactions with the environment can be expressed as

$$q_{LWR}^i = \sigma \varepsilon_1^i F_{sky}^i ((T_{sky})^4 - (T_1^i)^4) + \sigma \varepsilon_1^i F_{grd}^i ((T_{grd})^4 - (T_1^i)^4).$$

313 Using a linear approximation of the long-wave heat exchange term gives

$$\rho_1^i C_1^i w_1^i \frac{dT_1^i}{dt} = -(h_{cv,ex}^i + h_{cd|1}^R + h_{rad,ex}^i) T_1^i + h_{cd|1}^R T_2^i + q_1^i, \quad (7)$$

where

$$\begin{aligned} h_{rad,ex}^i &= 4\sigma\varepsilon_1^i (F_{sky}^i (\bar{T}_{sky})^3 + F_{grd}^i (\bar{T}_{grd})^3), \\ q_1^i &= h_{cv,ex}^i T_a + 4\sigma\varepsilon_1^i (F_{sky}^i (\bar{T}_{sky})^3 T_{sky} + F_{grd}^i (\bar{T}_{grd})^3 T_{grd}) + \alpha_1^i q_{SWR}^i, \\ \bar{T}_{sky} &= \frac{T_{sky} + T_1}{2}, \quad \bar{T}_{grd} = \frac{T_{grd} + T_1}{2}. \end{aligned}$$

314 The mean temperatures for long-wave exchange between the surface and sky

315 and surface and ground are assumed to be the same for all outside surfaces. Eq.

316 7 can be generalized and written in compact matrix form as

$$\tilde{C}_1 \frac{d\vec{T}_1}{dt} = -(\tilde{H}_{cd,1}^R + \tilde{H}_{rad,ex} + \tilde{H}_{cv,ex}) \vec{T}_1 + \tilde{H}_{cd,1}^R \vec{T}_2 + \vec{q}_1, \quad (8)$$

where

$$(\vec{q}_1)_i = h_{cv,ex}^i T_a + 4\sigma\varepsilon_1^i (F_{sky}^i (\bar{T}_{sky})^3 T_{sky} + F_{grd}^i (\bar{T}_{grd})^3 T_{grd}) + \alpha_1^i q_{SWR}^i.$$

317 3.3. Heat balance at the interior surfaces

318 For the n^{th} zone and i^{th} wall, the energy balance equation for the inside

319 surface is

$$\rho_n^i C_n^i w_n^i \frac{dT_n^i}{dt} = h_{cd|n}^L (T_{n-1}^i - T_n^i) - q_{net,rad}^i + q_{cv,in}^i, \quad (9)$$

320 where $q_{net,rad}^i$ is net radiative flux out of the inside wall and $q_{cv,in}^i$ the net heat

321 flux from the zone air and allows for the CFD coupling. The radiosity method

322 is utilized to express the net flux under the assumption that the walls are gray,

diffuse and opaque. The same linearization method used in Eq. 7 is employed

leading to

$$\vec{q}_{net,rad} = \tilde{A}^{-1}[\tilde{B}'\vec{T}_n - \vec{h}_o] \quad (10)$$

where

$$\tilde{A}_{ij} \equiv \frac{\delta_{ij}}{\varepsilon_j} - \frac{\rho_j}{\varepsilon_j} F_{ij},$$

and

$$\tilde{B}'_{ij} = 4\sigma(\delta_{ij} - F_{ij})\bar{T}^3.$$

Radiosity does not appear explicitly in the above expression, which is convenient for building simulations. Since h_0^i represents an external radiative source

acting on the i^{th} surface, the effects of internal sources and transmitted solar

energy through windows are treated in a consistent manner. For any shaped

room, the net radiative flux can be explicitly calculated as a function of surface

temperatures if the view factors and the external radiative sources are known.

By letting $\tilde{H}_{rad,in} \equiv \tilde{A}^{-1}\tilde{B}'$ and $\vec{q}_n \equiv \tilde{A}^{-1}\vec{h}_o$,

$$\tilde{C}_n \frac{d\vec{T}_n}{dt} = \tilde{H}_{cd,n}^L \vec{T}_{n-1} - (\tilde{H}_{cd,n}^L + \tilde{H}_{rad,in})\vec{T}_n + \vec{q}_n + \vec{q}_{n,cv,in}. \quad (11)$$

