Image-guided inversion in steady-state hydraulic tomography

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Abstract. In steady-state hydraulic tomography, the head data recorded during a series of pumping or/and injection tests can be inverted to determine the transmissivity distributions of an aquifer. This inverse problem is usually underdetermined and ill-posed. We propose to use structural information inferred from a guiding image to constrain the inversion process. The guiding image can be drawn from soft data sets such as seismic and ground penetrating radar sections or from geological cross-sections inferred from the wells and some geological expertise. The structural information is extracted from the guiding image through some digital image analysis techniques. Then, it is introduced into the inversion process of the head data as a weighted four direction smoothing matrix used in the regularizer. Such smoothing matrix allows applying the smoothing along the structural features. This helps preserving eventual drops in the hydraulic properties. In addition, we apply a procedure called image-guided interpolation. This technique starts with the tomogram obtained from the image guided inversion and focus this tomogram. These new approaches are applied on four synthetic toy problems. The hydraulic distributions estimated from the image-guided inversion are closer to the true transmissivity model and have higher resolution than those computed from a classical Gauss-Newton method with uniform isotropic smoothing.

Key words: Hydraulic tomography; image-guided-inversion; transmissivity.

1. Introduction

Hydraulic conductivity (or transmissivity) distribution is of course the holy grail of subsurface hydrogeology. Adequate knowledge of this parameter is required to model groundwater flow and the migrations of contaminants in aquifers. Hydraulic conductivity or transmissivity are usually determined by performing pumping tests (Bear, 1988; Vasco et al., 2000; Straface et al., 2007; Li et al 2007; Bohling et al., 2007). These tests consist in pumping water in one well (or one well interval) and monitoring the water level changes in a set of piezometers or well intervals. This analysis gives an averaged and localized estimate of the transmissivity in the vicinity of the wells. This estimate can be however erroneous in the case of heterogeneous aquifers (e.g., Wu et al., 2005; Illman et al., 2012).

Hydraulic tomography has been proposed as a method to investigate the distributions of the hydraulic parameters of heterogeneous aquifers. Hydraulic tomography consists in stimulating an aquifer by performing a sequence of pumping tests or injection/extraction tests in a set of wells, possibly at different depths between packers, and to record the hydraulic head response at the remaining wells. The response is then analyzed in terms of finding the transmissivity distribution on a grid (tomogram) as done typically in geophysics or medical imaging. Over the past two decades, several numerical and field studies have demonstrated how steady-state hydraulic tests can be used to image hydraulic conductivity or transmissivity (e.g., Gottlieb and Dietrich, 1995; Yeh and Liu, 2000; Liu et al., 2002; Cardiff and Barrash, 2011). Transient pumping tests can be also be used to estimate both of the specific storage and hydraulic conductivity fields (e.g., Zhu and Yeh, 2005; Hao et al., 2008; Berg and Illman, 2011a). Hydraulic conductivity tomography can also be combined with the temporal moment method to reconstruct the hydraulic parameters heterogeneities (e.g: Li et al., 2005; Li et al., 2007; Yin and Illman, 2009; Zhu and Yeh, 2006). This method has found a broad number of applications in the recent literature (e.g., Sharmeen et al., 2012; Berg and Illman, 2012). In

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5 75 steady-state conditions, such problem is similar to electrical resistivity tomography in
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7 76 geophysics (see recently Zhou et al., 2014). The inverse problem is generally non-unique and
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9 77 the resolution depends on the number of hydraulic data and prior information.

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12 78 To incorporate additional information in the inverse problem, various geostatistical
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14 79 frameworks have been introduced in order to use any known prior knowledge of the spatial
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16 80 variability of the transmissivity. Two approaches have been extensively used in hydraulic
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18 81 tomography: the quasi-linear geostatistical approach by Kitanidis (1995) and the Sequential
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20 82 Linear Estimator (SLE) by Yeh et al. (1996). Despite the differences in their mathematical
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22 83 formulations, these two approaches are similar in their spirit of using spatial statistics (mean,
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24 84 variance, and correlation length scales) of hydraulic properties as prior information. The
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26 85 quasi-linear geostatistical approach of Kitanidis (1995) is formally equivalent to the Tikhonov
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28 86 regularization family of methods. These approaches are also used in the realm of geophysical
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30 87 inversion with a generalized covariance functions included in a regularization term added to
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32 88 the model misfit term in the objective function to minimize. This approach is based on the
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34 89 linearization of the functional relationship between parameters (hydraulic conductivities) and
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36 90 measured quantities (hydraulic head) and it comprises a step of estimating the geostatistical
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38 91 parameters, such as the variance and correlation lengths, used for the prior covariance
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40 92 function (Michalak and Kitanidis, 2004; Cardiff et al., 2012; Nowak and Cirpka, 2006).

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46 93 The Sequential Linear Estimator (SLE) approach of Yeh (1996) treats the model
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48 94 parameter as a stochastic processes characterized by their spatial statistics (mean, variance,
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50 95 and correlation lengths). Afterwards, the SLE uses kriging to obtain the most likely
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52 96 parameters values given the data. It then updates (i) the estimated model parameters using the
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54 97 differences between the observed heads and (ii) the covariances to obtain residual
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56 98 covariances. In addition, the SLE goes beyond cokriging because it assumes a non-linear
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58 99 relationship between parameters fields and hydraulic data (Yeh et al., 1996). The SLE
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approach has proven to be an efficient tool in hydraulic tomography. Its success has been demonstrated in saturated flow synthetic tests (Yeh and Liu, 2000; Zhu and Yeh, 2005; Yeh and Zhu, 2007; Xiang et al., 2009; Sun et al., 2013), in variably-saturated flow synthetic tests (Mao et al, 2013), in pneumatic tomography (Ni and Yeh, 2009), in sandbox experiments (Liu et al., 2002; Liu et al., 2007; Illman et al., 2010, Berg and Illman, 2011a), and in field experiments (Straface et al., 2007; Illman et al., 2009; Huang et al., 2011; Berg and Illman, 2011b; Berg and Illman, 2015).

Another approach is the pilot point method, which is applied as a parameterization technique for reducing the number of unknown parameters to estimate by evaluating the model parameters only at few key-locations only in the aquifer (called pilot points) where the values of the transmissivity can be assessed to fit the hydraulic head measurements via deterministic or stochastic optimization processes (de Marsily et al., 1984; LaVenue et al., 1995; Lavenue and De Marsily, 2001; Vesselinov et al 2001a-b). The full spatial field is obtained by geostatistical interpolation (kriging) of the values of the transmissivity at the pilot points by assuming a cross-covariance function. However, one limitation of the pilot point method is due to the fact that the covariance function used for the interpolation of the model parameter values from those obtained at the pilot points is fixed (Huang et al., 2011). This means that the pilot point method does not optimally exploit the information provided by the hydraulic head data in the sense that it does not fully take advantage of the changes in the flow field due to changes of the pumping locations.

