Parameter sensitivity analysis of duplex Stirling coolers

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Abstract Duplex Stirling coolers are external heat-driven regenerative machines that obtain cooling power with high efficiency and reliability. Although the technology is promising, little progress has been reported since they were invented. This paper examines the influence of system parameters on the cooling performance of a duplex Stirling cooler. The study showed that the system was sensitive to a number of parameters, as slight changes in these parameters resulted in the cooling performance deviating significantly from the design. In practice, deviation of system parameters from the design is inevitable. Therefore, actual systems often produce unexpected performance, and this is likely the main obstacle to utilizing such a system in practical applications. To overcome this disadvantage, two methods were proposed to allow the power piston to move with a constant displacement, which may allow for duplex Stirling coolers to become a more widely used cooling system. A new

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17 configuration for duplex Stirling coolers was also considered, and it showed similar
18 characteristics as the typical configuration.

19 **Key words:** duplex Stirling; regenerative; parameter sensitivity

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Nomenclature			
A_{pis}	piston area (m ²)	i	$\sqrt{-1}$
I	current (A)	Im	imaginary part of a complex number
K_{pis}	spring constant (N/m)	M_{pis}	piston mass (kg)
P_{en}	fluctuanting pressure in engine (Pa)	P_{pis}	fluctuanting pressure in front of piston (Pa)
P_{re}	fluctuanting pressure in refrigerator (Pa)	Q_c	cooling power (W)
Q_h	heating power (W)	Re	real part of a complex number
R_{elec}	electric resistance (Ω)	R_{mech}	Mechanical resistance (kg/s)
T_c	cooling temperature (K)	T_h	heating temperature (K)
T_r	room temperature (K)	U_{en}	volume flow rate of engine (m ³ /s)
U_{pis}	volume flow rate of piston (m ³ /s)	U_{re}	volume flow rate of refrigerator (m ³ /s)
v	velocity of piston (m/s)	$W_{acou, en}$	acoustic work out of engine (W)
$W_{acou, re}$	acoustic work into refrigerator (W)	$W_{dissi, pis}$	acoustic work dissipated by the power piston (W)
X_{elec}	imaginary of electric impedance	Z_{elec}	electric impedance of the alternator (Ω)
Z_{en}	acoustic impedance of the engine (Pa·s/m)	Z_{mech}	mechanical impedance of piston (N·s/m)
Z_{re}	acoustic impedance of the refrigerator (Pa·s/m)	η_e	exergy efficiency
τ	transduction coefficient (N/A)	ω	angular frequency (rad/s)
$ $	magnitude of a complex number		

1. Introduction

In heat-driven regenerative coolers, high temperature thermal energy is used to generate cyclic pressure fluctuations in heat engines, which are used to produce cooling in refrigerators. The thermal energy can come from any heat source including concentrated solar energy, combustion of fossil fuels or waste material, or the waste thermal streams of industrial plants or power systems. The ability to use thermal energy to produce refrigeration makes these coolers especially attractive where electrical energy is deficient. One potential application is natural gas liquefaction, which is accomplished by burning part of the gas [1]. At present, the highest reported efficiency of heat-driven regenerative natural gas liquefaction has been achieved by burning about 30% to liquefy the rest [2, 3].

There are three kinds of heat-driven regenerative coolers: the Vuilleumier cooler, the heat-driven thermoacoustic cooler, and the duplex Stirling cooler. In a Vuilleumier cooler, pressure fluctuations are produced by shuttling gas periodically from an ambient temperature region to a high temperature region by the action of a reciprocating displacer in a hot cylinder. There is another displacer in the cold cylinder to pump heat. The hot and cold displacers are coupled by a kinematic mechanism to maintain proper phasing and synchronous operation. The friction between the displacers and the cylinders makes the Vuilleumier cooler susceptible to failure, which limits its application [4]. In a heat-driven thermoacoustic cooler, the engine and refrigerator are coupled acoustically and the pressure fluctuates spontaneously as long as the heating temperature is maintained [5-7]. There are no moving parts, which results in reliability and simplicity. The working frequency of this cooler is determined by the length of the resonant tube [8]. However, the flow loss in the resonant tube is significant, especially when the pressure ratio is high, so the efficiency of the thermoacoustic cooler is low [9]. In a duplex Stirling cooler, a power piston is employed to couple a free-piston engine

and a free-piston refrigerator. Two displacers are used to adjust the phase [10]. Compared with a heat-driven thermoacoustic cooler, there are no long tubes so the flow loss is significantly reduced. Furthermore, the moving parts are suspended by flexure or gas bearings, which eliminates frictional contact. Therefore a duplex Stirling cooler has the advantages of high efficiency, high reliability, and compactness.

