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2 **Theoretical aspects for estimating anisotropic saturated hydraulic**
3 **conductivity from in-well or direct-push probe injection tests**
4 **in uniform media**

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7 Harald Klammler^{a,b*}, Leif Layton^{a,b}, Bassel Nemer^c, Kirk Hatfield^{a,b}, Ana Mohseni^a.

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9 *Corresponding Author: <haki@gmx.at>, phone (352) 392-9537 x 1401, fax (352) 392-3394.

10 ^aEngineering School for Sustainable Infrastructure and Environment, University of Florida,
11 Gainesville, FL, 32611-6450, USA.

12 ^bInter-Disciplinary Program in Hydrologic Sciences, University of Florida, Gainesville, FL,
13 32611-6450, USA.

14 ^cDepartment of Engineering, University of Guelph, Guelph, ON, N1G 1Y4, Canada.

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Abstract

Hydraulic conductivity and its anisotropy are fundamental aquifer properties for groundwater flow and transport modeling. Current in-well or direct-push field measurement techniques allow for relatively quick determination of general conductivity profiles with depth. However, capabilities for identifying local scale conductivities in the horizontal and vertical directions are very limited. Here, we develop the theoretical basis for estimating horizontal and vertical conductivities from different types of steady-state single-well / probe injection tests under saturated conditions and in the absence of a well skin. We explore existing solutions and a recent semi-analytical solution approach to the flow problem under the assumption that the aquifer is locally homogeneous. The methods are based on the collection of an additional piece of information in the form of a second injection (or recirculation) test at a same location, or in the form of an additional head or flow observation along the well / probe. Results are represented in dimensionless charts for partial validation against approximate solutions and for practical application to test interpretation. The charts further allow for optimization of a test configuration to maximize sensitivity to anisotropy ratio. The two methods most sensitive to anisotropy are found to be (1) subsequent injection from a lateral screen and from the bottom of an otherwise cased borehole, and (2) single injection from a lateral screen with an additional head observation along the casing. Results may also be relevant for attributing consistent divergences in conductivity measurements from different testing methods applied at a same site or location to the potential effects of anisotropy. Some practical aspects are discussed and references are made to existing methods, which appear easily compatible with the proposed procedures.

Keywords: groundwater, aquifer, injection test, horizontal and vertical conductivity.

1. Introduction

Hydraulic conductivity including its spatial heterogeneity and anisotropy are fundamental hydrogeological properties impacting groundwater flow and contaminant transport in both saturated (Zlotnik et al., 2011) and unsaturated (Zaslavsky and Rogowski, 1969) soils. While heterogeneity refers to spatial variability between different locations, anisotropy refers to how conductivity at a given point may be dependent on direction. Under homogeneous conditions in a depositional environment, this directional dependence may be attributed to a preferentially horizontal orientation of non-spherical grains, such that horizontal conductivity may exceed the vertical by as much as one order of magnitude (Freeze and Cherry, 1979; Burger and Belitz, 1997 and references therein). For heterogeneous layering, the horizontal conductivity may exceed the vertical by as much as several orders of magnitude at both the local (Beckwith et al., 2003) and regional (Freeze and Cherry, 1979) scale. In contrast, in environments where roots, worms or cracks may cause preferential flow paths (secondary porosity) oriented downward, the vertical hydraulic conductivity may exceed the horizontal by several orders of magnitude (Bouma and Dekker, 1981).

Measurement of anisotropic conductivity in the laboratory using permeameter testing requires the collection of perpendicular (e.g., horizontally and vertically oriented) cylindrical core samples (Bathke and Cassel, 1991; Burger and Belitz, 1997). Alternatively, cubic soil samples can be collected and directly tested for conductivity in one, two or three directions (Bouma and Dekker, 1981; Beckwith et al., 2003; Surridge et al., 2005; Ivanek, 2008; Germer and Braun, 2015). Bagarello et al. (2009) provide an experimental comparison of laboratory testing methods for anisotropic hydraulic conductivity under saturated conditions.

1
2 In-situ measurement of hydraulic conductivity is often preferred due to limitations associated
3 with the collection of soil samples (e.g., disturbances, maximum size and depth). Within the
4 class of field methods, and with a focus on saturated conditions, classical pump tests only deliver
5 average conductivities at the scale of aquifer thickness (e.g., 10 to 100 m). In contrast, “in-well”
6 (or “single-well”) techniques have emerged as efficient tools for smaller scale characterization
7 (e.g., 0.1 to 1 m). They involve the installation of well screens, through which instrumentation
8 can be installed and removed for performing injection or slug tests from packer-off intervals at
9 different depths (Chen, 2012). More recently, “direct-push” probes (Cho et al., 2000; Liu et al.,
10 2012; Kobayashi et al., 2012) have been developed to avoid the need for well screen installation.
11 These probes are self-contained units consisting of a drive-point, casing, screen and
12 instrumentation, which can be pushed or hammered into unconsolidated media for test
13 performance at different depth.

14
15 A number of in-situ conductivity tests for unsaturated conditions (Reynolds and Elrick, 1985;
16 Amoozegar, 1989) and saturated conditions (Hinsby et al., 1992; Cardenas and Zlotnik, 2003;
17 Kobayashi et al., 2012; with comparative studies provided by Landon et al., 2001; Zlotnik and
18 Zurbuchen, 2003) assume isotropy or require independent knowledge of anisotropy ratio a priori.
19 However, in the absence of this information, some tests are known to deliver measurements
20 closer to horizontal (Zlotnik and Zurbuchen, 1998; Butler et al., 2007) or vertical (Kelly and
21 Murdoch, 2002) conductivity. The two-stage method of ASTM (2011) is capable of estimating
22 upper and lower limits of vertical and horizontal conductivities, respectively. The method of
23 Dietrich et al. (2008) is particularly useful for rapid profiling of relative conductivity contrasts

(heterogeneity) with depth, but it needs to be combined with other methods to produce absolute conductivity measurements (Liu et al., 2009). Numerical inversion techniques based on the method of Butler et al. (2007) have been applied by Liu et al. (2008) and Zschornak et al. (2013) for simultaneous estimation of hydraulic conductivity in multiple layers.