3.4. State-space representation of the thermal envelope network

Gathering the system of equations that represent heat balance equations for

external to internal wall elements, i.e. Eqs. 5, 8 and 11,

$$\tilde{C}_w \dot{\vec{T}}_w = \tilde{H}_{ww} \vec{T}_w + \vec{q}_w + \vec{q}_{cv,in}, \quad (12)$$

where

$$\begin{aligned}
w &= \begin{bmatrix} \tilde{C}_1 & & & \\ & \tilde{C}_2 & & \\ & & \ddots & \\ & & & \tilde{C}_n \end{bmatrix}, \tilde{H}_{ww} = \begin{bmatrix} -\tilde{H}_1 & \tilde{H}_{cd,1}^R & & \\ \tilde{H}_{cd,2}^L & -(\tilde{H}_{cd,2}^L + \tilde{H}_{cd,2}^R) & \tilde{H}_{cd,2}^R & \\ & \ddots & \ddots & \ddots \\ & & \tilde{H}_{cd,n}^L & -\tilde{H}_n \end{bmatrix}, \tilde{H}_1 = \\
&(\tilde{H}_{cd,1}^R + \tilde{H}_{cv,ex} + \tilde{H}_{rad,ex}), \tilde{H}_n = (\tilde{H}_{cd,n}^L + \tilde{H}_{rad,in}), \vec{T}_w = \begin{bmatrix} \vec{T}_1^T & \vec{T}_2^T & \dots & \vec{T}_n^T \end{bmatrix}^T
\end{aligned}$$

and $\vec{q}_w = \begin{bmatrix} \vec{q}_1^T & \vec{q}_2^T & \dots & \vec{q}_n^T \end{bmatrix}^T$. As expected, a tri-diagonal block matrix is formed with parameters that characterize heat transfer due to conduction in the walls (from node number 2 to node number $n-1$) and radiative/convective heat transfer at the boundaries (first and last nodes only). The terms $\vec{q}_j$ where j ranges from 2 to $n-1$ vanish if there are no heat flux sources inside the wall such as embedded radiant heating or cooling systems. The term, $\vec{q}_{cv,in}$, represents the set of net convective heat fluxes to the zone air and allows CFD coupling. All entries in the vector are zeros except for inner walls.

3.5. Coupling with the restaurant CFD model

The one dimensional heat balance equations, Eqs. 13, are coupled to a reduced-order indoor air model. Although it is possible to subdivide wall surfaces sufficiently to model temperature variations across the walls and solve three dimension heat equations, we chose the 1-D form for computational simplicity. However Zhai and Chen [22] showed that the isothermal wall dynamics coupled to a CFD model leads to reasonable accuracy in predicting spatial temperature distribution of zone air and cooling/heating load. The state-space modeling (Eq. 13) is applied to the restaurant walls. It results in the following

355 system of equations:

$$\dot{X}_e = A_e X_e + B_e u \quad (13)$$

$$Y_e = C_e X_e + D_e u,$$

356 where X_e is the state vector ($\vec{T}_w$ in Eq. 13), u is the input vector of 22 el-
357 ements comprising the outdoor weather variables and the fluxes through the
358 walls computed from the ROM. The finite difference scheme with 2 to 4 nodes
359 per material layer of a wall results in 35 elements in X_e . The number was se-
360 lected after checking the grid independence of a simulation result. The output
361 Y_e represents a vector of the computed wall temperatures that are inputs to the
362 ROM. This coupling is illustrated in a simplified scheme in Fig. 10.

363 4. Optimal control of the cooling energy

364 4.1. Problem formulation

365 This work studies the restaurant indoor-air model for use in performing op-
366 timal control calculations. Unless modified, the default values of all the input
367 variables are fixed at the nominal values as specified in Section 2. The supply
368 air temperatures, occupancy and cooking loads, walls, ceiling and floor temper-
369 atures coming from the envelope model are considered as perturbations to the
370 nominal configurations. Thus, due to the linearity of the model, their effects on
371 the ROM are additive. The control variables are the four RTU supply air tem-
372 peratures and the four RTU outside air fractions. All the variables and problem
373 parameters are summarized in Tables 3 and 4.

