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Outer Region Scaling Using the Freestream Velocity for Nonuniform Open Channel Flow over Gravel

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Abstract: The theoretical basis for outer region scaling using the freestream velocity for nonuniform open channel flows over gravel is derived and tested for the first time. Owing to the gradual expansion of the flow within the nonuniform case presented, it is hypothesized that the flow can be defined as an equilibrium turbulent boundary layer using the asymptotic invariance principle. The hypothesis is supported using similarity analysis to derive a solution, followed by further testing with experimental datasets. For the latter, 38 newly collected experimental velocity profiles across three nonuniform flows over gravel in a hydraulic flume are tested as are 43 velocity profiles previously published in seven peer-reviewed journal papers that focused on fluid mechanics of nonuniform open channel over gravel. The findings support the nonuniform flows as equilibrium defined by the asymptotic invariance principle, which is reflective of the consistency of the turbulent structure's form and function within the expanding flow. However, roughness impacts the flow structure when comparing across the published experimental datasets. As a secondary objective, we show how previously published mixed scales can be used to assist with freestream velocity scaling of the velocity deficit and thus empirically account for the roughness effects that extend into the outer region of the flow. One broader finding of this study is providing the theoretical context to relax the use of the elusive friction velocity when scaling nonuniform flows in gravel bed rivers; and instead to apply the freestream velocity. A second broader finding highlighted by our results is that scaling of nonuniform flow in gravel bed rivers is still not fully resolved theoretically since mixed scaling relies to some degree on empiricism. As researchers resolve the form and function of macroturbulence in the outer region, we hope to see the closing of this research gap.

47 **LIST OF SYMBOLS:**

48	D_x	=	particle diameter for which x percent of particles are finer
49	Fr	=	$U(gH)^{-1/2}$ is the Froude number
50	H	=	Flow depth
51	k_s	=	roughness height
52	P_∞	=	pressure
53	Q	=	flow rate
54	R	=	the hydraulic radius
55	Re	=	indicates the Reynolds number
56	R_{so}	=	outer region fluid stress scale
57	$R_x(\alpha)$	=	rotation matrix about the x-axis
58	$R_y(\alpha)$	=	rotation matrix about the y-axis
59	$R_z(\alpha)$	=	rotation matrix about the z-axis
60	S	=	bed slope
61	S_f	=	free surface slope
62	S_u	=	streamwise velocity spectrum estimated using fast Fourier transform
63	TKE	=	turbulent kinetic energy
64	U	=	Time averaged streamwise velocity
65	U_b	=	bulk velocity
66	U_{so}	=	outer region velocity scale
67	U_∞	=	free stream velocity
68	U_*	=	Shear velocity
69	V	=	mean wall normal velocity
70	V_{mag}	=	magnitude of the time average velocity vector
71	W	=	channel width
72	W/H	=	Aspect ratio
73	d,e,f	=	functions describing the velocity profile
74	g	=	gravitational acceleration
75	k_s	=	equivalent roughness height
76	k^+	=	$k_s U_* v^{-1}$ is the shear Reynolds number
77	p,q,r	=	functions describing the Reynolds stress profile
78	x	=	stream wise direction
79	y	=	wall normal distance above flume bed
80	$\overline{u'v'}$	=	streamwise-vertical Reynolds shear stress
81	$\overline{u'u'}$	=	streamwise Reynolds normal stress
82	$\mathcal{A}$	=	energy gradient parameter
83	$\mathcal{O}$	=	parameter representing a dependence on external conditions
84	δ	=	boundary layer height
85	δ^+	=	$\delta U_* v^{-1}$ is the local Reynolds number
86	δ^*	=	displacement thickness
87	η	=	y/δ normalized wall normal distance
88	ν	=	Kinematic viscosity
89	ω	=	frequency
90	κ	=	von Karman constant
91	ρ	=	fluid density
92			

INTRODUCTION:

The motivation of this paper is to present and test the theoretical basis for outer region scaling using the freestream velocity (U_∞) for nonuniform open channel flows over gravel. The open channel flows over gravel are classified as fully hydraulically rough turbulent flow with relatively large roughness elements that can be felt throughout much of, if not the entire, flow depth. We focus on a nonuniform case for these turbulent flows in which the fluid is gradually varied and expanding in the vertical due to an adverse pressure gradient. In application, the nonuniform flow case is general to any hydraulically rough channel flow with a backwater profile, and more specifically one such example is mountainous gravel and cobble bed rivers with decelerating pools that are constrained by streambank sidewalls.

Nonuniform open channel flows over gravel is classified as a subset of turbulent boundary layer flows with a number of notable characteristics. First, the authors focus on moderate to high flow conditions typical of mountainous gravel and cobble bed rivers where fluid momentum and sediment mass transport will be high. For such conditions, the fluid will contain both appreciable inner and outer regions, the latter of which is the focus herein, and will be hydraulically rough. Moderate to high flow conditions infer moderate to high values for the relative submergence ($R/D_{84} > 4$, where R is the hydraulic radius and D_{84} is the bed particle size for which 84% are finer). The relative submergence conditions will contain an inner region, termed the roughness layer for open channel flow over gravel and cobbles, that is indicative of the shedding process around particles and an outer region defined to include the overlap, wake and near-surface layers where viscous effects become small (Dittrich and Koll, 1997; Carollo et al., 2005). The hydraulically rough definition is indicated by a relatively high value for the roughness Reynolds number ($k_s U_* v^{-1} > 70$, k_s , U_* , and ν are the roughness height, friction velocity and kinematic viscosity) resulting in separation of the vertically-associated inner and outer regions with the outer region beginning one to two times the roughness height above the bed (Chang, 1998; Nikora et al., 2001; Belcher and Fox, 2011; Smits et al., 2011).

Second, due to the adverse pressure gradient impacting the open channel, the flow is nonuniform, decelerating and the outer region of the turbulent boundary layer is gradually expanding in the vertical direction as the boundary layer height, δ , increases while the freestream

velocity, U_∞ , at δ decreases as a function of x . The high Reynolds number and gradual nature of the expansion dictate that the turbulent structure of the flow maintains its general function in transferring potential and kinetic energy to dissipated heat as well as its general double-layer form. But also there exists the enhancement of the vertical velocity explicit in the continuity equation and it sustains the Reynolds shear stresses throughout the profile (Song, 1994).

Third, the outer flow structure does not fully lose memory of the shear structure initiated by the roughness elements at the bed. It has been argued in early literature using scaling and roughness laws and now recently using experimental results that bed initiated turbulence impacts the flow structure throughout its entire depth including the outer region (Einstein and Barbarossa, 1952; Vanoni and Nomicos, 1960; Rouse, 1965; Hurther et al., 2007; Belcher, 2009; Amir and Castro, 2011). While the connectivity between the outer region turbulent structure and near bed shedding has not been fully resolved, a fairly consistent paradigm has emerged that includes near-bed generation of hairpin-like vortices that eject away from the bed in the form of connected hairpin-like packets that evolve to alternating high momentum and low momentum cells (Hurther et al., 2007; Belcher, 2009).

In terms of outer region scaling of the nonuniform flow, we consider the two characteristics above including the high Reynolds number and consistency of the phenomenology of coherent structures throughout the gradual expansion, which suggest the flow be defined as an equilibrium turbulent boundary layer where the dynamical influences of the flow develop together (Wosnik, 2000). The nonuniform flow can be defined using the equilibrium solution based on the asymptotic invariance principle (AIP), which assumes that all velocity, turbulent length scale and pressure gradients maintain the same x dependence (George and Castillo, 1997). AIP equilibrium implies that the transformation mechanisms between potential energy and kinetic energy throughout the boundary layer are consistent, and therefore the turbulence structure is not changing form in the streamwise direction; the streamwise velocity is self-similar when properly scaled; and the energy gradient parameter is constant.

The AIP equilibrium definition has been shown to result in outer scaling of the velocity deficit, $U_\infty - U$, where U_∞ is the free-stream velocity and U is the streamwise mean velocity by U_∞ .

AIP equilibrium has been derived for flat plate boundary layers with zero, favorable and adverse pressure gradients and open channels with favorable gradients (George and Castillo, 1997; Castillo and George, 2001; Belcher and Fox, 2011), however, AIP equilibrium has not been derived or tested for the nonuniform decelerating open channel flow case. The gradual vertical expansion and consistent turbulent structure of nonuniform flow over gravel prompted us to hypothesize that the flow case can be defined by AIP equilibrium. One application of this scaling would be that nonuniform flow in gravel bed rivers could be predicted using the easily measurable U_∞ as compared to the often used, but elusive, friction velocity.

