1 Isolating parameter sensitivity in reach scale transient storage modeling

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25 **Abstract**

26 Parameter sensitivity analyses, although necessary to assess identifiability, may not lead to an
27 increased understanding or accurate representation of transient storage processes when
28 associated parameter sensitivities are muted. Reducing the number of uncertain calibration
29 parameters through field-based measurements may allow for more realistic representations and
30 improved predictive capabilities of reach scale stream solute transport. Using a two-zone
31 transient storage model, we examined the spatial detail necessary to set parameters describing
32 hydraulic characteristics and isolate the sensitivity of the parameters associated with transient
33 storage processes. We represented uncertain parameter distributions as triangular fuzzy numbers
34 and used closed form statistical moment solutions to express parameter sensitivity thus avoiding
35 copious model simulations. These solutions also allowed for the direct incorporation of different
36 levels of spatial information regarding hydraulic characteristics. To establish a baseline for
37 comparison, we performed a sensitivity analysis considering all model parameters as uncertain.
38 Next, we set hydraulic parameters as the reach averages, leaving the transient storage parameters
39 as uncertain, and repeated the analysis. Lastly, we incorporated high resolution hydraulic
40 information assessed from aerial imagery to examine whether more spatial detail was necessary
41 to isolate the sensitivity of transient storage parameters. We found that a reach-average hydraulic
42 representation, as opposed to using detailed spatial information, was sufficient to highlight
43 transient storage parameter sensitivity and provide more information regarding the potential
44 identifiability of these parameters.

45 **Keywords:** transient storage; stream hydraulics; parameter sensitivity; statistical moments

1. Introduction

A collective goal among transient storage model applications is to identify parameters that describe the individual solute transport processes represented by the model (e.g., [1-3]). However, identification of parameters through traditional calibration techniques proves problematic due to parameter interactions and the many distinct combinations of values yielding predictions that similarly reproduce stream tracer observations (e.g., [4-6]). Because only unique, identifiable parameters can be used to describe or predict solute transport processes (e.g., [2, 7]), traditional calibration techniques can leave the representation of transient storage residence times dubious or even incorrect (e.g., [8, 9]). Although parameter uniqueness is rarely tested, the individual effects, interactions, and identifiability of uncertain parameters can be assessed with sensitivity analyses (e.g., [10]). Through such analyses, parameters with the largest sensitivities are typically considered identifiable and will more likely return information regarding the dominant processes (e.g., [11]). Unfortunately, performing a sensitivity analysis may not provide an increased understanding or accurate representation of transient storage processes—especially if there are strong parameter interactions with dominant advective or dispersive processes muting their associated parameter sensitivities (e.g., [5]). To provide a more realistic representation of processes and improve predictive capabilities of stream solute transport, reduction in the number of uncertain parameters (e.g., set through field-based measurements) is necessary (e.g., [12-14]). However, there is currently no best practice for reducing the number of uncertain parameters included within a sensitivity analysis to better focus on understanding the sensitivity of the parameters describing the processes of interest.

The objective of this paper is to examine the spatial detail necessary to set parameters describing hydraulic characteristics and isolate the sensitivity of the parameters associated with

transient storage processes. Appropriately incorporating hydraulic detail into the model representation ultimately yields more information regarding the identifiability (i.e., uniqueness or potential to describe related processes) of these remaining uncertain parameters. To form a baseline for comparison, a sensitivity analysis was first performed with all model parameters. Then we set the parameters describing hydraulic characteristics (collectively channel width, depth, and the dispersion coefficient) using reach-averaged information and repeated the analysis. Finally, we incorporated a detailed data set describing the spatial variability of hydraulic characteristics and repeated the sensitivity analysis.

2. Methods

We performed a fuzzy number sensitivity analysis and used closed form statistical moment solutions to express parameter sensitivity on a two-zone transient storage model. To illustrate the utility of this approach, depth and the dispersion coefficient were estimated from channel widths assessed from high-resolution aerial imagery collected over a case study reach. By applying spatial width observations, we were able to investigate how different levels of hydraulic detail can influence the interpretation of transient storage parameter sensitivity.