In-situ methods for measuring both horizontal and vertical conductivities at a local scale are much less abundant and established. For example, and limiting attention to saturated conditions, Kabala (1993) proposed a transient version of the dipole flow test, which was shown to be largely insensitive to vertical conductivity, unless additional head observations are taken at some lateral distance from the dipole probe (Hvilshoj et al., 2000). Similarly, Kelly and Murdoch (2002) measured horizontal and vertical conductivities in a streambed by applying a steady-state injection test with laterally offset piezometers. Chen (2000) applied slug testing through standpipes with horizontal and vertical end pieces, thus limiting application to extremely shallow depths due to installation. A tomographic slug test method for inferring heterogeneity in hydraulic conductivity, anisotropy and specific storage between wells was field demonstrated by Paradis et al. (2016). All of these methods for measuring horizontal and vertical conductivities at a local scale are technically complicated by requiring installation and observations at more than a single-well or probe. The “single-well interference slug test” introduced by Paradis and Lefebvre (2013) appears to be the only currently available single-well technique for measurement of horizontal and vertical conductivities. It uses two separate screen intervals for injection and observation, over which transient changes in respective heads are monitored.

1 In the present work, we propose a number of methods / procedures for inferring the horizontal
2 and vertical conductivities from steady-state analysis of single-well or direct-push injection tests.
3 At any given test location, the aquifer is hereby assumed to be locally homogeneous and the
4 possible occurrence of a well skin (Van Peurse et al., 1999; Kabala, 2001) is neglected. Some
5 procedures are directly applicable to existing test or probe designs, while others may inspire their
6 optimization or the development of new techniques targeting applications under anisotropic
7 conditions. We make use of a semi-analytical solution for steady-state axisymmetric flow and
8 express different combinations of observed injection test parameters (e.g., injection head, flow
9 rate, or observed heads along probe casing) as a function of probe geometry and anisotropy ratio.
10 Results in the form of charts and approximate formulae immediately illustrate the potential of a
11 method or probe configuration to estimate the degree of anisotropy. This allows for optimizing
12 the choice of method and probe geometry in order to maximize sensitivity to expected anisotropy
13 ratios. It further provides the means for convenient interpretation of test results in terms of
14 horizontal and vertical conductivities.

15
16 The fundamental idea is that results from a single injection test (e.g., steady-state injection head
17 and flow rate) are insufficient for estimating horizontal and vertical conductivities separately.
18 Clearly, a second piece of information is required for this purpose. This may be in the form of a
19 second injection test imposing a different flow field geometry, or by making additional head or
20 flow rate observations during a single injection test. In the sequel, we lay out the theoretical
21 fundamentals of steady-state axisymmetric flow in homogeneous but anisotropic domains and
22 briefly describe the relevant aspects of the semi-analytical solution approach with some analytic

approximations. Subsequently, we present the results for a number of test scenarios and provide a discussion related to their significance for potential practical application.

2. Methods

2.1. Governing equation and scaling to isotropy

Consider a flow domain in cylindrical coordinates, where the radial and angular coordinates in the horizontal plane are denoted by r [L] and θ [rad], respectively, and the vertical coordinate by z [L]. It is assumed that the hydraulic conductivity of the domain is spatially uniform but anisotropic with principal values K_r [L/T] in all horizontal directions and K_z [L/T] the vertical direction. By further limiting attention to axisymmetric flow conditions, the flow field becomes independent of θ and the governing (Laplace) equation for steady-state potential flow based on the laws of continuity and Darcy is known to be

$$K_r \left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} \right) + K_z \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad (1)$$

with φ [L] being the hydraulic head. After dividing Equation 1 by K_z and substituting

$$r' = r \rho \quad (2)$$

where

$$\rho = \sqrt{\frac{K_z}{K_r}} \quad (3)$$

is the anisotropy ratio, one obtains

$$\frac{\partial^2 \varphi}{\partial r'^2} + \frac{1}{r'} \frac{\partial \varphi}{\partial r'} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad (4)$$

This is the steady-state governing equation of a radially scaled flow domain with spatially uniform and isotropic hydraulic conductivity K' [L/T].

By considering an infinitesimal element of the flow domain in cylindrical coordinates as shown in Figure 1, the magnitude of K' is determined by imposing continuity of flow between the original and the scaled flow domains. For example, for the vertical flow Q_z [L^3/T] through an infinitesimal element one finds the condition that $K_z(d\phi/dz)r d\theta dr = K'(d\phi/dz)r' d\theta dr'$ leading to $K' = K_r$. In the same way, for the radial flow Q_r [L^3/T] one finds $K_r(d\phi/dr)r d\theta dz = K'(d\phi/dr)r' d\theta dz$, which also leads to $K' = K_r$. This is a well known property (Hvorslev, 1951; Zlotnik, 1994) and forms the basis of the approach presented in the sequel. In more intuitive terms, the property expresses that axisymmetric flow domains with anisotropic hydraulic conductivity of principal axes aligned with the coordinate system may be scaled to an equivalent isotropic flow domain of conductivity $K' = K_r$ without invalidating flow field solutions. In order to non-dimensionalize the problem with respect to the coordinates (or probe size), the flow domain is further scaled to unit $a' = \rho a$, where a [L] is the injection probe radius. This means that r' and z are substituted by r'/a' and z/a' in Equation 4. While this scaling does not affect the solution of ϕ (as it cancels out of Equation 4), it does change the effective hydraulic conductivity of the flow domain from $K' = K_r$ to $a'K_r$, if continuity of flow is required. Similar to above, this can be shown using Figure 1.

2.2. Solution method and approximate formulas

The resulting isotropic axisymmetric and steady-state potential flow problem for the mixed type boundary condition of an injection screen (treated as an ideal constant head boundary; Klammler et al., 2014) located along an impermeable casing (treated as a no-flow boundary) is solved by the semi-analytical method of Klammler et al. (2011). For constant head top, bottom and lateral

boundaries of the flow domain as illustrated in Figure 2a, φ is expressed in terms of a finite series as

$$\varphi\left(\frac{r'}{a'}, \frac{z}{a'}\right) = \varphi_0 \sum_{n=1}^N B_n \frac{K_0\left(m \frac{r'}{a'}\right)}{K_0(m)} \sin\left(m \frac{z}{a'}\right) \quad \text{with } m = n\pi a'/d \quad (5)$$

Here, φ_0 [L] is a reference head (e.g., at an injection screen), $n = 1, 2, \dots, N$ [-] is a summation index and N [-] a number of equidistant control points along the injection device. The coefficients B_n [-] are determined by solving a linear system of equations in order to meet the boundary conditions at each control point along the probe imposing a known head along the screen and zero radial flow components along the casing. $K_0()$ is the modified Bessel function of the second kind and order zero (Dwight, 1947). As shown in Figure 2, d [L] is the vertical extent of the flow domain. For the present study, the top, bottom and lateral boundaries are located sufficiently far from the injection screens as to not influence results. Following Klammler et al. (2011), the number of control points N for each boundary value problem is repeatedly doubled to hyperbolically extrapolate to an exact solution for $N \rightarrow \infty$. The process is terminated when consecutive values of extrapolated results (e.g., flow rates or heads) are within 1 %. For computational efficiency, symmetry about $z = d/2$ can be explored by recognizing that $B_n = 0$ for even n , such that $m = (2n - 1)\pi a'/d$ may be used in Equation 5 with control points only located over $0 < z < d/2$ or $d/2 < z < d$. Similarly, for anti-symmetry (as in the case of recirculating flow between two equal screens) $B_n = 0$ for odd n , and using $m = 2n\pi a'/d$ allows limiting control point locations to either the top or bottom half of the probe surface.

