Hamiltonian Analysis of a Hydro-energy Generation System in the transient of sudden load increasing

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Abstract: This paper addresses the Hamiltonian mathematical modeling and dynamic analysis of a hydro-energy generation system in the transient of sudden load increasing. First, six dynamic transfer coefficients of the hydro-turbine for the transient of sudden load increasing are innovatively introduced into the hydro-energy generation system. Considering the elastic water-hammer model of the penstock and third-order model of the generator, we established a dynamic mathematical model of the hydro-energy generation system in the transient of sudden load increasing. Moreover, from the point of view of the transmission and dissipation of energy of the system, we propose the hydro-energy generation system into the theory frame of the generalized Hamiltonian system. A novel Hamiltonian model of the hydro-energy generation system is established utilizing the method of orthogonal decomposition. Finally, based on the data of a real hydropower plant, numerical simulations and

physical experiment are carried out, and the results indicates that the Hamiltonian system can reflect the essence of the nonlinearity of the hydro-energy generation system in the transient of sudden load increasing. More importantly, these methods and results will supply theoretical basis for designing and running a hydropower plant.

Keywords: hydro-energy generation system; Hamiltonian function; transient of sudden load increasing; dynamic transfer coefficients

1. Introduction

The development of economy and technology puts forward higher request to the reliability of electrical power supply [1-5]. The stability of the power system is directly related to the safety, reliability and availability of the power system [2-8]. The hydro-energy generation system (HEGS), as an important part of the power system, affects the power transmission and safe operation of the hydropower plant [9-15]. In the actual operation, due to the flow inertia of the penstock, the disturbance of the stochastic load and the sudden changing of the load, the vibration and the self-excitation of the HEGS will become more prominent [16-27]. Therefore, it is important to study the dynamic characteristics of the HEGS.

The Hamiltonian system, which is an open generalized system, has the energy dissipation and energy exchange with the outside environment. It can better describe the energy dissipation, the energy produced from the interior of the system and the energy supplied from the environment [10-11, 23]. The HEGS is a typical open system, which can better reflect the transmission and dissipation of energy [10, 28-31].

From the point of view of the transmission and dissipation of energy, putting the HEGS into the theory frame of the generalized Hamiltonian system is a new research approach to study the energy characteristics of the HEGS [23-40]. In previous studies, few published papers however focus on the investigation of the Hamiltonian model of the HEGS in large fluctuation processes. Most literatures address the Hamiltonian modeling in small fluctuation processes by utilizing the jump function or Gaussian function to indicate the dynamic characteristics and energy changing of the HEGS. According to Ref. [10-11, 18, 23-25, 29], Tab. 1 is made to indicate the benefits and drawbacks of the existing Hamiltonian models of the HEGS. Apparently, these methods cannot better reflect the essence of the nonlinearity of the HEGS in large fluctuation processes. Therefore, establishing a proper Hamiltonian mathematical model of the HEGS and investigating the corresponding dynamic characteristics are of great significance for theory and industry practice.

Tab. 1. Assessment of the existing Hamiltonian models of the HEGS

Existing Hamiltonian Models of the HEGS	
Benefits	Drawbacks
The model can better reflect the characteristics of the Hamiltonian system in the small fluctuation process.	The Hamiltonian model cannot describe the characteristics of large fluctuation processes well.
Some typical nonlinear elements including stochastic electrical load, hydraulic exciting force, etc., are considered.	The more complex hydro electromechanical relation cannot be contained.
The dynamic behaviors of the Hamiltonian system are researched by introducing random functions.	The introduced random function cannot truly describe the dynamic change of the hydropower system with time.

Motivated by the above discussions, we have three advantages which make our approach attractive, comparing with the prior work. First, we creatively introduce six

dynamic transfer coefficients of the hydro-turbine into the HEGS and consider the essence of the nonlinearity of the system, the dynamic mathematical model of the HEGS in the transient of sudden load increasing is established. Second, the mathematical model of the HEGS is converted to a novel Hamiltonian system by putting the HEGS to the theory frame of the generalized Hamiltonian system and using the method of orthogonal decomposition. Finally, we carry out numerical simulations and physical experiment based on the data of an existent hydropower plant. The detailed information of energy changing and the dynamic characteristics of the HEGS for the transient of sudden load increasing are exhaustively investigated.

