Modeling Heat Loss from the Udder of a Dairy Cow

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Abstract

A mechanistic model that predicts sensible and latent heat fluxes from the udder of a dairy cow was developed. The prediction of the model was spot validated against measured data from the literature, and the result agreed within 7% of the measured value for the same ambient temperature. A dairy cow can lose a significant amount of heat (388W/m^2) from the udder. This suggests that the udder could be considered as a heat sink. The temperature profile through the udder tissue (core to skin) approached the core temperature for an air temperature $\geq 37^oC$ whereas the profile decreased linearly from the core to skin surface for an air temperature less than 37^{o}C. Sensible heat loss was dominant when ambient air temperature was less than 37.5^{o}C but latent heat loss was greater than sensible heat loss when air temperature was $\geq 37.5^oC$. The udder could lose a total (sensible + latent) heat flux of 338 W/m^2 at an ambient temperature of 35^{o}C and blood-flow rate of $3.2\times10^{-3} m^3/(s\cdot m^3$ tissue). The results of this study suggests that, in time of heat stress, a dairy cow could be cooled by cooling the udder only (e.g., using an evaporative cooling jacket).

Keywords: *Udder, Dairy cow, Model, Heat flux, Heat stress*

Introduction

Heat stress is a major factor in the economics of dairy farming, especially with high-producing Holstein dairy cows. Economic losses due to reduced milk yield caused by heat stress have been estimated to be approximately 1.5 billion dollars per year nationally (St-Pierre et al., 2003). In addition, there are losses due to negative effects on reproductive performance and the associated costs of breeding a cow and losses in genetic gains (al-Katanani et al., 1999; Jordan, 2003). Needless to say, reducing the effect of summer heat stress on dairy cows is very important to the 40 billion-dollar U.S. dairy industry.

The udder can be characterized as a heat sink because looking at the thermal image (infrared thermography) of a cow, the eye and the udder are at higher temperature than at any exterior part of the cow (thecattlesite.com, 2010). Gebremedhin et al. (2013) also reported a uniform thermal

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image of the udder, which confirms the uniformity of blood flow within the skin surface of the udder. Bitman et al. (1984) also reported that the internal-body and udder temperatures were highly correlated (the correlation coefficient was 0.98-0.99 in each cow). They reported that the mean core and udder temperature was found to be 38.84^{o}C. This is because a large volume of blood circulates through the udder. Similarly, Smith et al. (1977) studied the role of heat loss from the udder in sheep and reported that the udder remained at a relatively constant temperature despite changes in air temperature. The same study found no correlation between time of day and udder temperature. This might be due to thermal regulation of blood flow in the udder.

Several other studies (Peeters et al., 1979; Mustafa, 2001; Anderson, 1985; Hurley, 2010) reported that as much as 400 to 500 liters of blood is needed to pass through the udder to produce 1 liter of milk. The rate of blood flow is, however, affected by breed (Al-Ani et al., 1985) and stage of lactation (Al-Ani and Vesweber, 1984). Given that these studies were conducted 20 years ago and that milk production of Holstein cows has been increasing at a rate of 1.5 -2.0% per year (Foreign Agricultural Service/USDA, 2015). It is, therefore, reasonable to assume that the volume of blood flow through the udder for current high-producing dairy cows could be higher.

The fact that the udder temperature is as high as the core-body temperature leads to the idea that cooling the udder is essentially cooling the large volume of blood passing through it. Cooling the udder only would be an effective and efficient way to cool a cow because less water would be required to wet a smaller surface area compared to wetting the entire body surface in time of heat stress. This was experimentally validated by Gebremedhin et al. (2013) who reported that there was no significant difference in mean rectal temperature between wetting the udder only and wetting the entire body of a cow. In addition to cooling the blood almost directly, there is much less hair on the skin of the udder compared to the rest of the body of a cow, and hair coat is an obstruction to evaporative cooling (Gebremedhin et al, 2010). The challenge to udder cooling only is to come up with an udder cooling system that works in a production setting. Gebremedhin et al. (2013) suggested the use of an evaporation cooling jacket that could be mounted onto the udder.

