Investigations of a new Free Piston Expander Engine Cycle

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Abstract

This work examines the design and operation of a new, small-scale Free Piston Expander (FPE) engine that operates using low temperature waste heat sources to produce useful power output. The FPE is based on a sliding-piston architecture that eliminates challenges associated with MEMS-based rotating systems. A nonlinear lumped-parameter model is derived to study the factors that control the performance of the FPE engine and its unique operating cycle. This basic analysis considers a closed cycle operation of the FPE with low thermal or heat inputs and dimensions on the order of several millimeters. Key system design and operating parameters such as piston mass, external load, and heat input are varied to identify conditions and trends for optimal performance. The model indicated the pressure-volume diagram resembles a constant pressure cycle for a certain set of operating conditions but is also condition dependent. Increased heat inputs to the FPE reduced the engine natural or operating frequency while increasing the power output. Thermal efficiencies of the FPE are shown to be predictably low, on the order of 0.2 % due to the small heat input and operating temperature gradients associated with waste heat. Key design features are identified that reveal FPE efficiency, operating frequency, and output power are dependent on piston mass, external load, input heat-rate, and duration of heat input.

Keywords: MEMS, reciprocating engine, thermal scavenge, steam system, volumetric expander

1. Introduction

There are many significant challenges that influence energy supply, policy, and future consumption trends. At the heart of these challenges is increased

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worldwide energy consumption. The U.S. Energy Information Agency (EIA)
5 has predicted that world consumption will increase 56% by 2050 compared to
2010 levels [1]. In the midst of the predicted energy consumption growth ex-
ists a competing need to reduce greenhouse gas (GHG) emissions [2]. There
is acknowledgement that renewable energy supplies will provide a growing con-
tribution to worldwide energy needs, however, traditional non-renewables will
10 remain a significant source for the foreseeable future [1]. This indicates a contin-
ued need for traditional cycle efficiency and thermal management improvement
[3].

Thermodynamic cycles operating from non-renewable sources include the
standard Otto cycle with average efficiency of about 35% [4]. The majority of
15 energy is lost to the surrounding environment as low temperature waste heat via
heat rejection. This reality is similar for other industrial-based uses of energy
[5, 6]. Through the capture and reuse of rejected ‘waste’ energy, it is possible
to both increase the operating efficiency of the base cycle as well as reduce
Greenhouse Gas (GHG) emissions [5].

20 The work in this article examines the design and operation of a new, small-
scale engine designed to operate from these low-temperature sources using a
unique thermodynamic cycle. The Free Piston Expander (FPE) engine operates
via modified steam cycle, similar in nature to an Organic Rankine Cycle (ORC),
or even direct thermal or heat input. Through focus on a small, millimeter-scale
25 engine, it is possible to use the device across multiple types of thermal sources
and applications. For example, a single device may be utilized to scavenge small
thermal energy amounts for individual sensor power. By contrast, many FPE
engines may be combined in parallel to scavenge thermal energy from larger,
industrial size sources.

30 A microfabricated boiler as well as heat exchangers and other required sys-

tem components are already under investigation to support the FPE. Thapa et al. investigated a unique MEMS-based boiler using both experiment and computational analysis [7]. Heat exchanger research has been conducted using traditional microfabrication materials and methods, like silicon [8]. In this work, the use of capillary channels was the primary means for fluid transport. Relatedly, unique materials and fabrication processes that included electroplated porous nickel [9] or copper [10] have been under development. The option of thermal energy storage for a system of this type has also been investigated using paraffin wax as the storage media [11]. The nature of the FPE and the harvesting system itself allows different working fluids to be substituted for application to a variety of sources and real-world applications.

As part of this scavenging system, the FPE produces useful power output that may be either mechanical or electrical depending on setup and application need. In final form, electrical output may be achieved through the use of piezoelectric materials that flex as a result of piston motion [12] or via linear generator output. The FPE represents a small-scale device with many key dimensions on the order of millimeters. Many components of the device, however, are consistent with common microfabrication techniques and advantages [13].

One of the traditional challenges of power generation on the MEMS or microscale has been the effective management of combustion processes [14]. The feasibility of sustained combustion has been limited by high surface area to volume ratios that exist within miniature combustion systems [15]. Wang et al. investigated the miniaturization limit on internal rotary combustion engines [16]. Miniaturization of these devices considered losses including mass loss by seals as well as thermal losses. Reduced compression ratio reduced mass loss, or “blow-by,” leading to increased efficiency as length scales diminished. The ability to maintain effective operating pressures and low-loss internal sealing

remains a key challenge.

For this reason, some research has pursued devices and approaches that
60 are based on external power sources that drive output. These approaches and
devices take many forms that have included phase-change actuators that rely on
external thermal inputs [17]. Pneumatic actuators have also been of interest on
the small scale as a further alternative [18]. Phase change steam engines have
also been investigated including a micro Newcomen device [19].

65 A secondary challenge has been the ability to fabricate and operate high-
speed rotational systems that include Brayton or Rankine cycle devices [20].
Key among the challenges to this approach is the limitation of decreasing rotor
size [21]. Below key threshold values, the Brayton cycle and Rankine cycle
require alternative designs for typical viscous flows.

70 Rankine cycle efforts have leveraged the advantage of external combustion
for the high speed system. System design and modeling [22] as well as fabri-
cation of critical components like microturbopumps [23] have been conducted.
Silicon was the material of choice for these efforts, relying on advanced MEMS
and microfabrication techniques. Limitations have paralleled those present in
75 high temperature Brayton cycle efforts and include layer stress reliefs in the
fabrication process, external thermal boundary controls, and gap geometry dur-
ing operation [24]. When applied to a lower temperature energy source, as is
the case with many waste heat sources, the use of the Organic Rankine Cycle
(ORC) has also been studied on the small scale [25]. This approach removes
80 challenges associated with elevated temperature operation, however, remains an
engineering challenge for fabrication and production.