The rest of the paper is organized as follows. In Section 2, a novel Hamiltonian mathematical model of the HEGS for the transient of sudden load increasing is established. The established Hamiltonian system is detailed discussed in Section 3. A comparison between the numerical and experimental simulations has been made to prove the validity of the proposed model of the HEGS in Section 4. Section 5 gives some useful conclusions. Discussion in Section 6 closes the paper.

Nomenclature			
M_t	the mechanical torque of the hydro-turbine, N.m	h_w	the characteristic coefficient of the pipeline
Q	the hydro-turbine flow, m ³ /s	T_{ab}	the mechanical starting time, s
H	the hydro-turbine head, m	P_m	the relative value of the output power of the hydro-turbine
n	the hydro-turbine speed, rad/s	P_{mc}	the relative value of the measured output power of the hydro-turbine
a	the guide vane opening, rad	e_{mx}, e_{my}, e_{mh}	the relative values of the hydro-turbine torque as regards the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head, p.u.
m_t	the relative value of the mechanical torque, p.u	e_{qx}, e_{qy}, e_{qh}	the relative values of the flow with respect to the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head, p.u.
h	the relative value of the hydro-turbine head, p.u	k_p	the proportional adjustment coefficient
x	the relative value of the hydro-turbine speed, p.u	k_i	the integral adjustment coefficient
q	the relative value of the hydro-turbine flow, p.u	k_d	the differential adjustment coefficient
q_c	the relative value of the measured hydro-turbine flow	u	the output of the regulator
y	the relative value of the guide vane opening, p.u.	m_e	the electromagnetic torque of the generator
x_1, x_2, x_3	the intermediate variables	P_e	the electromagnetic power of the generator
ω	the relative value of the generator rotor speed, p.u.	E'_q	the transient internal voltage of armature
δ	the relative value of the rotor angle, rad	V_s	the voltage of infinite bus
T_y	the engager relay time constant, s	x'_d	the direct axis transient reactance
T_r	the elastic water hammer time constant, s	x_q	the quartered axis reactanc
D	the damping coefficient	E_f	the output of excitation controller, p.u.
		H_1	the Hamitonian function for the hydro-turbine
		H_2	the Hamitonian function for the generator
		H	the Hamitonian function for the HEGS
		$J(x)$	the anti-symmetric matrix
		$P(x)$	the symmetric matrix
		X_f	the exciting winding reactance
		X_{ad}	the d axis armature reaction reactance

2 The Hamiltonian model of the HEGS in the transient of sudden load increasing

The HEGS composed of a control system and a controlled system as shown in

Fig. 1 [15, 23], contains the synchronous generator, the hydro-turbine, the governor and the penstock.

Fig. 1. The structure diagram of the HEGS.

2.1 The joint model of the hydro-turbine and the hydraulic speed regulation system

2.1.1 Dynamic transfer coefficients of the hydro-turbine

Generally, we obtain the mathematical model of the hydro-turbine utilizing the internal characteristics method, external characteristics method or simple analytical method. Here, the internal characteristics method is used due to its high accuracy [6, 18-19, 29].

The mathematical model of the HEGS including the transfer coefficients of the hydro-turbine is shown in Fig. 2 [15].

Fig. 2. The structure diagram of the HEGS including the transfer coefficients of the hydro-turbine.

Using the internal characteristics method, the dynamic characteristics of a Francis hydro-turbine [18] can be described as

$$\begin{cases} M_t = M_t(H, n, a) \\ Q = Q(H, n, a) \end{cases}, \quad (1)$$

where M_t , Q , H , n and a denote the mechanical torque of the hydro-turbine, the hydro-turbine flow, the hydro-turbine head, the hydro-turbine speed and the guide vane opening, respectively.

Using the Taylor series expansion, Eq. (1) can be rewritten as

$$\begin{cases} \frac{M_t - M_{t0}}{M_{tR}} = \frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{n}{n_R}} \frac{n - n_0}{n_R} + \frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{a}{a_{\max}}} \frac{a - a_0}{a_{\max}} + \frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{H}{H_R}} \frac{H - H_0}{H_R} \\ \frac{Q - Q_0}{Q_R} = \frac{\partial \frac{Q}{Q_R}}{\partial \frac{n}{n_R}} \frac{n - n_0}{n_R} + \frac{\partial \frac{Q}{Q_R}}{\partial \frac{a}{a_{\max}}} \frac{a - a_0}{a_{\max}} + \frac{\partial \frac{Q}{Q_R}}{\partial \frac{H}{H_R}} \frac{H - H_0}{H_R} \end{cases}, \quad (2)$$