The previous approaches provide some smoothness in the estimated hydrological parameters because they estimate the conditional means of such parameters. However, these smooth parameters successfully predict and reproduce the best unbiased mean data. These approaches are all adequate to characterize smoothly heterogeneous aquifers. However the distribution of the hydraulic parameters in the subsurface is not necessarily smooth. The

subsurface is generally characterized by a set of facies, each of them with their degree of heterogeneities. Inside each facies, the distribution of the material properties may be described with classical geostatistical indicators such as semi-variograms. That said, they may exist drops in the distribution of the transmissivity at the boundaries between the facies. Note that under some particular conditions (especially fractured media), one can imagine that the drop in the transmissivity is not necessarily determined by the geology alone and tectonic aspects need to be accounted for as well. Several approaches have been suggested to overcome the existence of discontinuities in the distribution of the hydraulic properties. Hu and Chugunova (2008) provided for instance a review of the multiple-point geostatistical approach. They studied the efficiency of using multiple-point simulations on multiple grids to recover the patterns observed in training images. Jafarpour et al. (2009) developed a new regularization approach, which has proven to be efficient for the reconstruction of geologic features, facies, and porosity distributions. Cardiff et al. (2009) used the level set approach to identify the shape and location of the boundaries between separate hydrofacies in some simple geometries.

The strategy we follow in this paper is to use additional information such as those provided by seismic or GPR cross-sections, or geological expertise in the hydraulic tomography problem. At the opposite of the methods derived by Brauchler et al. (2011, 2012), our approach does not require a statistical relationship between seismic / electromagnetic properties (e.g., P-wave velocity) and hydraulic properties. It only requires that GPR or seismic reflectors are associated with hydraulic conductivity drops that are not necessarily correlated in magnitude but are collocated with hydraulic conductivity or transmissivity drops. To take advantage of this collocation, we use an approach derived by Hale (2009a), who introduced two new techniques called image-guided inversion and interpolation methods. The first approach is to extract and use structural features from an image to guide the

inversion of geophysical data. The second approach is similar to focusing techniques in image processing starting with the fundamental question: how can we refocus a blurred image? There are several new ideas behind these methods. First, in contrast to classical interpolation methods requiring the definition of Euclidian distances from locations where data are unknown to locations where data are known, the image-guided interpolation uses orientations and shapes of features in images to predict unknown data values. This approach has proven to be very useful in geophysical tomography to obtain inverted models that honor geological structures and give more accurate results in terms of the value of the material properties. For instance, Ma et al. (2012) used the image-guided interpolation in waveform inversion of seismic data using as guiding image a migrated seismic section. They reduced the number of model parameters by discretizing the space parameters in a sparser space on which they apply the image-guided interpolation. Zhou et al. (2014) used the image-guided inversion to invert apparent resistivity data using guiding images derived from georadar (GPR) or geological cross-sections. In addition, they applied the image-guided interpolation on their inverted model in order to focus smooth resistivity tomograms using the structural constraints.

As mentioned above, the steady-state transmissivity inverse problem is mathematically similar to the electrical resistivity inverse problem. The main difference is that usually much more data are available in electrical resistivity inversion because of the easiness in collecting resistivity data. In this paper, we adapt the methodology used recently in Zhou et al. (2014) in order to perform image-guided inversion for transmissivity tomography. As far as we know, such method has never been used in hydrogeology. We solve the hydraulic inverse problem for four synthetic 2D-case studies in steady state conditions with the main goal to introduce these new techniques to simple case studies. The structural constraints will be derived from an image corresponding to the hydraulic pattern (assumed to be perfectly known in this paper for Case Studies 1, 2, and 4) and imposed via a four direction smoothing matrix. Finally, we

will study the effect of incorporating erroneous hydraulic pattern on the quality of the inverted model (Case Study 3).

2. Methodology

2.1. Forward Problem

The ground water flow in a saturated, heterogeneous, but locally isotropic confined aquifer under steady state regime is governed by the following partial differential equation

$$-\nabla \cdot (T_h \nabla h) = Q, \quad (1)$$

where T_h denotes the transmissivity (in $\text{m}^2 \text{s}^{-1}$), h denotes the hydraulic head (in m) and Q (in $\text{m}^3 \text{s}^{-1}$) is the pumping rate. Equation (1) is subject to the following boundary conditions

$$h = h_D \text{ on } \Gamma_D, \quad (2)$$

where h_D is the prescribed head at Dirichlet boundary Γ_D . Note that other boundary conditions can be eventually employed depending on field conditions.

To minimize the effects of the boundary conditions in our simulations, the modeling domain is encircled with a large buffering domain along which we impose a homogenous Dirichlet boundary condition. The forward problem is solved with the finite elements method using the commercial software Comsol Multiphysics 3.5a (Comsol Multiphysics User's Guide, 2008). The unknown transmissivity field is discretized into 900 cells for all the four test cases and a transmissivity value is assigned to each cell. We generate the hydraulic signal by simulating a series of dipole tests, i.e., we pump water in a well, we retrieve the water at the same rate in another well, and several permutations of dipoles are performed for each test.

2.2. Inverse Problem

The estimation of the transmissivity distributions can be formulated as a least-square optimization problem minimizing the following objective function L:

$$L = [\mathbf{d} - F(\tilde{s})]^T \mathbf{R}^{-1} [\mathbf{d} - F(\tilde{s})] + \lambda (\tilde{s} - \tilde{s}_0)^T \mathbf{C}_m (\tilde{s} - \tilde{s}_0), \quad (3)$$

where $\mathbf{d}$ denotes the vector of measured hydraulic data, $F(\cdot)$ is the forward problem operator representing the predicted data, $\mathbf{R}$ is the measurements error covariance matrix, $\tilde{s} = \log_{10}(T)$ is the decimal logarithm of the transmissivity field, λ is the regularization parameter, s_0 is the prior transmissivity model, and $\mathbf{C}_m$ is the smoothing matrix.

In Equation (3), the objective function is the sum of two terms. The first term represents the data misfit while the second term represents the model constraints. In the classical Tikhonov regularization, the matrix $\mathbf{C}_m$ can correspond to the Laplacian (smoothness) operator and be the same everywhere in the tomogram (uniform isotropic or anisotropic smoothing). In image-guided inversion, this matrix allows the inversion to be “structure oriented” and is therefore used to provide inverted models, which respect the structural information existing in an independent image.