The objectives of this study were: (1) to develop a mechanistic model that predicts sensible and latent heat losses from the udder of a dairy cow at varying ambient temperature, and (2) to conduct sensitivity analyses of heat loss with respect to changes in volumetric blood flow rate and blood circulation time to the udder. Based on our extensive search, there exists no mechanistic model that predicts sensible and latent heat losses from the udder. The only study we found was that by Janeczek et al. (1995) who reported specific measurements of heat flux and temperature changes of the udder during and after milking. The heat flux measurement from this study (Janeczek et al., 1995) was used to spot validate our predicted heat flux.

Model Development

Assumptions:

1. Cow udder is semi sphere
2. Radius of the semi sphere is 0.17m
3. An outer layer of 10-mm thickness of tissue was assumed for heat transfer between the udder and ambient air (Figure 1).
4. A one-dimensional transient (a function of time) heat flow was assumed through the thickness of the tissue (Figure 1).
5. Heat lost from the udder skin surface when the cow is standing is by evaporation and convection
6. Core temperature of udder tissue is 38.6^{o}C
7. Metabolic heat production rate in the tissue is 1,946W/m^3
8. Blood-flow time through the udder takes between 2-4min.
9. The physical and thermal properties are assumed to be constant
10. Temperature of blood entering the udder is equal to core temperature.

Heat transfer model

The energy equation for the tissue domain was based on *Penne's Bio-heat Equation* (Pennes, 1948). It is a nonlinear-conduction equation that is meant for biological tissues with added advection (convection) heating by infused blood flow. The equation given below is broken down into four terms: a transient term, a simple Fourier's Law conduction term, a nonlinear

advection term for the infused blood flow at a given temperature, and a source generation term, respectively, and is expressed as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) + \rho_b c_b \dot{V}_b (T_b - T) + Q_m \quad (1)$$

where, ρ is tissue density (kg/m^3), c is tissue specific heat (J/kg·K), T is tissue temperature (K), t is time (s), k is tissue thermal conductivity (W/m·K), ρ_b is blood density (kg/m^3), c_b is blood specific heat (J/kg·K), $\dot{V}_b$ is flow rate of blood into tissue (m^3 blood/s per m^3 tissue), T_b is blood inlet temperature (K), and Q_m is metabolic heat production rate in the tissue (W/m^3).

Equation (1) can be expressed in a simple form as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + S \quad (2)$$

where, the source term, S, is expressed as

$$S = \rho_b c_{p,b} \dot{V}_b (T_b - T) + Q_m \quad (3)$$

The discretization equation that is derived by integrating Eq. (2) over the control volume shown in Figure 2 and over the time interval from t to (t + Δt) can be expressed as (Patankar, 1980)

$$\rho c \int_w^e \int_t^{t+\Delta t} \frac{\partial T}{\partial t}\, dt\, dx = \int_t^{t+\Delta t} \int_w^e \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) dx\, dt + \int_t^{t+\Delta t} \int_w^e S\, dx\, dt \quad (4)$$

where,

$$\rho c \int_w^e \int_t^{t+\Delta t} \frac{\partial T}{\partial t}\, dt\, dx = \rho c \Delta x (T_P - T_P^0) \quad (5)$$

$$\int_t^{t+\Delta t} \int_w^e \frac{\partial}{\partial x}(k \frac{\partial T}{\partial x}) dx\, dt = [\frac{k_e (T_E - T_P)}{(\delta x)_e} - \frac{k_w (T_P - T_W)}{(\delta x)_w}] \Delta t \quad (6)$$

$$\int_t^{t+\Delta t} \int_w^e S\, dx\, dt = \bar{S} \Delta x \Delta t \quad (7)$$

where, T_P is temperature at Node P, T_W and T_E are temperatures at Node W and E, respectively, (See Fig. 2), T^0_P is the temperature at previous iteration, k is conductivity at the control surfaces (at e and w), $\bar{S}$ is the average value of S over the control volume, which can be expressed as

$$\bar{S} = S_C + S_P T_P \qquad (8)$$

where, S_C represents the constant part of $\bar{S}$, and S_P is the coefficient of T_P.