374 The details of loads and occupancy are summarized as follows:

- 375 • Kitchen temperature (KT): $24^\circ C$ rising to $28^\circ C$ between $t = 10h$ and
376 $t = 22h$.
- 377 • Dining load (DL): 0.2 rising to 1.2 between $t = 12h$ and $t = 22h$.
- 378 • Wine bar load (WBL): 0.2 rising to 1.2 between $t = 12h$ and $t = 22h$.
- 379 • Occupancy: the occupancy function is 1 between h_{min} and h_{max} , and 0
380 otherwise

381 4.1.1. Cost Function (Equipment Model)

382 In this analysis, we distinguish several mass flow rates, namely, $\dot{m}_s|_i$ as the
383 supply mass flow rate of each RTU, $\dot{m}_{oa}|_i$ as the mass flow rate supplied from
384 outside for each RTU and $\dot{m}_r|_i$ as the return mass flow rate to each RTU, where
385 the subscript i designates quantities related to the i^{th} RTU. Continuity of these
386 rates gives the following balance:

$$\dot{m}_s|_i = \dot{m}_{oa}|_i + \lambda_i \dot{m}_r|_i. \quad (14)$$

387 Where λ_i is the fraction of the return air supplied in each RTU. We denote by d_i
388 the fraction of the supply air from outside. This fraction satisfies the following
389 equation:

$$\dot{m}_{oa}|_i = d_i \dot{m}_s|_i. \quad (15)$$

390 Combining Eqs. 14 and 15, we get the following relation between d_i and λ_i ,
391 where d_i is a control variable and λ_i is computed as:

$$\lambda_i = (1 - d_i) \frac{\dot{m}_s}{\dot{m}_r}|_i. \quad (16)$$

392 Since λ_i is supposed to be less than or equal to unity, we distinguish two cases:

393 • if $d_i \leq 1 - \frac{\dot{m}_r}{\dot{m}_s}|_i$, then $\lambda_i = 1$

394 • if $d_i > 1 - \frac{\dot{m}_r}{\dot{m}_s}|_i$, then λ_i is computed using Eq. 16

395 The total cooling load at each RTU is divided into sensible $Q_s|_i$ and latent
396 $Q_l|_i$ loads.

397 The sensible load related to the temperature change is expressed as:

$$398 \quad Q_s|_i = C_p \dot{m}_s|_i (T_m|_i - T_{c|i}^*) \quad (17)$$

$$T_{c|i}^* = T_{c|i} - \frac{0.05}{1.004} (T_{c|i} - T_{roof}), \quad (18)$$

399 where C_p is the heat capacity of the air at constant temperature, $T_m|_i$ is the
400 RTU inlet air temperature, $T_{c|i}$ is the supply RTU temperature, and $T_{c|i}^*$ is the
401 modified supply RTU temperature obtained after the circulation of the supply
402 air in the attic. The RTU inlet air temperature or the mixing temperature can
403 be written as:

$$T_m|_i = \frac{\dot{m}_r|_i \lambda_i T_r|_i + \dot{m}_{oa}|_i T_{oa}}{\dot{m}_s|_i}, \quad (19)$$

404 where T_{oa} is the outside air temperature and $T_r|_i$ is the temperature at each
405 return.

406 The moisture formulation included in the CFD model allows us to easily com-
407 pute the latent load related to the change of state, which can be expressed
408 as:

$$Q_l|_i = L(\dot{m}_r|_i \lambda_i f_r|_i + \dot{m}_{oa}|_i f_{oa} - f_s \dot{m}_s|_i), \quad (20)$$

409 where $f_r|_i$, f_{oa} and $f_s|_i$ are the mass fraction of water respectively for the return,
410 outside air and supply flows, and L is the amount of cooling required to remove

411 1g of water. Thus, the total power expression is:

$$P_{t|i} = \frac{Q_{s|i} + Q_{l|i}}{cop_i}, \quad (21)$$

412 where cop_i is the coefficient of performance of RTU # i . We note that the
 413 RTUs are constant fan speed regardless of RTU stages. Therefore, fan powers
 414 are assumed to be constant and do not appear in the objective function. The
 415 final cost expression can be written as:

$$C = \int_0^T \sum_{i=1}^4 P_{t|i} dt \quad (22)$$

416 where T is a 24-hour period.

417 4.1.2. Constraints

418 Most of the constraints are applied for each RTU i ($i=1, 2, 3, 4$):

- 419 1. Specify the upper and lower bounds for the control temperature for dif-
 420 ferent RTUs. $10^\circ C \leq T_c|i \leq 30^\circ C$.
- 421 2. Set the upper comfort bound for the local temperature of the different
 422 zones at the occupancy time: for $h_{min} \leq t \leq h_{max}$, $T_z|i \leq T_a|_{max}$.
- 423 3. Set the upper comfort bound for the average zone temperature at the
 424 occupancy time. For $h_{min} \leq t \leq h_{max}$, $T_a \leq T_a|_{max}$.
- 425 4. Cooling coil can only decrease the temperature. $T_c|i^* \leq T_m|i$.
- 426 5. Set the upper and lower bounds for the outside air fraction. $0.1 \leq d_i \leq 1$.
- 427 6. Set the upper and lower bounds for the recirculated air fraction. $0 \leq \lambda_i \leq$
 428 1.