The regularization parameter λ in equation (3) ensures the balance between the two types of information. Several methods exist for the choice of the regularization parameter (e.g., Craven and Wahba, 1979; Golub and Von Matt, 1997; Hansen, 1992). We use the L-curve method among a range of methods such as the cross-validation approach. The L-curve method has proven to be an efficient method for computing the regularization parameter and is known for its robustness. It may fail in two cases (Hansen, 1999): (1) When the true model

is very smooth and (2) when we are dealing with a very large number of measurements. In this second case, the asymptotic behavior of the regularization parameter using the L-curve method can be large resulting in an over-regularization of the inverse problem. Neither one of these cases correspond to the numerical test cases studied in our paper. So we believe that L-curve method is suitable to our case studies.

The smoothing matrix can be computed from images inferred from several data sets such as geological cross-sections, seismic or GPR migrated cross-sections (known to contain a lot of structural information). In this study, the smoothing matrix is calculated using guiding images that are obtained from a fictitious geological cross-section representing the hydraulic pattern. The hydrofacies and the structure are assumed prior to the inverse modeling of the hydraulic head data. If the image used to guide the inversion contains some uncertainties, we can simulate several realizations of this image and use them to see the effect of the uncertainty of the guiding image with respect to the final hydraulic tomogram. This aspect will be treated in a separate contribution since it would encapsulate the present approach in a Bayesian theory to look at the effect of the uncertainty existing in the guiding image itself.

In hydraulic tomography, the inverse problem is non-linear. Therefore the minimization of the objective function defined in equation (3) requires the use of a gradient-based method, such as the Gauss-Newton approach. This method solves the vector of model parameters (log of the transmissivity for each cell) by iteratively updating iteratively this vector according to

$$\tilde{\mathbf{s}}_{i+1} = \tilde{\mathbf{s}}_i + \delta\tilde{\mathbf{s}} \quad (4)$$

where $\delta\tilde{\mathbf{s}}$ is defined as

$$\delta\tilde{\mathbf{s}} = (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \lambda \mathbf{C}_m)^{-1} \mathbf{H}^T \mathbf{R}^{-1} [\mathbf{F}(\tilde{\mathbf{s}}_i) - \mathbf{d}], \quad (5)$$

and where $\mathbf{H}$ denotes the sensitivity matrix.

Several studies have been conducted to derive the adjoint equations for the transmissivity inverse problem (Sun and Yeh, 1990; Zhu and Yeh, 2005; Cardiff and Kitanidis, 2008; Hsien-tsung et al., 2010). The adjoint state equation related to Equation (3) is

$$-\nabla \cdot (\mathbf{T}_h \nabla \phi) = \delta(\mathbf{x} - \mathbf{x}_m) \quad (6)$$

where δ denotes the delta function, $\mathbf{x}$ is the spatial distribution, and $\mathbf{x}_m$ is the measurement point spatial location, and ϕ denotes the adjoint state variable subject to the following boundary conditions

$$\phi = 0 \text{ on } \Gamma_d. \quad (7)$$

The sensitivity of the hydraulic head is given as

$$\frac{\partial h_p}{\partial \tilde{s}_k} = -\ln(10) \int_{\Omega_k} 10^{-\tilde{s}} \nabla h \nabla \phi d\Omega_k. \quad (8)$$

The subscript k denotes the position of the unknown parameters $\tilde{s}_k$ in the domain Ω . The subscript p denotes the location of the hydraulic head observation.

As in Farquharson (2008), we use a four direction smoothing matrix defined as

$$\mathbf{C}_m = \mathbf{C}_x^T \mathbf{C}_x + \mathbf{C}_y^T \mathbf{C}_y + \mathbf{C}_{d_1}^T \mathbf{C}_{d_1} + \mathbf{C}_{d_2}^T \mathbf{C}_{d_2}, \quad (9)$$

where $\mathbf{C}_x, \mathbf{C}_y, \mathbf{C}_{d_1}, \mathbf{C}_{d_2}$ denote the first or second order derivatives smoothing matrices in the x -direction, y -direction, d_1 -direction and d_2 -direction (d_1 and d_2 are the diagonal directions described in Figure 1). The smoothing matrix of equation (3), $\mathbf{C}_m$, compares each transmissivity value in a cell to its four neighbors in the x -direction, y -direction, d_1 -direction, and d_2 -direction.

In order to improve the level of structural constraints, we have to smooth in each of the four directions, depending on the orientations of the structures. To control the amount of the smoothing in each direction, we use four structural weighting diagonal matrices $\mathbf{W}_x, \mathbf{W}_y, \mathbf{W}_{d_1}, \mathbf{W}_{d_2}$, respectively assigned to aforementioned smoothing matrices. Using this approach, we can apply different weights factors to each model cell (Günther et al., 2006; Bouchedda et al., 2012). Thus, by introducing the weighting matrices, the smoothing matrix $\mathbf{C}_m$ entering in the regularizer of equation (3) becomes

$$\mathbf{C}_m = \mathbf{C}_x^T \mathbf{W}_x^T \mathbf{W}_x \mathbf{C}_x + \mathbf{C}_y^T \mathbf{W}_y^T \mathbf{W}_y \mathbf{C}_y + \mathbf{C}_{d_1}^T \mathbf{W}_{d_1}^T \mathbf{W}_{d_1} \mathbf{C}_{d_1} + \mathbf{C}_{d_2}^T \mathbf{W}_{d_2}^T \mathbf{W}_{d_2} \mathbf{C}_{d_2}. \quad (10)$$

Thanks to Equation (10), we can obtain models that are preferentially smooth in any of the x -direction, y -direction, d_1 -direction, and d_2 -direction depending on the orientation of the structural information provided by the guiding image. Small weights should be assigned to the edges, while larger weights should be assigned along the structures corresponding to the predominant directions. For example, in Figure 1 a large value should be assigned to the $\mathbf{W}_{d_2}$ matrix element and $\mathbf{W}_{d_1}$ matrix element corresponding to cell 8 and cell 11, respectively. This approach allows applying the smoothing along the structural features represented in the guiding image and then providing models with structural information that are consistent with known guiding images. The guiding image can come from any image bearing such structural information (see Pugnin et al., 2013 and Bradford and Babcock, 2013 for some recent examples). In Figure 2, we show the structure tensor (defined below) for a GPR cross-section exhibiting a lot of textures that can be to define the model covariance matrix.

2.3. Image-guided inversion and interpolation

The purpose of this section is to present the key concepts involved in the image-guided inversion and interpolation methodologies, the approach being summarized in the flowchart shown in Figure 3.