where subscripts R and 0 denote the actual value of rated condition and the actual

value of steady condition, respectively. $\frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{n}{n_{\max}}} = \frac{\partial m_t}{\partial x}$, $\frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{a}{a_{\max}}} = \frac{\partial m_t}{\partial y}$, $\frac{\partial \frac{M_t}{M_{tR}}}{\partial \frac{H}{H_{\max}}} = \frac{\partial m_t}{\partial h}$,

$$\frac{\partial \frac{Q}{Q_R}}{\partial \frac{n}{n_{\max}}} = \frac{\partial q}{\partial x}, \quad \frac{\partial \frac{Q}{Q_R}}{\partial \frac{a}{a_{\max}}} = \frac{\partial q}{\partial y} \quad \text{and} \quad \frac{\partial \frac{Q}{Q_R}}{\partial \frac{H}{H_{\max}}} = \frac{\partial q}{\partial h}. \quad \text{Thus, Eq. (2) can be expressed as}$$

$$\begin{cases} \frac{\Delta M_t}{M_{tR}} = \frac{\partial m_t}{\partial x} \frac{n - n_0}{n_R} + \frac{\partial m_t}{\partial y} \frac{a - a_0}{a_{\max}} + \frac{\partial m_t}{\partial h} \frac{H - H_0}{H_R} \\ \frac{\Delta Q}{Q_R} = \frac{\partial q}{\partial x} \frac{a - a_0}{n_R} + \frac{\partial q}{\partial y} \frac{a - a_0}{a_{\max}} + \frac{\partial q}{\partial h} \frac{a - a_0}{H_R} \end{cases}. \quad (3)$$

Suppose that the guide vane opening a is equal to the main servomotor stroke Y in Eq. (3). The transfer coefficients of the hydro-turbine can be rewritten as

$$\begin{cases} m_t = e_{mw}x + e_{my}y + e_{mh}h \\ q = e_{qw}x + e_{qy}y + e_{qh}h \end{cases}, \quad (4)$$

where m_t , q , h , x and y represent the corresponding relative values of the hydro-turbine torque, the hydro-turbine flow, the hydro-turbine head, the hydro-turbine speed and the hydro-turbine guide vane opening, respectively. e_{mw} , e_{my} and e_{mh} respectively denote the relative values of the hydro-turbine torque as regards the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head. e_{qw} , e_{qy} and e_{qh} are the relative values of the flow with respect to the hydro-turbine speed, the hydro-turbine guide vane and the hydro-turbine head, respectively.

According to Eq. (4), the dynamic transfer coefficients of the hydro-turbine with time t for the transient of sudden load increasing are introduced, which are expressed as

$$\begin{cases} e_{my} = \frac{1}{5} \cos 4\pi t + 2e^{-t} + \frac{8}{5} \\ e_{mw} = \frac{1}{10} \cos 4\pi t + \frac{9}{10} e^{-t} - \frac{7}{10} \\ e_{mh} = \frac{4}{25} \cos 4\pi t - \frac{19}{10} e^{-t} + \frac{17}{10} \\ e_{qy} = \frac{2}{25} \sin 4\pi t + \frac{4}{5} e^{-t} + \frac{7}{5} \\ e_{qw} = \frac{1}{50} \sin 4\pi t + \frac{21}{100} e^{-t} - \frac{3}{20} \\ e_{qh} = \frac{1}{25} \sin 4\pi t - \frac{13}{30} e^{-t} + \frac{3}{5} \end{cases} . \quad (5)$$

2.1.2 Dynamic equation of the hydro-turbine output torque

Fig. 3. The mechanism of the hydro-turbine and the penstock system.

As demonstrated in Fig. 3 [15], considering the elastic water-hammer impact of the penstock, the transfer function between the hydro-turbine guide vane and the hydro-turbine output torque [29] is written as

$$G_t(s) = -\frac{e_{my}}{e_{qh}} \cdot \frac{es^3 - \frac{3}{h_w T_r} s^2 + \frac{24e}{T_r^2} s - \frac{24}{h_w T_r^3}}{s^3 + \frac{3}{e_{qh} h_w T_r} s^2 + \frac{24}{T_r^2} s + \frac{24}{e_{qh} h_w T_r^3}}, \quad (6)$$

where e is an intermediate variable, $e = \frac{e_{qy} e_{mh}}{e_{my}} - e_{qh}$. h_w and T_r are the

characteristic coefficient pipeline and the elastic water hammer time constant, respectively.