The free-piston Stirling concept was introduced by Beal in 1969 [11]. In recent years, it has drawn worldwide attentions due to the urgent need for renewable energy technologies [12-18]. In the 1980s, several duplex Stirling machines were developed by Sunpower Inc. and while experimental results demonstrated their feasibility, detailed parameters of these machines and quantitative results were not presented [19-21]. With the exception of several analyses [22-25], very little progress regarding duplex Stirling machines has been reported since. Compared with a free-piston engine or cryocooler [26, 27], the duplex configuration is more complex, which inevitably increases the difficulty in building such a system. Steady state operation can be achieved only when the acoustic power dissipated by the power piston is equal to the difference between the powers delivered by the engine and absorbed by the refrigerator. This paper will quantify the energy balance and demonstrate how the performance is influenced when system parameters deviate from the designed values. Several methods will be introduced to keep the system working steadily. To find possible ways to avoid parameter sensitivity, a new configuration for the duplex Stirling cooler will be developed. Finally, conclusions will be drawn.

2. Parameters of the system

2.1 Design for the engine and refrigerator

Fig. 1(a) shows the schematic of a duplex Stirling cooler design, which includes three

main components: a free-piston Stirling engine, a free-piston Stirling refrigerator, and a power piston between the engine and refrigerator. The designed working frequency is 50 Hz and the charging pressure is 6 MPa. Sharing the same power piston, the engine and the refrigerator have the same swept volume. To ensure the system can work independently, the engine must output more acoustic power than the refrigerator absorbs since the power piston will dissipate some work. So the refrigerator was first designed independently, then the parameters of the engine is adjusted to satisfy this.

(a)

(b)

Figure 1 (a) Schematic of a typical duplex Stirling cooler, (b) phasor diagram of the pressures and volume flow rate

For the refrigerator with the parameters in Table 1, it absorbs an acoustic work $W_{acou,re}$ of 7247 W and produces a cooling power Q_c of 2043 W at 110 K, corresponding to a coefficient of performance ($Q_c/W_{acou,re}$) of 0.282. Its acoustic impedance Z_{re} is $7.80 \times 10^7 \angle -79.4^\circ$ Pa.s/m. The swept volume of the power piston is 638 cc. Given the diameter and mechanical resistance in Table 1, the piston displacement of 14 mm and dissipated power $W_{dissi,pis}$ of 970W are obtained. So the engine must output acoustic work $W_{acou,en}$ of 8217 W. After adjusting, the parameters for the engine are then obtained as shown in Table 1. It absorbs 16.1 kW heating power Q_h at 873 K, corresponding to a thermal efficiency

($W_{acou,en}/Q_h$) of 50.9%. Its acoustic impedance Z_{en} is $7.41 \times 10^7 \angle 77.3^\circ$ Pa.s/m. If the overall exergy efficiency is defined as

$$\eta_e = \frac{Q_c T_h (T_r - T_c)}{Q_h (T_h - T_r) T_c}, \quad (1)$$

the exergy efficiency for this system is 33.3%, which indicates that consuming about 11% of the gas is sufficient to liquefy the remainder; this is comparable to conventional large-scale liquefiers based on Joule-Thomson or Brayton cycles [28, 29].