Independent of the computational implementation, once the coefficients B_n are known, the head distribution along the device may be computed from Equation 5 by using $r' = a'$ as

$$\varphi\left(\frac{z}{a'}\right) = \varphi_0 \sum_{n=1}^N B_n \sin\left(m \frac{z}{a'}\right) \quad (6)$$

with the respective choice of m applied. Moreover, the injection flow Q [L^3/T] between locations z_1 and z_2 along the probe may be found from

$$Q = -2\pi a' K_r \int_{\frac{z_1}{a'}}^{\frac{z_2}{a'}} \frac{\partial \varphi}{\partial r'} \Big|_{r'=a'} dz = -2\pi a' K_r \varphi_0 \sum_{n=1}^N B_n \frac{K_1(m)}{K_0(m)} \left[\cos\left(m \frac{z_2}{a'}\right) - \cos\left(m \frac{z_1}{a'}\right) \right] \quad (7)$$

where $K_1()$ is the modified Bessel function of the second kind and order one (Dwight, 1947).

Thus, a general equation for a shape factor F [L] relating injection head and flow rate to K_r can be formulated as

$$F = \frac{Q}{\varphi_0 K_r} = 2\pi a' \sum_{n=1}^N B_n \frac{K_1(m)}{K_0(m)} \left[\cos\left(m \frac{z_1}{a'}\right) - \cos\left(m \frac{z_2}{a'}\right) \right] \quad (8)$$

which simplifies to

$$\frac{F}{a'} = 4\pi \sum_{n=1}^N B_n \frac{K_1(m)}{K_0(m)} \quad (9a)$$

for $z_1 = 0$ and $z_2 = d$. For a single injection screen of length s [L] and a large enough value of d , such that the outer boundaries of the flow domain do not affect results, Equation 9a becomes a mere function of s/a' (through coefficients B_n). This relationship is graphically represented by the bold black line in the log-log plot of Figure 3.

Various simplified equations have been developed to approximate Equation 9a over different ranges of s/a' (e.g., Hvorslev, 1951; Ratnam, 2001; Mathias and Butler, 2007). When considering anisotropy and typical values of ρ smaller than one, however, the required range of s/a' , over which an approximation should be accurate, expands. Moreover, for estimating anisotropy ratios as described in the sequel, the inverse of the ratio of two shape factors should

not only exist analytically, but also be simple enough for practical use. No single approximation meeting both criteria could be found in literature or developed. Instead, we apply the ellipsoidal approximation of Hvorslev (1951) for large $\log_{10}s/a' > 0.5$

$$\frac{F}{a'} \approx \frac{2\pi \frac{s}{a'}}{\ln \frac{s}{a'}} \quad \text{for} \quad \log_{10}s/a' > 0.5 \quad (9b)$$

which is shown by the thin blue line in Figure 3. Interestingly, while the absolute difference between Equations 9a and 9b increases with s/a' , the relative difference, as relevant for the present work, approaches zero. Additionally, for the lower range of $-1 < \log_{10}s/a' < 2$, we found

$$\frac{F}{a'} \approx 9.66 \left[\left(\frac{s}{a'} \right)^{0.175} \right] \quad \text{for} \quad -1 < \log_{10}s/a' < 2 \quad (9c)$$

to be accurate. Equation 9c is shown by the thin red line in Figure 3 and is based on a linear regression fit of $\log_{10}\log_{10}F/a'$ against $\log_{10}s/a'$ from Equation 9a.

For a given injection test geometry, Equation 8 or 9 may be evaluated, if the anisotropy ratio ρ is known. Provided this is the case and given measurements of Q and φ_0 , K_r may be estimated as usual from inverting Equation 8 into

$$K_r = \frac{Q}{\varphi_0 F} \quad (10)$$

Subsequently, K_z is obtained from Equation 3. The problem remaining is that the magnitude of ρ is generally not known before-hand and the following sections describe different methods of how ρ can be estimated by gaining additional information from injection tests. The methods are denoted by letters A through F and graphically represented in Table 1.

2.3. Theoretical evaluation for different test methods

2.3.1. Subsequent exposure to different flow field geometries

Method A – Subsequent injection from two different screen geometries

At a given depth, two subsequent injection tests may be performed using screens of common radius a and different lengths s_1 and s_2 . Knowing that K_r has to be the same in both cases, one finds $Q_1/(\varphi_1 F_1) = Q_2/(\varphi_2 F_2)$ from Equation 10 and

$$\frac{F_1}{F_2} = \frac{Q_1 \varphi_2}{Q_2 \varphi_1} \quad (11)$$

where the right-hand-side may be computed from injection test results at steady-state. Moreover, Equation 9 may be applied to each screen length to compute F_1/F_2 as a function of s_1/a' and s_2/a' or, alternatively, as a function of s_1/a' and s_2/s_1 . The latter is convenient, because the unknown ρ is not contained in s_2/s_1 . The exact relationship from Equation 9a for $s_1 < s_2$ is represented by the black continuous graphs in Figure 4, which may be entered with a measured value of $Q_1 \varphi_2 / (Q_2 \varphi_1)$ on the ordinate. Knowing s_2/s_1 and s_1/a as characteristics of the probe, the corresponding graph is chosen and ρ is computed from the reading obtained on the abscissa.