2.3.1 Structure tensor field

The structure tensor draws a map that describes the orientations of the structures of the image at each pixel. It provides the orientations of coherent structures by describing the directions of the gradients of the grey scale intensity in the vicinity of each pixel. It also gives the coherence of these directions. The structure tensor is evaluated at every pixel and is defined as the smooth outer product of the local gradients image (Van Vliet and Verbeek, 1995; Weickert, 1997; Fehmers and Höcker, 2003). In our case, the guiding image will be given in terms of shades of grey (see for instance in Figure 2). In such images, each pixel has an intensity distribution $\mathfrak{I}(x, y)$ varying from 0 to 1 (x and y denote the position on the image in the x - and y -directions, respectively, $\mathfrak{I}(x, y)=0$ is white and $\mathfrak{I}(x, y)=1$ is black). The gradient of an image $\nabla \mathfrak{I}$ indicates any change in the intensity of the image. The magnitude of the gradient tells us how quickly the image is changing while the direction of the gradient tells us on which direction our image is changing the most quickly. The gradient is computed using its first partial derivatives in the x - and y -directions of the intensity:

$$\nabla \mathfrak{I}(x, y) = (\mathfrak{I}_x, \mathfrak{I}_y) = \left(\frac{\partial \mathfrak{I}}{\partial x}, \frac{\partial \mathfrak{I}}{\partial y} \right). \quad (11)$$

In our case we use the discrete formulation of the gradient. In this case, the intensity $\mathfrak{I}$ of the image is discretized on the pixels of the image. We denote the discrete intensity as $\mathfrak{I}[p]$ where p is a given pixel of the image. The discrete x and y components of the gradient at the pixel p are denoted $\mathfrak{I}_x[p]$ and $\mathfrak{I}_y[p]$.

As in Deriche (1993), Hale (2006; 2009a), and Van Vliet et al. (1998), we used the Gaussian derivatives to approximate the image gradients and the Gaussian filters to smooth the product of those components. In 2D, the structure tensor $\mathbf{T}$ in each pixel is a 2×2 positive semi-definite symmetric matrix given by

$$\mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix}. \quad (12)$$

where

$$T_{xx} = \sum_r w[r] (\mathfrak{I}_x[p-r])^2, \quad (13)$$

$$T_{yy} = \sum_r w[r] (\mathfrak{I}_y[p-r])^2, \quad (14)$$

$$T_{xy} = T_{yx} = \sum_r w[r] \mathfrak{I}_x[p-r] \mathfrak{I}_y[p-r], \quad (15)$$

where the index r is ranging over a set of index pairs (for example a window of size $n \times [-n, n] \times [-n, n]$ centered on the pixel p) and $w[r]$ is a window weight depending on r . The decomposition of $\mathbf{T}$ in its eigenvalues and eigenvectors is given by

$$\mathbf{T} = \lambda_u \mathbf{u} \mathbf{u}^T + \lambda_v \mathbf{v} \mathbf{v}^T \quad (16)$$

where λ_u and λ_v denote the eigenvalues and $\mathbf{u}$ and $\mathbf{v}$ denote the corresponding eigenvectors of the tensor $\mathbf{T}$. Assuming $\lambda_u \geq \lambda_v \geq 0$, the eigenvectors $\mathbf{u}$ indicate the directions corresponding to the highest image gradients, i.e., perpendicular to the linear features of the image. Then, the eigenvectors $\mathbf{v}$ show the directions corresponding to the lowest gradients, indicating the directions parallel to the linear features of the image. In the next section, we use the orientations of the image which are given by the structure tensor field in order to perform the structure-oriented smoothing.

2.3.2. Structure oriented smoothing

Another idea used below is that the transmissivity tomogram is usually a smooth version of the real transmissivity field. One may wish to focus the tomogram like we focus a blurred image. The structure oriented smoothing allows one to apply the smoothing along the features instead of applying it across them. Such type of smoothing is performed using the eigenvectors of the structure tensor field determined in section 2.3.1. Weickert (1997) and Fehmers and Höcker (2003) proposed to parameterize a structure-oriented smoothing filter by solving an anisotropic diffusion equation defined as

$$\partial g(\mathbf{x}, \tau) / \partial \tau = \nabla \cdot [\mathbf{D}(\mathbf{x}) \nabla g(\mathbf{x}, \tau)], \quad (17)$$

subject to the following initial conditions

$$g(\mathbf{x}, \tau=0) = f(\mathbf{x}), \quad (18)$$

where g denotes the output smoothed image, $\mathbf{D}$ is the diffusion tensor (also called metric tensor), f is the initial input image to be smoothed, $\mathbf{x}$ represents the space coordinates and τ is the time. Note that for any time $\tau > 0$, the output image g is always a smoothed version of the initial image f . The diffusion tensor $\mathbf{D}$ that appears in Equation (17) is an anisotropic metric tensor sharing the eigenvectors of the structure tensor $\mathbf{T}$. As stated by Hale (2009a, b), the diffusion tensor field $\mathbf{D}$ is formulated as

$$\mathbf{D} = (\min \lambda_v / \lambda_u)^{p_0} \left[(\lambda_v / \lambda_u)^{p_1} \mathbf{u}\mathbf{u}^T + \mathbf{v}\mathbf{v}^T \right], \quad (19)$$

where p_0 and p_1 are constants that respectively balance the amplitudes of the eigenvalues and accentuate the effect of linearity. This is this tensor that is shown in Figure 2 for 6 pixels. Solving the time-dependent anisotropic diffusion equation (17) can be very time-consuming.

An alternative to this, was proposed by Hale (2009a) who replaced equation (13) by the following quasi-static partial differential equation

$$g(x) - \alpha \nabla \cdot [D(x) \nabla g(x)] = f(x), \quad (20)$$

where α denotes the constant filter parameter. In the case $\alpha=0$, no smoothing is applied and the input image f remains unchanged. Hale (2009a) developed several numerical schemes to efficiently solve equation (20).

2.3.3. Structure oriented semblance

We introduce now the structure oriented semblance used to extract the discontinuities and the coherences of the guiding image. Usually, the semblance is computed using equal smoothing filters with the same weights; this is referred to as conventional semblance. Hale (2009a) introduced the concept of weighted semblance which consists in assigning different weights to the smoothing filters. The weighted semblance is formulated as

$$s[i] = \frac{\left(\sum_j h[i;j] f[j] \right)^2}{\sum_j h[i;j] (f[j])^2} \quad (21)$$

where f is the input image, $h[i;j]$ the smoothing-filters coefficients, j is the index of image pixels and i denotes the shift. In the frame of simple smoothing, we apply the smoothing along the 1st and 2nd directions of the image. For structure-oriented semblance, the smoothing along the image directions is replaced by the structure-oriented smoothing. Depending on the order of smoothing, we use the following structure oriented semblances:

$$s_1 = s_{v,u} = \frac{\langle \langle f \rangle_v^2 \rangle_u}{\langle \langle f^2 \rangle_v \rangle_u} \quad (22)$$

$$s_2 = s_{u,v} = \frac{\langle \langle f \rangle_u^2 \rangle_v}{\langle \langle f^2 \rangle_u \rangle_v} \quad (23)$$

where $\langle \cdot \rangle_u$ and $\langle \cdot \rangle_v$ denotes the smooth along eigenvectors u and v , respectively. Note that $\langle \cdot \rangle_u$ performs the smoothing across image features while $\langle \cdot \rangle_v$ performs the smoothing along those features. For both equations (22) and (23), the smoothing is performed at two levels. The first smoothing is referred to as an inner smoothing while the second is referred to as an outer smoothing operator. For s_1 , we first smooth along the orientations of image features ($\langle \cdot \rangle_v$) then across the orientations of those features ($\langle \cdot \rangle_u$) and vice versa for s_2 . The role of the outer smoothing operator is to stabilize the semblance values when the inner smoothing outputs are too small. Depending on the values of the semblances s_1 and s_2 , we can identify the nature of the image structures. For instance, small values of s_2 indicate the presence of a discontinuity in the direction across the image features while small values of s_1 show discontinuity in the direction along the image features.