2.1. Model and solutions

Surface transient storage (STS) and hyporheic transient storage (HTS) processes can be important controls to reactive solute [15] and heat transport [16, 17]. STS is commonly defined as slower moving, recirculating stream water relative to main stream flow and HTS is generally defined as continuous exchange between the adjacent alluvial aquifer and stream water (e.g., [1]). We used the solute component of the two-zone temperature and solute (TZTS) transient

storage model [16, 17] because both STS and HTS are reflected and the physical layout is constrained by widths and depths in the parameterization. Although this parameterization doubles the number of parameters from the one-zone counterpart (e.g., [1, 18]), it allows for more direct incorporation of hydraulic characteristic measurements (e.g., widths and depths) into the model. Assuming flow is steady and the active stream channel and storage zones are rectangular, the governing equations are [16]

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - \frac{Q}{Y B_{tot}(1-\beta)} \frac{\partial C}{\partial x} + \frac{\alpha_{STS} Y_{STS}}{Y(1-\beta)\beta B_{tot}^2} (C_{STS} - C) + \frac{\alpha_{HTS}}{Y_{HTS} Y} (C_{HTS} - C), \quad (1)$$

$$\frac{dC_{STS}}{dt} = \frac{\alpha_{STS}}{(\beta B_{tot})^2} (C - C_{STS}), \quad (2)$$

$$\frac{dC_{HTS}}{dt} = \frac{\alpha_{HTS}}{Y_{HTS}^2} (C - C_{HTS}), \quad (3)$$

where C is the solute concentration (mg m^{-3}); Q is the volumetric flow rate ($\text{m}^3 \text{s}^{-1}$); D is the longitudinal dispersion coefficient ($\text{m}^2 \text{s}^{-1}$); B_{tot} is the total wetted channel width (m); Y is the mean depth of water in the channel (m); α_{STS} and α_{HTS} are the exchange rate coefficients between main stream flow and the STS and HTS zones ($\text{m}^2 \text{s}^{-1}$), respectively; Y_{STS} and Y_{HTS} are the STS and HTS mean depths (m), respectively; β is the STS fraction of B_{tot} ; x is distance (m); and t is time (s). There are eight parameters requiring calibration or estimation (B_{tot} , Y , D , β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS}).

Model simulations can be inefficient and subject to numerical instability or truncation error. Conversely, statistical moments can reflect temporal characteristics of a complex transport model with simple, closed form solutions that are functions of model parameters [19-21]. Furthermore, it has been determined that only the first two moments are needed to express more than 80% of the dynamic behavior of a model output [22]. As a result, such closed form solutions can be used to analyze the sensitivity of uncertain parameters on the model output

without requiring copious (typically on the order of 1,000 to 10,000 (e.g., [11])) model simulations. A Laplace-domain solution to the solute residence time probability density function (PDF) provides a moment generating function [23] to derive these closed form statistical moment solutions.

Assuming initial and boundary conditions are

$$\begin{aligned} C(x,0) = C_{STS}(x,0) = C_{HTS}(x,0) = 0, \\ C(0,t) = g(t), \text{ and } C(\infty,t) = 0, \end{aligned} \quad (4)$$

where $g(t)$ represents an inflow solute breakthrough curve, the Laplace-domain solution to stream solute concentration (Eq. (1)-(3)) is [20]

$$\bar{C} = \bar{g}(s) \exp \left[\frac{x}{2D} \left(U - \sqrt{U^2 + 4D\psi(s)} \right) \right], \quad (5)$$

where $\bar{C}(x,s) = \mathcal{L}\{C(x,t)\}$ is the stream solute concentration in the Laplace domain, s is the

Laplace variable, $\bar{g}(s) = \mathcal{L}\{g(t)\}$, $U = \frac{Q}{YB_{tot}(1-\beta)}$, $\psi(s) = \left(s + \kappa_1 + \kappa_2 - \frac{\kappa_1\kappa_3}{(s+\kappa_3)} - \frac{\kappa_2\kappa_4}{(s+\kappa_4)} \right)$,

$\kappa_1 = \frac{\alpha_{STS}Y_{STS}}{Y(1-\beta)\beta B_{tot}^2}$, $\kappa_2 = \frac{\alpha_{HTS}}{Y_{HTS}Y}$, $\kappa_3 = \frac{\alpha_{STS}}{(\beta B_{tot})^2}$, and $\kappa_4 = \frac{\alpha_{HTS}}{Y_{HTS}^2}$. Spatially variable parameters

are incorporated into the solution through a convolution,

$$\bar{C}(X,s) = \bar{g}(s) \cdot \bar{c}_1(\Delta x,s) \cdot \bar{c}_2(\Delta x,s) \cdot \dots \cdot \bar{c}_i(\Delta x,s), \quad (6)$$

where $\bar{c}_i(\Delta x,s) = \bar{C}_i(\Delta x,s) / \bar{g}(s)$, i is the reach segment index, Δx is the reach segment length,