Exploring the approximation of Equation 9b to express F_1/F_2 allows inverting for s_1/a' analytically, which is illustrated by the blue graphs in Figure 4. From this, and knowing F_1/F_2 from measurements and Equation 11, a closed form expression for estimating ρ over the range $\log_{10} s_1/a' > 1$ can be formulated as

$$\rho \approx \frac{s_1}{a} \left(\frac{s_2}{s_1} \right)^{\frac{1}{1 - \frac{F_1 s_2}{F_2 s_1}}} \quad \text{for} \quad \log_{10} s_1/a' > 1 \quad (12)$$

Similarly, based on Equation 9c, F_1/F_2 may be approximated as shown by the red lines in Figure 4, with a closed form expression for estimating ρ over the range $-0.5 < \log_{10} s_1/a' < 1$ as

$$\rho \approx 0.92 \frac{s_1}{a} \left[\frac{1 - \left(\frac{s_1}{s_2} \right)^{0.175}}{\log_{10} \frac{F_1}{F_2}} \right]^{5.7} \quad \text{for } -0.5 < \log_{10} s_1/a' < 1 \quad (13)$$

In case the injection screen lengths s_1 and s_2 are also associated with different screen radii a_1 and a_2 (e.g., different probes or a single probe with different diameter sections), above results remain directly applicable, if (1) Q_1 and Q_2 are substituted by Q_1/a_1 and Q_2/a_2 in Equation 11 and if (2) s_1/a and s_2/s_1 are substituted by s_1/a_1 and $s_2 a_1/(s_1 a_2)$ in Equations 12, 13 and Figure 4. This corresponds to a simple transformation of injection tests results to a unit probe radius, i.e., as if both screens had the same (unit) radius.

Method B – Injection from a lateral screen and through the bottom of a cased borehole

It is also common to perform injection tests through the open bottom of a cased borehole, for which Ratnam et al. (2001; their Equation 5 with $L = 0$) used a finite element model to determine a shape factor $F_{bot}/(2a) = 3.11$ for isotropic conditions. Silvestri et al. (2012; their Equation 5 with $N = 0$) apply a semi-analytical method to arrive at $F_{bot}/(2a) = 2.8$. Using Ratnam et al.'s solution and transforming it to anisotropic conditions gives

$$\frac{F_{bot}}{a'} = 6.22 \quad (14)$$

such that Equation 10 remains valid relating F_{bot} to K_r . Equation 14 is a constant, because in this case anisotropy only affects the magnitude of injection flow, but not the shape of the flow field. If a lateral injection test through a screen of length s and of shape factor F_{lat} is available at the same location, then Equation 11 may be written as

$$\frac{F_{lat}}{F_{bot}} = \frac{Q_{lat}\phi_{bot}}{Q_{bot}\phi_{lat}} \quad (15)$$

where the right-hand-side may again be evaluated from observed magnitudes. The left-hand side may be evaluated by combining Equations 9a and 14. This results in the bold continuous black graph depicted in Figure 5, which may be entered on the ordinate with a measured value of $Q_{lat}\phi_{bot}/(Q_{bot}\phi_{lat})$ to obtain an estimate of ρ from the reading on the abscissa.

Moreover, using Equations 9b for F_{lat} and Equation 14 for F_{bot} in Equation 15 (with $2\pi \approx 6.22$) and taking the natural logarithm, yields $\ln(F_{lat}/F_{bot}) = \ln(s/a') - \ln \ln(s/a') \approx W(s/a')$. The approximate equality holds for large s/a' with $W()$ denoting the Lambert W-function, the inverse of which is known to be $W()e^{W()}$ (Barry et al., 1995). Thus, s/a' may be expressed explicitly as $F_{lat}/F_{bot} \ln(F_{lat}/F_{bot})$, which is represented by the blue dotted line in Figure 5 after multiplication by a visually chosen adjustment factor of 1.25 to improve the approximation over the range of interest depicted. Further isolating ρ gives

$$\rho \approx 0.8 \frac{s}{a} \frac{\frac{F_{bot}}{F_{lat}}}{\ln \frac{F_{lat}}{F_{bot}}} \quad \text{for} \quad \log_{10} s/a' > 0 \quad (16)$$

as an accurate approximation for $\log_{10} s/a' > 0$, where F_{bot}/F_{lat} is known from Equation 15.

Similarly, from substituting Equations 9c and 14 into Equation 15 an approximate closed form expression for estimating ρ over the range $\log_{10} s/a' < 2$ may be found as

$$\rho \approx 0.92 \frac{\frac{s}{a}}{\left(\log_{10} 6.2 \frac{F_{lat}}{F_{bot}} \right)^{5.7}} \quad \text{for} \quad \log_{10} s/a' < 2 \quad (17)$$

which corresponds to the red dashed graph in Figure 5.

An alternative form of this test may consist of (1) injection through the bottom of a borehole of radius a and (2) injection of shape factor $F_{lat+bot}$ through a lateral screen (or open borehole wall) of radius a and length s immediately above an also open borehole bottom. While Equation 14 applies to the former case, Ratnam (2001; their Equation 5) give

$$\frac{F_{lat+bot}}{a'} = 6.22 + 3.41\sqrt{\frac{s}{a'}} + 1.19\frac{s}{a'} \quad (18)$$

for the latter case and over the range $s/a' \leq 16$. For $s = 0$, $F_{lat+bot}$ reduces to F_{bot} . A graph of $F_{lat+bot}/F_{bot}$ is shown by the thin continuous black line in Figure 5 and is well approximated by Equation 16 (blue dotted graph) over the range depicted. It correctly approaches F_{lat}/F_{bot} for growing s/a' , which is due to the decreasing influence of the injection flow through the borehole bottom, when the lateral injection length is large and / or K_z is small.

If lateral injection (regardless whether with or without borehole bottom) occurs through a cylindrical surface of radius a_{lat} , which is different to the radius a_{bot} for injection through the borehole bottom only, then results still remain applicable, if (1) Q_{lat} and Q_{bot} are substituted by Q_{lat}/a_{lat} and Q_{bot}/a_{bot} in Equation 15 and if (2) s/a in Equations 16, 17 and Figure 5 is substituted by s/a_{lat} . As above, this corresponds to a transformation of injection tests results to a unit probe / borehole radius.

Method C – Injection and recirculation test

Yet another straight-forward possibility to impose different flow fields is by simultaneous injection through two separate screens (here taken to be of equal length s and separated by distance p as shown in Figure 2b and Table 1) and subsequent recirculation of flow between the

same screens. Observing the flow rates and screen heads for injection and recirculation denoted by Q_{inj} , φ_{inj} , Q_{rec} and φ_{rec} , the right-hand-side of

$$\frac{F_{inj}}{F_{rec}} = \frac{Q_{inj}\varphi_{rec}}{Q_{rec}\varphi_{inj}} \quad (19)$$

becomes known. Applying Equation 9a to the boundaries of Figure 2b with B_n obtained for both screen heads equal to $\varphi_0 = \varphi_{inj}$ yields F_{inj}/a' as depicted by the bold continuous black lines in Figure 6a. An approximation by plotting twice the magnitude of Equation 9b is shown by the blue dotted line. In turn, applying Equation 8 with $z_1 = 0$ and $z_2 = d/2$ to the boundaries of Figure 2b with B_n obtained for one screen head at $\varphi_{rec}/2$ and the other screen head at $-\varphi_{rec}/2$ yields F_{rec}/a' as shown by the thin continuous black lines in Figure 6a. A more approximate solution of Zltonik and Zurbuchen (1998; their Equations 10 and 11) is shown as red dashed for comparison. From Figure 6a, F_{inj}/F_{rec} may be computed as a function of s/a' and for different values of p/s . Results are plotted in Figure 6b, which may be entered with a measured value of $Q_{inj}\varphi_{rec}/(Q_{rec}\varphi_{inj})$ on the ordinate to find an estimate of ρ from the respective reading on the abscissa.