Our objective was to derive the AIP equilibrium solution for the nonuniform flow case with gradual expansion indicated by wall normal velocity and flow deceleration; and test the applicability of the U_∞ scaling that results from the similarity analysis. For the latter, we collect 38 new experimental velocity profiles across three nonuniform flows over gravel in a hydraulic flume. We also use 43 published velocity profiles collected over gravel in a hydraulic flumes and rivers for testing the AIP equilibrium solution. Our own data was needed, beyond the data already available in the literature, to: (i) account for Froude number conditions approaching transcritical flows that are typical of decelerating pools in steep mountainous gravel and cobble bed rivers, and (ii) provide many velocity profiles for the same expanding boundary layer and thus allow further testing of AIP equilibrium by checking that the energy gradient parameter is constant.

As a secondary objective, we also considered and tested several empirical outer region scaling's that can be combined with U_∞ to provide a mixed scale. One characteristic of the AIP-equilibrium solution is that U_∞ alone may not collapse the velocity deficit due to variations in external flow conditions and boundary conditions, e.g., variations in the pressure gradient and roughness elements (Castillo and George, 2001). We test empirical scalings of Zagarola and Smits (1998) and Fox and Stewart (2014) using our newly collected experimental data and the previously mentioned literature data. Zagarola and Smits (1998) and Fox and Stewart (2014) were chosen due to the fact that their scalings have been shown to perform well for boundary layers with large roughness elements (Connelly et al., 2006; Fox and Stewart, 2014).

SIMILARITY ANALYSIS:

The theoretical basis for outer region using the freestream velocity (U_∞) for nonuniform open channel flows over gravel is presented by defining the equilibrium solution based on the asymptotic invariance principle (AIP) and performing similarity analysis of the streamwise momentum equation. By defining the equilibrium solution based on AIP, it is assumed that all velocity, turbulent length scale and pressure gradients maintain the same x dependence for the conditions to be equilibrium (George and Castillo, 1997). Similarity analysis that applies the AIP definition has been reported in a number of publications for flat plate boundary layers with zero, favorable and adverse pressure gradients and open channels with favorable gradients (George and Castillo, 1997; Castillo and George, 2001; Belcher and Fox, 2011; Fox and Stewart, 2014). Based on this earlier work coupled with considerations for nonuniform flow, we derive the AIP similarity analysis solution and thus extend the definition to the case of nonuniform open channel flow with an adverse pressure gradient manifested at the free surface. Thereafter, we consider several mixed scales reported in the literature that can potentially be combined with U_∞ and applied to nonuniform open channel flow over gravel to account for external conditions and roughness effects that AIP cannot collapse alone.

The AIP definition and resultant derivation can be applied to nonuniform open channel flow above the roughness layer in the outer region where viscous effects can be neglected. The streamwise momentum and continuity equations for the outer region of a turbulent open channel at high Reynolds number are

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = gS - \frac{1}{\rho} \frac{\partial P_\infty}{\partial x} + \frac{\partial}{\partial y} (\overline{u'v'}) + \frac{\partial}{\partial x} (\overline{u'u'}) , \text{ and} \quad (1)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 , \quad (2)$$

where x and y are streamwise and vertical coordinates, U and V denote streamwise and vertical mean velocity that can vary in the x and y directions, u' and v' denote the fluctuating components of the instantaneous velocities, g is gravitational acceleration, S is the bed slope, ρ is fluid density, P_∞ is pressure, $\overline{u'v'}$ is the streamwise-vertical Reynolds stress, and $\overline{u'u'}$ is the

streamwise normal stress. In Equation (1), the viscous terms are neglected for the outer region because they are small away from the roughness layer. The normal stress term is also neglected from Equation (1) based on the consideration that it is small relative to the Reynold's shear term. To justify the omission, the normal stress term was evaluated upstream of a large obstruction to provide a conservative case for investigation. Experimental analysis showed that the gradient of the normal stress in the streamwise direction was at least an order of magnitude smaller than the vertical gradient of shear stress in the outer region (e.g., the normal stress gradient was on the order of 0.03 cm s^{-2} and the shear stress gradient was 1.4 cm s^{-2}). Thus, consistent with the similarity solution of others (Castillo and George, 2001), it was considered reasonable to neglect the streamwise gradient of the normal stress component even in non-uniform flow. The flow evaluated experimentally represents a large obstruction and pronounced adverse energy gradient causing deviation from uniform flow; at normal depth the gradient would be identically zero and thus for most cases of obstructed flow this assumption should remain valid.

Similarity solutions for the deficit form of the mean velocity and fluid stress within the nonuniform open channel flow can be given as

$$U - U_{\infty} = U_{SO}(x) d(y, \delta, \nu, g, k_s, U_o, S, S_f) \text{ and} \quad (3)$$

$$\overline{u'v'} = R_{SO}(x) p(y, \delta, \nu, g, k_s, U_o, S, S_f), \quad (4)$$

where the freestream velocity, U_{∞} , is the maximum velocity in the water column and is associated with the extent of the boundary layer, δ . U_{SO} and R_{SO} are the velocity and fluid stress outer scales, d and p denote functions for the velocity and Reynolds stress profiles, respectively, in the outer region for which a solution is sought, U_o is an appropriate velocity scale for investigating the relative forces within the fluid, S_f is the gradient of the free surface, and all other variables are defined previously. Dimensional analysis allows grouping of the variables within the functions of d and p into well-known dimensionless numbers for the outer region of open channel flow over a gravel bed. The result of dimensional analysis is given as

$$U - U_{\infty} = U_{SO}(x) e\left(\frac{y}{\delta}, \frac{\delta U_*}{\nu}, \frac{k_s U_*}{\nu}, \frac{U_{\infty}}{\sqrt{g\delta}}, S - S_f, \frac{\delta}{k_s}\right) \text{ and} \quad (5)$$

$$\overline{u'v'} = R_{SO}(x) q\left(\frac{y}{\delta}, \frac{\delta U_*}{\nu}, \frac{k_s U_*}{\nu}, \frac{U_{\infty}}{\sqrt{g\delta}}, S - S_f, \frac{\delta}{k_s}\right). \quad (6)$$

where e and q denote the new functions for the velocity and Reynolds stress profiles, respectively, that are dependent upon the dimensionless numbers. The dimensionless numbers on the right hand side of Equations (5) and (6) include the relative distance from the boundary, the local Reynolds number, the roughness Reynolds number, the Froude number, the relative submergence, and the nonuniformity of the flow indicated by the difference between the boundary gradient and free-surface gradient.

The dependency of functions d and q in Equations (5) and (6) upon the dimensionless numbers may be revised when considering simplification based on the nonuniform case of open channel flow over a gravel bed as well as considering the equilibrium definition for which a solution is sought. First, based on the AIP equilibrium definition, d and q become independent of δ^+ as the local Reynolds number goes to infinity. Further, we consider only the case of hydraulically rough open channel flow where the roughness Reynolds number is greater than about 70 (Chang, 1998). Therefore, the Reynolds numbers can be removed from the brackets and the similarity functions are no longer dependent on the Reynolds numbers.

Second, consistent with the similarity solution of others (Clauser, 1952, 1954; George and Castillo, 1997; Belcher and Fox, 2011), the authors introduce the energy gradient parameter symbolized here as $\mathcal{A}$. The energy gradient parameter accounts for external energy inputs and reflects the ratio of gradients in the dominant energy terms. For an equilibrium boundary layer, the ratio of gradients should remain constant because the energy terms are evolving together and balancing with each other and the energy losses in the x -direction. For example, in pipeflow studies the energy gradient parameter reflects a ratio of the gradient in kinetic energy to the gradient in pressure energy which remains constant for a pipeflow equilibrium layer (Castillo and Wang, 2004). For uniform open channel flows, the energy gradient parameter has been found to reflect the ratio of the gradient in kinetic energy to the gradient in potential energy and remains constant for the equilibrium layer (Belcher and Fox, 2011). For the nonuniform open channel flow case considered here, the energy gradient parameter has yet to be derived in the literature, and its expression is found later in this section as one of the results of the authors' similarity solution. Specifically, the authors will show that the energy gradient parameter captures the external conditions associated with both the Froude number and the difference

between the boundary and free-surface gradients ($S - S_f$), and therefore these terms are coupled within Λ when expressing the boundary layer functions. The authors also show later that Λ is constant for a given nonuniform flow case where the velocity scale, length scale and pressure gradient exhibit the same x dependence.