$\mathcal{L}^{-1}\{\bar{c}_i(\Delta x,s)\} = c_i(\Delta x,t)$ is the solute residence time PDF of the i th reach segment, $X = N\Delta x$ is

the total reach length, and N is the total number of reach segments (i.e., the number of spatial

observations). Note that this convolution is valid for study reaches with sufficiently large Péclet

numbers (i.e., $(XU)/D \gg 1$) to ensure that the upstream reach segment solution can be used as

the boundary condition for the next downstream reach segment solution and mass is conserved [19]. Eq. (6) was normalized by the zeroth temporal moment to produce a Laplace-domain solute residence time PDF and derive closed form statistical moment solutions.

From an Eulerian perspective, the first temporal moment, μ_t (s), represents the mean residence time; the second temporal moment centered about the mean, σ_t^2 (s²), represents the variance, a metric of symmetrical spreading; the third temporal moment centered about the mean, S_t (s³), is related to skewness, a metric of symmetry, and reflects temporal information contained in the late-time tailing of the solute residence time PDF. We expressed the sensitivity of model parameters using the closed form solutions to these statistical moments from Schmadel et al. [20] (Table 1). These solutions were derived from Eq. (6) and as a result, the reach-averaged moment estimate is expressed as a summation of segment-wise solutions [19]. This summation method produced the spatial variability factors, which are dimensionless weights that reflect the spatial variation of parameters, such as channel width and depth, from their reach-averaged estimates. It is through these factors that the influence of spatially variable hydraulic characteristics was directly incorporated into the moment solutions.

2.2. Parameter sensitivity analysis

There are a variety of sensitivity analysis methods available to quantify the impact of uncertain model parameters on model outputs (e.g., see Tang et al. [11]). We chose to use a fuzzy number sensitivity analysis because fuzzy numbers can represent parameter uncertainty with very limited a priori knowledge. Specifically, while representing uncertainty as a PDF may require frequentist information, a fuzzy number can represent it similarly via different levels of membership (0 to 1) based only on possible ranges [24]. Lacking the information necessary to

statistically justify a particular parameter distribution over another results in an ambiguous distribution. We circumvent this problem by representing uncertain parameters as triangular fuzzy numbers with nothing more than ranges and a central tendency assumption. The general transformation method [25, 26] was used to implement these fuzzy numbers and identify the relative influence (i.e., sensitivity) of model parameters on the moment solutions. In short, use of this method consisted of: (1) generation of possible parameter combinations by sampling within the triangular fuzzy numbers (see Fig. 1A for example), (2) evaluation of the moment solutions for each parameter combination to construct fuzzy numbers for the moments that represent uncertainty in solutions (see Fig. 1B for example), and finally, (3) evaluation of the relative influence of each parameter by essentially normalizing each triangular fuzzy number input by the constructed moment fuzzy number output. See Hanss [25, 26] for more details. Note that the number of parameter combinations, p , grows exponentially with the number of uncertain parameters, $p = \sum_{k=1}^m k^n$, where m is the number of membership levels selected (four in this case) and n is the number of uncertain parameters. In the end, with nothing more than ranges of parameter values, use of the general transformation method provided the influence of each uncertain parameter on a scale of 0-100%.

2.3. Application example

We provide an example of employing this fuzzy number sensitivity analysis with the moment solutions (Table 1) when (1) considering all model parameters as uncertain, (2) setting reach-averaged hydraulic characteristics, and (3) incorporating spatially variable hydraulic characteristics. We based this example on available data within a 6.47 km study reach of the Virgin River located in southwestern Utah. This reach is a semi-low gradient desert river system

