Simulating the wake flow effect of wind turbines on velocity and turbulence using particle random walk method

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Abstract

This paper presents a novel way of simulating the effect of velocity decay and turbulence of wind turbine's wake flow. By decoupling the solving of wake flow from that of the velocity field, the proposed model treats the wake flow intensity as a kind of convective and diffusive virtual matter. The particle random walk method is utilized to simulate the motion of the virtual matter. Comparing to the existing linear model for turbine wake flow, the proposed model can predict the distributions of velocity decay and turbulence of wake flow in a non-uniform flow field above complex terrain. Experimental data from wind tunnel and real wind farm is used to validate the model, demonstrating its effectiveness on estimating the velocity decay and the turbulence intensity, and additionally, the power yield of a wind farm. The model proposed in this paper can be integrated into algorithms for numerical assessment and micro-siting optimization of wind farms on complex terrains.

Keywords: wind turbine, wake flow, particle random walk

1. Introduction

Using horizontal-axis wind turbines for wind power generation is one of the most mature technologies of utilization of clean and renewable energies. Besides the efficiencies of the wind turbines and the power grid, wind turbine positioning is remarkably influential to the amount of power yield of a wind farm. Since wind wake reduces wind speed and increases turbulence in the downstream region, irrational layout of wind turbines within a wind farm will lead to high interactions among the turbines, causing unnecessary power loss.

Multiple intelligent algorithms have been studied on solving the wind turbine positioning problem. Mosetti et al. [1] introduced Genetic Algorithm (GA) to

37 this problem, using a 10×10 Cartesian mesh. After them, various studies have
38 been reported in developing new approaches for optimization of wind turbine
39 positioning, such as using triangular-shaped grids instead of Cartesian mesh [2],
40 encoding the turbine layout into real number sequences instead of binary coding
41 [3], adopting the Particle Swarm Optimization (PSO) algorithm [4], Monte Carlo
42 algorithm [5] and greedy algorithm [6], etc.

43 In all optimization algorithms of wind turbine positioning, in order to search
44 for the best layout for wind turbines, it is essential for the process of evaluating
45 the layout's power yield to be run for a large number of times, leading to a big
46 challenge in developing an appropriate model for wake flow estimation. The
47 wake flow estimation model must run fast, otherwise the optimization process
48 will cost unacceptable time to obtain an optimized solution.

49 The most widely used model for wake flow estimation is the linear model
50 proposed by Jensen [7]. It treats the wake affected region as a conical area, inside
51 which the amount of velocity decay is unrelated to the radial position. Similar
52 but more complicated models are proposed by Larsen [8] and Ishihara et al.
53 [9] respectively. Although more factors have been taken under consideration in
54 their improved models, all the parameters of the wake flow are still determined
55 only by the flow states at the turbine. Downstream variations of airflow are
56 unable to influence the behavior and characteristics of the wake flow. These
57 simplified models are limited for flat terrain only.

58 On the other hand, researchers in the field of fluid mechanics performed
59 detailed numerical simulation using Computational Fluid Dynamics (CFD) [10,
60 11, 12, 13, 14], and experimented on sophisticated wind tunnel measurements
61 of wind turbine wake flow [15, 16]. Their results describe the nature of wake
62 flow accurately under many circumstances, providing better understanding on
63 how multiple wake flows interact and overlap with each other. However, in
64 the optimization of wind turbine positioning in a wind farm, any optimization
65 algorithm requires multiple times of wake flow evaluation for a large quantity of
66 different turbine layouts. An ordinary iteration of solving the flow field around
67 a wind turbine may takes hours to converge. Time cost on simulating wake flow
68 behaviors for multiple wind turbines inside a wind farm for multiple different
69 layouts is even more unacceptable. It is not practical to integrate CFD directly
70 into optimization approaches for wind turbine positioning.