Third, θ is introduced as an external parameter to account for initiation of the boundary layer shedding. Numerous approaches have been proposed to account for these external conditions (Zagarola and Smits, 1998, Belcher and Fox, 2011; Fox and Stewart, 2014), and these methods are termed empirical scaling laws and will also be tested later in this study. Fox and Stewart (2014) introduced an approach that explicitly accounts for the role of the relative submergence (i.e., the last term in Equations 5 and 6) due to the fact that the roughness particles at the bed have been suggested to impact the turbulent flow structure throughout the entire flow depth (Amir and Castro, 2011; Belcher and Fox, 2011). Based on the three mentioned considerations, Equations (5) and (6) can be updated as

$$U - U_\infty = U_{SO}(x) f(\eta, \Lambda, \theta) \text{ and} \quad (7)$$

$$\overline{u'v'} = R_{SO}(x) r(\eta, \Lambda, \theta), \quad (8)$$

where f and r denote the updated functions for the velocity and Reynolds stress profiles, respectively, and $\eta = \frac{y}{\delta}$ is the similarity variable in the outer region for which a solution is sought.

The similarity functions are substituted for U and $\overline{u'v'}$ in the momentum equation of Equation (1) to yield

$$(U_{so}f + U_\infty) \frac{\partial(U_{so}f + U_\infty)}{\partial x} + V \frac{\partial U_{so}f}{\partial y} + V \frac{\partial U_\infty}{\partial y} = gS - \frac{1}{\rho} \frac{\partial P_\infty}{\partial x} + \frac{\partial R_{so}r}{\partial y} \quad (9)$$

We note that the vertical gradient of U_∞ is zero since U_∞ is constant for any location in the streamwise direction and the vertical gradient of f can be concisely written as

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial \eta} \frac{\partial(y/\delta)}{\partial y} = \frac{f'}{\delta}. \text{ Expanding and representing derivatives of } f \text{ and } r \text{ with respect to } \eta \text{ as}$$

f' and r' allows Equation (9) to be rewritten as

$$U_{so} f \frac{\partial U_{so} f}{\partial x} + U_{\infty} \frac{\partial U_{so} f}{\partial x} + U_{so} f \frac{\partial U_{\infty}}{\partial x} + U_{\infty} \frac{\partial U_{\infty}}{\partial x} + \frac{V U_{so}}{\delta} f' = gS - \frac{1}{\rho} \frac{\partial P_{\infty}}{\partial x} + \frac{R_{so}}{\delta} r' . \quad (10)$$

The number of parameters in Equation (10) can be reduced by relating the pressure gradient to the freestream velocity (i.e. at the boundary layer edge) assuming that viscosity is negligible at δ using the Euler's equation applied to open channel flow as

$$\frac{\partial}{\partial x} \left(\frac{1}{2} U_{\infty}^2 \right) + \frac{1}{\rho} \frac{\partial P_{\infty}}{\partial x} + gS = 0$$

or

$$\frac{1}{\rho} \frac{\partial P_{\infty}}{\partial x} = gS - U_{\infty} \frac{\partial U_{\infty}}{\partial x} \quad (11)$$

Assuming inviscid flow at the boundary location is common practice as velocity gradients are theoretically non-existent and Euler's equation is a reasonable approximation to the equations of motion. The assumption does not restrict the limit of the solution space to the boundary; it simply allows us to make outer parameters comparable. By using Euler's equation to describe the pressure gradient in terms of freestream velocity and canceling terms, we arrive at

$$U_{so} f \frac{\partial U_{so} f}{\partial x} + U_{\infty} \frac{\partial U_{so} f}{\partial x} + U_{so} f \frac{\partial U_{\infty}}{\partial x} + \frac{V U_{so}}{\delta} f' = \frac{R_{so}}{\delta} r' . \quad (12)$$

Equation (12) is the momentum equation written in terms of similarity solutions for mean velocity and Reynolds shear stress. Next, the vertical velocity component, V , is written in terms of U using the continuity equation specified for the similarity solution as

$$\frac{\partial V}{\partial y} = - \frac{\partial U}{\partial x} = - \frac{\partial U_{\infty}}{\partial x} - f \frac{\partial U_{so}}{\partial x} - U_{so} \frac{\partial f}{\partial x} . \quad (13)$$

Integrating Equation (13), allows the vertical velocity to be expressed as

$$\int \frac{\partial V}{\partial y} \partial y = \int_0^y \frac{\partial V}{\partial y} \partial y = V(y) = - \frac{\partial U}{\partial x} = - \left(\int \frac{\partial U_{\infty}}{\partial x} \partial y + \int f \frac{\partial U_{so}}{\partial x} \partial y + \int U_{so} \frac{\partial f}{\partial x} \partial y \right) \quad (14)$$

Note it is technically incorrect to evaluate an integral at the variable for which integration was performed, however use of a dummy variable makes this technically correct. $V(y)$ in Equation (14) requires we enforce the no slip boundary condition at the wall. We can observe that the freestream velocity and outer velocity scale do not depend on y and can be moved out of the integral. These integrals can be written in terms of the similarity variable η using the following relations

$$\eta = y / \delta, \quad \partial y = \delta \partial \eta, \quad \frac{\partial f}{\partial x} = \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} = f' \frac{-y}{\delta^2} \frac{\partial \delta}{\partial x} = -\frac{\eta f'}{\delta} \frac{\partial \delta}{\partial x}, \quad (15)$$

and upon substitution

$$V(y) = -\frac{\partial U}{\partial x} = -\delta \eta \frac{\partial U_\infty}{\partial x} - \delta \frac{\partial U_{so}}{\partial x} \int_0^\eta f \partial \eta + U_{so} \frac{\partial \delta}{\partial x} \int_0^\eta \eta f' \partial \eta. \quad (16)$$

Using integration by parts, we can integrate with respect to η and evaluate to arrive at the function for the wall normal velocity component written in terms of similarity solutions and variable. We can then substitute the wall normal velocity into Equation (12) to arrive at

$$U_{so} f \frac{\partial U_{so} f}{\partial x} + U_\infty \frac{\partial U_{so} f}{\partial x} + U_{so} f \frac{\partial U_\infty}{\partial x} + \frac{U_{so}}{\delta} f' \left(-\delta \eta \frac{\partial U_\infty}{\partial x} - \delta \frac{\partial U_{so}}{\partial x} \int_0^\eta f \partial \eta + U_{so} \frac{\partial \delta}{\partial x} \left(\eta f \Big|_0^\eta - \int_0^\eta f' \partial \eta \right) \right) = \frac{R_{so}}{\delta} r' \quad (17)$$

The gradient of the similarity function in the streamwise direction goes to zero as per the definition of similarity solutions for equilibrium flows. Gathering like terms of f and multiplying Equation (17) by δU_{so}^{-2} yields

$$f^2 \left[\frac{\delta}{U_{so}} \frac{\partial U_{so}}{\partial x} \right] + f \left[\frac{\delta U_\infty}{U_\infty^2} \frac{\partial U_{so}}{\partial x} + \frac{\delta}{U_{so}} \frac{\partial U_\infty}{\partial x} \right] + \eta f' \left[-\frac{\delta}{U_{so}} \frac{\partial U_\infty}{\partial x} \right] + \eta f' f \left[\frac{\partial \delta}{\partial x} \right] - f' \left[\frac{\delta}{U_{so}} \frac{\partial U_{so}}{\partial x} + \frac{\partial \delta}{\partial x} \right] \int_0^\eta f \partial \eta = \frac{R_{so}}{\delta} r' \quad (18)$$

The result in Equation (18) is of similar form to the AIP similarity analysis by others (Castillo and George, 2001; Belcher and Fox, 2010) with one notable difference: the integral for vertical velocity at η was evaluated continuously in this analysis rather than at the boundary. The robustness of the result is it allows the limit as x goes to infinity of all bracketed terms to maintain the same x -dependence while allowing independence of the similarity variable η . For all terms in Equation (18) to maintain the same streamwise (x) dependence, the following relationship must hold

$$\frac{\delta}{U_{so}} \frac{dU_{so}}{dx} \sim \left(\frac{U_\infty}{U_{so}} \right) \frac{\delta}{U_{so}} \frac{dU_{so}}{dx} \sim \frac{d\delta}{dx} \sim \left(\frac{U_\infty}{U_{so}} \right) \frac{d\delta}{dx} \sim \frac{R_{so}}{U_{so}^2} \quad (19)$$

where $\sim$ indicates the idea of ‘same x dependence’. Full similarity is only possible if

$$U_{so} \sim U_\infty, \text{ and} \quad (20)$$

$$R_{so} \sim U_{so}^2 \frac{d\delta}{dx} \sim U_{\infty}^2 \frac{d\delta}{dx} . \quad (21)$$