Rearranging Eqs. (5), (6), and (7), the fully implicit discretization equation can be expressed as

$$a_P T_P = a_E T_E + a_W T_W + b \qquad (9)$$

where,

$$a_E = \frac{k_e}{(\delta x)_e} \qquad (10)$$

$$a_W = \frac{k_w}{(\delta x)_w} \qquad (11)$$

$$b = S_C \Delta x + a_P^0 T_P^0 \qquad (12)$$

$$a_P^0 = \frac{\rho c \Delta x}{\Delta t} \text{, and} \qquad (13)$$

$$a_P = a_E + a_W + a_P^0 - S_P \Delta x \qquad (14)$$

Boundary conditions

Two boundary conditions are required to solve Eq. (9) and they are:

(1) The temperature at x = 0, which is at the inside of the udder (Fig. 1) is equal to the core temperature, expressed as

$$T\big|_{x=0} = T_c \qquad (15)$$

(2) At x = L, which is at the skin surface (Fig. 1), the heat flux conducted is equal to the sum of the heat flux convected and evaporated, expressed as

$$-k\frac{\partial T}{\partial x}\bigg|_{x=L} = h_c(T_s - T_a) + [31.5 + e^{(T_s - 27.9)/2.19115}] \tag{16}$$

where, T_c is core temperature of the udder, T_s is temperature of the udder skin surface, T_a is ambient temperature, h_c is convective heat transfer coefficient of the udder when the cow is standing, and L is udder tissue thickness (Fig. 1). The second term on the right side of Eq. (16) is the evaporative heat loss from the udder skin (da Silva and Maia, 2011).

The convective heat transfer coefficient, h_c, can be obtained from Nusselt number (Nu) as (Gebremedhin and Wu, 2001)

$$Nu = \frac{h_c\ d}{k} = \begin{cases} (0.43 + 0.50\,\mathrm{Re}^{0.50})\,\mathrm{Pr}^{0.38} & 1 < \mathrm{Re} < 10^3 \\ 0.25\,\mathrm{Re}^{0.6}\,\mathrm{Pr}^{0.38} & 10^3 < \mathrm{Re} < 2\times 10^5 \end{cases} \tag{17}$$

where, Pr is the Prandtl number assumed to be 0.7, and Re is Reynold number calculated by

$$\mathrm{Re} = \frac{\rho_a u d}{\mu} \tag{18}$$

where, ρ_a is air density, u is air velocity across the udder, d is udder diameter, and μ is dynamic viscosity of air.

Numerical simulation

The algebraic Eq. (9) can be solved by tri-diagonal matrix algorithm (Patankar, 1980). The grid points are numbered 1, 2, 3, …, N, in which points 1 and N denote the boundary points. Equation (9) can be written as

$$a_i T_i = b_i T_{i+1} + c_i T_{i-1} + d_i \tag{19}$$

If T_{i-1} is expressed as

$$T_{i-1} = P_{i-1} T_i + Q_{i-1} \tag{20}$$

Then, Eq. (19) can be written as

$$a_i T_i = b_i T_{i+1} + c_i (P_{i-1} T_i + Q_{i-1}) + d_i \tag{21}$$

where, the coefficients P_i and Q_i are expressed as

$$P_i = \frac{b_i}{a_i - c_i P_{i-1}} \tag{22}$$

$$Q_i = \frac{d_i + c_i Q_{i-1}}{a_i - c_i P_{i-1}} \quad (23)$$

The values of P_1 and Q_1 for i = 1 are given by

$$P_1 = \frac{b_1}{a_1} \quad (24)$$

$$Q_1 = \frac{d_1}{a_1} \quad (25)$$

Since $b_N = 0$, then $P_N = 0$ for i = N, and the temperature, T_N can be obtained by

$$T_N = Q_N \quad (26)$$

The solution procedure for tri-diagonal matrix algorithm can be summarized as follows:

1. Calculate P_1 and Q_1 from Eqs. (24) and (25)
2. Solve P_i and Q_i from Eqs. (22) and (23)
3. Set $T_N = Q_N$
4. Solve T_{N-1}. T_{N-2}, …, T_3, T_2, T_1 from Eq. (20).

