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power generator with longitudinal vortex generators

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Abstract

This work investigates the feasibility of using longitudinal vortex generators (LVGs) to improve heat transfer in thermoelectric generator (TEG) systems. A coupled fluid-thermal-electric model is established with COMSOL Multiphysics[®] to study the effects of LVG height, LVG attack angle, and hot-side inlet gas temperature. We find that LVGs can significantly enhance the heat transfer performance, power output, and thermal conversion efficiency due to the generated longitudinal vortices, especially at small LVG attack angles. The performance of the thermoelectric generators with LVGs is best for LVGs that span the full height of the channel at the highest temperature examined (550 K), where the heat input, net power and thermal conversion efficiency are enhanced by 29%-38%, 90%-104% and 31%-36%, respectively, compared to smooth flow channel. As the hot-side inlet gas temperature decreases, the pumping power remains constant and requires a larger portion of the power output since the heat input and power output are significantly reduced. Therefore, it is not beneficial to use tall LVGs at lower hot-side inlet temperatures and higher inlet Reynolds numbers due to the large ratio of pressure drop to power output, but smaller LVGs are still useful under these conditions.

Keywords: Thermoelectric generator, longitudinal vortex generator, heat transfer enhancement, pressure drop, power output

Nomenclature

c_p	= Specific heat, J/(kg·K)
D	= Thickness, mm
D_h	= Hydraulic diameter, mm
$\mathbf{E}$	= Electric field intensity vector, V/m
H	= Height, mm
I	= Electric current, A
$\mathbf{J}$	= Electric current density vector, A/m ²
k	= Thermal conductivity, W/(m·K)
L	= Length, mm
p	= Static pressure, Pa
P_l	= Longitudinal pitch, mm
P_o	= Power output, W
P_t	= Transverse pitch, mm
Q_h	= Hot-side heat transfer rate, W
Re	= Reynolds number
T	= Temperature, K
U	= Area-averaged velocity in inlet section, m/s
$\mathbf{u}$	= Velocity vector, m/s
V	= Voltage, V
$\dot{V}$	= Volumetric flow rate, m ³ /s
W	= Width, mm
W_{net}	= Net power, W
$W_{\Delta p}$	= Pumping power, W

Greek

α	= Seebeck coefficient, V/K
β	= Attack angle, °
ρ	= Density, kg/m ³
σ	= Electrical conductivity, S/m
Π	= Peltier coefficient, V
μ	= Dynamic viscosity, Pa·s

η = Thermal conversion efficiency

Subscripts

c = Cold

f = Fluid

h = Hot

v = Longitudinal vortex generator

1. Introduction

The use of fossil fuels in automobiles is expensive in both monetary and environmental terms. In the Otto cycle, only about 30% of the energy released from the fuel is converted to useful work while the remaining 70% is lost primarily as heat discharged through the exhaust gases or expelled by the cooling system [1]. Recently, many automotive manufacturers have explored the use of thermoelectric generators in order to convert some fraction of the waste heat contained in the exhaust gas into useful electric power.

The potential use of thermoelectric devices for waste heat recovery has received considerable attention in recent years. Wang et al. [2] developed a general and three-dimensional numerical model for a thermoelectric device. Their results indicated that including the effects of temperature dependent properties and heat losses to the ambient gas had significant effects on the TEG system performance. Liang et al. [3] established a two-stage thermoelectric model using exhaust gas from an internal combustion engine as the heat source. They examined the effects of temperature and heat transfer coefficients for the hot and cold sides, and found that the performance increased with increasing heat transfer coefficient up to 400 W/m²-K, and that there was an optimum ratio of thermoelectric elements for the two stages to maximize power and efficiency. Nguyen and Pochiraju [4] developed a thermoelectric model to study a thermoelectric generator for a hot side subject to a transient heat source and natural convection on the cold side and compared their results with experiments. They showed that the Thomson effect played an important role on the power generation while it was insignificant with regard to the temperature distributions. Sandoz-Rosado and Stevens [5] examined the effects of solder, ceramic