(streambed slope of 0.0039 m m^{-1}) with sand to gravel substrate. The volumetric flow rate, Q , measured at the upper reach limit was $\sim 1.06 \text{ m}^3 \text{ s}^{-1}$ (this can range seasonally from $0.5\text{--}25 \text{ m}^3 \text{ s}^{-1}$) and is assumed steady based on continuous water levels recorded at this limit. Groundwater inflow effects on hydraulics (e.g., depth) are assumed negligible based on previous work [6, 12, 16, 17]. A Rhodamine WT tracer was instantaneously injected at the upper reach limit and measured at the downstream reach limit, which was considered to behave conservatively due to the low organic matter content in the streambed sediments [16]. See Bandaragoda and Neilson [6] for more details regarding this tracer experiment. To represent the spatially variable hydraulic characteristics in this application example, we used values of B_{tot} , Y , and D from Schmadel et al. [20]. Estimates of B_{tot} were assessed every five streamwise meters over the 6.47 km from the clipped raster (0.7 m pixel resolution) of observed water temperatures (banks, vegetation, and sandbars excluded) presented in Bingham et al. [12] (Fig. 2a). This clipped raster was produced from remotely sensed, high-resolution three-band and thermal infrared imagery collected from an aerial platform. The 5-m estimates of B_{tot} ranged from 3 to 42 m with a reach average of 16.8 m (Fig. 2b). Y and D were estimated spatially from the B_{tot} estimates using hydraulic principles and relationships (i.e., momentum, Manning's equation, and Fischer's [27] empirical relationship for D). See Schmadel et al. [20] for more details. A wider channel translated into shallower water depths where Y ranged from 0.11 to 0.55 m with a reach average of 0.21 m (Fig. 2c). Subsequently, wider and shallower locations resulted in higher dispersion coefficients, which ranged from 0.5 to $137 \text{ m}^2 \text{ s}^{-1}$ with a reach average of $24.1 \text{ m}^2 \text{ s}^{-1}$ (Fig. 2d).

As a baseline for comparison, the fuzzy number sensitivity analysis was performed where the sensitivities of all eight model parameters (B_{tot} , Y , D , β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS}) were expressed by the moment solutions. In this baseline analysis, feasible ranges of each uncertain

parameter were established by generating parameter sets using Latin Hypercube sampling, comparing model simulations (using Eq. (5)) to the observed tracer breakthrough curve at the downstream reach limit (Fig. 3), and determining which parameter sets result in a Nash-Sutcliffe Efficiency (E) greater than 0.9 after Neilson et al. [17] (Table 2). With a baseline established, we then investigated how different levels of hydraulic detail influenced the interpretation of transient storage parameter sensitivity. At the minimum level, B_{tot} , Y , and D were set as the reach averages quantified from the 5-m values, leaving β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS} as uncertain parameters, and the sensitivity analysis was repeated. Lastly, we incorporated the 5-m spatial estimates of B_{tot} , Y , and D into the moment solutions through the spatial variability factors (Table 1), again leaving β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS} as uncertain parameters, and repeated the sensitivity analysis.

3. Results

When considering all model parameters as uncertain, B_{tot} (~45%) had the largest influence on the mean residence time, μ_t (Fig. 4a). The next largest influences on μ_t were Y_{HTS} (~30%), Y (~15%), Y_{STS} (~7%), and β (~3%). The closed form solution to μ_t does not contain D , α_{STS} , or α_{HTS} and is, therefore, not influenced by these parameters (Table 1). For the higher-order moments, B_{tot} (~40%) also had the largest influence on the variance, σ_t^2 , and third central moment, S_t , specifying that symmetrical spreading and late-time tailing of the solute residence time PDF were mostly sensitive to the channel width for this reach. These higher-order moments were additionally influenced by the other hydraulic parameters, Y (~20%) and D (~10-15%). Most of the remaining influence on σ_t^2 and S_t was attributed to Y_{HTS} (~12-15%). For the parameter ranges and conditions of this reach, σ_t^2 and S_t were nearly insensitive to β , Y_{STS} , α_{STS} , and α_{HTS} . Although the parameters with the largest influences on the moment solutions were

recognized, it is unclear at this point how setting the hydraulic parameters (B_{tot} , Y , and D) may affect the interpretation of transient storage parameter (β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS}) sensitivity.

By setting B_{tot} , Y , and D as the reach averages quantified from the 5-m estimates, sensitivities of the transient storage parameters were better isolated; Y_{HTS} (~65-75%) now had the largest influence on all three statistical moments (Fig. 4b). With the influence of uncertain hydraulic parameters no longer a consequence, the influences of β and Y_{STS} (collectively ~20-30%) on μ_t , σ_t^2 , and S_t were more evident. However, σ_t^2 and S_t remained insensitive to α_{STS} and α_{HTS} .