Results and Discussion

In this study, eleven computational grids and one second time size were used to run the calculations for the temperature profile. In order to solve Eq. (1), the *Penne's Bio-heat Equation*, two fluids (blood and air) and one solid (tissue) were involved. The physical and thermal properties used in this study are given in Table 1.

Given the two boundary conditions specified previously, the temperature profile through the udder tissue was predicted by solving the algebraic equation (Eq. 9). Once the temperature profile was defined, heat flux was calculated. The temperature profile through the udder tissue approached the core temperature for an air temperature $\geq 37^oC$ but decreased linearly from the core to skin surface for an air temperature less than 37^oC (Fig. 3). The temperature of the udder skin surface was 33.2, 36.0 and 38.2^oC for ambient air temperature of 30, 34, and 37^oC, respectively, (Fig. 3). For these simulations, the core temperature was kept constant at 38.5^oC.

Sensible heat loss from the udder decreased linearly with increasing ambient air temperature but evaporative heat loss increased exponentially (Fig. 4). This is consistent with the boundary

conditions expressed by Eq. (16) where the convection term is a function of the temperature gradient between the skin surface and ambient (T_s-T_a) in a linear fashion whereas the evaporation heat loss term is an exponential function of the udder skin temperature (T_s). At an air temperature less than 37.5^{o}C, sensible heat loss from the udder was greater than latent heat loss (Fig. 4). At an air temperature greater than 37.5^{o}C, latent heat loss was larger than convective heat loss. Figure 4 also shows that the total heat flux from the udder decreased with increasing (from 30 to 37.5^{o}C) ambient air temperature. This was the range where sensible heat loss was dominant. When air temperature was greater than 37.5^{o}C, where latent heat loss was greater than sensible heat loss, the total heat loss started to increase.

The predicted total heat flux from the udder ranged from 292 to 360W/m^2 for blood circulation time of 2 and 4 min., respectively, while keeping the following parameters constant: ambient air temperature = 35 ^{o}C, average wind speed = 1.0m/s, blood-flow rate =3.2×10^{-3}m^3/(s·m^3 tissue) (Table 2). The blood-flow rate value used in this study was reported by Gorewit et al. (1989) who used a transit time ultrasonic blood flow metering system to study the arterial blood flow in the mammary glands of four lactating Holstein cows. Generally, sensible heat flux increases with increasing blood-circulation time through the udder. Based on Eq. (1), which is a transient heat transfer equation, the temperature at each computational node is directly proportional to time. Keeping constant ambient air temperature of 35^{o}C and an average air velocity of 1m/s, but increasing the udder-skin surface temperature through continuous blood flow results in a higher temperature gradient between the udder-skin surface and the environment, and therefore, increases the heat loss by convection. The increase in latent heat loss with blood-circulation time was, however, very small compared to the sensible heat loss because evaporative heat transfer is not a function of blood circulation time (Eq. 1).

Table 3 shows the sensitivity of heat flux with respect to blood-flow rate. Two arbitrary blood-flow rates, one smaller and another greater than 3.2×10^{-3} m^3/(s·m^3 tissue) were taken to see the sensitivity of heat flux. The total heat flux decreased by 8.7% for the lower blood-flow rate and increased by 6.8% for the higher blood-flow rate.