71 A wake flow model using the particle random walk technique has been pre-
72 viously proposed [17]. This model treats the wake flow effect as convective and
73 diffusive virtual particles carried by background airflow. As the background
74 flow field can be obtained from CFD calculation, wake flow predicted by the
75 particle model will bend according to local downstream airflow, for which it is
76 adaptable for non-uniform flow field above complex terrain. Additionally, since
77 the background flow field needs to be evaluated for only once, the model runs
78 fast enough to be integrated with optimizations. Subsequent studies [18, 19, 20]
79 demonstrated the usefulness of this model. Nevertheless, the relationship be-
80 tween the particles' diffusive motions and the incoming wind speed was not
81 considered, the attenuation process of the particles was not clearly defined, and
82 the overlapping phenomenon of multiple wake flows was not comprehensively

83 discussed.

84 This paper presents the advanced version of the wake flow model using parti-
 85 cle random walk technique. The limitations mentioned above are overcome, and
 86 additionally, a model to estimate turbulence intensity within wake regions is de-
 87 veloped. The remainder of the paper is organized as follows. Section 2 describes
 88 the linear wake model, which will be used in comparisons in the other sections.
 89 Section 3 overviews the previous version of the particle model and presents the
 90 improvements. Section 4 presents the model for turbulence estimation. Results
 91 are validated by experimental data in a wind tunnel and measurements on a
 92 real wind farm. Section 5 concludes the paper.

93 2. Overview of the linear wake model

94 The linear wake model was developed by Jensen et al. [7, 21]. Gonzalez et
 95 al. [22] considered that in most situations, wake flow regions do not cover entire
 96 rotors of the downstream turbines, and the model was modified by adding a term
 97 representing the overlapped area of the wake region and the affected turbine
 98 rotor. Therefore, the modified model is used in the wind farm evaluation in
 99 Section 3.4. In sections except Section 3.4, since our purpose is to calculate the
 100 velocities at certain coordinates within the field, not the wake effect to other
 101 turbines, the original model is used.

102 For the original model, the wake affected region of a single wind turbine is
 103 sketched in Figure 1. The wake flow is originated at the position of the turbine
 104 rotor, and develops in a conical shape. x indicates the distance from the turbine
 105 hub to a downstream position, projected to the streamwise direction. The radius
 106 of the wake affected region at distance x is

$$r_w = \alpha x + r_r, \quad (1)$$

107 where r_r is the radius of the turbine rotor (including blades), α is the entrain-
 108 ment constant calculated by

$$\alpha = \frac{0.5}{\log(h/z_0)}, \quad (2)$$

109 where h is the height of the turbine hub to the ground, z_0 is the ground roughness
 110 length. Let u be the speed of the incoming wind. The wind speed outside the
 111 wake region remains unchanged, while the wind speed inside the wake region is
 112 calculated through

$$u' = u \left[1 - \frac{2a}{(1 + \alpha x/r_1)^2} \right], \quad (3)$$

113 where a and r_1 are calculated by

$$a = \frac{1 - \sqrt{1 - C_T}}{2}, \quad r_1 = r_r \sqrt{\frac{1 - a}{1 - 2a}}, \quad (4)$$

114 where C_T is the thrust coefficient of the wind turbine.

For scenario with multiple turbines, the first step is to calculate the velocity in the wake region for each turbine separately using Equation 3, whose result is denoted as u'_i , where i stands for the i th turbine. Then the overall velocity in the wake region is

$$u' = u - \sqrt{\sum_{i=1}^N (u - u'_i)^2}, \quad (5)$$

where N is the total number of turbines.