Table 1 Parameters of the designed duplex Stirling cooler

engine	Heat rejecter	Tube-shell type, 80 mm in length, cooled by water of 300 K
	regenerator	158 mm in diameter and 70 mm in length, filled with random stainless fiber with diameter of 16 μ m and porosity of 90%
	heater	composed of 100 tube with diameter of 3 mm and length of 300 mm
	displacer	150 mm in diameter and 2 kg in weight, with mechanical resistance of 30 kg/s and flexure bearing stiffness of 3×10^5 N/m
	rod	47.4 mm in diameter
	compression chamber	volume of 2.27 L
power piston	piston	170 mm in diameter 25 kg in weight, with mechanical resistance of 100 kg/s and flexure bearing stiffness of 5.56×10^4 N/m
refrigerator	Heat rejecter	Tube-shell type, 80 mm in length, cooled by water of 300 K
	regenerator	180 mm in length and 75 mm in diameter, filled with 300 mesh stainless screens
	cold tip	composed of 100 tube with diameter of 3 mm and length of 200 mm
	displacer	140 mm in diameter 2 kg in weight, with mechanical resistance of 25 kg/s and flexure bearing stiffness of 2.75×10^5 N/m
	rod	37 mm in diameter
	compression chamber	volume of 1.45 L

Above calculation for the refrigerator and engine is carried out separately in two separate numerical programmes based on SAGE [30]. Once the engine and refrigerator are couple

together by the power piston, an energy balance will be built among $W_{acou,en}$, $W_{acou,re}$ and $W_{dissi,pis}$. In a practical system, many parameters such as the mechanical resistance of the power piston, charging pressure, and heating temperatures, et al may deviate from the designed values, which will destroy the designed energy balance. The system may change its working parameters to reach another balance instead of the designed. $W_{dissi,pis}$ is very small compared $W_{acou,en}$ and $W_{acou,re}$. Little change of $W_{acou,en} - W_{acou,re}$ may result great change of piston displacement since $W_{dissi,pis}$ is a function of piston displacement. So, how the system will respond to the parameter change must be further investigated.

2.2 Design for the total duplex Stirling cooler

With the Newton's law for the power piston, there is [31],

$$P_{en} - P_{re} = \frac{Z_{mec h}}{A_{pis}^2} U_{pis} \quad (2)$$

If the phase of U_{pis} is set to zero, the phasor diagram of the pressures and volume flow rate can be shown as in Fig. 1(b). The pressures in the engine and refrigerator are almost phase-opposite. The purpose of the power piston is to make the phases match. The phase of $Z_{mec h}$ was very close to 90° . Eq. (2) can be rewritten as

$$Z_{en} A_{pis}^2 - Z_{re} A_{pis}^2 = Z_{mec h} \quad (3)$$

Because

$$Z_{mec h} = R_{mec h} + i(\omega M_{pis} - K_{pis}/\omega), \quad (4)$$

two equations can be obtained from Eq. (3),

$$Re(Z_{en}) A_{pis}^2 - Re(Z_{re}) A_{pis}^2 = R_{mec h} \quad (5)$$

$$Im(Z_{en}) A_{pis}^2 - Im(Z_{re}) A_{pis}^2 = \omega M_{pis} - K_{pis}/\omega \quad (6)$$

If $\frac{1}{2}|v|^2$ is multiplied to Eq. (5), this equation will show the energy balance. The left two

terms represent the output work of the engine $W_{acou,en}$ and the absorbed work of the refrigerator $W_{acou,re}$, respectively. The right term represents the dissipated work by the power piston $W_{dissi,pis}$. Eq. (6) shows how to design the piston's mass and spring to make the system to work with a certain frequency. For a given duplex Stirling cooler, if parameters deviate from design, the system will automatically adjust the working frequency and the piston displacement to satisfy the equations.

With Eqs. (5) and (6), the duplex Stirling cooler can be simulated in one numerical programme based on SAGE. Fig. 2 shows the user interface panel of this program. It simulates all parts of the system and numerically determines the values of the working frequency and power piston's displacement to satisfy Eqs. (5) and (6). Therefore, the system performance and sensitivity to a number of system parameters can be investigated with this program.