Microengines without rotation have included microbubble actuators utilizing
capillary channels [26] and membrane-based thermal actuators that produced
mechanical power output through internal phase change of a low boiling point

85 working fluid. The unique nature of this membrane-type design was further studied for operation approaching the Carnot cycle [27].

In experimental efforts, the ability of the membrane-based actuator to operate at resonant frequency generated a unique working cycle as a result [28]. The general form of these devices utilized one lower membrane with thickness
90 on the order of $1\text{ }\mu\text{m}$ and an upper membrane of similar thickness, separated by a cavity volume about $75\text{ }\mu\text{m}$ thick. The cavity was filled with working fluid that boiled at low temperature. External heat was added through the lower membrane to generate liquid to vapor phase change. This resulted in increased device volume. When operated at resonant frequency, the pressure and vol-
95 ume changes inside the cavity rose and fell 90 degrees out of phase, producing the unique thermodynamic engine cycle. This unique engine and process was modeled by Bardaweel et al. [29].

Free Piston Engines have been the subject of research and general application for decades on the large scale and in a variety of applications [30]. These
100 engines traditionally consist of a piston within a bore that is free to move between the cycle top-dead-center (minimum volume) and the bottom-dead-center (maximum volume) without mechanical linkage or constraint that is typical of traditional reciprocating engines. This variable compression ratio feature allows fuel flexibility. Applications have varied on the larger scale with internal
105 combustion as the primary driving force. Mechanical outputs have included hydraulic pressure with the free piston used as a linear motion pump [31] as well as electrical power generation. Unique solutions to valve timing and modeling have been investigated due to the lack of rotating components in the basic FPE design [32].

110 This lack of rotation has made the FPE of interest to the small-scale because it works around many of the aforementioned micro-power challenges. Further,

the driving force may be either internal or external combustion. This makes its use attractive for sources like waste heat. Various prior approaches have been executed in both modeling and experimental efforts. Early efforts were presented
115 by Aichlmayr et al. and considered internal combustion in a homogeneous charge compression ignition (HCCI) approach [33, 34]. This test reaffirmed the challenge to provide precise control of the combustion environment to limit cold surface quenching and manage heat release even in this non-rotating, small-scale application. More recent work has considered the uses of a catalyst wall
120 within the combustion chamber of a micro-FPE that relies on HCCI [35]. This numerical effort showed a basic improvement in energy conversion though a decreasing compression ratio due to advanced ignition versus a non-catalytic design. Indicated power output was also increased via this approach. Bai et al. continued efforts at HCCI modeling and established a unique model that
125 predicted the required kinetic energy of the piston to achieve ignition [36]. As with other combustion-centered free piston work, controlling factors included projected sealing leakage as well as accurate equivalence ratio mixing.

Zhang et al. studied a novel ‘swing engine’ piston approach that moved the free piston in an arc [37]. Free Piston Stirling Engines (FPSE) have also been
130 considered and modeled with respect to scaling and operation [38]. Among the opportunities for this design was increased power density, however, energy loss through heat as well as sealing within the engine were among the challenges. Additional approaches include replacing the traditional piston-cylinder assembly with a compliant architecture to facilitate the free-piston motion [39]. In this
135 regard, nonlinear lumped parameter engine models and prototypes have been tested to predict engine dynamics and engine performance [40, 41].

Initial results of an external combustion, low-temperature steam cycle Free Piston Expander (FPE) engine have also been investigated with bore dimen-

sions on the order of a few millimeters [42]. A laser-vibrometer was used to
140 track piston position in the bore as a function of forcing pressure. The natural
frequency of the device was approximately 33 Hz.

The past decades have seen continuous progress and the inherent challenges
of power production on the small-scale. In the field of engines, these have in-
cluded the noted challenges to combustion efficiency and quenching, high speed
145 rotation, sealing, and fabrication itself. There remains significant opportunity,
however, to perform research on power generation that is both free of rotation
and operational from low-grade energy sources. Recent analysis has indicated
that the general use of volumetric expanders shows more significant promise
than other approaches due to the low flow rates and low grade heat conditions
150 associated with these sources [43]. In particular, analysis by Imran et al. indi-
cated Free Piston Expanders had potential to yield lower leakage and reduced
frictional losses versus other volumetric expanders [43].

With these challenges and opportunity in mind, this work presents a new
FPE model that is developed using experimental inputs specifically for use in
155 low temperature waste heat application. High speed rotation is avoided as are
the challenges of combustion quenching and combustion sealing.

The work contained in the present study follows the basic FPE dimensions
of those established by Champagne and Weiss [42]. In the present modeling
effort, the FPE is approached as a closed cycle device, dependent on external
160 combustion or direct heat input. If applied to a real-world situation, this would
require the use of a so-called ‘thermal switch.’ This thermal switch would intro-
duce thermal energy to the sealed FPE directly from a waste heat source and
without fluid crossing any FPE boundary. These switches have been previously
demonstrated in external combustion micro-engine development [44]. However,
165 as justified in this work, this approximation from an open to closed system in

FPE operation allows useful interpretation of the basic FPE operation even if applied to a future open system.

To guide future design and testing efforts, key parameters are studied in this work. Specific parameters include the effects of piston mass, engine loading, and working fluid enthalpy or heat input on the unique FPE operating cycle. These provide key insight into the design of a real small-scale FPE engine that is well suited to the important field of low-temperature waste heat scavenging.

2. Theory

The basic design of the Free Piston Expander (FPE) is based on the notion of mechanical or electrical power output from an external thermal source. The FPE operates without reliance on high speed rotation or internal combustion which has been difficult to achieve on the microscale as noted previously. Through a sliding-expander type of mechanical action, it is possible to achieve power output without the rotation required of cycles like the Rankine or Brayton. Figure 1 illustrates the setup and key components of the modeled FPE.