7. Set the upper and lower bounds for the cooling power of each RTU.

$$0 \leq Q_{s|i} + Q_{l|i} \leq cap_{max|i}$$

429 .

430 8. Latent heat is only removed during the occupancy period.

431 4.1.3. Numerical Approximation

432 In optimal control, one is concerned with minimizing the functional (22)
 433 from a given initial state over a forward planning horizon $([0, T])$. Here we
 434 briefly study a discretized version of the problem using the MATLAB/SIMULINK
 435 based tool: Berkeley Library for Optimization Modeling. At this time BLOM
 436 2.0 is restricted to discrete-time dynamics, so it is necessary to convert from
 437 a continuous to a discrete time model. Thus, for example, the envelope model
 438 (14) is replaced by

$$X_e|_{k+1} = \Phi_e X_e|_k + \Gamma_e, \quad (23)$$

439 where

$$\Phi_e = \exp(A_e * dt) \quad (24)$$

$$\Gamma_e = \int_0^{dt} \exp(A_e(dt - s)) B_e ds \quad (25)$$

440 and dt is the time step.

441 5. Results

442 5.1. Optimal control results

443 From the figures plotting the average zone temperature (Fig. 11), the local
444 zone temperatures (Fig. 12), the control temperatures (Fig. 13), the fraction
445 of outside air (Fig. 14), the fraction of recirculated air (Fig. 15), the sensible
446 load (Fig. 16) and the total power used in each RTU (Fig. 17) , we conclude
447 the following. The control process can be divided into three regions: the first
448 is from 0h to 11h where the comfort constraints are not active, the second is
449 from 11h to 22h where the comfort constraints are active, and the third is from
450 22h to 24h where the comfort constraints are not active again. Concerning the
451 first portion, all the units are off, and the supply temperatures are delivered
452 in a way that guarantees a zero sensible load. There is no need for equipment
453 pre-cooling here because the damper is fully open and the supplied mass flow
454 rate comes completely from outside. Therefore, the temperature of the indoor
455 air is already cool. Near the beginning of the occupancy period, the sensible
456 loads at most of the zones (1, 2 and 4) become positive and the RTUs start to
457 cool the area. This influences almost all time histories and shows an obvious
458 discontinuity. In fact, the fraction of outside air jumps from 1 to 0.1 because the
459 outside air is hot and it is more efficient to rely on the recirculated air. During
460 this period, RTU 1 delivers greater levels of power than the other RTUs since it
461 is the most efficient, while the sensible load at RTU 3 remains zero. Concerning
462 the last period, the comfort constraints are relaxed, which drives the sensible
463 load to return to zero at about 22h.

464 The total cost for this standard configuration is 689822 kJ.

465 5.2. Comparison with experimental conventional control

466 Experimental measurements of the temperature at four data loggers in differ-
467 ent zones of the restaurant were taken upon application of a classic conventional
468 control. RTUs 2, 3 and 4 operate in only one compressor stage, whereas RTU 1
469 operates in two equivalent compressor stages. We present the cooling results for
470 few days of August 2013. The data loggers sampling time is five minutes. The
471 data represent the temperature at each zone and the corresponding power used
472 to reach the given temperature. To find the power used, the data is integrated
473 over a time interval of 24 hours, and compared to the power used for the case
474 of conventional control.

475 We compare the two controllers for five consecutive days starting from August
476 25th 2013 in Table 5, assuming that the maximum temperature for the optimal
477 control is 24.5°C. These comparison values are very sensitive to the maximum
478 allowed indoor air temperature. The power reduction may reach 45 % for spe-
479 cific cases. We plot the temperature response to both conventional (Fig. 18)
480 and optimal controls (Figs. 19 and 20) on August 28th. The figures show that
481 the optimal control gives more regular temperatures, whereas the conventional
482 control shows a considerable difference between the zones and significant tem-
483 perature fluctuations. Therefore, the optimal control is both more comfortable
484 and energy efficient.