Similarity solutions exist when the U_{∞} and δ scales produce deficit profiles that are asymptotically independent of the local Reynolds number and thus independent of x . In terms of scaling, results from the similarity analysis using the AIP definition are that U_{∞} can be used as an appropriate velocity scale for the velocity deficit in Equation (7). In this manner, collapse of U_{∞} profiles *via* U_{∞} provides a first test for AIP defined equilibrium when applied to nonuniform flow where external conditions and boundary conditions are constant. In addition, independent constraints arise from Equations (19) through (21) if equilibrium conditions are satisfied as

$$\frac{d\delta}{dx} \sim \frac{\delta}{U_{\infty}} \frac{dU_{\infty}}{dx} \sim \frac{\delta g}{U_{\infty}^2} \left(S - \frac{d\delta}{dx} \right) . \quad (22)$$

Using Equation (22), equilibrium can only be satisfied when the energy gradient parameter is constant as

$$\Lambda \equiv \frac{\delta}{(d\delta/dx)} \frac{g}{U_{\infty}^2} \left(S - \frac{d\delta}{dx} \right) = \text{constant, and equivalently} \quad (23)$$

$$\Lambda \equiv \frac{\delta}{(d\delta/dx)} \frac{1}{U_{\infty}} \frac{dU_{\infty}}{dx} = \text{constant} \quad (24)$$

As seen in Equation (23), the dimensionless energy gradient parameter for the nonuniform flow condition includes both the effects of the Froude number and difference between the bed and free-surface gradients, as brought forth in Equations (5) and (6). Equation (24) can be integrated since Λ is constant and the integral of δ^{-1} is the natural logarithm of δ , and likewise for U_{∞} . Therefore, for non-zero values of Λ , Equation (24) yields

$$\delta \sim U_{\infty}^{-1/\Lambda} . \quad (25)$$

Equation (25) provides a second test for AIP defined equilibrium when applied to nonuniform flow where external conditions and boundary conditions are constant, and the second similarity condition is based on maintaining the constant energy gradient parameter.

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Results in Equation (18) admit solution to the boundary layer equations for the outer region of nonuniform open channel flow *via* AIP and in turn scale $U-U_{\infty}$ with U_{∞} . The solution encompasses Clauser-equilibrium but yet is more robust in that: theoretically AIP-equilibrium, unlike Clauser, provides a pressure gradient parameter that can remain constant for the

nonuniform flow case (i.e., the energy gradient defined by Clauser is dependent upon the changing Froude number for nonuniform open channel flows); in practice, U_∞ provides an outer region parameter that is measureable in comparison with U_* ; and within an experimental setting, AIP-equilibrium provides an equilibrium that can be maintained (George and Castillo, 1997).

However, external flow conditions and boundary conditions of nonuniform flows limit the AIP-equilibrium solution in Equation (18) and thus limit universal scaling of the velocity deficit *via* U_∞ alone due to variations associated with Λ and θ for different flow cases. Λ of a nonuniform adverse pressure gradient can vary due to U_∞ at the onset of the pressure gradient as well as the geometry of the hydraulic control inducing the adverse gradient. In the case of open channel flow over gravel, θ will be indicative of the moderate relative submergence of roughness particles at the bed that can impact the turbulent flow structure throughout the entire flow depth including the outer region (Einstein and Barbarossa, 1952; Vanoni and Nomicos, 1960; Rouse, 1965) as exemplified by recent visualization results from the field and laboratory (Roy et al., 2004; Fox and Belcher, 2011).

EMPIRICAL MIXED SCALING LAWS:

To account for variation of Λ and θ in the nonuniform case herein, we consider several empirical scalings that can be combined with U_∞ to provide a mixed scale for collapsing the outer region deficit. The empirical mixed scaling of Zagarola and Smits (1998) scales the velocity deficit with U_∞ and the ratio of the displacement thickness (δ^*) to the boundary layer height where δ^* is defined as

$$\delta^* = \int_0^\delta \left(\frac{U_\infty - U}{U_\infty} \right) \partial y \quad (26)$$

The displacement distance represents the velocity lost from the theoretical inviscid flow profile when the effects of viscosity, no slip boundary condition and roughness elements on the velocity distribution are considered (Zagarola and Smits, 1998). The AIP outer flow scaling derived in Equation (18) assumed the outer region was independent of viscosity, wall effects were only directly accounted for by enforcing the no slip at the wall boundary condition in the equation for the wall normal velocity, and there was no theoretical reasoning to advocate the removal of external conditions. The Zagarola and Smits (1998) scaling suggests that external conditions are

accounted by multiplying f by the ratio $\delta^*:\delta$. The Zagarola and Smits (1998) result has produced one of the most effective empirical scalings to normalize the mean velocity deficit profiles in rough boundary layers subject to pressure gradients and has been shown to remove the effects of roughness, freestream turbulence and Reynolds number (Brzek et al. 2010). However, estimation of δ^* requires detailed knowledge of the velocity distribution rendering it impractical for estimating velocity in a predictive sense for realistic flow scenarios.

A second empirical scaling that is tested for nonuniform open channel flow over gravel is that proposed by Fox and Stewart (2014) and uses a function of the relative submergence (R/D_{84}) for θ , where R is the hydraulic radius, and D_{84} is the particle diameter for which 84% of particles are finer. The R/D_{84} scaling considers that flow over roughness elements with moderate relative submergence will gradually modify turbulence throughout the entire flow depth (Belcher and Fox, 2010; Amir and Castro, 2011). Physically, shed vortices scale with D_{84} near the bed and throughout the roughness layer (Belcher, 2009) and moving away from the roughness layer, scaling with D_{84} remains important but gradually diminishes. The D_{84} dependence results from the turbulent structure consisting of connected vortices that eject away from the roughness layer, increase in streamwise length scale, and give way to macroturbulence closer to the free-surface that scales with outer variables (Roy et al., 2004; Hurther et al. 2007; Belcher, 2009; Fox and Belcher, 2011). Based on this knowledge, θ was parameterized as

$$\theta = \frac{1}{R} \left[\left(1 - \frac{D_{84}}{R} \right)^y + D_{84} \right] , \quad (27)$$

which allows for scaling *via* the external condition of D_{84}/R close to the bed; and the external bed condition diminishes when moving farther from the bed to scale with R . The scaling result enables the outer region of gravel flows to be scaled with U_∞ , R , and D_{84} . The latter two parameters are consistent with traditional river research and applications, as R/D_{84} has been historically used as an empirical parameter to understand flow roughness and velocity distributions in gravel- and cobble-bed rivers (e.g., Hey, 1979). Unlike the Zagarola and Smits (1998) scaling, the R/D_{84} scaling provides parameters that are easily measureable in the field making it practical for prediction of the streamwise velocity distribution in realistic flow scenarios.

EXPERIMENTAL DATASETS:

To test the results of the AIP equilibrium solution in Equation (18) as well as provide comparison with the empirical mixed scaling laws, newly collected as well as previously published datasets were applied. Published datasets for hydraulic rough nonuniform flow over gravels vary widely in terms of relative submergence, however, we found that there was an absence of subcritical Froude number ($Fr=U_b (gH)^{-1/2}$, where U_b is the bulk velocity, g is gravitational acceleration, and H is flow depth) flows between 0.5 to 1 that are more typical of decelerating pools in steep mountainous gravel and cobble bed rivers. The lack of published velocity profiles with these moderate Froude numbers provided one reason for collecting new experimental data, and therefore 38 new experimental velocity profiles across three nonuniform flows over gravel in a hydraulic flume were collected with Froude number ranging from approximately 0.5 to almost unity and relative submergence ranging from 8 to 17. A second reason for collecting additional data was that few published datasets provided enough repetition velocity profiles from nonuniform flows with the same external conditions (e.g., pressure gradient, roughness type) to test the uniqueness of the energy gradient parameter (λ). For this reason, we collected at least 10 profiles at different nonuniform flow depths with the same pressure gradient and roughness types in order that λ could be tested *via* Equation (25).

After reviewing the literature, 43 published velocity profiles were also included to test the scaling. To our knowledge, the 43 profiles represent all peer-reviewed, published data of fully hydraulically rough, nonuniform, gradual decelerating open channel flow over gravels in both gravel bed rivers and flumes. Table 1 summarizes the nine datasets collected in our hydraulic flume and from the literature. The datasets in Table 1 are detailed as follows: PS indicates datasets that were collected in a hydraulic flume in the present study; velocity measurements collected from previous studies in laboratory flumes reported by Afzalimehr & Anctil (1999) abbreviated as AA99, Ahmed et al. (1998) abbreviated as Ah98, Song & Chiew (2001) abbreviated as SC01, Song (1994) abbreviated as So94; and field datasets were collected in gravel-bed river field studies and reported by MacVicar & Roy (2007) abbreviated as MR07 and Afzalimehr & Rennie (2009) abbreviated as AR09. Table 2 lists the hydraulic conditions for the individual profiles in each dataset.