In order to perform the image-guided inversion, we discretize the domain of simulation into a regular or an irregular grid. To each cell of the grid, we assign a transmissivity value which will be optimized during the inversion process. After mapping the locations of the grid cells into the indices of the pixels of the guiding image, each cell is associated to a set of pixels. Each cell inherits the structural properties of the corresponding group of pixels. In other words, if a group of pixels contains some discontinuities or coherences, the associated cell should contain the same features. We consider that the orientation of features is unvarying within a cell and is only evaluated at the center of the cell.

392 To determine the orientation of the features in a cell, we calculate at the center of each cell,
 393 the slope of the eigenvectors v defined as v_1/v_2 and depending on the angle of the slope, we
 394 establish whether the cell is coherent or discontinuous in each of the x -direction, xy -direction,
 395 d_1 -direction, or d_2 -direction.

396 As mentioned before, the semblance can be used as an indicator on the nature of the
 397 structural features. Therefore, we compute the semblance for a group of pixels. Based on the
 398 values of the semblance we identify the structures i.e. we fix a threshold and when the values
 399 of s_2 are smaller than the threshold the structure is considered to be a discontinuity and the
 400 locations where s_1 is greater than another threshold correspond to coherences. The choice of
 401 the threshold is important. If we have too low or high values of the s_1 threshold, some bias
 402 appear along the structure features due to the undersmoothing or oversmoothing, while too
 403 low s_2 threshold will diminish the resolution of the inverted model by ignoring some edges. In
 404 this paper, we fixed the thresholds values as 0.9 for s_2 semblance and 0.9934 for s_1
 405 semblance.

406 **2.3.4. The weighting matrices**

407 The aim of the weighting matrices is to add different weighting factors to the smoothing
 408 directions. Those weighting factors represent the structural constraints. Assigning the weights
 409 to the smoothing matrix is done using the tensor field $\mathbf{D}$ and the semblance s . First of all,
 410 during the inversion process, the simulation domain is divided into a grid of cells. Each of
 411 these cells is associated with a transmissivity value. We then map the locations of these cells
 412 into the indices of the pixels of the guiding image. The tensor field and the semblance are
 413 computed for all the pixels of the guiding image. The values of the semblance help identifying
 414 the nature of the features. The locations on the guiding image where the s_2 semblance values

are smaller than an empirically chosen threshold are defined as edges while the locations on the guiding image where the s_1 semblance values are greater than a certain threshold are defined as coherences. Once all the features of the guiding image have been identified, we assign values of the weighting matrices in order to apply the smooth along the features. When dealing with edges, small weights are assigned to the three directions not parallel to the feature orientation while large weights are assigned to the feature direction in the case of coherences. One way to do this is to assign default large values for all coherent cells and small values by default for edges. An example of such default values can be 20 for weighting matrix entries corresponding to coherence and 0.05 weighting matrix entries corresponding to edges. All the remaining entries of the weighting matrices will be equal to one.

2.3.5. Image-guided interpolation

We first point out that the image-guided interpolation is not an inherent part of the image-guided inversion process. The image-guided interpolation is an optional complementary step, which is used to increase the resolution of the best transmissivity model estimated from the image-guided inversion (see Figure 3). Basically, the image-guided interpolation seeks to increase the resolution of the inverted transmissivity model by solving a diffusion-like equation and "back-diffusing" the values of known samples to the whole domain under the guidance of the tensor field $\mathbf{D}(\mathbf{x})$.

Generally in geosciences, the model parameters of an inverse problem should be discretized into a large number of unknowns in order to obtain good resolution for the inverted models. This requires a huge computational effort. To obtain transmissivity models of higher resolution honoring the structures, using a reasonable number of unknown parameters, we interpolate the final sparse transmissivity model resulting from the image-

guided inversion into a higher resolution model covering all the pixels of the guiding image.

To perform such interpolation, we use the so-called “image-guided blended neighbor interpolation” approach (Hale, 2009a). This interpolation is guided by an image that describes the structural features of the study domain (the same used to guide the inversion). The input of the image-guided interpolation is a set of scattered data consisting of m samples (number of known data values)

$$F = \{f_1, f_2, \dots, f_m\}, \quad (24)$$

with $f_k \in \mathbb{R}$, $\forall k \in \{1, 2, \dots, m\}$. The set of known data F is associated with a set of m sample points

$$X = \{x_1, x_2, \dots, x_m\}, \quad (25)$$

where $x_k \in \mathbb{R}^n$, n is the space dimension (in this work $n = 2$).

In our case, f_k represent the inverted transmissivity values and x_k are the coordinates of the cells of these values. The aim of the interpolation is to construct a function $q(x)$ such as

$$\begin{cases} q: \mathbb{R}^n \rightarrow \mathbb{R} \\ q(x_k) = f(x_k) = f_k \end{cases}, \quad (26)$$

Here x refer to the coordinates of all the points, both known and unknown. The function q has two properties: (i) the known values f_k remain unchanged while applying q ; (ii) the interpolated model obtained with q has higher resolution than the initial model F . In the image-guided interpolation frame, the nearest neighbor of a point is evaluated not in terms of the minimum Euclidian distance but it corresponds to the paths of the minimum time. The image-guided blended neighbor interpolation process can be summarized as following:

First, we solve the Eikonal equation

$$\begin{cases} \nabla t(x) \cdot \mathbf{D}(x) \cdot \nabla t(x) = 1, & x \notin X \\ t(x) = 0, & x \in X \end{cases} \quad (27)$$

where $\mathbf{D}(x)$ denotes the diffusion (metric) tensor field derived from the guiding image.

Solving equation (27) gives both the minimum travel time from a point x to its nearest

known neighbor x_k , $t(x)$ and the nearest neighbor interpolant of this point $p(x)$ which has

the value f_k (corresponding to x_k). The metric tensor field $\mathbf{D}(x)$ derived from the guiding

image allows computing the nearest neighbor interpolant $p(x)$.

The second step consists in performing the blended neighbor interpolation by solving

the following smoothing diffusion equation.

$$q(x) - \frac{1}{e} \nabla \cdot t^2(x) \mathbf{D}(x) \nabla q(x) = p(x), \quad (28)$$

where $e \geq 4$ is a constant controlling the smoothing (the larger e the smoother the results).