Figure 1: Setup and key components of the FPE

As the piston reacts to increased pressure at the minimum cycle volume (Thermodynamic state 2) it expands. This has the dual effect of reducing the forcing pressure, while increasing the reaction pressure that is building on the other side (nonthermal input side) of the piston. It is this reaction pressure

185 that returns the piston to its initial thermodynamic state 2, ready for the next thermal or heat input.

In this present effort, the FPE is modeled as a closed thermodynamic system, requiring thermal or heat input from an external source. However, future work and ongoing experimental efforts are also characterizing design and operation
 190 using an open arrangement where pressurized working fluid (supplied by an external boiler) moves in and out of the FPE as part of its operation.

Figure 2: Schematics sketch of FPE

Based on this basic design, a nonlinear lumped-parameter model was developed to study the expander behavior and performance. Figure 2 shows a schematic diagram of the free piston expander used in the model development.
 195 The working fluid of the expander is considered to be a sealed volume V_o of vapor in this closed system approach. The basic properties of the fluid are taken to match 3MTM HFE-7200 working fluid. This provides consistency with parallel experimental efforts and is well suited for low temperature operation given the boiling point of 76 °C. The heat transfer to and from the expander normally
 200 associated with super saturated vapor injection and exhaust is modeled by the heat-rate $q(t)$ in and out of expander cavity of volume V_o at the left end. The pressure inside the expander cavity oscillates about injection pressure of vapor

P_i .

The material enclosing the vapor, the cavity, is represented by a deformable,
 205 bellows corrugations. The deformable corrugations of stiffness, s , allow horizontal displacement $x(t)$ of the right end of the volume, modeled to have a lump-parameter inertia m . The stiffness s is introduced only for computation advantage, hence its numerical value is very low; $s = 0.1$ N/m. When the displacement of the mass, m , is zero, the volume V_o is $V_o = LS$, where L is the
 210 nominal length and S is the cross-sectional area. The nominal volume V_o is numerically equal to V_i and V_2 i.e. $V_o = V_i = V_2$. The movement of the mass, m , is impeded by the dampers b_f and b . These dissipative elements model energy loss to friction and converted to useful work respectively. In addition, a damper with negative damping coefficient b_c ($b_c < 0$) is incorporated to model the work
 215 done on the expander during the compression stroke.

For a steady state operation, the vapor temperature, $T_i + \Delta T(t)$, pressure, $P_i + \Delta P(t)$, density, $\rho_i + \Delta \rho(t)$, and engine volume, $V_i + \Delta V(t)$ undergo cyclic variation, where the nominal and time-varying cyclic components are denoted with the “i” subscript and Δ respectively.

220 The nonlinear lumped parameter model of the expander shown in Figure 2 is derived by applying conservation of mass, conservation of energy, an ideal gas model to a moving control volume containing the vapor as working fluid, and Newton’s second law for the mass m . Mathematical statements of these principles are:

$$\frac{dM}{dt} = \frac{d}{dt} [(\rho_i + \Delta \rho) (V_i + \Delta V)] = 0 \quad (1)$$

$$\begin{aligned} \frac{d}{dt} [(\rho_i + \Delta\rho) c_v (V_i + \Delta V) (T_i + \Delta T)] + (P_i + \Delta P) \frac{d(V_i + \Delta V)}{dt} \\ + h (T_i - T_o + \Delta T) = q(t) \end{aligned} \quad (2)$$

$$\rho_i + \Delta\rho = \frac{P_i + \Delta P}{R (T_i + \Delta T)} \quad (3)$$

$$\frac{m}{S^2} \Delta \ddot{V} + \frac{b + b_f + b_c}{S^2} \Delta \dot{V} + \frac{s}{S^2} \Delta V = P_i - P_o + \Delta P \quad (4)$$

225 where $M = \rho_i V_i$ is the mass of vapor in the expander chamber at any one time, c_v is the constant volume heat capacity of vapor and is assumed constant over the temperature range, h is a coefficient that models conduction/convection heat losses from the enclosed vapor volume to the outside environment, and R is the mass-specific gas constant of vapor. T_o and P_o represent ambient
230 temperature and pressure conditions, respectively. The damping coefficient b_c is zero for cases where no external work is done on the expander during the compression stroke, and/or for other processes in the expander thermodynamic cycle.

If the heat-rate, $q(t)$, duration of heat addition, t_q , duration of heat rejection, t_c , and initial conditions, $\Delta T(0)$, $\Delta P(0)$, and $\Delta V(0)$ are specified, Equations 1 -
235 4 constitute a nonlinear model for the determination of the intermediate thermodynamic state variables: $\Delta T(t)$, $\Delta P(t)$, and $\Delta V(t)$. As the heat-rate function $q(t)$ is periodic, it is assumed that the dependent variables $\Delta T(t)$, $\Delta P(t)$, and $\Delta V(t)$ will also be periodic at steady state. The heat-rate function $q(t)$ takes
240 the form as indicated in Equation 5 and graphically by Figure 3.

$$q(t) = \begin{cases} q_H, & 0 < t \leq t_q \\ 0, & t_q < t \leq t_1 \\ q_o, & t_1 < t \leq t_1 + t_c \\ 0, & t_1 + t_c < t \leq t_1 + t_2 \end{cases} \quad (5)$$

Figure 3: Heat rate function, $q(t)$

The heat-rate function, $q(t)$, consists of finite duration of heating and impulsive cooling. The heating and cooling pulses are of duration t_q and t_c ($t_c \rightarrow 0$) respectively. The heating pulse at a constant rate of q_H is applied at the end of the compression stroke at $t = 0$. At the time $t = t_1$, a cooling pulse at a constant rate q_o ($q_o < 0$) is then applied. At the time $t = t_1 + t_2 = t_p$, the cycle is repeated, so that t_p is the period of the cyclic heat-rate. This specification for the heat-rate $q(t)$ allows for asymmetry in the cycle, in that the times t_1 and t_2 , and the magnitude of the heating and cooling pulses, are not necessarily equal.