As shown in Table 1, the datasets collected in the present study as well as the literature datasets represent fully hydraulically rough, nonuniform open channel flow over gravel. All datasets are fully hydraulically rough as indicated by k^+ greater than 70. The data spans a wide range of relative submergence $R D_{84}^{-1}$ and three orders of Reynolds numbers ($Re=U_b H \nu^{-1}$, where ν is kinematic viscosity) from 10^4 to 10^6 . In addition, the datasets span a wide range of subcritical flows as the Froude number varies from 0.2 to almost one across datasets. In the following, we briefly describe the newly collected data and its quality as well as the previously published literature data. A comprehensive database of all data used in this paper is published online as part of this publication.

Newly Collected Velocimetry Data:

Velocity data was collected within a tilting, recirculating flume 12 m long and 0.61 m wide (Belcher, 2009; Belcher and Fox, 2011; Stewart, 2014; Stewart and Fox, 2015). To provide hydraulic rough flow over gravel, the flume bed was surfaced with angular gravel sized aggregates embedded on fiberglass-resin panels and channel side walls were acrylic plates. Particle size analysis found the flume bed material to have $D_{84}=5.7\text{mm}$, standard deviation 1.4 mm and $D_{50}=4.4\text{mm}$ (Belcher, 2009). The flume slope was adjustable as was the discharge *via* a valve on the supply line. An acoustic Doppler velocimeter (i.e., 3D Sontek MicroADV or just ADV from this point forward) measurement location was fixed to reduce any possible effects of varying upstream conditions due to slight discrepancies in bed roughness distribution. The ADV was mounted on a specially designed measurement carriage equipped with a mechanical screw mechanism for adjustment of ADV elevation. The combination of using a rigid ADV mounting rod and stabilizing the rod with the flume bed reduced ADV vibrations which can interfere with turbulence measurements (Dancey 1990).

ADV sensors also have a serious disadvantage: velocity measurements near the water surface are not possible with an ADV oriented to obtain near bed measurements. Therefore, following the collection of measurements with the probe in the down looking position, the probe was reoriented side-looking to obtain near surface measurements. The down looking probe collected measurements at 0.5 mm above the bed elevation to 54 mm above the bed when flow depths permitted. The down looking probe was unable to collect adequate data in the region from

1-2 cm and 3.3-3.8 cm above the bed as acoustic signals reflected from the bed resulted in poorly correlated velocity signals by the ADV. The side looking ADV was able to obtain measurements within 2 cm of the free surface. Velocities were measured with the probe oriented downward, and then the probe was rotated 90 degrees about the x -axis and translated so that probe tip was offset 5 cm from the channel centerline. The authors developed a control module and quality control method to ensure the orientation angle of the ADV and accurate positioning in the vertical (see Stewart, 2014).

Stabilization of the flow prior to collecting velocity measurements was ensured by measuring flow depth in the channel and verified by the stabilization of velocity statistics. The procedure involved using flow obstructions at the downstream end of the test section to induce the adverse pressure gradient effect. Stabilization time for the velocity field varied depending on obstruction size and flow rate but were generally achieved within 5 minutes; however velocity measurements commenced no sooner than 10 minutes following the placement of flow obstruction in a new location. The adverse pressure gradients increased the flow depth and significantly increased the volume of water stored in the channel. The channel storage volume was obtained from the flume reservoir thus decreasing the reservoir elevation which affects flume flow rate by increasing the head difference between the head box and reservoir. Therefore, the reservoir depth was measured frequently and maintained at a constant value for all obstruction sizes to maintain a constant flow rate. Further verification of constant flume flow rate was confirmed using acoustic Doppler transducers placed on the flume supply line. The stabilization time required following the change in obstruction size was further confirmed by stabilization of velocity statistical moments, which was less than 10 minutes. The pump used to circulate water dissipated heat energy into the system which increased water temperature. To prevent variations in fluid properties during experiments, the flume was started a minimum of one hour prior to data collection and water temperature were periodically monitored throughout experimentation to verify a constant temperature was maintained.

Velocity data was collected with the ADV. 4500 temporal data points were collected at each spatial location during testing to produce time series of three dimensional components of velocity recorded at 0.02 s. The ADV measures velocity in a small sampling volume (0.09 cm^3)

with resolution of 0.01 cm s^{-1} and accuracy of 1% of measured velocity. Data collected with the ADV was processed with the Bureau of Reclamations software WinADV32 which was used to filter data as suggested by the probe manufacturer. To ensure good quality data, filtering was performed to remove points with $\text{SNR} < 15$, where SNR indicates the signal to noise ratio, and correlation $< 70\%$ as described by Wahl (2000).

The overall quality of our ADV data is highlighted in Figure 1, which shows typical examples of spectra, timeseries, streamwise velocity distributions and Reynolds shear stress distributions. Figure 1a shows a typical example of streamwise velocity spectra plotted with Kolmogorov $-5/3$ law as dotted line indicating that the inertial subrange exists and sufficient scale separation likely exists to seek independent similarity solutions to the inner and outer boundary layer equations. Figure 1b shows a typical segment of streamwise velocity timeseries. The time series signal displays fast and slow fluctuating velocity variations indicating the passing small and large scale turbulent structures typical of fully turbulent gravel bed flows. Figure 1c shows typical velocity profiles collected at a number of different streamwise locations for a single flow rate. The velocity distributions decrease as flow depth increases to satisfy continuity. Figure 1d is typical Reynolds shear stress distributions collected at various distance from the flow obstruction for a single flow rate. The Reynolds stress distribution increases from the bed to the maximum value attained around $y/H=0.2$ then decreases as the free surface is approached. The Reynolds shear stress distribution is convex in the region from $y/H=0.2$ to the free surface, typical of adverse pressure gradient open channel flows (Song & Graf 1994; Kironoto & Graf 1995).

The present study (PS) dataset consists of 38 velocity profiles obtained from three flowrates. For each flow rate three blockage ratios from the obstructions were used to create the adverse pressure gradients. The highest flow rate contains 13 velocity profiles, the middle flow rate 14 profiles and lowest flow rate contains 11 profiles. Each velocity profile presented in this manuscript consists of 7 to 28 velocity points. The number of data points collected for each profile depended on flow depth while the number of data presented here was further reduced as some velocity data were not considered usable for estimating time average velocity (e.g., if 30% (1200) of data remained after filtering time series based on SNR and signal correlation).

Previously Published Velocimetry Datasets:

Song (1994) (So94) performed tests in a 16.9 m long and 0.6 m wide recirculating flume with glass sidewalls and fixed gravel bed. The size distribution of the gravel was analyzed by the method of Wolman, being $D_{50} = 1.23$ cm, $D_{16} = 0.9$ cm and $D_{84} = 1.65$ cm. The adverse pressure gradient was developed by changing the bed slope and by regulating the tailgate located downstream of the measuring section. Flow rate was measured using an electromagnetic flow meter installed in the conduit supplying the flume. The flume flow depth was measured using two ultrasonic limnimeters with an accuracy of 1 mm. 3D velocity measurements were made using an ADV profiler. Flow equilibrium was verified by plotting UU^1_∞ and $-\overline{u'v'} U_*^{-2}$ and observing a nearly self-similar distributions over the measuring reach.

Ahmed et al. (1998) (Ah98) conducted experiments in a 20-m-long and 1.22-m-wide recirculating flume with a fixed natural sand glued to a wooden board which formed the bed. The sand grains had a mean size of 1.84 mm and a geometric standard deviation of 1.2 mm. Velocity measurements were obtained using a Prandtl tube and data acquisition system sampling at 10 Hz to collect at least 2000 samples which were averaged to estimate mean velocity. Experimental uncertainties in estimated velocities were 1% at the 95% confidence level. The adverse pressure gradient was developed by placing a cylindrical obstruction downstream of the sampling locations.

Afzalimehr & Ancil (1999) (AA99) performed tests in 8.8 m long and 0.61 m wide recirculating flume with glass sidewalls and layer of immobile gravel which formed the bed. The adverse pressure gradient was developed by adjusting slope, flow rate Q and opening of the tailgate downstream of the measuring location. Velocity measurements were recorded using an ADV and water depth measured using mobile point gage limnimeter.