When the 5-m estimates of B_{tot} , Y , and D were incorporated into the moment solutions (Table 1), the relative influences (i.e., sensitivities) of transient storage parameters were redundant to the previous analysis using reach-averaged hydraulics (Fig. 4c). This redundancy indicates that the sensitivity of the parameters defining the overall reach scale behavior of transient storage can be isolated by setting hydraulic characteristics to their reach averages. For the conditions of this reach, Y_{HTS} is the most likely identifiable parameter as it had the largest influence on the mean residence time, symmetrical spreading, and late-time tailing of the solute residence time PDF. While μ_t , σ_t^2 , and S_t are additionally sensitive to β and Y_{STS} , the sensitivities of α_{STS} and α_{HTS} remained small and are, therefore, considered not identifiable for the conditions of this reach when all transient storage parameters are considered uncertain.

4. Discussion

Prior work has suggested that more confident transient storage parameter estimation may result from setting hydraulic parameters (e.g., B_{tot} , Y , and D) through measurements (e.g., [14]). Otherwise, transient storage parameters may not be confidently identifiable through inverse

solute tracer techniques due to sensitivities lower than, and interactions with, uncertain hydraulic parameters [5]. When hydraulic parameters were set and no longer considered uncertain, we found that the sensitivities of transient storage parameters were better isolated (Fig. 4). When the influence of transient storage parameters becomes clearer, there is improved understanding of which parameters are potentially identifiable, whether STS or HTS processes are dominant, and whether further reduction in the number of uncertain parameters may still be necessary to more accurately represent processes in model predictions.

Incorporating high-resolution spatial detail to describe hydraulic characteristics did not impact the interpretation of the reach scale transient storage parameter sensitivity (Fig. 4). This result suggests that average hydraulic information is adequate to conduct a sensitivity analysis and reveal the transient storage parameters with the largest influences on the model output. To further test whether this conclusion holds for different hydraulic conditions, we repeated the sensitivity analysis using subsets of the 5-m estimates of width, depth, and the dispersion coefficient that fall within individual 1-km sections of the original study reach. These subsets have reach averages above, below, and similar to the original study reach. The results, although not shown, were nearly identical to those from the sensitivity analysis of the entire study reach shown in Fig. 4. Therefore, the conclusion that a reach-averaged hydraulic representation is sufficient to isolate transient storage parameter sensitivity holds at different spatial scales within this study reach. However, we recommend that the robustness of this conclusion be further tested with similar hydraulic datasets of different stream systems.

Selection of parameter ranges can influence sensitivity results [5] and should be considered when applying these and other sensitivity analysis methods. For example, when the threshold of $E > 0.9$ is changed to $E > 0.95$ to define parameter ranges, the ranges narrow slightly

and the magnitudes of sensitivities change (particularly in the higher-order moments). Despite the influence of parameter ranges on sensitivities, we consistently found that a reach-average representation of hydraulics was sufficient to isolate transient storage parameter sensitivity for this study reach. While establishing feasible parameter ranges is possible with simple approaches (e.g., Latin Hypercube sampling), determining detailed information about parameter distributions can be more difficult. Accordingly, fuzzy numbers provide a versatile way to investigate the impact of uncertain parameters without detailed knowledge of this distribution. Furthermore, the general transformation method used to implement parameter fuzzy numbers and identify relative influences is a robust multivariate approach that is not constrained by traditional assumptions associated with probabilistic approaches [26, 28]. For example, more common variance-based approaches require normally distributed model error (e.g., [11]). A limitation to this transformation method is that the influence of parameter interactions is embedded in the result and not directly exposed.