interfaces, electrical contact thickness and leg spacing on the performance of thermoelectric modules. These effects were studied with standard one-dimensional constant property models that were commonly used in thermoelectric module design along with a new three-dimensional, device-level model using COMSOL[®]. Their results showed that the one-dimensional model resulted in 10-20% error compared to the three-dimensional multiphysics model. Al-Merbati et al. [6] used finite element methods to evaluate the impact of thermoelectric generator geometry on thermal stress, thermal efficiency, and output power, and found that the geometric configuration had an impact on the thermal efficiency and the thermal stresses could be reduced to improve device lifetime. Fraisse et al. [7] compared the benefits and drawbacks of simplified models, finite element methods, and one improved simplified model using two different Seebeck coefficients with a constant Thomson coefficient. Their work demonstrated the importance of proper accounting for the Thomson coefficient.

Unlike the above mentioned studies, Chen et al. [8] focused more attention on the effect of the fluid flowing in the channel, and developed a fluid-thermal-electric multi-physics coupled model based on a FLUENT[®] platform for the purpose of supporting a more realistic thermal boundary condition for the thermoelectric generator rather than using constant temperature or constant heat flux boundary conditions. Suzuki et al. [9] investigated the effects of flow patterns, i.e., counter-flow and split-flow, on the fluid velocity, temperature distribution, pressure drop and electromotive force, and showed that the power required to circulate fluids was not negligible. Rezanian and Rosendahl [10] evaluated the impact of the location of the thermoelectric legs, applied heat flux, variation of pressure drop along the heat sink, and the arrangement of inlet plenum in a thermoelectric generator with parallel microchannel heat sinks. They showed that with proper arrangement the pumping power could be kept small compared with the improvement in thermal performance. Lu et al. [11] developed a test bench to examine different locations for the placement of a TEG in the exhaust system of an automobile. They found that the best location for a TEG system was between the catalytic converter and the muffler since this

location provided more uniform flow, lower backpressure, and higher surface temperature. Zhou et al. [12] compared a simple TE model derived from Carnot efficiency with a detailed model based on an interfacial energy balance, and determined that the more rigorous coupled-field model was more accurate in comparison with previous experiments. They also showed that it was important to maintain turbulent flow within the channels of the heat exchanger. Reddy et al. [13] proposed an integrated thermoelectric device with the interconnector designed as an internal heat exchanger, which could reduce the thermal stress, materials usage, and thermal resistances. The effects of load circuitry resistance, hot-side inlet temperature, semiconductor material size, and flow rate on the performance were discussed using a developed numerical model based on a FLUENT® platform, and they also recommended that coupled field simulations be used for accurate modeling of TEG systems [14].

To further improve the power output and efficiency of TEGs, researchers have placed a greater emphasis on heat transfer enhancement from the heat exchanger in the thermoelectric generator system. Lesage et al. [15] and Amaral et al. [16] investigated the thermoelectric power improvement by adding turbulating inserts into the fluid channel in a liquid-to-liquid thermoelectric generator, and found that the inserts could improve performance but should be tailored for the desired thermal input parameters for a particular application. Pandit et al. [17] applied three-dimensional partial pin fin arrays of circular, triangular, hexagonal, and diamond shapes on the walls of a rectangular channel to improve heat transfer from the hot gas side of a thermoelectric generator system. They showed that significant improvements could be made with pin fins that were only 15% of the channel height, and that the shape, configuration, and height of the pin fins were all important parameters.

In addition to the aforementioned examples for TEGs, there are many other heat transfer enhancement methods that have been used in a variety of heating, ventilation, air-conditioning, and refrigeration systems. One specific example worthy of further study is a longitudinal vortex generator (LVG). As the name suggests, an LVG produces longitudinal vortices in the cross section in addition to transverse vortices in

the streamwise direction, creating a more three-dimensional phenomena than commonly used boundary layer trips or ribs. Several groups have conducted numerical and experimental studies on heat transfer and pressure drop effects of LVGs, and significant heat transfer enhancements have been obtained [18-20].