2.3.2. *Single injection with additional flow or head observations*

Method D – Separate flow measurements through central and extreme portions of screen

Running two separate injection tests at the same location (whether by a single instrument or not) may be inconvenient because of the additional time required. As a first alternative we investigate the possibility of injecting through a single screen of length s , while separately monitoring injection flow rate Q_{ctr} through the central portion of the screen of length s_{ctr} and flow rate Q_{ext} through the extreme portions of the screen of combined length s_{ext} . This is motivated by the fact that Q_{ctr} is predominantly radial near the screen and, thus, more sensitive to K_r , while Q_{ext} ,

including the flow singularities at the screen ends, possesses significant vertical flow components and, thus, sensitivity to K_z . Knowing that φ_0 and K_r are the same for Q_{ctr} and Q_{ext} , Equation 10 leads to

$$\frac{F_{ctr}}{F_{ext}} = \frac{Q_{ctr}}{Q_{ext}} \quad (20)$$

where F_{ctr} is found from Equation 8 with $z_1 = (d - s_{ctr})/2$ and $z_2 = (d + s_{ctr})/2$. Likewise, $F_{ext}/2$ is obtained from Equation 8 with $z_1 = (d - s)/2$ and $z_2 = (d + s)/2$, or from subtracting F_{ctr} from F for the full screen. Results of F_{ctr}/a' for $s_{ctr}/s = 0.8, 0.5$ and 0.33 are plotted as thin black lines in Figure 3. Moreover, ratios F_{ctr}/F_{ext} are shown in Figure 7 after normalization to s_{ctr}/s_{ext} for improving graphical representation. Figure 7 may be entered on the ordinate with observed values of $Q_{ctr}s_{ext}/Q_{ext}s_{ctr}$ to find ρ from the corresponding reading on the abscissa.

Method E – Single head observation along probe casing

As illustrated in Figure 2a and Table 1, the hydraulic head φ_{t1} [L] at a distance t_1 [L] from the end of a single injection screen at head φ_0 may be monitored. From Equation 6 it is evident that φ_{t1}/φ_0 only depends on the probe geometry and ρ , but not on the absolute values of K_r or K_z . Thus, by using $z = (d + s)/2 + t_1$ in Equation 6, φ_{t1}/φ_0 may be plotted as a function of s/a' and t_1/s . This is shown in Figure 8a, from which ρ may be found when used in the inverse direction.

Alternatively, when expressing the injection head φ_0 in Equation 6 in terms of flow rate Q using Equation 10, one may express

$$\frac{\varphi_{t1} K_r s}{Q} = \frac{\frac{s}{a'}}{\frac{F}{a'}} \frac{\varphi_{t1}}{\varphi_0} \approx \frac{1}{4\pi \left(\frac{1}{2} + \frac{t_1}{s} \right)} \quad (21)$$

after multiplication by s to non-dimensionalize. Knowing F/a' as a function of s/a' from Equation 9a or Figure 3, the first equality is represented by the continuous lines in Figure 8b. The second approximate equality is based on the spherical flow assumption of Butler et al. (2007; their Equation 2 with their r_i equal to $s/2 + t_1$ in our notation, representing the distance of the pressure transducer to the screen center) and is shown by dashed lines.

Method F – Two head observations along probe casing

As a further extension, assume the hydraulic heads φ_{t1} and φ_{t2} [L] at distances t_1 and t_2 [L] from the screen edge are monitored (Figure 2a, Table 1). Based on the results of Figure 8a the ratio $\varphi_{t2}/\varphi_{t1}$ is represented in Figure 9 as a function of s/a' , t_1/s and t_2/s , from which ρ may be estimated without requiring the injection head φ_0 [L]. Alternatively, Equation 21 and Figure 8b may be used to express the difference $\Delta\varphi = \varphi_{t1} - \varphi_{t2}$ [L] after normalization as

$$\frac{\Delta\varphi K_r s}{Q} = \frac{\frac{s}{a'}}{\frac{F}{a'}} \left(\frac{\varphi_{t1}}{\varphi_0} - \frac{\varphi_{t2}}{\varphi_0} \right) \approx \frac{1}{4\pi} \left(\frac{1}{\frac{1}{2} + \frac{t_1}{s}} - \frac{1}{\frac{1}{2} + \frac{t_2}{s}} \right) \quad (22)$$

The first equality of Equation 22 is depicted by the continuous graphs in Figure 10. The second approximate equality is based on the spherical flow assumption of Butler et al. (2007; their Equation 1b) and it is represented by the dashed lines in Figure 10.

3. Results and discussion

Figures 4 through 9 summarize results for determination of anisotropy ratio ρ based on measurements obtained from Methods A through F. Knowing ρ and independent of the method applied, a shape factor F may be estimated from Equations 8, 9 or Figure 3. In combination with measured injection head φ_0 and injection flow rate Q , Equation 10 delivers an estimate of horizontal conductivity K_r . Vertical conductivity K_z is then found from Equation 3. This meets the objective stipulated for this study with more detailed aspects of each method being discussed in the following subsections.