Figure 2 The front panel of the numerical program for the duplex Stirling cooler

3. Parameter sensitivity

Because of uncertainties in the machining and assembly accuracy, it is difficult to predict the mechanical resistance of the power piston in the design stage. Its actual value may deviate

from the design. Fig. 3 shows the influence of the power piston's mechanical resistance on the piston displacement and overall exergy efficiency of the system. Changing the mechanical resistance of the power piston disrupted the balance between the output work of the engine, the absorbed work of the refrigerator, and the power dissipation of the piston. The system changed the piston displacement to find a new energy balance. It can be seen in Fig. 3 that the power piston's displacement changed significantly with the mechanical resistance, which resulted in a significant decrease of the heating and cooling power, as shown in Fig. 4. The decrease of piston displacement makes the efficiency of the engine and refrigerator increase slightly, so the exergy efficiency in Fig.3 increases a little bit when the resistance increases from 50 to 100 kg/s. When the resistance further increases, the output work of the engine decreases very fast and the dissipated work by the piston become a relative big part, so the exergy efficiency decreases a little bit. The sensitivity of the system to the piston's mechanical resistance is a disadvantage because a piston displacement greater than the design limit may destroy the flexure bearings, while a piston displacement less than the design limit will deteriorate the cooling performance.

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Figure 3 Influence of the power piston's mechanical resistance on the piston displacement and overall exergy efficiency.

Figure 4 Influence of the power piston's mechanical resistance on the heating and cooling power.

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Figure 5 Influence of the displacer's mechanical resistance in the engine on the power piston displacement and output work of the engine.

Figure 6 Influence of the displacer's mechanical resistance in the refrigerator on the power piston displacement and absorbed work of the refrigerator.

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With the power piston's displacement and working frequency of the system fixed at 14 mm and 50 Hz, respectively, it was found that the output work of the engine dropped from 8478 to 7927 W if the displacer's mechanical resistance in the engine was increased from 5 to 60 kg/s as shown in Fig. 5(a). So, if the displacer's actual resistance deviated from the designed value in the duplex Stirling cooler, the power piston's displacement would have to

adjust significantly to find the new energy balance as shown in Fig. 5(b). However, as shown in Fig. 6, if the displacer's mechanical resistance in the refrigerator increased from 5 to 50 kg/s, the work absorbed by the refrigerator varied over a smaller range, from 7169 to 7236 W, so it was not necessary for the piston to change its displacement as much to find the new energy balance in this case.

In a practical system, the heating and cooling temperatures may fluctuate. It was necessary to investigate the influence of the temperatures on the power piston's displacement. In Fig. 7, it can be seen that the piston displacement increased rapidly with increased heating temperature. With the heating temperature 50 K higher than the designed value, the engine would output more acoustic work, so the piston displacement increased to 20 mm from 14 mm. At a heating temperature below 570 K, the piston displacement was zero, indicating the system would be nonfunctional. In Fig. 8, it can be seen that the piston displacement was also sensitive to the cooling temperature because the cooling temperature would change the absorbed work of the refrigerator. At a cooling temperature of 300 K, if the heating temperature was lower than 420 K, the engine could not output enough acoustic work, so the piston would not move as shown in Fig. 9. This indicates the onset temperature of this system is greater than 420 K. These results also suggest that the heating temperature should not be high when the cold tip has not been cooled down, otherwise the piston's displacement will exceed the design limit.

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Figure 7 Influence of the heating temperature on the power piston displacement.

Figure 8 Influence of the cooling temperature on the power piston displacement.

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When the working gas was heated or cooled, the average charging pressure of the system changed. Therefore the influence of the system charging pressure on the power piston's motion (displacement and frequency) was also investigated. The results are shown in Fig. 10. Because higher charging pressure is helpful for the engine to output more acoustic work, so it can be seen, the motion of the power piston was sensitive to the charging pressure.

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Figure 9 Influence of the heating temperature on the power piston displacement.

Figure 10 Influence of the charging pressure on the power piston displacement and working frequency.

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Figure 11 Influence of the power piston's diameter on its displacement and frequency.

Figure 12 Influence of the power piston's diameter on the charging pressure and working frequency, while holding the piston displacement constant.