Here, our study investigates the feasibility of using LVGs with thermoelectric generator systems. We use the novel integrated thermoelectric device proposed by Reddy et al. [13-14], in which the flow channel is smooth, as a benchmark, and we examine the effects of LVG height, inclined angle, and hot-side inlet temperature on the heat transfer, pressure drop, power output, and efficiency of the system.

2. Physical model and boundary condition

Figure 1(a) shows a schematic diagram of an integrated thermoelectric device with LVGs. This new device is based on the system proposed by Reddy et al. [13-14], which originally did not include LVGs and had a smooth channel. Unlike a conventional thermoelectric device, a copper inter connector is sandwiched between the P-type and N-type semiconductors and a flow channel is formed for the hot fluid in the center of the inter connector. One copper connector is mounted on the top surface of the P-type element, and another connector is attached to the bottom surface of the N-type element. The P-type and N-type semiconductors have a $W \times D$ (5 mm $\times$ 5 mm) square cross section with a height of $H_N = 5$ mm. The upper and lower connectors have a $2W \times D$ (10 mm $\times$ 5 mm) rectangular cross section with a height of $H_L = 1.5$ mm. The inter connector has a $W \times D$ (5 mm $\times$ 5 mm) square cross section with a height of $H_I = 10$ mm, while the size of the inner flow channel is $W_{in} \times D_{in} \times H_{in}$ (5 mm $\times$ 3 mm $\times$ 8 mm). In order to enhance heat transfer to the TEG and thus increase the electric power generated, four pairs of LVGs are mounted symmetrically on the top and bottom surfaces of the flow channel, as shown in Fig. 1(b). The LVGs have a $L_v \times W_v$ (0.5 mm $\times$ 0.1 mm) rectangular cross section with a height of H_v and an attack angle of β . Four different heights, H_v , (i.e., 1 mm, 2 mm, 3 mm, and 4 mm) and three different attack angles, β , (i.e., 30°, 45°, and 60°) are compared. The transverse pitch, P_t , and longitudinal pitch, P_l , of the LVGs are 1.2 mm and 2.25 mm, respectively. The

distance from the edge of the channel to the center of the first row of LVGs is $W = 1$ mm. The N-type semiconductor is made of 75% Bi_2Te_3 and 25% Bi_2Se_3 while the P-type semiconductor is made of 25% Bi_2Te_3 , 75% Sb_2Te_3 (1.75% excess Se). The physical properties of the copper and thermoelectric materials are considered to be temperature dependent with the same values found in Refs. [13, 22].

Figure 2(a) shows the numerical model and boundary conditions. The numerical model includes the integrated thermoelectric device with LVGs and the inner flow channel. In order to support a uniform flow for the inlet and suppress backflow at the outlet, the lengths of the inlet and outlet of the flow channel, respectively, are extended to be 5 times and 20 times that of the inner flow channel width, W . The hot air enters from the left side and exits from the right side. Here, the physical properties of air are assumed to be constant. We assume uniform velocity and temperature at the inlet, while a pressure outlet boundary condition is applied at the outlet. The top surface of the upper connector and the bottom surface of the lower connector are both kept at a temperature of T_c . The voltage is assumed to be 0 on the right side of the lower connector, and the distributed impedance with optimum load resistance is defined on the left side of the upper connector. The remaining outer surfaces are assumed to be adiabatic. The mesh used for the numerical model is shown in Fig. 2(b).