Song & Chiew (2001) (SC01) performed tests in a 18 m long and 0.6 m wide recirculating flume with glass sidewalls and fixed sand sized bed material. 3D velocity measurements were made with SonTek ADV sampling at 25 Hz for a minimum of 120 seconds. The adverse pressure gradient was developed by varying the flow rate Q , bed slope S , and the

opening of the tailgate downstream of the measuring location. The upper half of the profile was measured using an uplooking probe. The lower half was measured with a downlooking probe.

MacVicar & Roy (2007) (MR07) performed velocity measurements in Moras Creek, a gravel-bed river located in eastern Quebec, Canada. The test section had a riffle-pool geometry and was relatively straight and narrow section representative of riffle-pool dynamics. A Wolman pebble count was used to survey the bed and estimate a representative grain size distribution. Velocity measurements were collected from temporary bridges using electromagnetic current meters (ECMs). Velocities were measured during bankfull flow conditions where rainfall throughout the sampling maintained sufficiently long time periods of stable water levels and thus unsteadiness of the flow data with respect to a hydrograph was minimized for the field data (MacVicar & Roy 2007). Marsh-McBirney 523 ECMs were used to measure velocity in two-dimensions at a sampling rate of 20 Hz for 120 seconds.

Afzalimehr and Rennie (2009) (AR09) reported velocity measurements for the Ghamasiab River, a gravel-bed river located in western Iran. The selected reaches were straight and devoid of in-stream vegetation and bed forms. All measurements were made during periods of constant low-flow discharge. A Wolman pebble count was used to survey the bed and estimate a representative D_{50} and D_{84} within each cross section. The primary evidence of non-uniform flow was the longitudinal variation in width, depth, grain size, and velocity. However the authors indicate flow non-uniformity was clear they were unable to quantify the pressure gradient responsible. Velocity measurements were made using a micro-current propeller meter (propeller diameter of 5 cm).

COMPARISON OF SCALING LAWS:

In Figure 2, the velocity deficit is scaled using the three scaling approaches for each experimental study. In the left column, $U-U_\infty$ is scaled with U_∞ according to the AIP similarity analysis. Velocity distributions within each dataset collapse reasonably well with U_∞ alone providing evidence that flows presented here generally satisfied AIP equilibrium conditions. Exceptions to collapse by U_∞ in column one of Figure 2 have a common theme that the velocity profiles not displaying self-similarity likely have the presence of an external factor which could

perturb the turbulent flow structure.

The PS data scaled by U_∞ shows a few profiles (see hollow symbols) plotting slightly below the rest of this dataset. The profiles in PS dataset plotting below the remainder of the dataset where profiles measured just upstream of the flow obstruction used to create the adverse pressure gradient; it is possible that these velocity distributions were under the influence of localized perturbations as flow was directed around the obstruction. The variation of H at each flow rate changes the relative submergence of bed particles that may represent external conditions not accounted for by AIP similarity analysis. For each of the three flow rates represented by PS, velocity profiles display self-similarity regardless of distance upstream of the flow obstruction indicating that each flow satisfied AIP equilibrium conditions. AIP equilibrium is further supported for the PS datasets by using the second test identified via Equation (25). As shown in Figure 3, U_∞ plotted versus δ in logarithmic form for the three flow rates in the present study indicate a straight line for each condition. The parameters are related by the power function given in Equation (25) with the slope of the line indicating λ . We find that λ is constant and the second test for AIP equilibrium conditions is satisfied for these conditions. We can see that each of these three equilibrium flows had separate constant energy gradient parameter in the stream wise direction and that self-similar velocity distributions developed confirm that AIP equilibrium conditions were satisfied.

The Song (1994, So94) dataset for which much recent research on the effects of AGP in open channel flows is based collapses well when scaled by U_∞ . Flows in So94 were shown by the author to satisfy the more stringent Clauser equilibrium conditions and it appears that these flows also satisfy the AIP conditions for equilibrium as indicated by self-similarity of the velocity distributions. The Ahmed et al. (1998, Ah98) and Song & Chiew (2001, SC01) datasets collected in flumes with bed material at the sand and gravel threshold (i.e., about 2 mm) with high relative submergence showed an even tighter collapse than those of the present study and Song (1994). Comparison of the studies tends to support the idea that U_∞ alone may not account for the impact of relative submergence across the flow depth, which has been shown to impact the flow structure (Nikora et al, 2001).

The MacVicar & Roy (2007, MR07) and Afzalimehr and Rennie (2009, AR09) datasets do not show the degree of self-similarity displayed by the previously mentioned datasets suggesting that at least a subset of these flows may not satisfy AIP equilibrium conditions, at least when comparing within the studies. The solid symbols are profiles that do not collapse with the remaining datasets while the open symbols display self-similarity with the datasets collected by other researchers. The MacVicar & Roy (2007) and Afzalimehr and Rennie (2009) datasets were collected from natural gravel bed rivers. For example, the MacVicar & Roy (2007) data was collected by flow entering a pool in a natural gravel bed river where the presence of strong secondary currents was likely as was noted in the paper by the inflected profiles near the free surface. The spatial heterogeneity of bed roughness common to gravel bed rivers also suggests some of these velocity distributions may have been affected by individual roughness elements representing an external condition not accounted for in the AIP similarity analysis.

By observing the center column of Figure 2, the velocity profiles from all datasets display self-similar distributions when scaled using the Zagarola and Smits (1998) scaling. The ratio $\delta^*:\delta$ represents an average distance of velocity lost from each profile by the effects of viscosity and bed roughness on the resistance and flow structure. The collapse of these datasets indicates that the ratio of $\delta^*:\delta$ accounts for changing external conditions in each of these datasets to eliminate the effects of perturbations to the turbulent flow structure. The present study's datasets scaled by Zagarola and Smits (1998) is compared with the scaling using U_∞ alone: it can be seen that deviations from the self-similar distribution is virtually removed suggesting that for those flume conditions the effects of adverse pressure gradient and relative roughness is removed. The remaining datasets each show self-similar velocity distributions when the outer scaled velocity deficit is adjusted by the ratio $\delta^*:\delta$. These results indicate that AIP equilibrium is satisfied and that the ratio $\delta^*:\delta$ accounts for external conditions associated with bed roughness, which were not considered in the AIP similarity analysis alone.

The right column of Figure 2 shows the mixed scale of Fox and Stewart (2014) for $U-U_\infty$ parameterized using inner variables to account for the decreasing effect of bed roughness further into the core of the flow. The scaling is an improvement over using free stream velocity alone which is an expected result since this parameterization of external conditions was designed to

account for change in turbulent structure as the bed normal distance increases. The comparison shows that parameterization that makes use of gravel bed flow parameters to account for lost velocity due to the impact of bed roughness on the velocity distributions improves the collapse over U_∞ alone.

The velocity deficit profiles scaled by the free stream velocity collapse reasonably well to a single curve (see Fig 4a). The collapse of profiles from individual datasets was better than the collapse for all data sets combined which is not surprising. It is more remarkable that profiles collapse as well as they do given the wide range of flow conditions from which profiles were obtained. This illustrates the robustness for the AIP equilibrium conditions to be satisfied over a wide range of flows. The deviation of velocity profiles from the self-similar distribution indicate the presence of external conditions affecting the flow structure which has not been accounted for by scaling with U_∞ alone. The deviation of laboratory collected datasets alone is such that profiles with large relative submergence tend to plot toward the top of the graph while profiles in flows with low relative submergence plot on the bottom. This finding is consistent with reports from boundary layer flows over smooth and rough surfaces (Castillo et al. 2004). This reflects the impact of roughness elements across the entire depth of open channel flow. In Figure 4b, the outer scaled velocity deficit is shown parameterized using the ratio $\delta^*:\delta$ which had been shown in turbulent boundary layer flows to remove the effects of Reynolds number and upstream conditions from the velocity profiles. Figure 4b clearly shows that the Zagarola and Smits (1998) scaling removes the effect of energy gradient and roughness from the velocity distributions that was not removed by free stream velocity alone. This scaling was able to account for a wide range of bulk flow parameters as described in Table1 and any additional unknown external perturbation applied on the flow. Figure 4c shows outer scaled velocity deficit parameterized according to Fox and Stewart (2014). The collapse of velocity profiles when scaled in this manner is an improvement over freestream velocity alone. The velocity deficit no longer displays a dependence on relative roughness as was seen in Figure 4a. This indicates that parameterization of external perturbations to the flow structure is well described by the gravel bed parameters R and D_{84} and that flow structure becomes less dependent on bed conditions further away from the bed.