485 5.3. Parametric study of the optimal results

486 In this section, we study the impact of varying problem loads and variables
487 on the final cost function. We start with the time convergence. All the previous
488 simulations are done for $dt = 7.5min$. Table 6 shows the effects of the time
489 step dt on the cost. It is shown that when the time step decreases, the cost
490 also decreases but appears to converge. Fig. 21 plots the control supply air
491 temperatures and shows the same trend as with the time step change.

492
493 Next, we study the effects of some load variations on the final cost. Table 7
494 shows the effects of the dining load on the cost. For the reference configuration,
495 the load rises from 0.2 to 1.2 during the occupancy period. We reduce the step
496 size by 20%, then by 40%. The corresponding changes in cost are respectively
497 a reduction of 6% and 12%. This result is predictable since reducing the load,
498 requires less cooling power to reach the given constraint. Moreover, the ROM is
499 linear. So when input (a) is reduced twice as much as input (b), the reduction
500 in output (a) is twice as much as that of output (b). The same findings are
501 revealed in Table 8 which shows a study of variations in the kitchen load. In
502 fact, the kitchen load rises from $24^{\circ}C$ to $28^{\circ}C$ during occupancy time in the
503 reference configuration. If we reduce the step size by 20%, then 40%, the change
504 in the final cost is respectively -0.73% and -1.46%.

505
506 Then, we check the effects of the occupancy period on the cost by simulating
507 three starting hours: 11 (reference case), 12 and 13 as shown in Table 9. This

508 variation is highly dependent on the external weather data and therefore is not
509 expected to be linear.

510 **Conclusion**

511 A CFD model has been developed to simulate the indoor-air temperature
512 and moisture of a restaurant. Different loads such as human occupancy and
513 cooking emissions, and boundary conditions were modeled based on on-site and
514 conventional data. After bringing the simulation to a steady state, single step
515 input changes were applied to construct a linear time invariant representation of
516 the coupled system. The reduced order model serves as a basis for the optimal
517 control of the power required to operate the cooling rooftop units. The results
518 show that our optimal control is more efficient and comfortable than a classic set-
519 point conventional control. However, the real implementation of such controllers
520 is not straightforward because it includes the actuation of the damper position
521 and it assumes that the RTU stages in a continuous fashion, which requires
522 the modification of the current cooling settings. This analysis can be used
523 to determine if equipment upgrade would be worth the investment to reduce
524 cooling power consumption.

525 **Acknowledgment**

526 This research was supported by the Air Force Office of Scientific Research
527 under grant FA9550-10-1-0201 and by the DOE contract DE-EE0004261 under
528 subcontract # 4345-VT-DOE-4261 from Penn State University. The authors

529 gratefully acknowledge the constructive discussions exchanged with BLOM de-
530 velopers.

531 **References**

- 532 [1] Naidu D, Rieger C. G. Advanced control strategies for HVAC&R systems–
533 An overview: Part I: Hard control. HVAC&R Research 2011; 11:216.
- 534 [2] Naidu D, Rieger C. G. Advanced control strategies for HVAC&R systems–
535 An overview: Part II: Soft and fusion control. HVAC&R Research 2011; 17
536 (2): 144.
- 537 [3] Sane H, Haugstetter C, Bortoff S. Building HVAC control systemsrole of
538 controls and optimization. Proceedings of the 2006 American Control Con-
539 ference, Minneapolis, MN 2006; 17 (1): 2-21.
- 540 [4] House J, Smith T. Optimal control of building and HVAC systems. Pro-
541 ceedings of the American Control Conference. Vol. 6 Seattle, WA, 432630.
- 542 [5] Yu F, Chan K. Economic benefits of optimal control for water-cooled chiller
543 systems serving hotels in a subtropical climate. Energy and Buildings 2014;
544 42(2): 2039.
- 545 [6] Oldewurtel F, Parisio A, Jones C, Gyalistras D, Gwerder M, Stauch V,
546 Lehmann B, Morari M. Use of model predictive control and weather fore-
547 casts for energy efficient building climate control. Energy and Buildings
548 2012; 45: 15–27.