3. Numerical model and data reduction

The flow in the channel is considered to be three dimensional, laminar, incompressible, and steady. Our assumption of laminar flow in the channel is supported by several recent experimental and computational studies. An experiment conducted by Ma et al. [23] indicated that the critical Reynolds number for the transition from laminar flow to turbulent flow was around 1650 in a rectangular channel with LVGs. Wu and Tao [24] performed numerical simulations for a rectangular channel with LVGs using a laminar model, and they found that the deviation of heat transfer between numerical and experimental results was less than 10% when the velocity ranged from 0.2 m/s to 0.9 m/s. Saha et al. [25] and Tian et al.

[26] also used a laminar model to study the thermal hydraulic performance of a rectangular channel with LVGs in the low Reynolds number regime. In the present study, the velocity at the inlet varies from 0.24 m/s to 1.18 m/s, and the corresponding Reynolds number ranges from 100 to 500, thus it is reasonable to use a laminar model.

The continuity, momentum, and energy equations for the fluid domain are described as follows:

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho_f (\mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \mu_f \nabla^2 \mathbf{u} \quad (2)$$

$$\rho_f c_p \mathbf{u} \cdot \nabla T = \nabla \cdot (k_f \nabla T) \quad (3)$$

where ρ_f , k_f , μ_f , c_p , T , p , and $\mathbf{u}$, are the density, thermal conductivity, dynamic viscosity, heat capacity, temperature, static pressure, and velocity vector of the fluid, respectively.

Thermoelectric effects include the Seebeck, Peltier, and Thomson effects. In short, the Seebeck effect results in the generation of electricity from a temperature gradient in a thermoelectric material, while the Peltier effect governs cooling when an electric current is passed through a thermoelectric element. The Thomson effect is the reversible change of the heat content within a conductor in a thermal gradient when an electric current is applied. Within the thermoelectric module, the heat flux also includes Joule heating and heat conduction. Thus, the heat transfer equations in the N-type and P-type semiconductors are shown as follows [21]:

$$\nabla \cdot (-k_s \nabla T + \Pi \mathbf{J}) = \mathbf{J} \cdot \mathbf{E} \quad (4)$$

where,

$$\mathbf{J} = -\sigma (\nabla V + \partial \nabla T) \quad (5)$$

$$\mathbf{E} = -\nabla V \quad (6)$$

and the Peltier coefficient Π can be calculated from the following thermodynamical relationship with the Seebeck coefficient:

$$\Pi = \alpha T \quad (7)$$

In the above equations, k_s , Π , σ , and α are the thermal conductivity, Peltier coefficient, electrical conductivity, and Seebeck coefficient of the solid, respectively. $\mathbf{J}$, $\mathbf{E}$, and V are the electric current density vector, electric field intensity vector, and voltage, respectively.

The conservation of electrical current in the immobile solid is described as

$$\nabla \cdot \mathbf{J} = 0 \quad (8)$$

The commercial software COMSOL 4.4[®] is used as the tool for this study. In this software package, the governing equations are solved with a finite element method. The geometric multigrid iterative solver is used for the fluid flow calculations, and the algebraic multigrid iterative solver is used for the heat transfer and electric current calculations. The criterion of convergence is that the residual for every variable should be less than 10^{-5} .

The Reynolds number is defined as:

$$Re = \frac{\rho_f U D_h}{\mu} \quad (9)$$

where D_h is the hydraulic diameter based on the inlet rectangular channel, U is the corresponding inlet velocity, and ρ_f , and μ_f , are the density and dynamic viscosity of fluid, respectively.

The net power, W_{net} , supplied by the thermoelectric generator is defined as the difference between the total power output, P_o , and the pumping power $W_{\Delta p}$:

$$W_{net} = P_o - W_{\Delta p} \quad (10)$$

where,

$$W_{\Delta p} = \dot{V} \cdot \Delta p \quad (11)$$

where $\dot{V}$ is the volumetric flow rate, and p is the static pressure.