3.1. Sensitivity to anisotropy ratio ρ

Figure 11 represents a graphical summary of the methods considered in terms the sensitivity S_ρ [-] of each method to estimate the anisotropy ratio ρ , which we define as

$$S_\rho = \left| \frac{d[\log_{10} y]}{d[\log_{10} s / (\rho a)]} \right| \quad (23)$$

where y [-] is an observed magnitude representing the labels on the ordinates of Figures 4 through 9. Equation 23 is the absolute magnitude of the slope (derivative) of y versus $s/(\rho a)$ on double-logarithmic scale. As such, relative measurement errors e_y [-] in y can be shown to propagate into relative estimation errors e_ρ [-] in ρ through $e_\rho = (1 + e_y)^{1/S_\rho} - 1$. The magnitude of e_y generally depends on y , and since y is composed of multiple head and / or flow observations, e_y is a product of multiple and possibly correlated measurement errors. While the quantification of e_y is not investigated here, we adopt $e_y \approx \pm 5$ % as an exemplary value, which is conservative with respect to typical accuracies of pressure measurements below 1 %. In combination with values of $S_\rho \approx 0.1, 0.2$ and 0.5 , which we will refer to as low, moderate and large sensitivity below, this results in $e_\rho \approx 60, 30$ and 10 %. Since ρ is here defined as the square root of the ratio between vertical and horizontal conductivities, this further translates into relative errors in this

ratio of approximately 165, 60 and 20 %. As summarized in Table 2, this shows that all methods possess the potential to estimate ρ with moderate or large sensitivity, or at least its order of magnitude, when sensitivity is low. Methods B and E are identified as most sensitive to ρ . While Method B is best suited for moderate and small values of ρ ($K_z \leq K_r$), Method E is most sensitive for values of ρ near and above one ($K_z \geq K_r$). In contrast, Method D is least sensitive over the full range, and Method F becomes completely blind to small values of anisotropy. Methods A and C are moderately sensitive, if anisotropy ratios are not too small.

Moreover, knowing $e_y(y)$ for a potential test equipment and a range of expected anisotropy ratios to be tested, Figure 11 (with an approximate summary given in Table 2) also allows for optimizing design parameters (e.g., s/a , t_1/s , etc.) in order to maximize sensitivity and minimize uncertainty in resulting estimates of ρ . Assuming that ρ is not significantly larger than one, Method A performs best, when minimizing the aspect ratio s/a of one screen to practical limits (e.g., $s \approx a$), while maximizing the aspect ratio of the other screen. This may be explained by the increasing difference in flow field geometry imposed by injection from each screen. If both screens are long, Figure 4 illustrates that F_1/F_2 approaches s_1/s_2 and dependency on ρ is lost. Similarly, gains to already moderately good sensitivity may be achieved for Method B, if the length of the screen for lateral injection is maximized. For Method C, short and nearby screens are seen to be beneficial. If screens are too far apart, injection and recirculation become identical to injection (or extraction) from each screen separately. This leads to redundant information (essentially same flow field geometries during injection and recirculation) and F_{inj}/F_{rec} in Figure 6b approaches a value of 4 for the definition of ϕ_{rec} adopted.

The sensitivity of Method D is maximized by using long injection screens with the central screen portion for separate flow measurement almost reaching the screen ends (i.e., simultaneously increasing s/a and s_{ctr}/s). However, for the parameter ranges investigated there, sensitivity remains low in all cases. The good sensitivity of Method E may be further maximized to an anticipated anisotropy range by simultaneously adjusting s/a and t_1/s within practical ranges. Note however, that locating the head observation points distant from the screen (large t_1/s) maximizes S_ρ , but also leads to small heads to be measured (Figure 8a). This may inflate e_y and taking head observations closer to a screen of possibly increased s/a is an alternative. Method F is only useful for estimating ρ , if ρ is not too small and the injection screen is short enough with one of the head observation points close enough to the screen (e.g., $t_1/s = 1$ as shown in Figure 11). Increasing the distance t_2 between the screen and the second head observation point has the potential to increase S_ρ . This is again to the price of reducing the magnitude of the head ϕ_{t2} to be measured (Figure 9).

3.2. Comparison of test procedures

Table 2 further summarizes the effort involved in applying each method of estimating ρ at a single location in terms of the numbers of injection tests and steady-state head and flow measurements needed. A major drawback of Methods A, B and C may be the requirement of performing two tests at each location. Methods A and C seem to be easily implemented by a direct-push probe or an appropriate packer arrangement moving in a screened well. For Method B, however, which possess the largest sensitivity to ρ , the bottom injection required may pose an additional hurdle to time-efficient field application at variable depths. If Method A is implemented by a single direct-push probe, the screens need to be sufficiently distant from each

other, such that injection from one screen is not influenced by the passive presence of the other screen. Methods D, E and F only require a single injection test to the cost of additional head or flow measurements. Comparing Methods D and E shows that an additional head observation along the probe / well casing (besides providing information on the location of the water table) also has a larger potential for accurately determining ρ than an additional flow measurement. However, the mechanical robustness of pressure transducers in direct-push probes may create problems at greater depths (Butler et al., 2007).

Method F collects a second head measurement along the casing, which allows for estimating ρ from Figure 9 without making use of the injection head. This has the advantage of eliminating effects of uncontrolled head losses inside the probe and / or across the screen due to clogging or smearing. Knowing ρ and using one of the head observations, an effective injection head (i.e., applied injection head minus head losses through probe and screen) may then be inferred from Figure 8 for subsequent estimation of K_r and K_z . The down-side, however, is that Method F performs worst in terms of sensitivity to ρ over the most common range of interest ($K_r \geq K_z$).

Methods A and D may also possess a certain robustness with respect to screen losses. This is due to Equations 11 and 20, where both the numerators and denominators decrease in the presence of screen losses, thus limiting the impact on the resulting ratio used for finding ρ . Methods B, C and E are considered most susceptible to screen losses. In the presence of a vertical hydraulic gradient, Methods C, E and F require correction by subtracting measured heads prior to injection from those measured at steady-state during injection.

3.3. Relationship to existing methods

For each method, Table 2 also gives references to related studies and test configurations, which may be modified or generalized to account for the possibility of anisotropy. Method A is most related to classical injection tests through lateral screens on direct-push probes or using packered-off intervals in screened boreholes. Method B is recommended by ASTM (2011), where test interpretation under anisotropic conditions yields a maximum value of K_z from injection through the borehole bottom and a minimum value of K_r from injection through the borehole wall including the bottom circle. By applying the present results, direct estimates of K_z and K_r may be obtained from the same testing procedure. Moreover, results for Methods A and B may be applied to investigate anisotropy as a possible cause for systematically divergent conductivity estimates from different probe types and geometries used at a same site (Kobayashi et al., 2012; Chapuis and Chenaf, 2003). In other words, in the presence of anisotropy, two test types imposing different flow field geometries may possess different sensitivities to K_r and K_z , thus delivering consistently different conductivities at equal locations. Method C is closely related to the dipole probe (Kabala, 1993; Zlotnik and Zurbuchen, 1998). Graphs of F_{rec} in Figure 6a allow for directly assessing the impact of assuming isotropy for conductivity estimation (as discussed by Zlotnik and Zurbuchen, 1998). Method D appears easily compatible with any type of test configuration using a long enough lateral screen for injection or extraction. Method E is related to the single head method of Butler et al. (2007) and also represents a steady-state counterpart of Paradis and Lefebvre (2013), where the head observation screen is reduced to a point. The test configuration for Method F is identical to that of Butler et al. (2007) and Liu et al. (2008, 2009, 2012) using two pressure transducers. As such, measurements at steady-state, rather than steady-shape may speed up field testing (Lui et al., 2008) and present results may provide a convenient complement for test interpretation in terms of both K_r and K_z .