For the modified model, Equation 5 is modified into

$$u' = u - \sqrt{\sum_{i=1}^N \left[\frac{A_i}{\pi r_r^2} (u - u'_i)^2 \right]}, \quad (6)$$

where r_r is rotor radius of the turbine, A_i is the overlapped area of the wake region generated by the i th turbine and the turbine being affected. Figure 2 plots the wake region and the turbine rotor being affected as a projection onto the plane of the turbine rotor. O_T is the center of the turbine being affected, while $O_{w,i}$ is the center of the wake region generated by the i th turbine. $r_{w,i}$ is the radius of the wake affected region of the i th turbine, calculated by Equation 1. According to basic geometry, the overlapped area of the two circles is

$$A_i = \begin{cases} \pi r_r^2, & d_i \leq r_{w,i} - r_r \\ \frac{1}{2} r_r^2 (\theta_1 - \sin \theta_1) + \frac{1}{2} r_{w,i}^2 (\theta_2 - \sin \theta_2), & r_{w,i} - r_r < d_i < r_{w,i} + r_r \\ 0, & d_i \geq r_{w,i} + r_r \end{cases}, \quad (7)$$

where

$$\theta_1 = 2 \arccos \frac{r_r^2 + d_i^2 - r_{w,i}^2}{2 r_r d_i}, \quad (8)$$

$$\theta_2 = 2 \arccos \frac{r_{w,i}^2 + d_i^2 - r_r^2}{2 r_{w,i} d_i}. \quad (9)$$

3. The present model for velocity

One of the primary effects caused by wake flow is velocity decay in the downstream region of a wind turbine. The common concept of the widely used wake models is to decouple the calculation of velocity decay from the background flow field, as mentioned in the introduction. In other words, a background flow field without any influence of wake flow is pre-obtained by other means, and the amount of velocity decay at each position is superposed onto the background flow field, producing the wake-influenced velocity field. The model proposed in this paper imitates this similar idea of decoupling, but calculates the amount of velocity decay in a more accurate way.

140 3.1. Velocity decay and random walk particles

141 The particle model, as described in a previous paper [17], assumed that the
 142 behavior of the ratio of velocity decay acts like a kind of virtual convective and
 143 diffusive matter. The matter is produced by rotating turbine blades, and carried
 144 to downstream regions by airflow in the background flow field. Chen et al.
 145 demonstrated that this phenomenon of scalar transportation can be numerically
 146 calculated through particle random walk method [23]. Using this technique,
 147 the calculation of velocity decay is performed in two steps. First, multiple
 148 particles are generated in the area of the rotating blades of each turbine, and
 149 the convective and diffusive motions of the particles are simulated for a certain
 150 length of time period. During the simulation, the number of particles passing
 151 through each concerned region is counted. Second, the counted numbers are
 152 transformed into magnitudes of velocity decay, which are then superposed to
 153 the background flow field, producing the final result. The details are explained
 154 as follows.

155 In every time step of the simulation, particles are generated randomly with
 156 uniform distribution within the circular area. For each newly generated particle,
 157 its radial and angular position with respect to the rotor center is calculated by

$$r = \sqrt{R_1} r_r, \quad \alpha = 2\pi R_2, \quad (10)$$

158 where r_r is the rotor radius of the turbine (including the blades), R_1 and R_2
 159 are two independent random numbers with uniform distribution on $[0, 1)$. Then
 160 the birthplace of the particle is calculated by

$$\mathbf{x} = \mathbf{x}_T + \begin{pmatrix} 0 \\ r \cos \alpha \\ r \sin \alpha \end{pmatrix}, \quad (11)$$

161 where $\mathbf{x}_T$ is the coordinate of the center of the turbine rotor.

162 Within each time step of the simulation, the displacement of a particle con-
 163 sists of its convective displacement and its diffusive displacement:

$$\Delta x_i = \Delta x_{i,c} + \Delta x_{i,d}, \quad (12)$$

164 where its convective displacement is identical to the motion of the local fluid,
 165 which is

$$\Delta x_{i,c} = u_i \Delta t, \quad (13)$$

166 while the diffusive displacement is approximated by a Gaussian random number.
 167 Box and Muller suggested that a Gaussian random number can be generated by
 168 $\sqrt{-2 \log R_3} \cos(2\pi R_4)$, where R_3 and R_4 are two independent random numbers
 169 with uniform distribution on $[0, 1)$ [24]. Einstein proposed that in Brownian
 170 movement, the average length of the square of diffusive displacement is pro-
 171 portional to time [25], that is $\overline{x_{i,d}^2} \propto \Delta t$. Then the diffusive displacement is
 172 calculated by

$$\Delta x_{i,d} = \sigma \sqrt{-2 \Delta t \log R_3} \cos(2\pi R_4), \quad (14)$$

173 where σ characterizes the strength of the diffusive motion, Δt is the length of
 174 the time step, and R_3 and R_4 are two independent random number with uniform
 175 distribution on $[0, 1)$.