For the subsequent analysis, a nondimensional scaling was adopted. Equa-

tion 6 shows the specific scales as applied to the independent and dependent variables.

$$\begin{aligned}
\Delta \bar{V} &= \frac{\Delta V}{V_i} \\
\Delta \bar{P} &= \frac{\Delta P}{P_i} \\
\Delta \bar{T} &= \frac{\Delta T}{T_i} \\
\Delta \bar{\rho} &= \frac{\Delta \rho}{\rho_i} \\
\bar{q} &= \frac{q}{q_H} \\
\bar{t} &= t\omega
\end{aligned} \tag{6}$$

In Equation 6, $\omega = \sqrt{s_h/m}$ is a reference frequency associated with the Helmholtz stiffness $s_h = \gamma P_i S^2/V_i$ of the vapor within the expander and the overbar indicates a nondimensional independent or dependent variable. After application of these scales, the nonlinear model [Equations 1 - 4] becomes:

$$\Delta \bar{\rho} = -\frac{\Delta \bar{V}}{1 + \Delta \bar{V}} \tag{7}$$

$$C\Delta \dot{\bar{T}} = -(1 + \Delta \bar{P})\Delta \dot{\bar{V}} - H\left(1 - \frac{T_o}{T_i} + \Delta \bar{T}\right) + \Gamma \bar{q} \tag{8}$$

$$\Delta \bar{P} = (1 + \Delta \bar{\rho})(1 + \Delta \bar{T}) - 1 \tag{9}$$

$$\Delta \ddot{\bar{V}} + 2(\zeta + \zeta_c)\Delta \dot{\bar{V}} + \Lambda \Delta \bar{V} = \frac{1}{\gamma}\left(1 - \frac{P_o}{P_i} + \Delta \bar{P}\right) \tag{10}$$

where:

$$\begin{aligned}
C &= \frac{\rho_i c_v T_i}{P_i} \\
H &= \frac{h T_i}{\omega P_i V_i} \\
\Gamma &= \frac{q_H}{\omega P_i V_i} \\
\zeta &= \frac{1}{2} \frac{b + b_f}{m \omega} \\
\zeta_c &= \frac{1}{2} \frac{b_c}{m \omega} \\
\Lambda &= \frac{s}{s_h}
\end{aligned} \tag{11}$$

For analysis, the set [Equations 7 - 10] is placed into state space format, the system model, and becomes:

$$\frac{d\Delta\bar{V}}{dt} = \Delta\dot{\bar{V}} \tag{12}$$

$$\frac{d\dot{\bar{V}}}{dt} = -\Lambda\Delta\bar{V} - 2(\zeta + \zeta_c)\Delta\dot{\bar{V}} + \frac{1}{\gamma} \left(\frac{1 + \Delta\bar{T}}{1 + \Delta\bar{V}} - \frac{P_o}{P_i} \right) \tag{13}$$

$$\frac{d\Delta\bar{T}}{dt} = -\frac{1}{C} \left(\frac{1 + \Delta\bar{T}}{1 + \Delta\bar{V}} \right) \Delta\dot{\bar{V}} - \frac{H}{C} \left(1 - \frac{T_o}{T_i} + \Delta\bar{T} \right) + \frac{\Gamma}{C} \bar{q} \tag{14}$$

In this format, the state variables are $\Delta\bar{V}$, $\Delta\dot{\bar{V}}$, and $\Delta\bar{T}$.

To characterize the pressure-volume behavior of the expander in this work,
the model [Equations 7 - 10] was integrated numerically. It was assumed that
the duration of the heating pulse t_q is finite such that $q_H t_q = E$, where E is
the finite energy supplied to the expander per cycle during the heating pulse.
In this circumstance the pressure-volume behavior of the expander at steady
state takes the form shown in Figure 4. Thermodynamic states 1-2-3-4 on
the diagram correspond to the end of impulsive cooling and beginning of the

compression stroke, end of the compression stroke and beginning of the heating, end of heating and beginning of expansion stroke, and the end of the expansion stroke respectively. The expander volume and the temperature inside it before and after the heat rejection can be computed from Equations 13 and 14 and are given by Equations 15 and 16.

$$\Delta \bar{V}_1 = \Delta \bar{V}_4 \quad (15)$$

$$\Delta \bar{T}_1 = \Delta \bar{T}_4 + \frac{\Gamma t_c \omega}{C} \left(\frac{q_o}{q_H} \right) \quad (16)$$

Figure 4: Steady state P-V diagram

It was assumed that the impulsive cooling pulses occur at zero piston velocity, $\Delta \dot{\bar{V}}_4 = \Delta \dot{\bar{V}}_1 = 0$. The resulting volumes and temperatures $\Delta \bar{V}_4, \Delta \bar{T}_4$ and $\Delta \bar{V}_2, \Delta \bar{T}_2$ from expansion and compression processes 3-4 and 1-2 were computed by numerically integrating [Equations 12 - 14] from the initial conditions $\Delta \bar{V}_3, \Delta \bar{T}_3, \Delta \dot{\bar{V}}_3$ and $\Delta \bar{V}_1, \Delta \bar{T}_1, \Delta \dot{\bar{V}}_1 = 0$. Similarly, the resulting volume $\Delta \bar{V}_3$ and temperature $\Delta \bar{T}_3$ from heat addition process 2-3 were com-

puted by numerically integrating [Equations 12 - 14] from the initial condition $\Delta\bar{V}_2 = 0, \Delta\bar{T}_2 = 0, \Delta\dot{\bar{V}}_2 = 0$.