DISCUSSION AND CONCLUSIONS:

The AIP similarity solution in Equation (18) for nonuniform open channel flow over gravel supports the hypotheses that the flow case can be defined by AIP equilibrium. The hypothesis is further supported by comparison with newly collected and previously published experimental data for nonuniform open channel flow over gravel from flumes and rivers as well as the condition in Equation (25) for the newly collected dataset. The ability to define the nonuniform flow case as AIP equilibrium is attributed to the consistent form and function of the double-layer turbulent structure through a gradual vertical expansion. While the wall normal velocity likely can increase interaction between the near wall roughness and outer regions, the expansion is gradual enough such that transformation mechanisms between potential energy and kinetic energy throughout the boundary layer are consistent, and therefore the turbulence structure is not changing form in the streamwise direction. The self-similarity of the streamwise velocity and the constant energy gradient parameter support this concept.

The empirical scalings of Zagarola and Smits (1998) and Fox and Stewart (2014) are shown to better collapse all data when comparison with U_∞ alone. The result is attributed to ability of the empirical methods to account for variable pressure gradients and roughness types, which AIP similarity solution alone does not consider. The mixed scaling approach of Zagarola and Smits (1998) provides the best collapse to the data, however the practicality of measuring the displacement thickness negates the applicability of this approach outside of the research setting, which has also been highlighted by others. Nevertheless, the single ratio of $\delta^*:\delta$ and lack of dependence on the vertical highlight potential for further extension of the Zagarola and Smits (1998) method, e.g., using empirical connectivity between δ^* and measureable quantities. The Fox and Stewart (2014) mixed scaling approach accounts for the variation of turbulent length in the vertical to parameterize the external boundary impact on the flow for the outer region. These results suggest turbulence growth in adverse pressure gradient open channels is similar to that of uniform flows; this hypothesis should be validated in future research. Using easily obtained parameters the Fox and Stewart (2014) scaling allows estimation of velocity profiles applicable to field research where detailed velocity measurements is either not possible or difficult to obtain.

One significant application of this contribution is the relaxing the use of the friction velocity for scaling nonuniform flows in gravel bed rivers. In this sense, U_∞ provides an alternative to the traditional Clauser defined equilibrium based on scaling with the friction velocity. The AIP definition is expected to provide a more achievable definition of equilibrium and Clauser equilibrium is in fact a subset of AIP (Maciel et al. 2006). Further, friction velocity can be rather difficult to estimate in practice with numerous publications prescribing and comparing methods for its estimation, with different methods showing estimates varying by as much as 30% (Biron et al. 2004; Song & Graf 1995; Afzalimehr & Rennie 2009). For example, the Clauser method of estimating friction velocity by fitting a logarithmic velocity distribution to measurements near the bed is highly subject to errors in estimations of the location of zero velocity and requires numerous measurements which may be difficult to obtain in realistic flows. Extrapolation of Reynolds stresses to bed is the preferred method of estimating U_* in turbulence research (Nezu & Nakagawa 1993), however this requires numerous multi-component velocity measurements where large errors are introduced by improper probe orientation with the flow direction. Estimation of U_* with the St. Venant equation requires estimation of water surface slope using measurements of flow depth which display small changes over the rather large streamwise distances between them. On the other hand, estimation of U_∞ requires only a single velocity measurement at a location where velocity gradients are small and thus errors introduced by improperly estimating the location of maximum velocity is not likely to have significant impact on estimates or the accuracy of predicted velocity distributions. Further, in the nonuniform case, such measurements could be carried out in pools within gravel bed rivers using velocity sensors, such as inexpensive velocity bend sensors (Stewart et al., 2013), in order to provide a longitudinal velocity distribution within a reach. The ability to accurately estimate U_∞ compared to the friction velocity may provide for the improved collapse of the velocity distributions. The field researcher can obtain relatively accurate measurements of U_∞ and flow depth to estimate the shape of the velocity distribution and calculate a depth average velocity. This method for obtaining depth average velocity will likely improve the accuracy of these estimates as compared to traditional one and two-point methods that assume a logarithmic velocity profile.

However, the scaling of nonuniform gravel bed rivers is not fully resolved in a theoretical

sense. In open channel flows over intermediate to large roughness, the relative motions and energy containing components of turbulent motions throughout the flow are dependent on surface roughness which represents θ for these flows in the AIP similarity analysis. For this reason, parameterization of θ for surface roughness is necessary in open channel flows similar to that seen in boundary layers where roughness elements are relatively large (Fox and Stewart, 2014; Amir and Castro, 2011). This result presents a bit of a theoretical conundrum in that inner and outer regions are clearly separated consistent with high Reynolds number flows, yet roughness impacts are retained to some degree in the outer region thus disagreeing with Townsend's wall similarity hypothesis (Townsend, 1976). Yet, Townsend's hypothesis for flows with moderate relative submergence remains an open, debated question in the literature. For example, the work by Flack, Schultz and co-workers tends to argue for the wall similarity hypothesis, even with larger roughness heights and the relative submergence as low as 6 (Flack et al., 2005; Connelly et al., 2006). Amir and Castro (2011) argue against the wall similarity hypothesis suggesting that the roughness particles impact the entire flow depth for moderate relative submergence. Resolution to this problem should focus on elucidating the form and function of the relatively weak but persistent macroturbulence within the outer region of the flow and thereafter integrating its imprint within the boundary layer equations.

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TABLE CAPTIONS

Table 1. Range of hydraulic conditions for the 9 datasets of nonuniform flow.

Table 2. (Supplemental Table to be printed online). Hydraulic conditions for the individual profiles in each dataset.

FIGURE CAPTIONS

Figure 1. Examples of time series, spectra, streamwise velocity and Reynolds stresses for subset of the newly collected data. PS-###=Present Study-Data Set.(flow depth (cm))

Figure 2. The velocity defect of nonuniform open channel flows over gravel for the present study (PS) and published datasets scaled with U_∞ (left column), Zagarola & Smits scaling (center column), and Fox and Stewart scaling (right column).

Figure 3. Estimation of $\mathcal{A}$ for the experimental datasets of the present study.

Figure 4. Scaling of the velocity defect for nonuniform open channel flows over a gravel bed using (a) U_∞ , (b) the mixed scaling of Zagarola and Smits (1998) and (c) the mixed scaling of Fox and Stewart (2014).

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Table 1. Range of hydraulic conditions for the 9 datasets of nonuniform flow.

Data Set	Q (l s ⁻¹)	H (cm)	U_{∞} (cm s ⁻¹)	δ (cm)	D_{50} (mm)	D_{84} (mm)	$W H^{-1}$	Fr	$Re \times 10^{-5}$	$k^+ = \frac{k_s U_*}{\nu}$	$\frac{U_{\infty}}{U_*}$	$R D_{84}^{-1}$
PS-1	43	9.6-14.5	62-85	8-12	4.4	5.7	4-6	0.51-0.81	0.5-0.5	257-361	12-14	13-17
PS-2	23	5.8-9.8	50-70	5-8	4.4	5.7	6-11	0.47-0.81	0.3-0.3	225-354	11-15	9-13
PS-3	17	5.6-7.9	44-63	5-7	4.4	5.7	8-11	0.45-0.74	0.2-0.2	208-282	11-13	8-11
So94	55-90	14.5-20.5	78-116	15-21	12.3	16.5	3-4	0.46-0.6	3.8-6.2	810-1140	15-18	6-7
Ah98	65	18.0	26-33	17-18	1.82	2.2	9-9	0.22-0.22	0.2-0.2	144-144	12-16	52-52
AA99	52	27.3-27.3	42-42	21-21	25.4	0	2-2	0.2-0.2	2.9-2.9	838-838	13-13	6-6
SC01	42.00	13.1-16.2	50-51	13-16	2.6	0	4-5	0.34-0.47	3.5-3.5	78-97	13-16	35-40
MR07	4900	81-134	167-223	66-105	60	190	4-7	0.31-0.59	11.3-15	15960-60040	7-20	3-5
AR09	NA	18-63	62-161	13-70	14-23	27-40	16-49	0.46-0.77	1.1-7.3	2432-6031	7-12	5-18

Table 2 (Supplemental Table to be printed online). Hydraulic conditions for the individual profiles in each dataset.