The thermal conversion efficiency η of thermoelectric generator is defined as follows:

$$\eta = \frac{W_{net}}{Q_h} \quad (12)$$

where Q_h is the heat input to the TEG. Q_h is evaluated by:

$$Q_h = mc_p(T_h - T_{h,out}) \quad (13)$$

where m , c_p , T_h , and $T_{h,out}$ are the mass flow rate, specific heat, inlet temperature and outlet temperature of fluid.

4. Grid Independence and Code Validation

In order to verify that the grid size was large enough to ensure accuracy of the numerical results, four grid systems were examined with elements ranging from 204,005 to 5,053,097 for thermoelectric generators with LVGs of $H_v=H$, and $\beta=45^\circ$ at $Re=500$. Compared to the densest grid system (5,053,097 cells), the relative deviation of the hot-side heat transfer rate, total power output, and pressure drop of the grid system with 2,171,831 cells is less than 1%, as shown in Table 1. Considering the computational resources and calculation time, the grid system with 2,171,831 cells was selected as the final grid in the later calculations.

For the purpose of validating the present numerical method, a thermoelectric generator without a fluid flow channel shown in Ref. [22] was established and compared. The temperatures of the hot-side and cold-side surfaces are kept at 600.15 K and 300.15 K, respectively. The thermoelectric generator is composed of a couple of P-type and N-type semiconductors with a length of 1 cm and with cross sectional areas of 1.24 cm^2 for P-type semiconductor and 1 cm^2 for N-type semiconductor. The cold sides are connected to a load resistance of $3.92 \times 10^{-3} \Omega$. Table 2 displays a comparison of results from the present study with prior work in the literature [22,24]. If constant physical properties are considered for the thermoelectric legs, the present results are coincident with those obtained by an analytical analysis [22] and an ANSYS numerical model [24] for the same TEG. When considering temperature-dependent physical properties for the thermoelectric legs, we find excellent agreement between the present results and the ANSYS numerical model [24] as the maximum relative difference for parameters such as Q_h , P_o , η , and I is less than 1.5%.

Next, a thermoelectric generator was modeled with a smooth fluid flow channel

as shown in Fig. 1 in order to compare with the results of Reddy et al. [14]. It should be noted that there are no LVGs in the fluid flow region during this comparison. The results from our comparison are shown in Fig. 3, and it is clear that the present study agrees well with the results from Reddy et al. [14]. The maximum relative deviations of the power output and heat input are less than 6% over a range of Reynolds numbers. The above performance comparisons of the TEGs with and without the fluid flow channel validate the present numerical method.

5. Results and discussion

5.1 Effect of LVG height

The effects of varying the LVG height on the heat input and pressure drop of the thermoelectric generator are plotted in Fig. 4. The attack angle, β , for all of the LVGs is 45° and the inlet temperature of the hot air is kept at 550 K. The heat input to the TEG increases with increasing height for the LVG; however, the pressure drop also increases. The physical reason for these trends is that the larger LVG height creates stronger disturbances for the flow in the channel that increase fluid mixing, but also create greater backpressure. For a fixed LVG height, the heat input increases with increasing Reynolds number while the pressure drop also increases. However, for large Reynolds numbers, the heat input increases slowly, while the pressure drop increases at a much greater rate. Compared to flow in a smooth channel, the heat input to the thermoelectric generator with LVGs ($H_v=H$) is enhanced by 29%-38% while the pressure drop is increased by 126%-136% for inlet Reynolds numbers ranging from 100 to 500.

The corresponding velocity profiles of the middle cross sections of the thermoelectric modules are shown in Fig. 5. First, in Fig. 5(a), we observe that there are no longitudinal vortices in the smooth channel. When the LVGs are mounted on the top and bottom surfaces of the otherwise smooth channel, two pairs of longitudinal vortices are generated along the top and bottom surfaces. As the LVG height increases, the longitudinal vortices become larger and the local velocity increases simultaneously. When the height of the LVGs reaches the full height of the

channel, the longitudinal vortices are reduced to a single pair but they fill the entire channel and produce the greatest local velocity. These velocity profiles show that the longitudinal vortices increase the degree of fluid mixing, thus disrupting the thermal boundary layer and enhancing the heat transfer. These results help explain the heat input and pressure drop trends observed in Fig. 4.