The system [Equations 12 - 14] can only reach steady state when the proper
 280 cooling pulse magnitude q_o/q_H and compression work modeled by damper b_c
 are chosen for cycle integration. Improper choice of q_o/q_H and b_c would lead to
 monotonically drifting state variables $\Delta\bar{V}$ and $\Delta\bar{T}$ precluding periodic behavior.
 To determine the steady-state behavior of the expander $q_H, \Delta\bar{V}_2 = 0, \Delta\bar{T}_2 =$
 0 , and $\Delta\bar{P}_2 = 0$ were specified, the ratio q_o/q_H and damping coefficient b_c
 285 were chosen, and integration was started from the quiescent initial state of
 $\Delta\bar{V}_2 = \Delta\dot{\bar{V}}_2 = \Delta\bar{T}_2 = 0$. An example integration illustrating the first cycle in a
 transient dwell-up from this initial state is shown in Figure 5.

Figure 5: Transient state P-V diagram

First, temperature and volume were computed for the thermodynamic cycle
 1-2-3-4-1 as explained below and later Equations 7 and 9 were used to calculate
 290 corresponding pressure. The volume $\Delta\bar{V}_3$ and temperature $\Delta\bar{T}_3$ were obtained
 by integrating the set [Equations 12 - 14] from the initial condition $\Delta\bar{V}_2 =$
 $\Delta\dot{\bar{V}}_2 = \Delta\bar{T}_2 = 0$ until a time t_q . The volume $\Delta\bar{V}_4$ and temperature $\Delta\bar{T}_4$ were

obtained by integrating the set [Equations 12 - 14] from the initial conditions $\Delta\bar{V}_3, \Delta\bar{T}_3, \Delta\dot{\bar{V}}_3$ until a time, $\bar{t}_1$, such that $\Delta\dot{\bar{V}}_4 = 0$. Note that the time, $\bar{t}_1$ is not
 295 known ‘*a priori*’, and is determined during the integration. Then, the volume $\Delta\bar{V}_1$ and temperature $\Delta\bar{T}_1$ were determined from Equations 15 and 16. Finally, a volume $\Delta\bar{V}_{2'}$ and $\Delta\bar{T}_{2'}$ were obtained by integrating the set [Equations 12 - 14] from the initial conditions $\Delta\bar{V}_1, \Delta\bar{T}_1, \Delta\dot{\bar{V}}_1 = 0$ until a time, $\bar{t}_2$, such that $\Delta\dot{\bar{V}}_{2'} = 0$. Like the time of expansion, $\bar{t}_1 - \bar{t}_q$, the time of compression, $\bar{t}_2$, is not
 300 known ‘*a priori*’, and is determined by the conditional integration. Since the system undergoes a transient phase, the cycle is not closed and $\Delta\bar{V}_2 \neq \Delta\bar{V}_{2'}$ and $\Delta\bar{T}_2 \neq \Delta\bar{T}_{2'}$. However, if the process is repeated, and the proper ratio q_o/q_H and damping coefficient, b_c , are chosen, the cycle eventually reaches steady state, the cycle closes, and $\Delta\bar{V}_2 = \Delta\bar{V}_{2'}$ and $\Delta\bar{T}_2 = \Delta\bar{T}_{2'}$. The integration stops when
 305 the equality has been satisfied to a specified tolerance. In practice, a function-minimization computational procedure was used, which determined the proper choice of q_o/q_H and b_c for a specified q_H that would result in a steady-state periodic cycle.

3. Results and Discussion

310 The small-scale free piston expander shown in Figures 1 and 2 is modeled with $h = 0$ W/K, $b_f = 0$ N-s/m, $b = 3$ N-s/m, $V_o = 56.5$ mm³ (4.8 mm in diameter and 3.2 mm of nominal length), $s = 0.1$ N/m, and $m = 0.34$ g treated as the reference case. Standard temperature and pressure conditions of $T_o = 298$ K and $P_o = 101$ kPa are chosen for the ambient state. The injection
 315 temperature and pressure of the working fluid are selected to be $T_i = 373$ K and $P_i = 105$ kPa respectively. The properties of working fluid used are $R = 24.6$ J/kg-K, $\rho = 11.4$ kg/m³, $c_v = 1069$ J/kg-K, and $c = 96.9$ m/s. These values are based on real 3MTM HFE-7200 properties. The nonlinear model [Equations 7 -

10] is used to predict the performance of the free piston expander.

3.1. Thermodynamic Cycle

Figure 6 shows a plot of expander pressure and volume for one thermodynamic cycle 1-2-3-4-1. It is obtained using the nonlinear model [Equations 7 - 10] for a steady state operation by supplying heat at a rate $q_H = 2$ W for a duration $t_q = 1.9$ ms at the end of the compression process 1-2 to the reference case. The ratio $q_o/q_H = -0.99$ and $b_c = -3.27$ N-s/m are required for steady state operation of the nonlinear model. The figure shows that the pressure inside the expander throughout the heat addition process remains nearly constant at about 105 kPa, indicating a constant pressure process. Following the heat addition process 2-3, the expander expands isentropically 3-4 to about 59.5 mm³. Next, the heat rejection process 4-1 takes place instantaneously by interacting with the surroundings. Finally, the expander undergoes isentropic compression 1-2 due to external work resulting in a minimum cycle volume $V_2 = V_o = 56.5$ mm³.

Figure 6: P-V diagram for the reference expander for input heat-rate $q_H = 2$ W

To determine the work done per cycle, a numerical integration of the P-V

335 diagram (Figure 6) for one thermodynamic cycle was performed and found to be
 3.9 μJ . From the figure, it has been found that the work done by the expander
 during the overall expansion and compression processes 2-4 and 1-2 are 309.2
 μJ and -305.3 μJ respectively. The corresponding expander thermal efficiency
 for one thermodynamic cycle was calculated to be 0.1 %.