Data Set	S	H (cm)	U_{∞} (cm s ⁻¹)	δ (cm)	D_{50} (mm)	D_{84} (mm)	W (m)	$W H^{-1}$	Fr	$Re \times 10^{-5}$	k^+	$H D_{84}^{-1}$
PS-1.1	0.60	13.3	69	11.1	4.4	5.7	0.61	4.6	0.58	0.5	301	23.3
PS-1.2	0.60	13.7	67	10.4	4.4	5.7	0.61	4.5	0.56	0.5	260	24.0
PS-1.3	0.60	13.9	69	11.7	4.4	5.7	0.61	4.4	0.57	0.5	286	24.4
PS-1.4	0.60	14.3	65	11.7	4.4	5.7	0.61	4.3	0.53	0.5	257	25.1
PS-1.5	0.60	14.5	62	11.2	4.4	5.7	0.61	4.2	0.51	0.5	270	25.4
PS-1.6	0.60	10.4	81	7.7	4.4	5.7	0.61	5.9	0.74	0.5	358	18.2
PS-1.7	0.60	10.9	78	9.5	4.4	5.7	0.61	5.6	0.71	0.5	361	19.1
PS-1.8	0.60	11.2	79	9.5	4.4	5.7	0.61	5.4	0.70	0.5	334	19.6
PS-1.9	0.60	11.6	79	9.5	4.4	5.7	0.61	5.3	0.70	0.5	312	20.4
PS-1.10	0.60	11.6	77	9.5	4.4	5.7	0.61	5.3	0.67	0.5	273	20.4
PS-1.11	0.60	9.6	85	8.1	4.4	5.7	0.61	6.4	0.81	0.5	352	16.8
PS-1.12	0.60	10.0	85	8.1	4.4	5.7	0.61	6.1	0.79	0.5	334	17.5
PS-1.13	0.60	10.1	84	7.8	4.4	5.7	0.61	6.0	0.78	0.5	340	17.7
PS-2.1	0.60	5.8	70	4.5	4.4	5.7	0.61	10.5	0.81	0.3	318	10.2
PS-2.2	0.60	8.6	56	7.0	4.4	5.7	0.61	7.1	0.55	0.3	250	15.1
PS-2.3	0.60	8.9	54	7.2	4.4	5.7	0.61	6.9	0.52	0.3	235	15.6
PS-2.4	0.60	9.2	54	6.8	4.4	5.7	0.61	6.6	0.52	0.3	233	16.1
PS-2.5	0.60	9.6	51	7.5	4.4	5.7	0.61	6.4	0.49	0.3	231	16.8
PS-2.6	0.60	9.8	50	7.8	4.4	5.7	0.61	6.2	0.47	0.3	225	17.2
PS-2.7	0.60	7.0	69	5.7	4.4	5.7	0.61	8.7	0.74	0.3	300	12.3
PS-2.8	0.60	7.4	66	5.9	4.4	5.7	0.61	8.2	0.69	0.3	298	13.0

Table 2 (continued)

PS-2.9	0.60	7.5	66	5.7	4.4	5.7	0.61	8.1	0.69	0.3	320	13.2
PS-2.10	0.60	7.9	62	6.3	4.4	5.7	0.61	7.7	0.63	0.3	276	13.9
PS-2.11	0.60	8.0	61	6.5	4.4	5.7	0.61	7.6	0.62	0.3	281	14.0
PS-2.12	0.60	6.5	70	5.3	4.4	5.7	0.61	9.4	0.78	0.3	272	11.4
PS-2.13	0.60	6.8	70	5.7	4.4	5.7	0.61	9.0	0.76	0.3	306	11.9
PS-2.14	0.60	6.9	70	4.5	4.4	5.7	0.61	8.8	0.75	0.3	354	12.1
PS-3.1	0.60	6.9	54	5.9	4.4	5.7	0.61	8.8	0.58	0.2	239	12.1
PS-3.2	0.60	7.3	50	6.2	4.4	5.7	0.61	8.4	0.53	0.2	229	12.8
PS-3.3	0.60	7.5	50	6.4	4.4	5.7	0.61	8.1	0.52	0.2	219	13.2
PS-3.4	0.60	7.8	47	6.6	4.4	5.7	0.61	7.8	0.48	0.2	213	13.7
PS-3.5	0.60	7.9	44	6.7	4.4	5.7	0.61	7.7	0.45	0.2	208	13.9
PS-3.6	0.60	6.0	56	5.1	4.4	5.7	0.61	10.2	0.64	0.2	282	10.5
PS-3.7	0.60	6.2	55	5.3	4.4	5.7	0.61	9.8	0.62	0.2	276	10.9
PS-3.8	0.60	6.5	53	5.5	4.4	5.7	0.61	9.4	0.59	0.2	244	11.4
PS-3.9	0.60	6.6	53	5.6	4.4	5.7	0.61	9.2	0.58	0.2	254	11.6
PS-3.10	0.60	5.6	63	4.8	4.4	5.7	0.61	10.9	0.74	0.2	273	9.8
PS-3.11	0.60	5.6	59	4.8	4.4	5.7	0.61	10.9	0.69	0.2	279	9.8
So94-1	0.25	20.0	89	20.0	12.3	16.5	0.6	3.0	0.54	6.2	845	12.1
So94-2	0.90	18.0	78	18.0	12.3	16.5	0.6	3.3	0.49	4.8	875	10.9
So94-3	0.50	14.5	78	14.5	12.3	16.5	0.6	4.1	0.53	3.8	858	8.8
So94-4	0.50	16.5	96	16.5	12.3	16.5	0.6	3.6	0.56	4.8	967	10.0
So94-5	0.50	18.5	111	18.5	12.3	16.5	0.6	3.2	0.6	6.2	1140	11.2

Table 2 (continued)

So94-6	0.75	17.0	84	17.0	12.3	16.5	0.6	3.5	0.46	4.1	810	10.3
So94-7	0.75	20.5	82	20.5	12.3	16.5	0.6	2.9	0.46	5.5	817	12.4
So94-8	0.90	18.0	116	18.0	12.3	16.5	0.6	3.3	0.6	6.2	1140	10.9
Ah98-1.1		18.0	33	17.4	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.2		18.0	26	17.3	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.3		18.0	28	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.4		18.0	29	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.5		18.0	30	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.6		18.0	31	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.7		18.0	32	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
Ah98-1.8		18.0	33	18.0	1.82	2.2	1.22	8.7	0.22	0.2	144	81.8
AA99-1	0.70	27.3	42	21.1	25.4		0.61	2.2	0.2	2.9	838	10.7
SC01-1.1	0.75	16.2	51	16.2	2.6		0.6	3.7	0.342	3.5	78	62.1
SC01-1.2	0.75	14.6	51	14.6	2.6		0.6	4.1	0.397	3.5	85	56.2
SC01-1.3	0.75	13.1	50	13.1	2.6		0.6	4.6	0.47	3.5	97	50.2
MR07-1.1		81	223	66	60	190	6	7.4	0.59	13.4	60040	4.3
MR07-1.2		92	187	75	60	190	6	6.5	0.41	11.3	32908	4.8
MR07-1.3		118	167	105	60	190	6	5.1	0.37	14.8	33991	6.2
MR07-1.4		134	171	84	60	190	6	4.5	0.31	15.0	15960	7.1
AR09-1.1		28	108	28	18	31	8.4	29.7	0.66	3.1	3038	9.1
AR09-1.2		32	113	31	18	36	8	24.8	0.64	3.7	3816	9.0
AR09-1.3		25	90	17	21	40	7	27.7	0.57	4.8	4160	6.3

Table 2 (continued)

AR09-1.4	37	86	70	22	35	6	16.1	0.66	4.7	4130	10.7
AR09-1.5	28	97	34	20	40	7.5	26.5	0.58	2.8	3280	7.1
AR09-1.6	45	161	44	20	37	8.5	18.8	0.77	7.3	6031	12.2
AR09-1.7	18	62	13	21	38	9	49.2	0.46	1.1	2432	4.8
AR09-1.8	26	88	25	17	33	11.3	43.0	0.55	2.3	2607	8.0
AR09-1.9	38	121	37	22	33	10	26.1	0.62	2.3	3498	11.6
AR09-1.10	37	128	36	18	37	11	29.5	0.67	4.8	4255	10.1
AR09-1.11	30	98	29	14	30	12	39.6	0.57	3.0	2520	10.1
AR09-1.12	40	137	39	19	37	11	27.3	0.69	5.5	4588	10.9
AR09-1.13	34	104	36	16	30	10	29.2	0.57	3.6	2670	11.4
AR09-1.14	63	87	25	20	31	10	15.9	0.55	5.5	2480	20.3
AR09-1.15	33	108	32	16	27	10.5	31.5	0.60	3.6	2565	12.3
AR09-1.16	30	102	29	19	32	8	26.4	0.60	1.9	2944	9.5
AR09-1.17	23	80	22	23	40	8.9	38.2	0.53	1.9	3000	5.8
AR09-1.18	22	89	22	17	34	8.9	39.9	0.59	2.0	2584	6.6
AR09-1.19	26	86	25	18	33	7.5	28.5	0.54	2.3	2607	8.0