The effect of LVG height on the power output of the thermoelectric generator is presented in Fig. 6. Similar to the heat input, the LVGs also improve the power output for all Reynolds numbers. Additionally, as the LVG height increases, the power output increases for a fixed inlet Reynolds number. Since the Seebeck voltage is proportional to the temperature difference between the hot and cold sides of the thermoelectric modules, it follows that the output power should increase given the increases in heat input with the addition of LVGs observed previously. However, as noted earlier, the increase in heat input with the LVGs comes with the penalty of additional pressure drop that should be addressed. This additional pressure drop requires an increase in pumping power. Thus, the ratio of pumping power to power output is also shown in Fig. 6, and the ratio is never more than 0.55% since the pumping power is much less than the power output for all of the cases. Therefore, the net power is almost the same as the power output.

As shown in Fig. 7, the net power output of the thermoelectric generator with LVGs ($H_v=H$) is improved by 90%-104% compared to a TEG with a smooth flow channel. The thermal conversion efficiency shows a trend similar to the heat input and power output as the efficiency also increases with the addition of the LVGs. Compared with the smooth flow channel, the thermal conversion efficiency of the thermoelectric generator with LVGs ($H_v=H$) increases by 31%-36%. The thermoelectric generator with LVGs ($H_v=H$) at $Re=500$ obtains the best performance, with maximum net power of 0.134 W and maximum thermal conversion efficiency of 5.38%.

5.2 Effect of hot-side inlet temperature

The inlet temperature has a significant effect on the heat transfer performance of

the heat exchanger and thus the power output of the thermoelectric generator. Therefore, the effect of the LVG height on the heat input and pressure drop of the thermoelectric generator at a hot-side inlet temperature of 350 K is also examined. The cold-side surface temperature is kept at 300 K and the attack angle, β , for all of the LVGs remains at 45°. As shown in Fig. 8, the heat input increases with inlet Reynolds number for air at 350 K in the same manner as it did for air at 550 K. Again, under the same inlet Reynolds number, the heat input to the thermoelectric generator with LVGs is greater than that for the smooth channel case, and larger LVGs give greater heat input. However, of course, the heat input at 350 K is much less than that at 550 K for all of the cases with the average reduction in heat input at 350 K being about 80%. At 350 K, the heat input to the thermoelectric generator with LVGs ($H_v=H$) is enhanced by 40%-46% compared to smooth flow channel. There is little change for the pressure drop at 350 K compared with 550 K because of the assumption of constant physical properties for air.

Due to the obvious reduction in heat input, the power output at 350 K is significantly smaller than the power output for the 550 K case. As shown in Fig. 9, the power output at 350 K is only 4.4% of that at 550 K. However, since there is little change for the pressure drop at this lower temperature, at an inlet Reynolds number of 500, the ratio of pumping power to power output is increased from 8.3% to 11.4% when adding LVGs to the thermoelectric generator.

The net power and thermal conversion efficiency of the thermoelectric generator for the 350 K case is presented in Fig. 10. The net power does not increase as rapidly with increasing Reynolds number at 350 K as it did for the 550 K case because the pumping power consumes a larger portion of the power output. These results show that the benefits of using LVGs are small for low hot-side inlet temperatures due to the large ratio of pressure drop to power output. The maximum net power and thermal conversion efficiency at 350 K are 5.37 mW and 1.19%, respectively.