- 549 [7] University of Wisconsin–Madison. Solar Energy Laboratory and Klein, San-
550 ford A. TRNSYS, a transient system simulation program. Solar Energy
551 Laboratary, University of Wisconsin–Madison 1979.
- 552 [8] Crawley DB, Lawrie LK, Winkelmann FC, Buhl WF, Huang WJ, Pedersen
553 CO, Stran RK, Liesen RJ, Daniel E. Fisher DE, Witte MJ, Glazer J. En-
554 ergyPlus: creating a new-generation building energy simulation program.
555 Energy and Buildings 2001; 33(4): 319–331.
- 556 [9] Ma Y, Kelman, A, Daly Y, Borrelli F. Predictive control for energy ef-
557 ficient buildings with thermal storage. IEEE Control Systems Magazine
558 2012; 32(1):44 - 64.
- 559 [10] Ma Y, Borrelli F, Hencsey B, Coffey B, Benga S, Haves P. Model Predictive
560 Control for the Operation of Building Cooling Systems. IEEE Transactions
561 on Control Systems Technology 2012; 20(3): 796-803.
- 562 [11] Siroky J, Oldewurtel F, Cigler J, Prvara S. Experimental analysis of model
563 predictive control for an energy efficient building heating system 2011. Ap-
564 plied Energy 2011; 88: 30793087.
- 565 [12] Ma J, Qin J, Salsbury T, Xu P. Demand reduction in building energy
566 systems based on economic model predictive control. Chemical Engineering
567 Science 2012; 67: 92-100.
- 568 [13] Zlatanovi I, Gligorevi K, Ivanovi S, Rudonja N. Energy-saving estimation
569 model for hypermarket HVAC systems applications. Energy and Buildings
570 2011; 43: 33533359.

- 571 [14] Srebric J, Chen Q, Glicksman LG. A coupled airflow-and-energy simulation
572 program for indoor thermal environmental studies. ASHRAE Transactions
573 2000; 106(1): 465-476.
- 574 [15] Zhai Z, Chen Q, Hayes P, Klems J. On approaches to couple energy sim-
575 ulations and computational fluid dynamics programs. Building and Envi-
576 ronment 2000; 37: 857-864.
- 577 [16] Borggaard J, Cliff E, Gugercin S. Model reduction for indoor-air behavior
578 in control design for energy-efficient buildings. Proceedings of 2012 ACC
579 2012.
- 580 [17] Fluent user's manual 2006.
- 581 [18] Gugercin S, Antoulas AC, Beattie CA. $\mathcal{H}_2$ model reduction for large-scale
582 linear dynamical systems. SIAM J. on Matrix Analysis 2008;30:609-638.
- 583 [19] Kung SY. A new identification and model reduction algorithm via singular
584 value decomposition. Proc. 12th Asilomar Conf. Circuits, Syst. Comput
585 1978.
- 586 [20] Cliff EM, Borggaard JT, Braun JE, Gugercin S, Kim D. Coupled
587 CFD/building envelope model for the Purdue Living Lab. 2012 Interna-
588 tional High Performance Buildings Conference at Purdue. Purdue Univer-
589 sity, West Lafayette, IN 2012.

- 590 [21] Kim D and Braun JE. A general approach for generating reduced-order
591 models for large multi-zone buildings. *Journal of Building Performance*
592 *Simulation* 2015; 8(6): 435-448
- 593 [22] Zhai ZJ and Chen QY. Performance of coupled building energy and CFD
594 simulations. *Energy and buildings* 2005; 37 333-344

Number	Name	Nominal Value	Increment
1	north_temp	27.00°C	5.00°C
2	east_temp	25.58°C	5.00°C
3	west_temp	28.43°C	5.00°C
4	floor_temp	24.92°C	5.00°C
5	ceiling_temp	25.46°C	5.00°C
6	rtu_1_temp	16.60°C	−5.00°C
7	rtu_2_temp	16.60°C	−5.00°C
8	rtu_3_temp	16.60°C	−5.00°C
9	rtu_4_temp	16.60°C	−5.00°C
10	rtu_1_h2o	7.00 g/kg	1.40 g/kg
11	rtu_2_h2o	9.00 g/kg	1.80 g/kg
12	rtu_3_h2o	9.00 g/kg	1.80 g/kg
13	rtu_4_h2o	9.00 g/kg	1.80 g/kg
14	rtu_1_mass	2.9320 kg/s	0.5864 kg/s
15	rtu_2_mass	0.5320 kg/s	0.1064 kg/s
16	rtu_3_mass	0.3990 kg/s	0.0798 kg/s
17	rtu_4_mass	0.5320 kg/s	0.1064 kg/s
18	load_dining	100%	20%
19	load_winebar	100%	20%
20	kitchen_inflow_temp	25.00°C	5.00°C
21	kitchen_inflow_h2o	16.00 g/kg	3.20 g/kg
22	kitchen_inflow_mass	0.5910 kg/s	0.1180 kg/s

Tab. 1: Input Variables: Nominal and Perturbation Values.