176 Former models of turbine wake flow imply that the shape of the wake af-
 177 fected region is unrelated to the incoming velocity and the scale of the turbine.
 178 Therefore we choose $\sigma = \sigma_d \sqrt{\|\mathbf{u}\| r_r}$ (where σ_d is a constant) in order to make
 179 the scale of $\Delta x_{i,d}/\Delta x_{i,c}$ unrelated to the velocity and the rotor radius. The
 180 final formula to calculate the overall motion of a particle within a simulation
 181 time step is

$$\Delta x_i = u_i \Delta t + \sigma_d \sqrt{-2 \|\mathbf{u}\| r_r \Delta t \log R_3} \cos(2\pi R_4). \quad (15)$$

182 A regular attenuation process is added to the particles to compensate for
 183 the deviations caused by the decoupling approximation. A half-life period is as-
 184 signed to each particle, denoted as τ . Therefore in each time step of simulation,
 185 the probability of each particle to vanish is

$$\gamma(\Delta t) = 1 - \left(\frac{1}{2}\right)^{\Delta t/\tau}. \quad (16)$$

186 After a certain length of simulation is performed, the counted number of par-
 187 ticles per unit volume in each concerned region (denoted as $n(\mathbf{x})$) is transformed
 188 into particle concentration through

$$c(\mathbf{x}) = \frac{n(\mathbf{x})u_0}{p}, \quad (17)$$

189 where u_0 is the wind speed in the background flow field at the position of the
 190 center of the turbine rotor, and p is the number of particles generated per unit
 191 area and unit time span. With the particle concentration calculated, the wind
 192 velocity with velocity decay superposed is obtained by

$$u'(\mathbf{x}) = u(\mathbf{x}) (1 - \beta c(\mathbf{x})), \quad (18)$$

193 where β is a constant. Theoretically $c(\mathbf{x})$ is 1 near the rotor center in the
 194 downstream direction [23].

195 On the other hand, the traditional linear model calculates the wake-influenced
 196 velocity through Equation 19.

$$u'(x) = u(x) \left[\frac{2a}{(1 + \alpha x/r_1)^2} \right], \quad (19)$$

197 where x is the distance originated from the rotor center towards the downstream
 198 direction along the rotor axis, α is the entrainment constant, r_1 is a constant
 199 related to the rotor radius, and a is the axial induction factor of the turbine,
 200 which has the following relationship with its thrust coefficient C_T .

$$C_T = 4a(1 - a). \quad (20)$$

201 In order to maintain consistency with the linear model at $x = 0$, we can deduce
 202 that $\beta = 2a$, leaving σ_d and τ to be determined.

203 3.2. Calibration for a single turbine

204 σ_d and τ can differ for different types of turbines. In the previous study,
 205 we took the Nibe-B 630 kW wind turbine as an example to calibrate the par-
 206 ticle model [17]. This type of turbine has its rotor diameter at 40 m (including
 207 blades), and its hub height at 45 m. However, in the previous study, the wake
 208 shape is not unrelated with velocity and rotor radius, and the half-life attenu-
 209 ation process is not revealed. Calibration in the previous study concludes that
 210 $\sigma = 0.3$ and $\gamma = 0.005$ are proper for this type of turbine, with $u_0 = 8.5$ m/s
 211 and $\Delta t = 1$ s. Therefore, according to the definitions of σ and γ in the previous
 212 section, it can be deduced that

$$\sigma_d \approx 0.23, \quad \tau \approx 138 \text{ s}. \quad (21)$$

213 The results of the calibration on $u_0 = 8.5$ m/s are plotted in Figures 3, 4
 214 and 5, where the horizontal axis is the radial position normalized by the rotor
 215 diameter, and the vertical axis is the wind speed normalized by the incoming
 216 wind speed. X is the streamwise coordinate, and D is the diameter of the
 217 turbine rotor. Experimental data is provided by Taylor et al. [26]