5.3 Effect of LVG attack angle

The effect of the LVG attack angle on the heat input and pressure drop of a

thermoelectric generator at an inlet Reynolds number of 300 and inlet hot-side temperature of 550 K is shown in Fig. 11. Interestingly, the LVG angle of attack has little influence on the power output. However, the larger attack angles created a greater pressure drop. It may be because the small attack angles are sufficient to generate a vortex strong enough to produce sufficient fluid mixing, and any additional mixing generated by the larger attack angles cannot increase heat transfer further. Thus, the large attack angles simply block more of the flow and create a greater pressure drop. As shown in Fig. 12, the impact of the attack angle on power output and thermal conversion efficiency is small. Therefore, the best attack angle for LVGs is 30° in the present study.

6. Conclusions

In this study, the feasibility of using LVGs with thermoelectric generator systems is examined in an integrated thermoelectric device. A fluid-thermal-electric multiphysics coupled model based on the COMSOL[®] platform was established for the present study, in which the Seebeck, Peltier, Thomson, and Joule heating effects are considered. The effects of LVG height, LVG attack angle, and hot-side inlet temperature on the heat transfer, pressure drop, power output, and thermal efficiency were analyzed. The main conclusions can be summarized as follows:

- (1) The LVGs can enhance the heat transfer performance and thus improve the power output and thermal conversion efficiency, but the pressure drop also increases with the addition of LVGs. As the LVG height increases, the heat input, power output and thermal conversion efficiency increase under a fixed inlet Reynolds number and inlet temperature. At a hot-side inlet temperature of 550 K, compared to smooth channel flow, the heat input to the thermoelectric generator with LVGs ($H_v=H$), net power, and thermal conversion efficiency are enhanced by about 29%-38%, 90%-104%, and 31%-36%, respectively, while the pressure drop is increased by 126%-136% at the same time. However, the ratio of pumping power to power output is less than 0.55% so that the pressure drop can be neglected at 550 K. The thermoelectric generator with LVGs ($H_v=H$) at $Re=500$ obtains the

best performance, in which the maximum net power is 0.134 W and the maximum thermal conversion efficiency is 5.38%.

- (2) As the hot-side inlet temperature decreases from 550 K to 350 K, the heat input and power output are obviously reduced, and the LVGs only give a modest improvement over the smooth channel. However, the penalty associated with the pressure drop does not decrease with decreasing inlet temperature, and the ratio of pumping power to power output increases to 11.4% so the gains in net power and thermal efficiency are small. Thus, it is unsuitable to use LVGs at low hot-side inlet temperatures and large inlet Reynolds numbers due to the large ratio of pressure drop to power output.
- (3) The LVG attack angle has little effect on the heat input and power output, but large LVG attack angles produce greater pressure drop. The best attack angle for LVGs in this study is 30°.

In an actual thermoelectric generator, rather than using a single thermoelectric pair, multiple modules of this novel integrated thermoelectric power generator would be connected electrically in series and thermally parallel. As an upper limit to device performance, if the materials properties, as well as all of the operating conditions on the hot and cold sides, are kept as constants, the ideal power of a large module with N couples of TE legs is approximately N times the power generated by a single pair of TE legs [28], while its ideal thermal conversion efficiency is similar to the efficiency from that same pair of TE legs. If the temperature drop of the hot side along the streamwise direction as well as the variations of physical properties of the semiconductors are considered, the power and thermal conversion efficiency are lower than the ideal values [29]. Given the computational challenges of simulating a large thermoelectric system with many legs, a single pair of thermoelectric uncouple assembly is often used to predict the performance of a TEG [30]. In the literature, only one pair, or a few pairs, of thermoelectric legs are typically simulated when using this kind of coupled fluid-thermal-electric method [8, 9, 13-14]. Therefore, the present study can still serve as a qualitative guide for the design of TEGs with LVGs including the effects of LVG height, inlet temperature, and inclined angle. In the near

future, we plan to develop a multi-scale model coupled with the present fluid-thermal-electric method to consider the variations of physical properties, and to conduct an experiment based on a large-scale module, to give a more accurate prediction for a large-scale thermoelectric generator with LVGs.