The solution of this equation is the interpolated model $q(x)$. This model respects the

structures and has a higher resolution. Furthermore it does not modify the structure of the

original inverted model, i.e., $(f_k(q(x_k)) = f_k)$.

The image guided inversion/interpolation process can be divided in three main steps

(Figure 3): (1) Step 1 (initialization): We derive the structural information from the guiding

image and store it in the final smoothing matrix $\mathbf{C}_m$. (2) Step 2 (image-guided inversion): we

optimize the transmissivity field using the smoothing matrix $\mathbf{C}_m$ as a constraint term. This

step is an iterative process, which stops once the convergence criterion in terms of data misfit

is met. (3) Step 3 (image-guided interpolation): the aim of this optional step is to increase the

resolution of the transmissivity tomogram obtained from step 2 by interpolating in the whole domain under the guidance of the tensor field $D(x)$.

3. Numerical Case Studies

In this section, we explore the efficiency of the image-guided inversion for estimating the transmissivity field from the hydraulic head data of a shallow unconfined aquifer. Four synthetic cases are studied. In order to demonstrate the advantage of our guided inversion approach, we compare its result to the conventional Gauss-Newton method (using simple isotropic smoothing along the x - and y -directions) for each of the three test cases. For the first two cases, we suppose that the image used to guide the inversion and interpolation exactly reflects the hydraulic pattern. In the third test case, we use both exact and erroneous guiding images that do not totally reflect the hydraulic pattern in order to see how this error can affect the resolution of the inverted model. In all the test cases, the transmissivity field is a binary heterogeneous distribution and the water head measurements are collected by performing a series of pumping/injection tests in a set of wells intersecting the aquifer. The information concerning the three cases is summarized in Table 1. In addition, a fourth case study is investigated using a vertical distribution of transmissivity for a set of layers displaced by a normal fault.

The results of the tomographies for the four case studies are shown in Figures 4 to 12. The regularization parameters computed using the L-curve approach are shown in Figure 13. Figure 14 show the convergence for the case studies. Finally, Figure 15 shows a comparison between the true head data and the head data computed from the final solution. We now come back on these results case by case.

3.1. Case Study 1

This is the simplest case study in which we just assume that the simulation domain consists of a central square surrounded by a larger domain. Both entities have different transmissivity values (Figure 4a). We simulate four dipole tests. The results of the inversion are shown in Figure 4b (inverted model obtained with the conventional Gauss-Newton method) and Figure 4c (inverted model obtained from the image-guided structure-constrained inversion method). We see that the Gauss Newton method gives a smoother model but it correctly reproduces the position of the heterogeneity. This is due to the fact that the transmissivity field is not complex and a sufficient number of piezometers are available. The image-guided inversion also provides results that are better than the Gauss Newton inverted results in the sense that the amplitudes of the true model are better respected and no smoothing effect is visible at the interfaces of the two medium. The plot of the true hydraulic heads data against the simulated head data shows an excellent match (see Figure 15a). Figure 14b shows the decrease of the objective function with the number of iterations. The image used to guide the inversion, its semblance and diffusion tensor field $\mathbf{D}$ are shown in Figure 5. Figure 5a shows the guiding image in grey scales. This image is used to provide the structure used to smooth in different directions. In Figure 5b, we see that the semblance s_2 clearly identifies the structures and s_2 values smaller than 0.5 correspond to its edges. The tensor field is illustrated with yellow ellipses that are horizontal and vertical along the edges in Figure 4c. Actually, the long and thin ellipses reflect the more coherent and linear structures, which are associated to the large eigenvalues of the vector $\mathbf{v}$ of the diffusion tensor $\mathbf{D}$. We note that this is surprising to see the image-guided inversion performing better than the Gauss-Newton inversion because more information is provided to the inversion in the second

case. That said, it is interesting to see how the image-guided inversion nearly allows to recover the real T_h -field.

3.2. Case Study 2

The goal of this test is to underscore the weakness of the Gauss–Newton method when the true model is slightly more complex and there is lack of data to characterize its full complexity. The true model used in this simulation is shown in Figure 6a and the inverted results in Figures 6b and 6c. The guiding image (in grey scales) is illustrated in Figure 7a. The small values of the semblance (Figure 7b) clearly show the contours of the hydraulic pattern. The tensor field map (Figure 7c) shows the ellipses that are going along the linear features of the guiding image. We note that the transmissivity field reconstructed from the image-guided inversion (Figure 6c) has values that are better recovered and the artifacts that appear on the Gauss-Newton inverted model (Figure 6b) are absent. The simulated data obtained from the image guided inversion are in total accordance with the true hydraulic head data as shown in Figure 15b while Figure 14b highlights the fact that the RMS error in the image guided inversion is decreasing much faster than for the Gauss-Newton one.

3.3. Case Study 3

In this case, we used a slightly more complicated transmissivity distribution (see Figure 8a). Such type of models is similar to the transmissivity of river estuaries in which paleochannels can be observed. In this case, electrical resistivity can be used to map the boundaries between these paleochannels and the surrounding sediments (e.g., Revil et al., 2005). We added a Gaussian noise corresponding to 2 percent of the hydraulic data. Seven dipole tests are performed to stimulate the synthetic aquifer and identify the most sensible

region of this aquifer. The guiding image used to perform the inversion is shown on Figure 9a, it approximates the stair shape corners of the true model with straight lines. This approximation is quite realistic since it is very hard to initially know the exact geometry of the hydraulic pattern.

The semblance and tensor field are shown in Figure 9b and Figure 9c, respectively. The semblance values smaller than 0.65 correspond to the edges; this is in accordance with the ellipses of the tensor field which are longer and thinner along the linear features of the image. Once the image-guided structure-constrained inversion has been performed, we obtain a 30×30 transmissivity model. This sparse model is represented in Figure 8d. The inverted model obtained after applying the image-guided interpolation is shown in Figure 8c. The heterogeneities are apparent in the estimated model. Although we observe some bias in the vicinity of the corners due to the mismatch between the guiding image and the true model, the image-guided inversion results are excellent since it correctly approximate the shape of the hydraulic pattern as well as the magnitudes of the transmissivity field. The comparison between the true data and the computed data obtained from the image-guided inversion shows a perfect fit between the data (see Figure 15). The convergence criteria are reached after seven iteration for both the image-guided inversion and the Gauss-Newton algorithm but the objective function of the image-guided inversion decreases much faster than the Gauss-newton one. The exact values taken by the objectives functions are reported in Table 2.

The Gauss-Newton inversion result shown in Figure 8b has lower resolution and does not correctly reflect the structures. In addition, it oversmooths the transmissivity distribution. This brings out the rationale for using the four smoothing direction to assess the transmissivity field instead of the classical $x - y$ smoothing. Nevertheless, if a higher density of piezometers was used, the Gauss Newton method is expected to give better results.