Janeczek *et al.* (1995) reported a measured mean heat flux of 1234 $\pm$ 247.5W/m^2 at the back of the udder for mean air temperature of 15.8^{o}C, a mean relative humidity of 70.8% and core temperature of 38.6^{o}C. Using our model, the predicted heat flux was 1325W/m^2 for the same air temperature and core temperature, blood-flow rate of 3.2×10^{-3}m^3/(s·m^3 tissue) and blood-flow time of 3min. The blood-flow rate and blood-flow time for the Janeczek *et al.* (1995) study was not known. The spot validation show that the predicted result agreed within 7% of the measured heat flux. We recognize, however, that the parts of the udder (front and side surfaces) that are protected from wind by body parts (legs and belly) would be exposed to lower wind speed than the back of the udder, and thus would have less convective heat loss.

Conclusions

The following specific conclusions could be drawn from this study:

1. A mechanistic model that predicts sensible and latent heat fluxes from the udder of a dairy cow was developed. A prediction of the model was spot validated against measured flux data from the literature and the result agreed within 7% of the measured data. It was predicted that a cow could lose 388W/m^2 of total (sensible +latent) heat flux from the udder at 35^{o}C ambient air temperature and 3.2×10^{-3}m^3/(s·m^3 tissue) blood-flow rate. This suggests that the udder could be considered as a heat sink, and thus, in time of heat stress, a dairy cow could be cooled by cooling the udder only using an evaporation cooling jacket that could function in a production setting.

2. The temperature profile through the udder tissue approached the core temperature for an air temperature $\geq$37^{o}C but decreased linearly from the core to the skin surface for an air temperature less than 37^{o}C. The temperature of the udder skin was 33.2, 36.0 and 38.2^{o}C for air temperature of 30, 34, and 37^{o}C, respectively. For these simulations, the core temperature was kept constant at 38.5^{o}C.
3. Sensible heat loss was dominant when ambient air temperature was less than 37.5^{o}C but latent heat loss was greater than sensible heat loss when air temperature was $\geq$37^{o}C.

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Table 1 Model inputs

Name	Symbol	Value	Unit
Tissue thermal conductivity	k	0.6	W/m·K
Tissue density	ρ	998.2	kg/m^3
Tissue specific heat	c	3600	J/kg·K
Blood density	ρ_b	1060	kg/m^3
Blood specific heat	c_b	4181.3	J/kg·K
Blood flow rate	$\dot{V}_b$	3.2×10^{-3}	$m^3/s \cdot m^3$ tissue
Metabolic heat production rate	Q_m	1946	W/m^3
Air density	ρ_a	1.2	kg/m^3
Dynamics viscosity of air	μ	1.79×10^{-5}	Pa·s

Table 2. Predicted heat flux of the udder versus blood circulation time [1]

Heat flux	Blood circulation time (minute)		
(W/m^2)	2	3	4
Sensible	212	252	271
Latent	80	86	89
Total	292	338	360

[1] Ambient temperature = 35°C, and blood-flow rate = 3.2×10^{-3} $m^3/(s \cdot m^3$ tissue).

Table 3. Predicted heat flux of the udder versus blood-flow rate [1]

Heat flux (W/m^2)	Blood-flow rate ($m^3/(s\cdot m^3$ of tissue))		
	2.2×10^{-3}	3.2×10^{-3}	4.2×10^{-3}
Sensible	229	252	272
Latent	82	86	89
Total	311	338	361

[1] Ambient temperature = 35 ^{o}C, and blood-circulation time = 3 minutes.