340 Figure 7 shows a plot of expander temperature and volume for one ther-
 modynamic cycle. It is obtained for the same operating conditions used for
 generating P-V diagram in Figure 6. It is evident from the figure that the
 constant rate heat addition process 2-3 is not a constant temperature process.

Figure 7: T-V diagram for the reference expander for input heat-rate $q_H = 2 \text{ W}$

Though the expansion and compression processes 3-4 and 1-2 are isentropic,
 345 the temperature change in them are small $\Delta T = 0.5 \text{ K}$ due to the small change
 in expander cavity volume. However, during the heat rejection process 4-1, the
 temperature drops significantly by $\Delta T = 5.5 \text{ K}$. From the figure, the temper-
 ature inside the expander over one thermodynamic cycle varies between 372.5
 K and 378.5 K indicating that the working fluid is always superheated based
 350 on $3M^{TM}$ provided thermodynamic tables. Ideal gas behavior is assumed as a

result, as has been successful in other modeling approaches [45].

3.2. Expander Performance

The expander performance is evaluated in terms of thermal efficiency and power output. Output power from the expander is computed assuming that the only dissipation mechanism present is the energy conversion damping b (also called loading). The nonlinear model [Equations 7 - 10] is used to study the effects of design and operating parameters such as enthalpy of the working fluid, piston mass, and load on the expander performance.

3.2.1. Enthalpy of the working fluid

The enthalpy of the working fluid depends on its mass, thermodynamic properties, and thermodynamic state variables. Since the thermodynamic properties and mass of the closed system remain unchanged, the enthalpy depends solely on the thermodynamic state variables: temperature and pressure. Studying the effect of enthalpy provides insight in to injection temperature and pressure of the working fluid for achieving greater expander performance. Therefore to study the effect of enthalpy on expander performance, the injection temperature and pressure are varied. The temperature variation is accomplished by adding heat at the end of the compression process 1-2, while the injection pressure is varied by setting the expander pressure at state 2, P_2 . The heat addition to the expander depends on input heat-rate and duration of heat addition process 2-3. For our analysis, only heat-rate is varied, while the duration of heat addition is kept constant i.e. $t_q = 1.9$ ms.

Input Heat Rate. The effect of working fluid injection temperature on the expander performance is evaluated by adding heat at the end of the compression process 1-2. For a fixed duration of heat addition $t_q = 1.9$ ms, the input heat-rate q_H is varied. Figure 8, shows the expander behavior in the form of a P-V

diagram plotted for various input heat-rates $q_H = 1, 2$, and 3 W, piston mass $m = 0.34$ g, load $b = 3$ N-s/m, and injection pressure $P_i = P_2 = 105$ kPa. It can be observed that for a constant input heat-rate $q_H = 2$ W, the heat addition process 2-3 models a constant pressure process. However, for higher or
 380 lower input heat-rates the model departs from constant pressure process. This indicates that there exists a computational domain in which the closed cycle, constant heat-rate modeling approach models a constant pressure cycle. Also, it can be observed that for increasing input heat-rate the P-V diagram expands,
 385 implying an increase in work output.

For an increase in input heat-rate from $q_H = 1$ to $q_H = 2$ W, the work output increased from 1.6 to 3.9 $\mu\text{J}/\text{cycle}$ which translates to an increase in efficiency from 0.09% to 0.13% . On the other hand, the operating frequency of the expander decreased from $f = 39.8$ Hz to $f = 38.6$ Hz owing to the resulting
 390 larger stroke lengths. Despite the reduction of operating frequency, the output power increased from 66 μW to 150 μW due to the significant rise in work output per cycle. Therefore, it is desired to operate the expander with higher input heat-rate to achieve higher efficiency and output power. However, in practice there exists an upper limit on the input heat-rate determined by enthalpy of
 395 the vapor or working fluid as well as the actual real-world heat source itself.

Injection pressure. As previously noted, injection pressure of the working fluid also determines the enthalpy of the working fluid. To study the effects of enthalpy, the injection pressure is varied while maintaining input heat-rate constant. For this, the expander pressure at state 2 is set to be equal to the injection
 400 pressure P_i . Figure 9 shows a P-V diagram generated for three injection pressures; $P_i = 105, 110$, and 115 kPa, while maintaining the input heat-rate $q_H = 2$ W, input heat $E = 3.8$ mJ/cycle, piston mass $m = 0.34$ g, and load $b = 3$ N-s/m. From the figure, despite same minimum volume V_2 , an increase in

Figure 8: P-V diagram for the reference expander and varying input heat-rates $q_H = 1, 2,$ and 3 W.

injection pressure increased the swept volume $V_2 \rightarrow V_4$. For instance, the swept
405 volume $V_2 \rightarrow V_4$ for $P_i = 105$ kPa is about 3 mm^3 , while for $P_i = 110$ kPa
the swept volume $V_2 \rightarrow V_4$ is about 6 mm^3 . Though the heat input for both
conditions is maintained at $E = 3.8 \text{ mJ/cycle}$ there is an increase in output
work. This implies that higher injection pressure is desirable for higher output
work. It is clear from the figure that the constant input heat-rate departs from
410 constant pressure cycle for higher injection pressure. It is also worth noting
that the expander pressure equilibrates with ambient pressure at the end of the
expansion process 3-4 irrespective of the injection pressure P_i . The nonlinear
model predicted that the operating frequency decreases from $f = 38.6 \text{ Hz}$ to $f =$
 33.2 Hz for an increase in injection pressure P_i from 105 kPa to 110 kPa . This
415 means that the model predicts a softening behavior of the expander system.
A similar trend was noted in direct study of the input heat rate as discussed
previously.