However, since the drilling cost may be high, in practice we may rely on a small number of wells. It is for this reason that we limited the number of wells in our simulation.

So far, in all the test cases studied above, the guiding images exactly reflected the structures or only slightly mismatched a part of those features. Because it is quite common to not have any prior knowledge or just broad information about the structures or the hydraulic pattern, we propose to guide the inversion with a false image that does not totally reflect the structures (see Figure 10a). We can see that the inverted model (Figure 10b) follows the correct structures inferred from the guiding image but does not preserve some of the features of the true model which are not present in the guiding image. We can also notice as stated in the previous sections that the main purpose of the image-guided interpolation is to increase the resolution of the inverted model and make it better respect the structural constraint (see Figure 10c). To study the uncertainty induced by the use of a wrong or incomplete guiding image we compute the posterior covariance matrix defined as follows:

$$\mathbf{P} = (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} + \mathbf{C}_m)^{-1}. \quad (29)$$

We can compute now the vector of the standard deviations of the estimated parameters σ_{post} by evaluating the square root of the diagonal elements of the matrix $\mathbf{P}$:

$$\sigma_{post} = \sqrt{\text{diag}(\mathbf{P})}. \quad (30)$$

By plotting the vector σ_{post} , we draw a map corresponding to the uncertainties involved in the reconstruction of each transmissivity cell (Figure 11). The lowest values σ_{post} represent the well reconstructed areas while the largest values of σ_{post} represent the areas where the estimation is less reliable. These areas correspond to the locations where there is a lack of information in the guiding image.

At this stage, we can say that the use of an erroneous guiding image doesn't make the inversion fail but the reconstruction of the transmissivity field is poorer in some areas of the domain, compared to the case where the hydrofacies were assumed to be perfectly known. The use of a guiding image that does not exactly respect the structural information will be studied in more details in another paper and we plan to use a stochastic generator to associate the uncertainty in the guiding to an uncertainty in the T_h -field. Furthermore, we plan to incorporate a level set method which will allow for correcting the guiding image using the hydraulic head measurements and thus improve the estimation of the transmissivity in the vicinity of hydrofacies. We believe also that an abundance of data (such as in transient hydraulic tomography) can add additional information to a wrong guiding image and therefore improve the inversion.

3.4. Case Study 4

This case study concerns a vertical distribution of transmissivity with 4 layers displaced by a fault and five wells with 4 small intervals from which the pumping tests are performed (Figures 12a, b). The modeling of the pumping tests includes the effect of gravity. The characteristics of the domain and pumping tests are reported in Table 1 and the model misfit in Table 2 for both the Gauss Newton and image-guided inversion. The image used for the image-guided inversion is shown in Figure 12c and the results of the image-guided inversion and image interpolation are shown in Figures 12 d and Figure 12e, respectively. In all cases, the starting (prior) model is totally homogeneous in the transmissivity with a constant value everywhere corresponding to the median of the true transmissivity distribution.

We can recover very well the transmissivity distribution by comparison with the Gauss-Newton inversion based on isotropic smoothing. An empirical initial value was used for the

regularization parameter. This value is updated during the iterative inversion process according to the objective function values. In other words, the value of the regularization parameter is divided by 10 if the objective function decreases and multiplied by 10 if the objective function increases. This procedure is repeated until the convergence criteria are met. Although this technique is simple, it provided good computation of the regularization parameter. After the third iteration, Figure 14d shown that the objective function misfit is smaller for the image guided inversion by comparison with the Gauss-Newton inversion based on isotropic smoothing. Figure 14d shows that the head data are perfectly recovered with the image guided inversion.

4. Conclusion

We have investigated the advantage of using the image-guided inversion to assess the transmissivity field of heterogeneous aquifers. We use the structural information contained in a guiding image (either obtained from seismic data, GPR data or from some geological expertise) to guide the inversion of the transmissivity field through image-processing. The image-guided approach offers a promising alternative to incorporate geophysical or geological information to reconstruct the hydraulic parameters. First, structural information is extracted from the guiding image (typically in grey scales) using image-processing techniques. Then, the image-guided inversion is used to apply the smoothing along the structural features instead of applying it across those features. This helps to obtain an estimated model of the transmissivity that better respects the structural constraints and that better recovers the true amplitudes of the transmissivity. The image-guided interpolation helps to increase the resolution of the inverted model as demonstrated in three numerical test cases.

Future studies could include the 3D estimation of the hydraulic parameters heterogeneities from the joint inversion of geophysical and transient hydraulic data using the

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5 651 image-guided method. In such applications, structural information such as geological cross-
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7 652 sections, ground penetrating radar images or migrated seismic sections could be used to
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9 653 constrain the inversion. We could also use a stochastic generator of geological models to test,
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11 654 which of many geological models, would impose the best structural information in recovering
12
13 655 the optimal transmissivity field.
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Tables

Table 1. Summary of model information for the three test cases. N_d : number of data, N_c denotes the number of cells, N_w the number of wells, N_t the number of dipole tests, B_c denotes the boundary conditions, Γ are the boundaries of the simulation domain, Γ_D denotes a Dirichlet-type boundary and Γ_N denotes a Neumann-type boundary ($\hat{\mathbf{n}}$ unit vector normal to the interface). The measurement error corresponds to a standard deviation of 1 cm on the hydraulic head data.

Tests	Geometry (m×m)	N_d	N_c	N_w	N_t	B_c
Case 1	100×100	60	900	8	4	$h = 0$ on Γ
Case 2	48×27	24	900	12	6	$h = 0$ on Γ
Case 3	100×100	84	900	13	7	$h = 0$ on Γ
Case 4	50×25	218	900	15	12	$h = 0$ on Γ_D $K\nabla h \cdot \hat{\mathbf{n}} = 0$ on Γ_N

Table 2. Computed Mean Squared parameters errors. MSE denotes the transmissivity misfit defined by $MSE = (1/M) \sum_{i=1}^M (\log_{10}(K)_{\text{True}} - \log_{10}(K)_{\text{Inverted}})^2$ where M is the number of transmissivity model unknowns.

	Inversion method	MSE
Case Study 1	Image guided	0.0003
	Gauss Newton	0.9065
Case Study 2	Image guided	0.0323
	Gauss Newton	0.9083
Case Study 3	Image guided	0.5021
	Gauss Newton	0.9339
Case Study 4	Image guided	0.1007
	Gauss Newton	0.1582

Table 3. Objective functions values for each study case. IGI stands for “ image guided inversion” while GN stands for “Gauss-Newton”.