Number	Name	Value	Number	Name	Value
1	q_north	11.5095 w/m ²	23	dk_4_temp	21.7974°C
2	q_east	10.4278 w/m ²	24	dk_2_h2o	9.9237 g/kg
3	q_west	19.3538 w/m ²	25	dk_4_h2o	9.8813 g/kg
4	q_floor	15.8151 w/m ²	26	rtu1_z1_temp	21.1414°C
5	q_ceiling	4.9463 w/m ²	27	rtu2_z1_temp	22.2903°C
6	rtn_1_mass	−2.4959 kg/s	28	rtu3_z1_temp	21.3036°C
7	rtn_2_mass	−0.8248 kg/s	29	rtu4_z1_temp	22.0641°C
8	rtn_3_mass	−0.8248 kg/s	30	rtu1_z1_h2o	8.8190 g/kg
9	rtn_4_mass	−0.8392 kg/s	31	rtu2_z1_h2o	9.903 g/kg
10	rtn_1_temp	22.1082°C	32	rtu3_z1_h2o	9.4225 g/kg
11	rtn_2_temp	22.3562°C	33	rtu4_z1_h2o	9.9617 g/kg
12	rtn_3_temp	21.7175°C	34	dining_room_temp	21.9751°C
13	rtn_4_temp	22.6413°C	35	dining_room_h2o	9.2293 g/kg
14	rtn_1_h2o	9.3741 g/kg	36	winebar_temp	22.3145°C
15	rtn_2_h2o	9.2538 g/kg	37	winebar_room_h2o	10.0578 g/kg
16	rtn_3_h2o	9.2147 g/kg	38	winebar_skirt_temp	21.65525°C
17	rtn_4_h2o	10.5528 g/kg	39	winebar_skirt_h2o	9.8574 g/kg
18	thermo_1_temp	21.5048°C	40	fireplace_temp	21.7940°C
19	thermo_2_temp	21.80948°C	41	fireplace_h2o	9.6071 g/kg
20	thermo_3_temp	21.7650°C	42	room_avg_temp	22.0661°C
21	thermo_4_temp	21.8446°C	43	room_avg_h2o	9.4163 g/kg
22	dk_2_temp	21.7961°C			

Tab. 2: Selected Nominal Steady Outputs.

Tab. 3: Model variables.

Variable	Significance	ROM Input/Output
$T_c _i(i = 1 : 4)$	RTU control temperature	IN(6)to IN(9)
$\dot{m}_s _i(i = 1 : 4)$	RTU supply mass flow rate	IN(14)to IN(17)
$f_s _i(i = 1 : 4)$	Supply water mass fraction	IN(10)to IN(13)
DL	Dining load	IN(18)
WBL	Wine bar load	IN(19)
KT	Kitchen temperature	IN(20)
$\dot{m}_r _i(i = 1 : 4)$	RTU return mass flow rate	OUT(6)to OUT(9)
$T_r _i(i = 1 : 4)$	Return temperature	OUT(10) to OUT(13)
$T_z _i(i = 1 : 4)$	Local zone temperature	OUT(26)to OUT(29)
$f_r _i(i = 1 : 4)$	Return water mass fraction	OUT(14)to OUT(17)
T_a	Average zone temperature	OUT(42)
d_i	Fraction of outside air	NA
λ_i	Fraction of recirculated air	NA

Tab. 4: Model parameters.

Parameter	Significance	Value
$T_{a max}$	Upper bound for comfort	23°C
$h_{min,max}$	Occupancy bounds	11h, 22h
C_p	Heat capacity	1 kJ/(kg K)
$d_{min,max}$	Air fraction bounds	0.1, 1.0
$T_{min,max}$	Bounds for control temp	10°C, 30°C
$cop_i(i = 1 : 4)$	Total coefficient of performance	3.5, 2.6, 2.6, 2.6
$cap_{max i}(i = 1 : 4)$	Total cooling capacity of each RTU	53, 14, 10.5, 14 kW
f_{oa}	Outside water mass fraction	17 g/kg
dt	Time step	7.5 min
L	Cooling power for removal of 1g of water	2.26 kW