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Figure Captions

Fig. 1 Representative model of an integrated thermoelectric device with LVGs: (a) Novel integrated thermoelectric device with LVGs. The hot gas passes through the copper interconnector in the center of the device; (b) Schematic diagram of the LVGs mounted on the top and bottom surfaces of the inter connector. The LVGs are intended to increase heat transfer from the hot gas to the TEG.

Fig. 2 Numerical model, boundary conditions and mesh: (a) Numerical model and boundary conditions; (b) Mesh for numerical model.

Fig. 3 Performance comparison of the integrated thermoelectric device with Reddy et al. [14] for flow in a smooth channel with no LVGs: (a) Electric power output comparison; (b) Heat input comparison.

Fig. 4 Effect of LVG height on heat input and pressure drop ($\beta=45^\circ$, $T_h=550$ K). The taller LVGs give the greatest heat transfer and pressure drop.

Fig. 5 Velocity profiles for the middle cross section of the channel: (a) smooth channel; (b) LVGs, $H_v=1/4H$; (c) LVGs, $H_v=2/4H$; (d) LVGs, $H_v=3/4H$; (e) LVGs, $H_v=H$. The key on the right of each image gives the velocity in units of m/s.

Fig. 6 Effect of LVG height on power output ($\beta=45^\circ$, $T_h=550$ K), and the ratio of pumping power to power output. For this high inlet temperature, the penalty associated the increase in pumping power for large LVGs is small.

Fig. 7 Effect of LVG height on net power and thermal efficiency ($\beta=45^\circ$, $T_h=550$ K).

Fig. 8 Effect of LVG height on heat input and pressure drop ($\beta=45^\circ$, $T_h=350$ K).

Fig. 9 Effect of LVG height on power output and the ratio of pumping power to power output ($\beta=45^\circ$, $T_h=350$ K). At this lower inlet temperature, the pumping power becomes a significant fraction of the power output from the TEG.

Fig. 10 Effect of LVG height on net power and thermal efficiency ($\beta=45^\circ$, $T_h=350$ K).

Fig. 11 Effect of attack angle on heat input and pressure drop ($Re=300$, $T_h=550$ K).

Increasing the attack angle produces negligible improvement in heat transfer, but a significant increase in pressure drop.

Fig. 12 Effect of attack angle on power output and efficiency ($Re=300$, $T_h=550$ K).

Table Captions

Table 1 Grid independence test ($H_v=H$, $\beta=45^\circ$, $Re=500$)

Table 2 Performance comparison of thermoelectric generator without flow channel

(a) Novel integrated thermoelectric device with LVGs. The hot gas passes through the copper interconnector in the center of the device.

(b) Schematic diagram of the LVGs mounted on the top and bottom surfaces of the inter connector. The LVGs are intended to increase heat transfer from the hot gas to the TEG.

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Table 1 Grid independence test ($H_v=H$, $\beta=45^\circ$, $Re=500$)

Grid	Elements	Q_h (W)	Error	P_o (W)	Error	Δp	Error
Grid 1	204,005	2.491	-	0.149	-	14.426	-
Grid 2	841,068	2.404	3.62%	0.140	6.43%	13.095	10.16%
Grid 3	2,171,831	2.348	2.39%	0.134	4.24%	11.695	11.97%
Grid 4	5,053,097	2.344	0.17%	0.1343	0.22%	11.755	0.51%

Table 2 Performance comparison of thermoelectric generator without flow channel

Cases	Parameter	Analytical [22]	ANSYS [24]	Present
Case 1 (Constant properties)	Q_h/W	13.04	13.03	12.99
	P_o/W	1.44	1.43	1.42
	$\eta/\%$	11.00	11.00	10.93
	I/A	19.20	19.10	19.03
Case 2 (Temperature dependent properties)	Q_h/W	-	11.07	11.06
	P_o/W	-	1.05	1.05
	$\eta/\%$	-	9.50	9.47
	I/A	-	16.40	16.35