218 3.3. Wake overlap for multiple turbines

219 In a wind farm with multiple wind turbines, particles generated by each
 220 turbine are simulated and counted separately. Denote the particle concentration
 221 at position $\mathbf{x}$ caused by the i th turbine as $c_i(\mathbf{x})$. In the previous study of the
 222 particle model [17], the overall particle concentration referred by Equation 18
 223 is the sum of all concentrations:

$$c(\mathbf{x}) = \sum_{i=1}^N c_i(\mathbf{x}), \quad (22)$$

224 where N is the number of turbines within the wind farm. This manner of wake
 225 overlapping may lead to high values of $c(\mathbf{x})$ and low values of $u'(\mathbf{x})$, which is
 226 not like the real situation that happens in a real wake overlapped region.

227 In the present study, the overall particle concentration is simply assigned as

$$c(\mathbf{x}) = \max_{i=1,2,\dots,N} c_i(\mathbf{x}). \quad (23)$$

228 Using this way of wake overlapping, wake flow generated by each turbine is cal-
 229 culated separately, and at any position $\mathbf{x}$, only the concentration of the strongest
 230 wake flow is considered to be the overlapped value.

231 Experimental data obtained by Chamorro et al. [15] is used to validate the
 232 particle model. In their study, 25 miniature turbines are placed in a staggered
 233 pattern inside a wind tunnel, as plotted in Figure 6, where wind blows along
 234 the positive direction of the X -axis. The rotor diameter (including blades) is
 235 $D = 128$ mm and the height of the hub is $h = 104$ mm. Column spacing and
 236 row spacing of the layout are $4D$ and $5D$ respectively. Surface roughness is
 237 $z_0 = 0.05$ mm.

238 The experimental data contains normalized measured velocity along the
 239 mainstream direction (X direction), in the 2nd column of turbines ($Y = 2D =$
 240 0.256 m), at the heights of hub ($Z = h = 0.104$ m), bottom tip ($Z = h - D/2 =$
 241 0.04 m) and top tip ($Z = h + D/2 = 0.168$ m) respectively.

242 The incoming velocity profile is specified by the logarithmic law:

$$u(z) = \frac{\ln(z/z_0)}{\ln(h/z_0)}. \quad (24)$$

243 The characteristics of the miniature turbine are different from those of the
 244 turbine used in the calibration in the previous section, and its thrust coefficient
 245 is unknown. Among the 2nd column of turbines, which have been experimented
 246 on, the region marked by blue dashed line in Figure 6 is the region with mini-
 247 mum wake overlapping. Measured velocities in this region are used to calibrate
 248 for thrust coefficient C_T . Different values of C_T produce different velocity dis-
 249 crepancies between the calculated result and the experimental data within this
 250 region, as listed in Table 1. According to the result, it can be obtained that the
 251 best fit occurs when $C_T = 0.59$, with discrepancy at 4.6674%.

252 With all the coefficients in the particle model determined, the model is ap-
 253 plied to calculate the velocity distributions along the X -axis at $Z = 0.104$ m,
 254 0.168 m and 0.04 m respectively for the 2nd column of turbines. Results are
 255 plotted in Figures 7, 8 and 9, where the vertical coordinate is the streamwise
 256 velocity normalized by incoming wind speed at the height of the hub.

257 Results calculated using the linear wake model are also presented as black
 258 dotted lines for comparison. The algorithm of the linear model is briefly de-
 259 scribed in 2.

260 It can be seen from the curves that the previous overlapping method over-
 261 estimates the strength of the overlapped wake flow, especially in the further
 262 downstream region. The present overlapping method produces better results
 263 than the previous one. The errors for the hub, the rotor tip and the rotor bot-
 264 tom are all lower than the ones of the previous model, as listed in Table 2 and
 265 Table 3. And all the errors of the present model are below the errors of the
 266 linear model.