Tab. 5: Conventional / Optimal control cost

Date (August 2013)	25	26	27	28	29	30
Conventional cost(10^5 kJ)	7.0083	6.9046	6.8156	11.279	7.2406	9.4188
Optimal cost(10^5 kJ)	5.7264	5.4820	5.3574	6.1908	6.1410	5.6805
Gain (%)	-18.29	-20.60	-21.39	-45.11	-15.18	-39.69

Tab. 6: Time-step sensitivity

Time-step dt (min)	7.5	3.75	1.875
Cost (kJ)	689822	686909	685600
Percentage	0 %	-0.4 %	-0.6 %

Tab. 7: Dining load sensitivity

Dining load	1.2	1.0	0.8
Cost (kJ)	689822	648175	606561
Percentage	0 %	-6 %	-12 %

Tab. 8: Kitchen load sensitivity

Kitchen load ($^{\circ}C$)	28	27.2	26.4
Cost (kJ)	689822	684773	679723
Percentage	0 %	-0.73 %	-1.46 %

Tab. 9: Occupancy period sensitivity

Control starting time (h)	11	12	13
Cost (kJ)	689822	656182	605763
Percentage	0 %	-4.9 %	-12.2 %

Fig. 1: Grid with zones - the left (green) wall is North.

Fig. 2: Steady response; Zone Average Temperatures.

Fig. 3: RTU1 supply air temperature

Fig. 4: RTN1 (left), RTN2 (right)

Fig. 5: Shifted comparisons: RTN1 (upper left), RTN2 (upper right) RTN3 (lower left), RTN4 (lower right)

Fig. 6: Comparison of FLUENT and ROM T_{east} responses.

Fig. 7: Comparison of FLUENT and ROM $T_{rtu,1}$ responses.

Fig. 8: Comparison of FLUENT and ROM h_kitchen responses.

Fig. 9: Notation for conduction through walls.

Fig. 10: Simplified scheme of ROM-envelope coupling.

Fig. 11: Average zone temperature profile compared to the outside temperature °C.

Fig. 12: Local zone temperatures (°C).

Fig. 13: RTU control temperatures ($^{\circ}\text{C}$).

Fig. 14: Fraction of outside air in each RTU.

Fig. 15: Fraction of recirculated air in each RTU.

Fig. 16: Sensible load in each RTU (kW).

Fig. 17: Cooling coil power used in each RTU (kW).

Fig. 18: Zone temperatures under conventional control for August 28th.

Fig. 19: Zone temperatures under optimal control for August 28th.

Fig. 20: Average temperature under optimal control for August 28th compared to the outside air temperature.

(a) $dt=7.5$ min

(b) $dt=3.75$ min

(c) $dt=1.875$ min

Fig. 21: Supply control temperatures using different time steps.

595 List of Figures

596	1	Grid with zones - the left (green) wall is North.	38
597	2	Steady response; Zone Average Temperatures.	38
598	3	RTU1 supply air temperature	39
599	4	RTN1 (left), RTN2 (right)	39
600	5	Shifted comparisons: RTN1 (upper left), RTN2 (upper right)	
601		RTN3 (lower left), RTN4 (lower right)	40
602	6	Comparison of FLUENT and ROM T _{east} responses.	41
603	7	Comparison of FLUENT and ROM T _{rtu1} responses.	42
604	8	Comparison of FLUENT and ROM h _{kitchen} responses.	43
605	9	Notation for conduction through walls.	44
606	10	Simplified scheme of ROM-envelope coupling.	45
607	11	Average zone temperature profile compared to the outside tem-	
608		perature °C.	46
609	12	Local zone temperatures (°C).	46
610	13	RTU control temperatures (°C).	47
611	14	Fraction of outside air in each RTU.	47
612	15	Fraction of recirculated air in each RTU.	48
613	16	Sensible load in each RTU (kW).	48
614	17	Cooling coil power used in each RTU (kW).	49
615	18	Zone temperatures under conventional control for August 28 th . . .	50
616	19	Zone temperatures under optimal control for August 28 th	51

617	20	Average temperature under optimal control for August 28 th com-	
618		pared to the outside air temperature.	52
619	21	Supply control temperatures using different time steps.	53

620 List of Tables

621	1	Input Variables: Nominal and Perturbation Values.	33
622	2	Selected Nominal Steady Outputs.	34
623	3	Model variables.	35
624	4	Model parameters.	36
625	5	Conventional / Optimal control cost	36
626	6	Time-step sensitivity	37
627	7	Dining load sensitivity	37
628	8	Kitchen load sensitivity	37
629	9	Occupancy period sensitivity	37