Although a reach-average understanding of hydraulics may provide transient storage parameter sensitivity, some hydraulic spatial detail may still be necessary to accurately predict solute residence times in model applications. Schmadel et al. [20] found that reach scale predictions of solute residence times were sensitive to the level of spatial detail used to represent the hydraulic characteristics. To achieve convergence in a solute prediction (i.e., where it no longer changes with increasing levels of detail), they found that the natural correlation structure of channel width dictated the level of spatial detail necessary in the model representation. It may also be important to better understand and represent transient storage at even finer spatial scales for certain modeling objectives. For example, predicting ecological implications may require an understanding of transient storage heterogeneity finer than what is necessary to represent

hydraulics (e.g., [14, 29]). Direct measurements of hydraulics (e.g., channel width and depth), however, may still provide a way to scale some transient storage parameters at small feature and reach scales [14, 30]. In a flume study, Aubeneau et al. [31] recently found that velocity fields identified from the interaction between open-channel flow and streambed morphology can be used to scale HTS residence times. In larger scale field settings, while imagery can be useful to estimate high-resolution hydraulic information, it can also be used to estimate some transient storage parameters (e.g., STS widths [12]), further reducing the number requiring calibration. Remote sensing techniques may also be useful for studying relationships between morphologic structure (e.g., sinuosity) and solute residence times at long reach scales.

In any sensitivity analysis, some combinations of parameter values can produce erroneous solutions and potentially bias results. We examined whether the sensitivity results on the moment solutions were potentially biased by omitting parameter combinations that resulted in unrealistic solutions. While we do not present these results, we found negligible differences indicating that the moment solutions effectively expressed parameter sensitivity on the model output. Therefore, we conclude that the fuzzy number sensitivity analysis with moment solutions provides an expedient and simple approach to test the value of spatial information, estimate the influence of setting measurable parameters, and expose parameters with the largest influences on the model output.

While the goal in model applications is to represent important processes, it is well recognized that there is a tradeoff between model complexity and data requirements. At the extremes of complexity (i.e., overly simple to overly complex), important processes are rarely realistically represented. Rather than only reducing model complexity through lumping processes (e.g., a one-zone versus a two-zone representation of transient storage) to enhance parameter

identification, some focus should be directed towards the reduction of the number of uncertain parameters through field-based measurements (e.g., [12, 13]). By using the fuzzy number sensitivity analysis with the moment solutions (Table 1), direct incorporation of such measurements is possible to provide information regarding which measurements may be necessary to isolate sensitivity and, therefore, identify parameters. When hydraulic information was incorporated into the moment solutions, we found that the sensitivities of STS and HTS exchange coefficients were still limited and the model output was mostly sensitive to sizes of the STS and HTS zones (Fig. 3). In particular, the HTS depth had the largest influence on the output. Choi et al. [32] found that for cases where one storage process is dominant, a one-zone modeling approach would suffice to represent transient storage processes. Although transient storage parameter sensitivity for both a one-zone and two-zone transient storage model would be similar in one-zone dominated systems, the individual extent of STS and HTS and identifiability of associated parameters would remain unclear by using a one-zone approach. Only by using a two-zone approach was it clear that the STS and HTS exchange coefficients and sizes of the STS zone may not be identifiable, which can be important information for certain modeling objectives. For example, if heat and reactive solute transport process are of interest, a two-zone representation of transient storage would likely still be necessary (e.g., [15-17]) and other methods would be needed to estimate STS parameters, further reducing the number considered uncertain (e.g., [30]). The next steps to improve predictive capabilities of both heat and reactive solute transport may include approximating the spatial distributions of STS and HTS parameters. Nonetheless, the fuzzy number sensitivity analysis with moment solutions approach in this present work will be useful to estimate the influence of spatial detail regarding STS and HTS parameters.

5. Conclusions

In this paper, we determined how incorporating different levels of information regarding hydraulic characteristics can better isolate the sensitivity of parameters describing transient storage processes. We used a fuzzy number sensitivity analysis with closed form solutions to the statistical moments of a two-zone transient storage model to express parameter sensitivity on the model output. These solutions allowed for direct incorporation of hydraulic spatial information while avoiding the need for extensive model simulations. We found that the sensitivity of transient storage parameters can be better isolated by setting hydraulics through measurements rather than through calibration techniques. Specifically in terms of the sensitivity of parameters describing reach-average transient storage processes, reach-average hydraulics were sufficient where incorporating a detailed spatial representation of hydraulics into the moment solutions provided redundant sensitivity information. However, improved predictive capabilities of solute residence times may still require detailed spatial distributions of transient storage parameters for specific modeling objectives. The fuzzy number sensitivity analysis with moment solutions provides an expedient and simple approach to test the value of spatial information necessary to isolate parameter sensitivity and better understand their potential identifiability.