Figure 9: P-V diagram for the reference expander for input heat-rate $q_H = 2$ W and varying injection pressures $P_i = 105, 110,$ and 115 kPa.

3.2.2. Load

In the nonlinear model [Equations 7 - 10], the load b models the work output
 420 or useful work while the frictional load b_f models friction loss. In this study, the
 frictional damping b_f is assumed to be negligible. This is based on prior exper-
 imental efforts that monitored the effect of various lubricants on a small-scale
 piston / bore assembly as well as the piston cyclic response [42]. These results
 allowed determination of a frictional damping coefficient to be approximately
 425 $.014 \frac{N-s}{m}$, well below the minimum engine loadings considered in this present
 work.

The effect of load on the engine performance is captured in the form of a
 P-V diagram shown in Figure 10.

The P-V diagram shown in Figure 10 is generated for three loads, $b = 3, 4,$
 430 and 5 N-s/m at constant input heat-rate $q_H = 2$ W and injection pressure P_i
 $= 105$ kPa. It can be observed that when the load on the expander is increased
 from $b = 3$ N-s/m to $b = 5$ N-s/m the heat addition process 2-3 begins to depart
 from a constant pressure process. This can be explained via the ideal gas law

Figure 10: P-V diagram for the reference expander for input heat-rate $q_H = 2$ W and varying loads $b = 3, 4$, and 5 N-s/m

and treating the expander as a spring-mass-damper system.

435 For a fixed injection pressure P_i , input heat-rate q_H , and higher loads b , the piston velocity decreases due to the increased damping on the expander; thus resulting in a positive rate of change of pressure. Based on a spring-mass-damper system analogy it is assumed that the stroke length gets smaller for higher load b . Despite increasing load b , the expander stroke length remains
440 nearly constant. This is due to the fact that higher load in turn results in higher expander pressure at state 3 that can push the piston further outward (thermodynamic state 4).

It is evident from the figure, the increase in load on the expander from $b = 3$ N-s/m to $b = 5$ N-s/m did not significantly increase the area enclosed by the P-V
445 diagram. The increased area during the heat addition process 2-3 is nullified by the decrease during the expansion process 3-4. However, a significant change in natural frequency of the expander was observed for different loads $b = 3, 4$, and 5 N-s/m. The natural frequency dropped from 38.6 Hz for $b = 3$ N-s/m to 27.1 Hz for $b = 5$ N-s/m. This behavior was anticipated and can be explained along

the lines of a simple oscillator. A simple oscillator's natural frequency reduces in the presence of a damper given by the equation $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, where ω_n is the natural frequency of the oscillator, ω_d is the damped natural frequency of the oscillator, and ζ is the damping ratio. Table 1 presents expander performance for various loads and fixed heat input $E = 3.8$ mJ/cycle.

Load (N-s/m)	Efficiency (%)	Operating Freq. (Hz)	Output Power (mW)
3	0.103	38.6	15.1
4	0.106	31.6	12.7
5	0.108	27.1	11.1

Table 1: Output power for various loading conditions for fixed input heat $E=3.8$ mJ/cycle

3.2.3. Piston Mass

The effect of piston mass on the engine performance is crucial as it is one of the key factors that dictates expander operating frequency. It is presumed that a smaller piston mass will result in higher natural or operating frequency that can result in higher output power. Also, studying the effect of piston mass can help identify an optimum piston size and type of material.

Figures 11 and 12 show a P-V diagram and volume-velocity diagram for three piston masses $m = 0.34, 1.7,$ and 3.4 g, input pressure $P_i = 105$ kPa, load $b = 1$ N-s/m, heat addition time $t_q = 1.9$ ms, and input heat $E = 10$ mJ/cycle, respectively. It is evident from the figure that upon increasing the piston mass from 0.34 g to 1.7 g, the inertia effect on the engine behavior becomes evident. Though higher piston mass implies higher resistance to motion, the pistons with higher mass unusually have longer stroke lengths. This can be understood by analyzing the expander total energy as discussed below.

The expander can be assumed to be a spring-mass-damper system whose total energy is the sum of thermal energy in the expander cavity, dissipative energy in the form of output work, kinetic energy of the piston, and potential

Figure 11: P-V diagram for load $b = 1$ N-s/m, input heat $E = 10$ mJ/cycle, and varying mass m

energy stored in the spring. For analysis of piston mass effects directly, the change in thermal energy of the working fluid and the dissipative energy are assumed to be negligible. Therefore, the total energy comprises of kinetic and potential energies. Since the heat addition per cycle $E = 10$ mJ/cycle is constant, the total energy is distributed between the piston and the spring. It is known that the kinetic energy in the piston assists the piston to travel a longer distance due higher momentum, while the potential energy in the spring retards the piston travel due to its opposing nature resulting in shorter stroke lengths. Therefore, for higher piston masses the kinetic energy dominates the potential energy resulting in larger stroke lengths and vice-versa. The effect of the piston mass on the operating frequency is schematically represented in Figure 13. The increasing piston mass resulted in decreasing natural frequency f and can be explained using the relation $f = \sqrt{s/m}$, where s is the stiffness of spring and m is the piston mass. Such similar trends have been observed by Weiss [12].