267 3.4. Validation in real wind farm

268 The particle model is then validated in a real wind farm at Wattle Point
 269 of Australia, whose geolocation is around 137.71E, 35.13S. 55 wind turbines of
 270 the type Vestas V82 (1.65 MW) were installed. The hub height is 68.5 m and
 271 the rotor diameter is 82 m. Layout of the wind farm is sketched in Figure 11,
 272 where contours represent terrain altitude. Wind speed measurement is obtained
 273 from meteorological station ID022046, which is public data provided by the
 274 Australian government. Observed records for WSW wind direction between
 275 March 19 to April 15 of the year 2013 are picked for validation. Data of actual
 276 power yield is public data provided by Australian Energy Market Operator.

277 The presented particle model is a model that evaluates the influence of the
 278 wake flow to the downstream velocity. In order to validate the model with

measured data of the power yield, the relationship between the incoming wind speed and the power output of the turbine is necessary. The theoretical power curve of Vestas V82 provided on its product brochure is plotted in Figure 10. Croonenbroeck et al. compared the theoretical power curve of Vestas V90 with the actual measured data from wind farms. They pointed out that the theoretical power curve differs from the real cases due to air density changes or the prudent estimation [27]. Therefore, in order to evaluate the power product of each turbine accurately, it is required to have a better model calibrated by using the actually measured wind speeds and power products on each turbine for various situations. Since the data with these details are not available for us at the present, we could only validate our model by using the theoretical power curve in this section, as a preliminary study. Further investigations are essential before the present model can be put into real application.

Figure 12 shows the relationship between the overall power output of the wind farm and the incoming wind speed for the picked records. The linear model overestimates the wake effect obviously, leading to lower estimation on the power output. The previous particle model is the most accurate one among the three models in the high speed range, and the present particle model acts better in the medium wind speed range. All the three models produce larger errors in the low wind speed range. The mean absolute error (MAE) of the linear model is 11000.86 kW, while the ones of the previous particle model and the present particle model are 6854.598 kW and 6749.856 kW respectively. The root mean square errors (RMSE) of the linear model, the previous particle model and the present particle model are 13362.14 kW, 8832.453 kW and 8113.797 kW, respectively.

4. The present model for turbulence

Although not exactly precise, the $k-\varepsilon$ turbulence model implies that the behavior of turbulent energy can be approximated to be following the same law as a convective and diffusive scalar, and it has the same diffusion coefficient as the one for velocity or momentum in the wind farm scenarios [28]. Based on this phenomenon, idea similar to the model of velocity is implemented to simulate the turbulence intensity within the wake flow region of a wind turbine. Turbulence intensity is treated as a convective and diffusive virtual matter, and its developing process is simulated through the particle random walk method.

4.1. Single turbine

Differing from the velocity decay, most of the turbulent flow is generated by the blade tips, rather than being uniformly generated within the circular area. With consideration of this nature of turbulence generation, particles representing turbulence intensity (referred to turbulence particle for short below) are generated only on the perimeter of the circular area of the rotating blades.

319 That is, the birthplace of the turbulence particle is

$$\mathbf{x} = \mathbf{x}_T + \begin{pmatrix} 0 \\ r_r \cos \alpha \\ r_r \sin \alpha \end{pmatrix}, \quad (25)$$

320 where $\mathbf{x}_T$ is the position of the center of the turbine rotor, r_r is the radius of the
321 turbine rotor (including blades), and α is a random angle uniformly distributed
322 within $[0, 2\pi)$.

323 The rest of the simulation steps are the same as the ones for the velocity
324 model. With the concentration of the particles obtained, turbulence intensity
325 is calculated through the linear transformation:

$$I(\mathbf{x}) = \zeta \cdot c(\mathbf{x}) + I_0, \quad (26)$$

326 where ζ and I_0 are transformation coefficients that need to be determined.