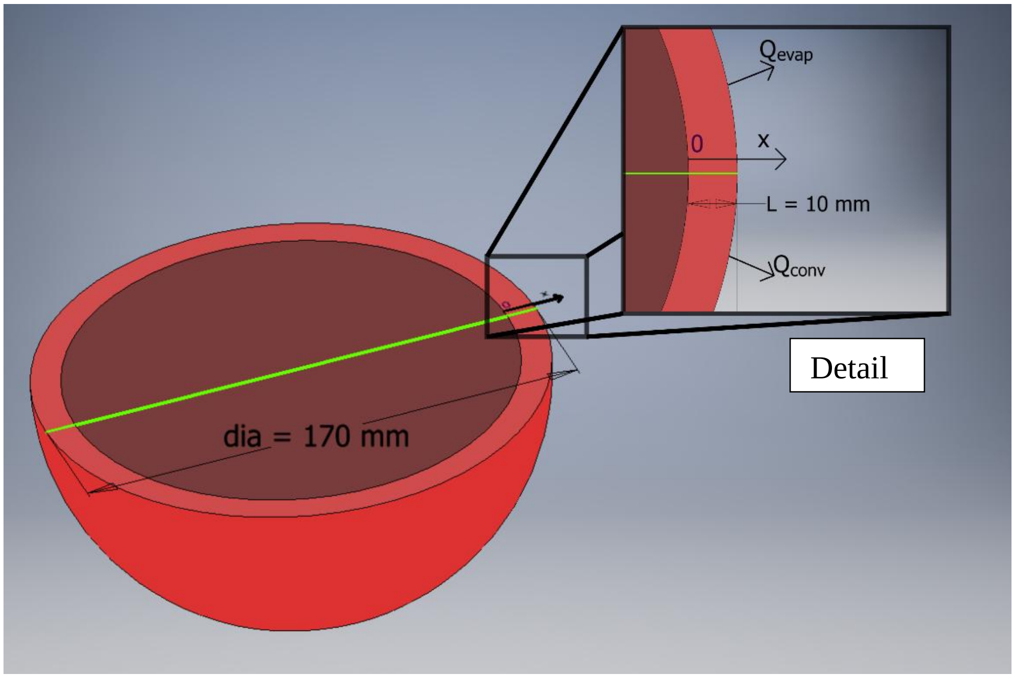

Figure 1. Semi spherical representation of the udder and a detail of 1-D heat flow through a 10-mm outer layer which represents the assumed tissue thickness of the semi sphere.

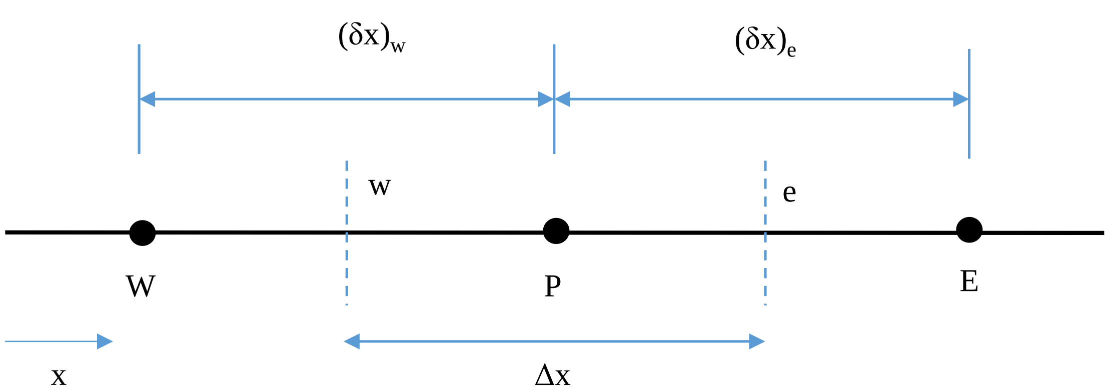

Figure 2. Grid-point cluster for one-dimensional problem. *P* represents center node of the control volume, *E* and *W* represent nodes at east and west of Node *P*, respectively, *e* and *w* are control surfaces.