The state-space equations of Eq. (6) are shown as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -a_0x_1 - a_1x_2 - a_2x_3 + y \end{cases} . \quad (7)$$

The dynamic equation of the hydro-turbine output torque is described as

$$m_t = b_3y + (b_0 - a_0b_3)x_1 + (b_1 - a_1b_3)x_2 + (b_2 - a_2b_3)x_3, \quad (8)$$

and its power is

$$P_m = \omega m_t, \quad (9)$$

where $a_0 = \frac{24}{e_{qh}h_wT_r^3}$, $a_1 = \frac{24}{T_r^2}$, $a_2 = \frac{3}{e_{qh}h_wT_r}$, $b_0 = \frac{24e_{my}}{e_{qh}h_wT_r^3}$, $b_1 = \frac{24ee_{my}}{e_{qh}T_r^2}$,

$b_2 = \frac{3e_{my}}{e_{qh}h_wT_r}$ and $b_3 = -\frac{ee_{my}}{e_{qh}}$. x_1 , x_2 and x_3 represent the intermediate variables.

2.1.3 The mathematical model of the hydraulic speed regulation system

From Refs. [15, 18-19], the dynamic characteristics of the hydraulic speed regulation system can be expressed as

$$T_y \frac{dy}{dt} + y = u, \quad (10)$$

where T_y and u respectively denote the engager relay time constant and the output of the regulator.

Here, a universal parallel PID governor [6, 25-27] is considered. Subsequently, the dynamic characteristics of the hydraulic speed regulation system can be shown as

$$\frac{dy}{dt} = \frac{1}{T_y} \left(-k_p \omega - \frac{k_i}{\omega_0} \delta - k_d \dot{\omega} - y \right). \quad (11)$$

2.1.4 The Hamiltonian model of the hydro-turbine and the hydraulic speed regulation system

From the above discussions, the joint model of the hydro-turbine and the hydraulic speed regulation system for the transient of sudden load increasing is

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -\frac{24}{(\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})h_w T_r^3}x_1 - \frac{24}{T_r^2}x_2 - \frac{3}{(\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})h_w T_r}x_3 + y \\ \dot{q} = (\frac{1}{50}\sin 4\pi t + \frac{21}{100}e^{-t} - \frac{3}{20})\frac{\dot{\omega}}{2\pi} + (\frac{2}{25}\sin 4\pi t + \frac{4}{5}e^{-t} + \frac{7}{5})\dot{y} + (\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})\dot{h} \\ \quad + ((2\pi\cos(4\pi t))/25 - 21/(100e^t))\frac{\omega}{2\pi} + ((8\pi\cos(4\pi t))/25 - 4/(5e^{-t}))y \\ \quad + (13/(30e^t) + (4\pi\cos(4\pi t))/25)h \\ \dot{h} = ((\dot{m}_l + (9/(10e^{-t}) - (2\pi\sin(4\pi t))/5))\frac{\omega}{2\pi} - (\frac{1}{10}\cos 4\pi t + \frac{9}{10}e^{-t} - \frac{7}{10})\frac{\dot{\omega}}{2\pi} \\ \quad + (2/e^{-t} - (4\pi\sin(4\pi t))/5)y - (\frac{1}{5}\cos 4\pi t + 2e^{-t} + \frac{8}{5})\dot{y})(\frac{4}{25}\cos 4\pi t - \frac{19}{10}e^{-t} + \frac{17}{10}) \\ \quad - (19/(10e^{-t}) - (16\pi\sin(4\pi t))/25)(m_l - (\frac{1}{10}\cos 4\pi t + \frac{9}{10}e^{-t} - \frac{7}{10})\frac{\omega}{2\pi} \\ \quad - (\frac{1}{5}\cos 4\pi t + 2e^{-t} + \frac{8}{5})y)/(\frac{4}{25}\cos 4\pi t - \frac{19}{10}e^{-t} + \frac{17}{10})^2 \\ \dot{y} = \frac{1}{T_y}(u - y) \end{array} \right. \quad . \quad (12)$$