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Table 1

Closed form solutions to the mean residence time, μ_t (s), variance, σ_t^2 (s²), and third central moment, S_t (s³), of the solute residence time probability density function (taken from Schmadel et al. [20]). The spatial variability factors represent the effects of spatially variable model parameters, $\langle \dots \rangle$ denotes the reach average, and i is the reach segment (i.e., observation) index.

Closed form solution	Spatial variability factor
$\langle \mu_t \rangle = \mu_t(x=0) + \frac{X}{\langle U \rangle} (1 + \langle \Lambda \rangle) \lambda_1$	$\lambda_1 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle}{U_i} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]$
$\langle \sigma_t^2 \rangle = \sigma_t^2(x=0) + 2 \frac{X}{\langle U \rangle} \langle \Theta \rangle \lambda_2 + 2 \frac{X \langle D \rangle}{\langle U \rangle^3} (1 + \langle \Lambda \rangle)^2 \lambda_3 + 2 \frac{X^2 \langle D \rangle}{\langle U \rangle^2} (1 + \langle \Lambda \rangle)^2 \lambda_4$	$\lambda_2 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle}{U_i} \frac{\Theta_i}{\langle \Theta \rangle}$ $\lambda_3 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle^3}{U_i^3} \frac{D_i}{\langle D \rangle} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^2$ $\lambda_4 = \sum_{i=1}^N \frac{\sigma_x^2}{X^2} \frac{\langle U \rangle^2}{U_i^2} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^2$
$\langle S_t \rangle = S_t(x=0) + 6 \frac{X}{\langle U \rangle} \langle \Phi \rangle \lambda_5 + 12 \frac{X \langle D \rangle}{\langle U \rangle^3} \langle \Theta \rangle (1 + \langle \Lambda \rangle) \lambda_6 + 12 \frac{X \langle D \rangle^2}{\langle U \rangle^5} (1 + \langle \Lambda \rangle)^3 \lambda_7 + X^2 \left[\frac{3 \mu_t(x=0) (1 + \langle \Lambda \rangle)^2}{\langle U \rangle^2} \lambda_4 + \frac{6 \langle \Theta \rangle (1 + \langle \Lambda \rangle)}{\langle U \rangle^2} \lambda_8 + \frac{6 (1 + \langle \Lambda \rangle)^3}{\langle U \rangle^4} \lambda_9 \right] + X^3 \frac{6 (1 + \langle \Lambda \rangle)^3}{\langle U \rangle^3} \lambda_{10}$	$\lambda_5 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle}{U_i} \frac{\Phi_i}{\langle \Phi \rangle}$ $\lambda_6 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle^3}{U_i^3} \frac{D_i}{\langle D \rangle} \frac{\Theta_i}{\langle \Theta \rangle} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]$ $\lambda_7 = \sum_{i=1}^N \frac{\langle \Delta x \rangle}{X} \frac{\langle U \rangle^5}{U_i^5} \frac{D_i^2}{\langle D \rangle^2} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^3$ $\lambda_8 = \sum_{i=1}^N \frac{\sigma_x^2}{X^2} \frac{\langle U \rangle^2}{U_i^2} \frac{\Theta_i}{\langle \Theta \rangle} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^2$ $\lambda_9 = \sum_{i=1}^N \frac{\sigma_x^2}{X^2} \frac{\langle U \rangle^4}{U_i^4} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^3$ $\lambda_{10} = \sum_{i=1}^N \frac{\sigma_x^3}{X^3} \frac{\langle U \rangle^3}{U_i^3} \left[\frac{(1 + \Lambda_i)}{(1 + \langle \Lambda \rangle)} \right]^3$

where $\langle \Lambda \rangle = \frac{\langle Y_{STS} \rangle \langle \beta \rangle}{\langle Y \rangle (1 - \langle \beta \rangle)} + \frac{\langle Y_{HTS} \rangle}{\langle Y \rangle}$, $\langle \Theta \rangle = \frac{(\langle B_{tot} \rangle \langle \beta \rangle)^2}{\langle \alpha_{STS} \rangle} \frac{\langle Y_{STS} \rangle \langle \beta \rangle}{\langle Y \rangle (1 - \langle \beta \rangle)} + \frac{\langle Y_{HTS} \rangle^2}{\langle \alpha_{HTS} \rangle} \frac{\langle Y_{HTS} \rangle}{\langle Y \rangle}$,