327 As mentioned before, the diffusive motion of the turbulence particles is with
328 the same strength as the one for the velocity model, but its strength of atten-
329 uation differs, leaving its half-life period τ to be determined. Calibration for τ
330 is also done using the experimental data from Taylor et al. [26]. For any value
331 of τ , best values of ζ and I_0 can be found to achieve the minimum discrepancy
332 between the simulated results and the experimental data, using the Successive
333 Quadratic Programming (SQP) algorithm. Different values of τ produce differ-
334 ent minimum discrepancies. For the Nibe-B wind turbine, some of the results
335 are listed in Table 4. Then the best value of τ can be chosen to be the one that
336 produces the overall minimum discrepancy, which is

$$\tau = 31 \text{ s}, \quad \zeta = 0.3373, \quad I_0 = 0.0784. \quad (27)$$

337 Fitted results are plotted in Figures 13 and 14. Discrepancies between our
338 simulated results and the experimental data along the streamwise positions and
339 the radial positions are 0.0057 and 0.0054 respectively.

340 4.2. Multiple turbines

341 Experiments done by Chamorro et al. discover that the turbulent flow gen-
342 erated by turbines in a staggered layout spreads more extensively upwards than
343 downwards [15]. In order to compensate for this phenomenon in the present
344 model, the birthplace of turbulence particles is amended from Equation 25 into

$$\mathbf{x} = \mathbf{x}_T + \begin{pmatrix} 0 \\ \pm r_r \sqrt{1 - R_z^2} \\ r_r R_z \end{pmatrix}, \quad (28)$$

345 where

$$R_z = 2 \left(\frac{1}{2} \sin \alpha + \frac{1}{2} \right)^m - 1. \quad (29)$$

346 m is an adjustable coefficient between 0 and 1, having the ability of concentrat-
347 ing the birthplaces of turbulence particles towards the top of the circular area
348 of the rotating blades.

349 Since the particles are generated along the perimeter of the circular area
350 rather than inside it, particle concentration can be extremely high near the
351 birthplace. Therefore, simulated data points within $1D$ in the downstream
352 direction of each turbine are discarded.

353 According to Equation 26, I_0 is the turbulence intensity where no turbulence
354 particle exists. So measured data before the most upstream turbine in the
355 measured column is used to determine the values of I_0 , which are listed in the
356 row labeled “Measured incoming flow” in Table 5. Similarly to the model for
357 velocity, measured data at positions marked by the blue dashed line in Figure
358 6 is used to calibrate for m and ζ . It obtains that the best fit occurs when
359 $m = 0.15$ and $\zeta = 0.0651$.

360 With the coefficients determined, the present model is applied to calcu-
361 late the distribution of turbulence intensity along the mainstream direction at
362 $Z = 0.168\text{ m}$, 0.104 m and 0.04 m respectively. Comparisons with Chamorro’s
363 experimental data are plotted in Figures 15, 16 and 17.

364 It can be noticed from Figure 16 that, even when simulated data points
365 within $1D$ in the downstream direction of each turbine are discarded, sharp
366 peaks still exist after each turbine’s top tip. The reason is that the generation
367 process of the turbulence particles and that of the real turbulent flow is not
368 exactly the same after all. Nevertheless, the present model is aimed at providing
369 a fast yet accurate model for wind farm numerical assessment and micro-siting
370 optimization, errors in the near-wake regions are irrelevant.

371 The errors are listed in Table 5. The relative errors of the present model
372 are around 7.6% to 12.5%. Considering that the turbulent phenomenon is a
373 rather complicated problem even for the field of fluid mechanics, the present
374 model developed for fast assessment and optimization of wind farms can be
375 acceptable.

376 5. Conclusions

377 This paper presented a fast model to calculate velocity decay and turbulence
378 of the wake flow of wind turbines. The most significant improvement comparing
379 to existing traditional models is that the wake flow simulated by the proposed
380 model can be distorted by downstream local airflow, making it capable to pre-
381 dict wake flow’s intensity and shape in non-uniform flow fields above complex
382 terrains. The proposed wake flow model simulates the wake flow decoupled from
383 the velocity field, and utilizes the meshless particle random walk method. It has
384 balanced performance on efficiency and accuracy.