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Generally, when the diameter changes, the piston displacement would change correspondingly, which would further change the dissipated work. So it can be seen that the piston displacement was sensitive to the diameter. A certain amount of error is inevitable in the simulation of a duplex Stirling cooler because of complications arising from the

oscillating flow and heat transfer. If the actual phase angle of the refrigerator Z_{re} was 78.3° instead of the calculated value of 77.3° (assuming there is no error with the other parameters), the piston diameter calculated from Eq. (5) was 222 mm. According to the result in Fig. 11, the piston displacement and operating frequency would increase to a point that would put the flexure bearings in extreme stress if this piston were employed.

4. Methods to keep the displacement constant

4.1 Adjusting the charging pressure

Generally, the working frequency was influenced by the charging pressure and power piston diameter, as seen in Figs. 10 and 11. The working frequency affects the energy balance between the engine, refrigerator, and power piston, which is the main reason that the piston's displacement is influenced by the charging pressure and power piston diameter. Therefore, actively changing the working frequency of the system should allow for a constant piston displacement. For a given system adjusting the charging pressure is easier than adjusting the piston diameter, which is fixed after machining. In Fig. 11, the piston's displacement varied very fast with the piston diameter. By adjusting the charging pressure, the piston displacement could be kept constant at 14 mm as shown in Fig. 12. If the piston's diameter was less than 170 mm, the charging pressure should be increased. If the piston's diameter was greater than 170 mm, the charging pressure should be decreased. In this way the working frequency was kept close to the desired 50 Hz value. The corresponding cooling power and exergy efficiency are shown in Fig.13 and remained relatively constant.

If the piston's mechanical resistance deviates from the designed value, adjusting the charging pressure can also ensure the piston displacement remained constant, without significantly reducing the cooling power as shown in Fig. 14. The same method can be

applied to a system in which the displacer's mechanical resistance deviates from the designed value.

Figure 13 Influence of the power piston's diameter on the cooling power and exergy efficiency, while holding the piston displacement constant.

Figure 14 Influence of the power piston's mechanical resistance on the charging pressure and cooling power, while holding the piston displacement constant.

4.2 Employing a linear alternator

For instantaneous deviations in the heating and cooling temperatures, it is not possible to adjust the charging pressure instantly. Employing a linear alternator is another method to keep the power piston's displacement constant. The alternator is connected to the power piston so that the alternator can convert energy between the acoustic work and electric power. For the piston and alternator,

$$Z_{en} A_{pis}^2 - Z_{re} A_{pis}^2 = R_{mec h} + i(\omega M_{pis} - K_{pis} / \omega) + \tau I / v \quad (7)$$

$$Z_{elec} I = \tau v \quad (8)$$

Combining Eqs. (7) and (8),

$$Re(Z_{en})A_{pis}^2 - Re(Z_{re})A_{pis}^2 = R_{mec h} + R_{elec} \tau^2 / |Z_{elec}|^2 \quad (9)$$

$$Im(Z_{en})A_{pis}^2 - Im(Z_{re})A_{pis}^2 = \omega M_{pis} - K_{pis} / \omega - X_{elec} \tau^2 / |Z_{elec}|^2 \quad (10)$$

For simplicity, let X_{elec} be zero. The program then optimized R_{elec} and working frequency to satisfy Eqs. (9) and (10).

Figure 15 Influence of the heating temperature on the alternator's electrical resistance and power, while holding the piston displacement constant.

Figure 16 Influence of the cooling temperature on the alternator's electrical resistance and power, while holding the piston displacement constant.

When the transduction coefficient τ equaled 50 N/A, the influence of the heating temperature on the alternator's electric resistance and power is shown in Fig. 15. If the heating temperature was 873 K, the electric resistance was infinite and there was no conversion between acoustic work and electric power. If the heating temperature was higher, the electric resistance was positive, which means the alternator would convert the acoustic work into electric power. If the heating temperature was lower, in theory the alternator could act as a compressor to convert electric power into acoustic work (the negative electric resistance and electric power mean electric power is converted into acoustic work). Of course, it was not necessary as long as the piston displacement was less than the designed value. In Fig. 16, it can be seen that the alternator always converted acoustic work into electric power with high or low cooling temperature if the piston displacement was kept constant.