$\langle \Phi \rangle = \frac{(\langle B_{tot} \rangle \langle \beta \rangle)^4}{\langle \alpha_{STS} \rangle^2} \frac{\langle Y_{STS} \rangle \langle \beta \rangle}{\langle Y \rangle (1 - \langle \beta \rangle)} + \frac{\langle Y_{HTS} \rangle^4}{\langle \alpha_{HTS} \rangle^2} \frac{\langle Y_{HTS} \rangle}{\langle Y \rangle}$, $\sigma_x^2 = \langle \Delta x^2 \rangle - \langle \Delta x \rangle^2$ is the spatial variance,

and $\sigma_x^3 = \langle \Delta x^3 \rangle - \langle \Delta x \rangle^3$ is the spatial skewness.

Table 2

Model parameter descriptions, symbols, and ranges. Feasible ranges of B_{tot} , Y , D , β , Y_{STS} , Y_{HTS} , α_{STS} , and α_{HTS} were established from parameter sets that resulted in a Nash-Sutcliffe Efficiency (E) greater than 0.9 after Neilson et al. [17].

Parameter Description	Symbol	Range ($E > 0.9$)
Total channel wetted width (m)	B_{tot}	11.4-29.8
Mean channel water depth (m)	Y	0.11-0.37
Longitudinal dispersion coefficient ($\text{m}^2 \text{s}^{-1}$)	D	2.7-15.9
STS fraction of B_{tot} (%)	β	5-27
STS mean depth (m)	Y_{STS}	0.02-0.85
HTS mean depth (m)	Y_{HTS}	0.02-0.83
STS exchange rate coefficient ($\text{m}^2 \text{s}^{-1}$)	α_{STS}	1.3×10^{-7} -0.08
HTS exchange rate coefficient ($\text{m}^2 \text{s}^{-1}$)	α_{HTS}	1.1×10^{-7} -0.03

Fig. 1. (a) Example of an uncertain parameter represented by a triangular fuzzy number established with nothing more than a range of values and central tendency assumption. In this example, total channel width (B_{tot}) is the parameter and four levels of membership are selected. The general transformation method was applied to the parameter fuzzy numbers like this one to sample and generate possible parameter combinations. The dotted lines and arrows illustrate how parameter values were sampled. (b) The statistical moment solution (e.g., the first temporal moment (μ_t)) was then evaluated for each combination of the sampled parameter values. The minimum and maximum moment estimates at each membership level were tabulated to construct the fuzzy number representing uncertainty in the moment solution. Using this constructed fuzzy number output and the parameter fuzzy number input, the relative influence of each uncertain parameter to the uncertain moment solution was evaluated. See Hanss [25, 26] for details.

Fig. 2. (a) The 6.47 km study reach of the Virgin River and clipped raster of only water temperature pixels from Bingham et al. [12]. This clipped raster was used to assess (b) total channel width (B_{tot}) estimates every 5 streamwise meters (modified from Schmadel et al. [20]). These B_{tot} estimates were used to estimate the mean depth of water in the channel (Y) and the dispersion coefficient (D) every 5 streamwise meters using hydraulic principles and relationships (see Schmadel et al. [20] for details). Hydraulic characteristics of the study reach are collectively described by estimates of B_{tot} , Y , and D . The corresponding reach averages are shown.

Fig. 3. Observed conservative solute tracer at the downstream reach limit following an instantaneous injection at the upstream reach limit. Model simulations where the Nash-Sutcliffe Efficiency (E) is greater than 0.9 are shown, which provided feasible parameter ranges for the baseline sensitivity analysis.

Fig. 4. (a) Relative influence (i.e., sensitivity) of all model parameters (see Table 2) expressed by the closed form solutions to the mean (μ_t), variance (σ_t^2), and third central moment (S_t) of the solute residence time probability density function (see Table 1). In this baseline case, all model parameters are considered uncertain. (b) Sensitivity of only the uncertain transient storage parameters expressed by the moment solutions after setting hydraulic parameters (i.e., B_{tot} , Y , and D) as reach averages. (c) Sensitivity of only the uncertain transient storage parameters expressed by the moment solutions after incorporating spatially variable hydraulic information (i.e., 5-m estimates of B_{tot} , Y , and D).