	Iterations	Objective function	
		IGI	GN
Case Study 1	1	354907.94	266944.37
	2	51026.17	2904.42
	3	5342.57	624.18
	4	179.10	554.81
	5	35.62	554.34
	6	21.99	554.20
	7	21.19	554.23
	8	21.27	554.22
Case Study 2	1	118904.89	147370.40
	2	1513.68	26720.03
	3	627.47	14481.76
	4	357.06	10945.85
	5	337.04	8706.49
	6	326.26	4506.59
	7	329.25	4489.91
	8	331.02	4488.15
Case Study 3	1	42067.95	85558.56
	2	8659.30	5949.72
	3	45.99	3874.83
	4	14.33	3455.80
	5	0.96	3406.57
	6	0.14	3388.72
	7	0.01	3385.46
	8	0.0004	3384.52

Figures

Figure 1. Illustration of the four direction smoothing on a 2D-piece-wise constant transmissivity model. The guiding image will always be in different level of grey scales. The d_1 and d_2 are the diagonal directions added to the horizontal and vertical directions (x and y) in order to better honor the orientations of structural features and to smooth along these features. The grey and white cells denote two hydrofacies with a (transmissivity) discontinuity along the d_1 and d_2 directions on both sides of the image. The guiding image can be obtained from the reflectors in a GPR or seismic migrated sections.

Metric tensor field $\mathbf{D}(\mathbf{x})$ at 6 pixels

Figure 2. Diffusion tensor fields obtained for six pixels on a migrated georadar vertical cross-section. This section is used as a grey-scales coded image (grey intensity from 0 to 1). The diffusion tensor field $\mathbf{D}(\mathbf{x})$ defined by equation (19) is here determined at six pixels $\mathbf{x}_i$ (red filled circles). The tensor components are shown as ellipses determined from the intensity of the grey scales using the image-gradient at these points. The long axe of each ellipse, parallel to the linear features of the image, denotes the eigenvector $\mathbf{v}$. This direction indicates the smallest image gradient. The short axe of the ellipses corresponds to the eigenvector $\mathbf{u}$, normal to the image structures. The lengths of the two axes of the tensor ellipses are determined by the corresponding eigenvalues of $\mathbf{D}(\mathbf{x})$. A circle corresponds to an isotropic structural element and therefore implies isotropic smoothing for the inversion.

Figure 3. Flowchart summarizing the image guided inversion methodology.

Figure 4. Results for Case Study 1. **a.** True model with two square shaped mediums. The filled circles correspond to the locations of the wells used to simulate the dipole hydraulic tests. The color bar denotes the decimal logarithm of the transmissivity (in m s^{-1}). **b.** Inverted model obtained with the conventional Gauss-Newton method. **c.** Inverted model obtained from the imaged-guided structure-constrained inversion method.

Figure 5. Structural information used to perform the imaged-guided structure-constrained method in the horizontal plane. **a.** The image used to guide the inversion in grey scale. **b.** Calculated semblances₂, the low values of the semblance corresponds to the dark lines denoting the discontinuities. The axes show the pixel indices. **c.** Metric tensor field inferred from the guiding image. We see the ellipses along the two media.

Figure 6. Results for Case Study 2. **a.** True model showing the hydraulic pattern. The black filled circles correspond to the locations of the wells. The model has two entities with different transmissivity ranging from 10^{-6} ms^{-1} to 10^{-4} ms^{-1} . **b.** Results obtained with the conventional Gauss-Newton method. The inverted model is oversmooth making the contrast between the heterogeneities of the true model not well-recovered. **c.** Inverted model obtained from the imaged-guided structure-constrained method. The hydraulic pattern is well respected and the magnitudes of the transmissivity are similar to those of the true model.

Figure 7. Structural information used to perform the imaged-guided structure-constrained method. **a.** Guiding image. **b.** Calculated semblances s_2 . The semblance map shows clearly the discontinuities as dark lines (low semblance values). The axes show the pixel indices. **c.** Calculated metric tensor field. The ellipses lie along the edges of the two domains.

Figure 8. Results for Case Study 3. **a.** True model of transmissivity. Such type of heterogeneities can correspond to paleochannels in estuaries. The black filled circles correspond to the locations of the wells used to do the dipole tests. The true model is discretized into 900 cells. **b.** Results of the inversion using the Gauss-Newton method. The inverted model is too smooth and the shape of the channels is not recovered. **c.** Results of the inversion using the image-guided inversion. The heterogeneities are well-recovered in terms of low and high values of the transmissivity. **d.** Image-guided inverted model before interpolation.

Figure 9. Structural information used to perform the imaged-guided structure-constrained inversion. **a.** Guiding image. **b.** Calculated semblances s_2 . The dark lines on the semblance map show the discontinuities between the two domains. The axes show the indices of the pixels of the guiding image. **c.** The tensor field illustrates the features and their orientations. The ellipses lie along the discontinuities.

Figure 10. Influence of using an erroneous guiding image on hydraulic tomography. **a.** Guiding image with erroneous description of the structure. **b.** Nearest neighbor interpolation of the inverted model reconstructed with image guided inversion. **c.** Image guided interpolation of the inverted model reconstructed with image guided inversion. Compared to the nearest neighbor interpolation, the image-guided interpolation increases the resolution of the inverted model respecting the structural constraints. The model is therefore sensitive to the correctness of the information given by the guiding image but this information does not fully dominate the reconstruction of the inverted model.

Figure 11. Estimate of the uncertainties. **a.** Posterior standard deviation for Case Study 3 using a correct guiding image. **b.** Posterior standard deviation for Case Study 3 using a wrong guiding image. Lower values correspond to areas where the reconstruction is better while the larger values represent areas that not well-reconstructed.

Figure 12. Results from Case Study 4 with 5 piezometers and 4 depths intervals per well (the filled circles). **a.** Simulation domain with the locations of the measurements ports for the pumping tests. **b.** True transmissivity model. **c.** Guiding image used to guide the image guided inversion. **d.** Results of the image guided inversion (the prior transmissivity model is homogeneous, see main text). **e.** Results of the image guided inversion using the image guided interpolation. **f.** Results from the Gauss-Newton inversion with isotropic smoothness (same prior model than the image-guided inversion).

Figure 13. L-curve graphs for each of the study cases. **a.** L-curve analysis for Case Study 1, the optimal value of the regularization parameter is $\lambda_o = 0.0015532$. **b.** L-curve analysis for Case Study 2, the optimal value of the regularization parameter is 0.0050026. **c.** L-curve analysis for Case Study 3, the optimal value of the regularization parameter is 0.034127.

Figure 14. Plot of the values of the MSE at each iteration (see Table 2) for both the Gauss Newton and image guided inversion methods. **a.** Case Study 1. **b.** Case Study 2. **c.** Case Study 3. **d.** Case Study 4.

Figure 15. Comparison between the true hydraulic head data and the simulated data. **a.** Case Study 1. **b.** Case Study 2. **c.** Case Study 3. **d.** Case Study 4.