3.4. Direct estimation of K_r

Here we compare our results to Butler et al. (2007), who use a spherical flow approximation to estimate K_r (without finding ρ or K_z) from one or two head observations along the direct-push permeameter. In Figures 8b and 10, their approach (dashed graphs) is compared to our Methods E and F (continuous graphs). It is seen that agreement exists for head observation points far enough from the screen, where the spherical flow approximation becomes more reliable. Agreement, however, is also limited to some upper range of $s/(\rho a)$, over which estimates of K_r can be obtained independent of ρ , if observations of Q and ϕ_t (or $\Delta\phi$) are available. A theoretical explanation of this dependence on K_r and independence on ρ is given by Zlotnik and Zurbuchen (2000). Graphs analogous to those of Figures 8b and 10 were also generated for Methods A through D (not shown here), but similar ranges of independency with respect to ρ could not be found for K_r nor for K_z .

3.5. Role of heterogeneity and scale

Aquifer homogeneity or heterogeneity is a matter of scale of interest. Statistically uniform heterogeneities at a small scale emerge as a homogeneous picture at a sufficiently large scale, while large scale heterogeneous features can be locally homogenous at sufficiently small scales. Similarly, anisotropy of hydraulic conductivity depends on the scale of interest with a typical example being the heterogeneous layering of isotropic materials (Freeze and Cherry, 1979). At the sub-layer scale, conductivity appears homogeneous and isotropic. At the scale of layer thickness, the picture becomes heterogeneous and isotropic, while a further enlargement of scale leads to increased homogeneity and anisotropy (conductivity across multiple layers is always

smaller than along them). Based on a detailed numerical analysis of Liu et al. (2008) for Method F, typical well / probe diameters at the centimeter scale and screen lengths at the centimeter to tens of centimeter scale, we estimate that the representative scale of the methods proposed here is $< 1 \text{ m}^3$. From this, we conclude that our assumption of homogeneity and, hence, test interpretation is most appropriate under heterogeneity at scales outside the 0.1 to 1 m range. Otherwise, test interpretation assuming heterogeneous layering of isotropic materials in combination with simultaneous inversion of multiple measurements at different depths (Liu et al., 2008; Zschornack et al., 2013; Paradis et al., 2013) may be more appropriate.

Another type of heterogeneity to potentially affect injection test results can be caused by the formation of a skin (high or low conductivity zone; Van Peursem et al., 1999; Kabala, 2001) around the well or probe. While Liu et al. (2008) show that Method F is insensitive to low-conductivity skins, the presence of high-conductivity skins is expected to lead to vertical flow channeling and overestimation of vertical conductivity, regardless of the method. However, a more detailed investigation of the influence of well skins on anisotropic conductivity measurements is left for the future.

4. Summary

In-situ characterization of the hydraulic conductivity of an aquifer at the local ($< 1 \text{ m}$) scale is fundamental for reliable groundwater flow and transport modeling. This includes the measurement or estimation of directional (anisotropic) conductivities, which are often times the largest in the horizontal plane and the smallest in the vertical direction. A number of single-well and direct-push techniques have been developed to rapidly measure depth profiles of hydraulic

conductivity. However, the vast majority of these methods either yield the horizontal conductivity alone or some effective value of conductivity based on the assumption of isotropy.

In the present work, we present the theoretical basis of several procedures (listed as Methods A through F in Table 2) for estimating horizontal and vertical conductivities from existing or slight modifications of existing steady-state single-well / probe injection tests. We hereby neglect the possible occurrence of high or low-conductivity skin zones around the well / probe and assume the aquifer to be locally homogeneous. Instead of a single injection test providing a single piece of information towards the magnitude of conductivity, the methods proposed are based on collecting a second independent piece of information. This can be in the form of an additional injection (or recirculation) test imposing a different flow field geometry in the aquifer, or in the form of additional head or flow observations along the well / probe during a single injection test. In all cases, measurements are combined by simple equations to form variables that only depend on the anisotropy ratio. Knowing the anisotropy ratio, shape factors may be computed for subsequent estimation of horizontal and vertical conductivities in the common way. The proposed methods may also be applied to slug injection after performing a mathematical conversion into equivalent steady-state results (Peres et al., 1989).

Results are based on applying a semi-analytical solution approach to steady-state axisymmetric potential flow and dimensionless charts are presented for (1) simple graphical estimation of anisotropy ratios, and (2) illustrating and comparing the sensitivity of each method to anisotropy ratio. Approximate equations and validation by comparison to established approximate solutions are provided in some cases. The charts are also useful for optimizing packer / probe design by

maximizing sensitivity to anisotropy ratio, for quantifying systematic errors involved in assuming isotropy, or for propagating confidence bounds in observed magnitudes into confidence bounds of anisotropy ratio and, subsequently, horizontal and vertical conductivity. Overall, no uniformly best method for measuring hydraulic conductivity under anisotropic conditions could be identified. Methods A, B and C may become time consuming as the required number of injection tests (and steady-states to be reached) is doubled. Besides, although highly sensitive to anisotropy, the borehole bottom injection involved in Method B may not be easily realized by direct-push probes or even less so by packer arrangements in screened wells. Methods D, E and F only require a single injection test at a given location, but in turn ask for additional instrumentation to measure heads and flows. Method E using one head observation along the casing near the screen presents a large sensitivity to anisotropy and may be a preferred method, if the influence of screen losses due to clogging and / or smearing is negligible. The influence of screen losses is eliminated by Method F, which, however, completely loses sensitivity to small anisotropy ratios. Under certain conditions, Methods E and F have the interesting potential for estimating only the horizontal conductivity without requiring knowledge of anisotropy ratio (as currently implemented by Butler et al., 2007).

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Figure 1. Infinitesimal element of flow domain in cylindrical coordinates before scaling to isotropy. After scaling to isotropy by Equation 2, r and dr become $r' = \rho r$ and $dr' = \rho dr$, respectively. Flow is assumed axisymmetric, such that the tangential flow component $Q_\theta = 0$.