5. Discussions about a new configuration

In the original configuration shown in Fig. 1(a), a single power piston was used, which will induce strong vibrations in the system. Fig. 17(a) shows a new configuration for a duplex Stirling cooler. With the dual-opposed pistons, vibrations induced by the pistons can be avoided. Different from the original, the engine and refrigerator have the fluctuating pressure instead of having the same swept volume. For each piston,

$$Z_{pis} A_{pis}^2 = Z_{mec} h \quad (11)$$

With the law of mass conservation,

$$U_{en} = U_{re} + 2U_{pis} \quad (12)$$

Combining them, we have

$$U_{en} = U_{re} + \frac{2P_{pis} A_{pis}^2}{Z_{mec} h} \quad (13)$$

Eq. (13) can also be written as

$$\frac{1}{Z_{en}} = \frac{1}{Z_{re}} + \frac{2A_{pis}^2}{Z_{mec h}} \quad (14)$$

If the phase of P_{pis} is zero, the phasor diagram of the pressure and volume flow rates can be shown as in Fig. 17(b). The volume flow rates in the engine and refrigerator were almost phase-opposite. The purpose of the power piston is to make the volume flow rates match. The phase of $Z_{mec h}$ was very close to 90° . Fig. 17(b) also tells that the total swept volume of the two pistons is almost twice of that in the original configuration. Two equations can be obtained from Eq. (11),

$$Re(Z_{pis})A_{pis}^2 = R_{mec h} \quad (15)$$

$$Im(Z_{pis})A_{pis}^2 = \omega M_{pis} - K_{pis}/\omega \quad (16)$$

With the same engine and refrigerator as in the original configuration, we obtains a piston diameter of 145 mm and a piston mass of 7.28 kg from Eqs. (15) and (16) (the piston's mechanical resistance is set to 50 kg/s instead of 100 kg/s because two pistons are employed here. The end bell volume of the pistons is 10 liter). The cooling power is 1.9 kW with an overall exergy efficiency of 30.6%. If the charging pressure is optimized, the efficiency could be as high as the efficiency of the original configuration shown in Fig. 1(a). This proves that the new configuration is theoretically feasible.

Although the advantage of the dual-opposed pistons is cancellation of the system vibration, it was found that the new configuration had the same parameter sensitivity as the single piston configuration. Adjusting the charging pressure and employing a linear alternator can ensure the system performs at a steady state.

Figure 17 (a) Schematic of a new duplex Stirling cooler, (b) phasor diagram of the pressure and volume flow rates

6. Conclusion and discussion

In a duplex Stirling cooler, steady state operation can be achieved only when the acoustic power dissipated by the power piston is equal to the difference between the powers delivered by the engine and absorbed by the refrigerator. This study finds that the system's performance is sensitive to a number of parameters such as the mechanical resistance of the piston, the heating temperature, and the piston diameter. Small variations in these parameters cause the power piston's displacement to deviate significantly from the designed value, which will subsequently alter the working state of the engine and refrigerator. Therefore, in practice the system often operates with an unexpected performance, which may be the main obstacle to build and utilize such a system.

To make the power piston's displacement constant, two methods were presented. First, the system's working frequency varies with the charging pressure; therefore it was shown that changing the charging pressure could help the system offset the influence of variations in system parameters. Second, by converting energy between acoustic work and electric power, a linear alternator can also ensure that the power piston's displacement remains constant,

especially when the parameters are instantaneously changed. These methods may endow the duplex Stirling cooler with reliable performance outcomes.

A new configuration for duplex Stirling coolers with two dual-opposed pistons was presented. It has the same system parameter sensitivity as the single piston configuration. Adjusting the charging pressure and employing a linear alternator are also applicable methods to keep the power pistons displacement constant in this system.

In this paper, the analyses are conducted in a way by individually investigating the parameter influence while fixing all the other parameters. In real applications, it is more common that several parameters deviate from the designed values, which will make the system more difficult to reach a steady state. But the mechanism of multi-parameter influence is the same as the single parameter. The influence can also be obtained with this method.

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