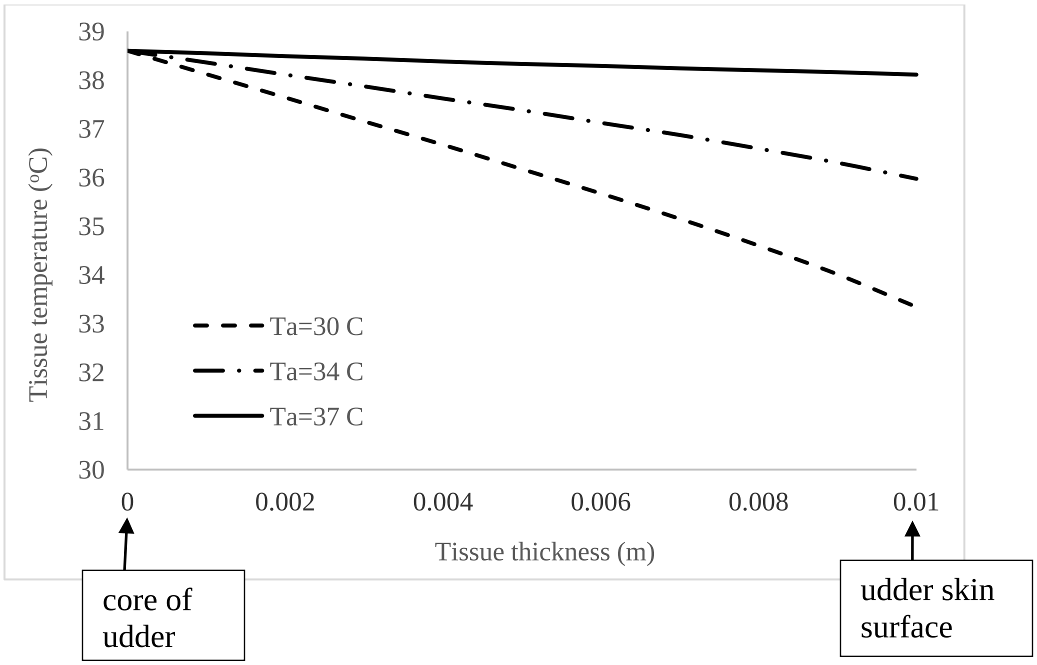

Figure 3. Temperature profile through the tissue for blood-circulation time of 3min. and blood-flow rate of 3.2×10^{-3}(m^3/s· m^3 of tissue).

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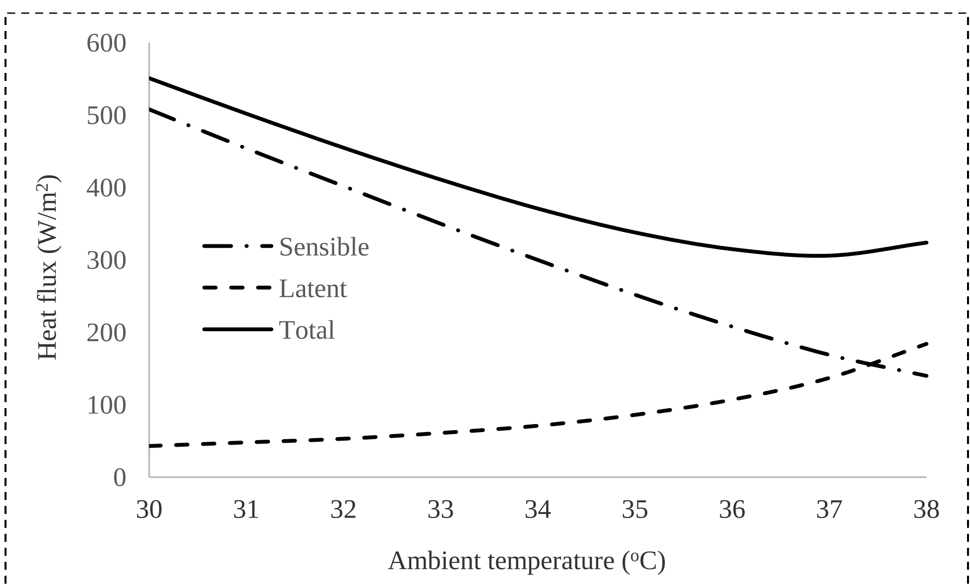

Figure 4. Heat flux (sensible, latent and total) versus ambient air temperature for blood-circulation time of 3min. and blood-flow rate of 3.2×10^{-3}(m^3/s· m^3 of tissue).