Figure 2. Schematics of axisymmetric flow domains with boundary types showing constant head boundaries as bold dotted and impermeable boundaries as bold continuous lines. (a) Injection from a single screen of length s with possible head observations at distances t_1 and / or t_2 from screen. (b) Injection or recirculation from two screens of equal length s and separated by distance p . In all cases, the lateral boundary is at infinity and d is chosen large enough for each case ($> 100s$) to not impact results.

Figure 3. Shape factors F as a function of injection screen length s , probe radius a and anisotropy ratio ρ . Bold black line corresponds to full screen length and Equation 9a, with approximations from Equations 9b (blue dotted) and 9c (red dashed). Thin black graphs correspond to Equation 8 and different center portions of the screen of lengths s_{ctr} .

Figure 4. Equation 11 (for $F_1 < F_2$) as a function of anisotropy ratio ρ for subsequent injection from screens of radius a , but different lengths s_1 and s_2 (Method A). Graphs based on Equation 9a are bold black, with approximations of Equations 12 (blue dotted) and 13 (red dashed). If s_1 and s_2 are associated with different screen radii a_1 and a_2 , then s_1/a and s_2/s_1 become s_1/a_1 and $s_2 a_1/(s_1 a_2)$, respectively, while Q_1 and Q_2 in Equation 11 become Q_1/a_1 and Q_2/a_2 .

Figure 5. Logarithm of Equation 15 as a function of anisotropy ratio ρ for subsequent injection from a lateral screen of length s and through the bottom of a cased borehole of radius a (Method B). Bold black continuous, blue dotted and red dashed graphs correspond to Equations 9a, 16 and 17, respectively. The thin black continuous line considers lateral injection including the borehole bottom, while the bold black continuous line does not include the borehole bottom.

Figure 6. (a) Shape factors F_{inj} (bold black) and F_{rec} (thin black) for injection from and recirculation between two screens of equal length s and separated from edge to edge by distance p . Black lines are from Equation 8, while blue dotted line is from doubling Equation 9b and red dashed lines from Zltonik and Zurbuchen (1998; their Equations 10 and 11). (b) Equation 19 as a function of anisotropy ratio ρ (Method C). Graphs are for $p/s = \{0.5, 1, 2, 5\}$.

Figure 7. Equation 20 (after normalization to s_{ctr}/s_{ext} for improving graphical representation) as a function of anisotropy ratio ρ for single injection from a lateral screen of length s with separate flow monitoring through the central portion of the screen of length s_{ctr} and the two extremes of the screen of combined length s_{ext} (i.e., $s = s_{ctr} + s_{ext}$; Method D).

Figure 8. (a) Normalized head φ_{t1}/φ_0 from Equation 6 as a function of anisotropy ratio ρ for single injection from a lateral screen of length s with separate head monitoring at a distance t_1 from the screen edge (Method E). (b) Continuous lines are the same as (a), but with φ_0 expressed in terms of Q , K_r , and F using first equality of Equation 21. Dashed lines are from spherical approximation in Equation 21. All graphs from top down are for $t_1/s = \{0.5, 1, 2, 5, 10, 20\}$.

Figure 9. Ratio $\varphi_{t2}/\varphi_{t1}$ as a function of anisotropy ratio ρ for single injection from a lateral screen of length s with separate head monitoring at distances t_1 and t_2 from the screen edge (Method F).

Figure 10. Normalized difference $\Delta\phi = \phi_{t1} - \phi_{t2}$ for single injection from a lateral screen of length s with separate head monitoring at a distances t_1 and t_2 from the screen edge. Continuous lines are from exact equality in Equation 22 or the continuous graphs in Figure 8b. Dashed lines are from spherical approximation in Equation 22 or dashed graphs in Figure 8b.

Figure 11. Direct graphical comparison of all methods in terms of sensitivity S_ρ to anisotropy ratio ρ .

Table 1. Graphical representation of Methods A through F for measuring anisotropic hydraulic conductivity. Methods A, B and C require subsequent injection imposing different flow field geometries, while Methods D, E and F require single injection with additional flow or head measurements.

Method (Figure)	Graphical representation	Method (Figure)	Graphical representation
A (4)		D (7)	
B (5)		E (8)	
C (6)		F (9, 10)	

*If $a_1 = a_2$, both radii are denoted by a in Section 2.3.1.

**If $a_{lat} = a_{bot}$, both radii are denoted by a in Section 2.3.1.

1 Table 2. Overview of method properties for measuring anisotropic hydraulic conductivity.

Method (Figure)	Description	Sensitivity S_ρ^* to ρ	Approximate design ranges for moderate to large S_ρ	No. of tests	Heads measured	Flows measured	Influence of screen losses	Correction for vertical gradient	Related to
A (4)	Separate injection through two lateral screens of different geometries	Low to moderate	$s_2 a_1 / (s_1 a_2) \geq 10$ $0.1 \leq s_1 / (\rho a_1) \leq 10^{**}$	2	2	2	Limited	No	Hinsby et al. (1992) Cho et al. (2000) Dietrich et al. (2008) Kobayashi et al. (2012) Cardenas and Zlotnik (2003)
B (5)	Separate injection through lateral screen and borehole bottom	Moderate to large	$s / (\rho a_{lat}) \geq 0.1^{***}$	2	2	2	Large	No	Chapuis and Chenaf (2003) ASTM (2011)
C (6b)	Injection and recirculation	Low to moderate	$p/s \leq 1$ $0.1 \leq s_1 / (\rho a) \leq 5$	2	2	2	Large	Yes	Kabala (1993) Zlotnik and Zurbuchen (1998)
D (7)	Separate flow measurements through screen center and extremes	Low	-	1	1	2	Limited	No	Hinsby et al. (1992) Cho et al. (2000) Kobayashi et al. (2012)
E (8a)	Single head measurement along probe casing	Moderate to large	$t_1/s \geq 1$ $0.1 \leq s_1 / (\rho a) \leq 500$	1	2	1	Large	Yes	Paradis and Lefebvre (2013) Butler et al. (2007)
F (9)	Two head measurements along probe casing	Zero to moderate	$t_1/s \leq 1$ $t_2/s \geq 5$ $0.1 \leq s_1 / (\rho a) \leq 2$	1	3	1	Zero	Yes	Butler et al. (2007) Liu et al. (2009, 2012)

2 *Low, moderate and large are used to refer to $S_\rho \approx 0.1, 0.2$ and 0.5 in Figure 11.

3 **If $a_1 = a_2$, both radii are denoted by a in Section 2.3.1.

4 ***If $a_{lat} = a_{bot}$, both radii are denoted by a in Section 2.3.1.