Subsequently, Eq. (12) is converted to an affine nonlinear function, which is

$$\dot{X} = f(x) + g(x)u, \quad (13)$$

where $\dot{X} = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{q}, \dot{h}, \dot{y}]^T$, $g(x) = [0, 0, 0, 0, 0, \frac{1}{T_y}]^T$,

and

$$f(x) = \begin{bmatrix} x_2 \\ x_3 \\ -\frac{24}{(\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})h_w T_r^3}x_1 - \frac{24}{T_r^2}x_2 - \frac{3}{(\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})h_w T_r}x_3 + y \\ (\frac{1}{50}\sin 4\pi t + \frac{21}{100}e^{-t} - \frac{3}{20})\frac{\dot{\omega}}{2\pi} + (\frac{2}{25}\sin 4\pi t + \frac{4}{5}e^{-t} + \frac{7}{5})\dot{y} + (\frac{1}{25}\sin 4\pi t - \frac{13}{30}e^{-t} + \frac{3}{5})\dot{h} \\ + ((2\pi\cos(4\pi t))/25 - 21/(100e^t))\frac{\omega}{2\pi} + ((8\pi\cos(4\pi t))/25 - 4/(5e^{-t}))y \\ + (13/(30e^t) + (4\pi\cos(4\pi t))/25)h \\ ((\dot{m}_t + (9/(10e^{-t}) - (2\pi\sin(4\pi t))/5))\frac{\omega}{2\pi} - (\frac{1}{10}\cos 4\pi t + \frac{9}{10}e^{-t} - \frac{7}{10})\frac{\dot{\omega}}{2\pi} \\ + (2/e^{-t} - (4\pi\sin(4\pi t))/5)y - (\frac{1}{5}\cos 4\pi t + 2e^{-t} + \frac{8}{5})\dot{y})(\frac{4}{25}\cos 4\pi t - \frac{19}{10}e^{-t} + \frac{17}{10}) \\ - (19/(10e^{-t}) - (16\pi\sin(4\pi t))/25)(m_t - (\frac{1}{10}\cos 4\pi t + \frac{9}{10}e^{-t} - \frac{7}{10})\frac{\omega}{2\pi} \\ - (\frac{1}{5}\cos 4\pi t + 2e^{-t} + \frac{8}{5})y)/(\frac{4}{25}\cos 4\pi t - \frac{19}{10}e^{-t} + \frac{17}{10})^2 \\ - y\frac{1}{T_y} \end{bmatrix}.$$

Utilizing the formula $g(x)^T \frac{\partial H_1}{\partial X} = -P_m$, the Hamiltonian function of the hydro-turbine is obtained as

$$H_1 = (T_y y(2b_0 x_1 + 2b_1 x_2 + 2b_2 x_3 + b_3 y - 2a_0 b_3 x_1 - 2a_1 b_3 x_2 - 2a_2 b_3 x_3))/2. \quad (14)$$

From Ref. [23], Eq. (13) is transformed into the Hamiltonian mathematical model, which is

$$\dot{X} = [J(x) + P(x)] \frac{\partial H_1}{\partial x} + g(x)u, \quad (15)$$

where $J(x)$ is the anti-symmetric matrix,

$$J(x) = \begin{bmatrix} 0 & \dot{J}_{12} & \dot{J}_{13} & \dot{J}_{14} & \dot{J}_{15} & \dot{J}_{16} \\ -\dot{J}_{12} & 0 & \dot{J}_{23} & \dot{J}_{24} & \dot{J}_{25} & \dot{J}_{26} \\ -\dot{J}_{13} & -\dot{J}_{23} & 0 & \dot{J}_{34} & \dot{J}_{35} & \dot{J}_{36} \\ -\dot{J}_{14} & -\dot{J}_{24} & -\dot{J}_{34} & 0 & \dot{J}_{45} & \dot{J}_{46} \\ -\dot{J}_{15} & -\dot{J}_{25} & -\dot{J}_{35} & -\dot{J}_{45} & 0 & \dot{J}_{56} \\ -\dot{J}_{16} & -\dot{J}_{26} & -\dot{J}_{36} & -\dot{J}_{46} & -\dot{J}_{56} & 0 \end{bmatrix}; \quad P(x) \text{ is the symmetric matrix,}$$

$$P(x) = \begin{bmatrix} p & 0 & 0 & 0 & 0 & 0 \\ 0 & p & 0 & 0 & 0 & 0 \\ 0 & 0 & p & 0 & 0 & 0 \\ 0 & 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 & 0 & p \end{bmatrix}. \text{ The detailed computational method for } J(x) \text{ and}$$

$P(x)$ can be obtained in Refs. [10-11, 23].