385 Comparisons with measurements in a wind tunnel validate the effectiveness
386 of the proposed wake flow model. The maximum MAE of the dimensionless
387 velocity is lower than 0.06, and the maximum relative error of the turbulence
388 intensity is lower than 13%. Comparison with wind farm data proves that the

present model has higher agreement with experiments than the widely used linear model on real terrain. The relative error of the predicted power yield with respect to the total capacity drops from 12.1% to 7.4%.

It is also notable that multiple improvements have been achieved since the previous version of the particle model: (1) adapted for wind turbines with different sizes; (2) has a more explicit definition on the attenuation process of wake particles; (3) overlapping of the velocity decay has been improved to have better agreement with the experimental data; (4) simulation of the turbulence intensity of the wake flow is capable using the same particle random walk technique.

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Figure 1: Schematic of the linear wake model.

Figure 2: Overlapped area of the wake region and the turbine being affected

Figure 3: Velocity at $X = 2.5D$ for the single turbine case.

Figure 4: Velocity at $X = 6D$ for the single turbine case.

Figure 5: Velocity at $X = 7.5D$ for the single turbine case.

Figure 6: Positions of the miniature turbines in a wind tunnel.

Figure 7: Velocity along the streamwise direction at the height of the hub ($Z = 0.104$ m) for the multiple turbines case.

Figure 8: Velocity along the streamwise direction at rotor top ($Z = 0.168$ m) for the multiple turbines case.

Figure 9: Velocity along the streamwise direction at the rotor bottom ($Z = 0.040$ m) for the multiple turbines case.

Figure 10: Power curve of the Vestas V82 wind turbine.

Figure 11: Altitude contours and turbine positions of the Wattle Point wind farm.

Figure 12: Total power of the Wattle Point wind farm under WSW wind direction.

Figure 13: Turbulence along the axis of the turbine for the single turbine case.

Figure 14: Turbulence at $X = 2.5D$ for the single turbine case.

Figure 15: Turbulence along the streamwise direction at the height of the hub ($Z = 0.104$ m) for the multiple turbines case.

Figure 16: Turbulence along the streamwise direction at rotor top ($Z = 0.168$ m) for the multiple turbines case.

Figure 17: Turbulence along the streamwise direction at the rotor bottom ($Z = 0.040$ m) for the multiple turbines case.

Table 1: Averaged velocity discrepancies between the result calculated by the present model and the experimental data within the region marked by blue dashed line.

C_T	Discrepancy
0.40	6.4026%
0.50	4.9895%
0.58	4.6903%
0.59	4.6674%
0.60	4.7091%
0.61	4.7755%
0.70	6.2361%

Table 2: Mean absolute errors of the evaluated dimensionless velocity for the multiple turbines case.

	Hub	Top	Bottom
Previous model	0.096	0.082	0.064
Present model	0.059	0.043	0.022
Linear model	0.172	0.060	0.040

Table 3: Root mean square errors of the evaluated dimensionless velocity for the multiple turbines case.

	Hub	Top	Bottom
Previous model	0.120	0.101	0.073
Present model	0.085	0.050	0.028
Linear model	0.200	0.072	0.048

Table 4: Minimum discrepancies of turbulence intensity between the simulated results and the experimental data for determination of τ .

τ	Discrepancy
10	0.009657
20	0.006243
30	0.005543
31	0.004664
32	0.005650
40	0.006814
50	0.007086

Table 5: Turbulent intensities and evaluated errors for the multiple turbines case.

	Hub	Top	Bottom
Measured incoming flow	0.076	0.066	0.091
Measured average in wake	0.102	0.118	0.092
Evaluated average error	0.0080	0.0147	0.0070
Relative error	7.8%	12.5%	7.6%