The effect of the piston mass on the expander efficiency and output power for a fixed input heat $E = 10$ mJ/cycle, load $b = 1$ N-s/m, and injection pressure P_i

Figure 12: Volume-velocity diagram for load $b = 1$ N-s/m, input heat $E = 10$ mJ/cycle, and varying mass m

Figure 13: Operating frequency dependency on mass m for fixed input heat $E = 10$ mJ/cycle and load $b = 1$ N-s/m

490 $= 105$ kPa is schematically represented in Figure 14. From Figure 14 it is clear that the efficiency steadily rises while the output power rises and then drops with the increase in piston mass. The input heat $E = 10$ mJ/cycle and duration of heat input $t_q = 1.9$ ms were held constant, and piston mass m was varied to find the optimum output power and efficiency. The results presented in Table

2 show that the piston mass $m = 1.02$ g produces a maximum output power of 148 mW at an efficiency of 0.16 %. Despite the trend that lower piston masses
 495 in the range $m \in (0.34 - 2.04$ g) produce higher output power, higher efficiencies $\eta > 0.2$ % are achieved for piston masses greater than $m > 2.38$ g. Given the low temperature operating conditions imposed on the FPE, it is recommended to use higher piston masses to achieve respectable thermal efficiencies.

Figure 14: Efficiency and output power dependency on piston mass m

Piston Mass (g)	Efficiency (%)	Operating Freq. (Hz)	Output Power (mW)
0.34	0.13	102.1	137
0.68	0.15	99.2	145
1.02	0.16	93.4	148
1.36	0.17	85.7	146
1.70	0.18	79.0	141
2.04	0.19	73.5	138
2.38	0.20	68.9	134
2.72	0.20	65.0	130
3.06	0.21	61.7	127
3.40	0.21	58.8	123

Table 2: Output power for various loading conditions for fixed input heat $E=3.8$ mJ/cycle

4. Conclusions

500 The ability to find new uses for low temperature energy sources like waste heat continues to grow in importance due to factors like fossil fuel use and coupled greenhouse gas emissions. This work examined the operating cycle and characteristics of a new, small-scale Free Piston Expander (FPE) engine designed to operate from these sources. The basic analysis considered a closed
505 cycle operation of the FPE with suitably low thermal inputs and dimensions on the order of several millimeters.

A new, nonlinear lumped-parameter model was created to provide initial evaluation of the FPE and the FPE cycle. The essential expander behavior and performance was captured through this approach. The effects of several key
510 system design and operating parameters were considered. Of specific interest were the cycle power output, projected efficiency, and critical design parameters. Variables considered included various piston masses, external loads, and input heat-rates.

There were several key results. First, a new operating cycle was indicated
515 that was similar in form to a constant pressure cycle but highly dependent on operating inputs. These included the input heat-rate, duration of heat input, and load on the expander itself. Second, the nature of the piston being driven by surrounding pressure balances gave the FPE the potential for a variety of closed cycle loops and shapes. Third, indicated efficiencies were low and ranged from
520 about 0.1 to 0.2 %. This was driven by the assumed low temperature gradient driving FPE operation as guided by waste thermal energy itself. Fourth, the efficiency was shown to be additionally dependent on design parameters, most notably the piston mass and heat input rate. Increased heat input had the effect of both reducing the FPE operating frequency while increasing the total power
525 output. This was due to the increased stroke length resulting from increased

energy input. The resulting stroke length increase also resulted in an expanded work output per cycle, however, a reduction in the frequency. Despite this operating frequency reduction, the expanded work output from the pressure-volume diagram produced a total increase in power output of about 127 %.

530 Despite the low efficiency result, good scope for improving basic design and performance output was generated from the analysis. Additional gains in power output and efficiency are easily achieved through operation of the engines in parallel or series as well as an increased operating temperature gradient specific to any targeted real world application. This work provides the basis for further
535 nonlinear modeling in an open cycle approach. This work also provides valuable insight into the operation and characteristics of an experimental FPE, designed to operate in a small-scale thermal scavenging application.

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Nomenclature

b	: Load (N-s/m)
b_c	: Damping coefficient for external work on the engine (N-s/m)
b_f	: Frictional damper (N-s/m)
c	: Speed of sound (m/s)
C	: Nondimensional heat capacity of vapor
c_v	: Constant volume heat capacity of vapor (J/kg-K)
E	: Finite energy supplied to the expander per cycle (mJ/cycle)
f	: Engine operating frequency (Hz)
h	: Heat loss coefficient (W/K)
H	: Nondimensional heat loss coefficient
L	: Cylinder nominal length (mm)
m	: Piston mass (g)
M	: Working fluid mass (kg)
P	: Pressure (kPa)
P_i	: Injection pressure of vapor (kPa)
$q(t)$	: Heat-rate (W)
q_H	: Heating pulse at constant rate (W)
q_o	: Cooling pulse at constant rate (W)
R	: Mass-specific gas constant of vapor (J/kg-K)
s	: Stiffness of deformable corrugations (N/m)
S	: Cylinder cross-sectional area (mm ²)
s_h	: Helmholtz stiffness (N/m)
t	: time (s)
T	: Temperature (K)
t_1	: Time at end of expansion process (ms)
t_2	: Duration of compression process (ms)
t_c	: Duration of heat rejection (ms)
t_p	: Engine cycle time period (ms)
t_q	: Duration of heat addition (ms)
V	: Volume (mm ³)
V_0	: Expander nominal cavity volume (mm ³)
$x(t)$	: Piston displacement (mm)
γ	: Ratio of specific heats
Δ	: Time-varying cyclic component
ζ	: Damping ratio
ζ_c	: Damping ratio of external work on the engine
Λ	: Nondimensional stiffness of deformable corrugations
ρ	: Density of working fluid (kg/m ³)
ω	: Reference frequency (rad/s)
Γ	: Nondimensional heating pulse
<u>Other Subscripts and Superscripts</u>	
<i>Numerical Subscript</i>	: Thermodynamic state in the engine cycle
<i>Overbar</i>	: Nondimensional term
<i>Overdot</i>	: Differentiation with respect to time
<i>Prime</i>	: Transient thermodynamic state in the engine cycle
<i>Subscript i</i>	: Nominal condition
<i>Subscript o</i>	: Ambient condition

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