The symmetric matrix $P(x)$ is further decomposed into the following form [11]

$$P(x) = \frac{1}{\|\nabla H_1(x)\|^2} \langle f(x), \nabla H_1 \rangle = S(x) - R(x), \quad (16)$$

where

$$S(x) = \begin{bmatrix} s(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & s(x) & 0 & 0 & 0 & 0 \\ 0 & 0 & s(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & s(x) & 0 & 0 \\ 0 & 0 & 0 & 0 & s(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & s(x) \end{bmatrix}$$

and

$$R(x) = \begin{bmatrix} r(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & r(x) & 0 & 0 & 0 & 0 \\ 0 & 0 & r(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & r(x) & 0 & 0 \\ 0 & 0 & 0 & 0 & r(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & r(x) \end{bmatrix}. \quad \text{Therefore, the Hamiltonian}$$

mathematical model of the hydro-turbine is rewritten as

$$\dot{X}_1 = [J(x) + S(x) - R(x)] \frac{\partial H_1}{\partial x} + g(x)u. \quad (17)$$

2.2 The Hamiltonian mathematical model of the generator

In this paper, we consider a third-order mathematical model of the generator [18-19, 29], which is shown as

$$\begin{cases} \dot{\delta} = \omega_0(\omega - 1) \\ \dot{\omega} = \frac{1}{T_{ab}}(m_t - P_e - D(\omega - 1)) \\ \dot{E}'_q = -\frac{\omega_0}{T_d} \frac{x_{d\Sigma}}{x'_{d\Sigma}} E'_q + \frac{\omega_0}{T_d} \frac{x_{d\Sigma} - x'_{d\Sigma}}{x'_{d\Sigma}} V_s \cos \delta + \frac{\omega_0}{T_d} E_f \end{cases}, \quad (18)$$

where δ , ω , D , T_{ab} , E'_q , V_s , T_d , E_f , x'_d and x_q denote the rotor angle, the deviation of the generator rotor speed, the damping coefficient, the mechanical starting time, the transient internal voltage of armature, the voltage of infinite bus, the transient time constant of axis d , the output of excitation controller, the direct axis transient reactance and the quartered axis reactance, respectively.

Here, the electromagnetic power of the generator P_e is expressed as

$$P_e = \frac{E'_q V_s}{x'_{d\Sigma}} \sin \delta + \frac{V_s^2}{2} \frac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}} \sin 2\delta. \quad (19)$$

$x'_{d\Sigma}$ and $x_{q\Sigma}$ are calculated with the following formulas

$$\begin{cases} x'_{d\Sigma} = \dot{x}_d + x_T + \frac{1}{2} x_L \\ x_{q\Sigma} = x_q + x_T + \frac{1}{2} x_L \end{cases}, \quad (20)$$

where x_T and x_L represent the short-circuit reactance of transformer and the transmission line reactance, respectively.

From Ref. [10], the Hamiltonian mathematical model of the third-order generator is

$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \\ \dot{E}'_q \end{bmatrix} = \begin{bmatrix} 0 & C_1 & 0 \\ -C_1 & -C_D & 0 \\ 0 & 0 & -C_G \end{bmatrix} \begin{bmatrix} \frac{\partial H_2}{\partial \delta} \\ \frac{\partial H_2}{\partial \omega} \\ \frac{\partial H_2}{\partial E'_q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A & 0 \\ 0 & \frac{\omega_0}{T_d} \end{bmatrix} \begin{bmatrix} m_t \\ E_f \end{bmatrix}. \quad (21)$$

The Hamiltonian function of the generator is expressed as

$$H_2 = \frac{T_{ab}}{2} \omega^2 + \frac{V_s^2}{2} \frac{x_{q\Sigma} - x_{d\Sigma}}{x_{q\Sigma} x_{d\Sigma}} \cos^2 \delta + \frac{V_s^2}{2x_{q\Sigma}} + \frac{1}{2x'_{d\Sigma} x_{d\Sigma} X_f} (X_{ad} V_s \cos \delta - x_{d\Sigma} \frac{X_f}{X_{ad}} E'_q)^2, \quad (22)$$

where X_f and X_{ad} denote the exciting winding reactance and the d axis armature reaction reactance, respectively.

2.3 The Hamiltonian mathematical model of the HEGS

From the above analyses and discussions, we combine the different components into a single system. Thus, the mathematical model of the HEGS in the transient of sudden load increasing is obtained as

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -\frac{24}{(\frac{1}{25} \sin 4\pi t - \frac{13}{30} e^{-t} + \frac{3}{5}) h_w T_r^3} x_1 - \frac{24}{T_r^2} x_2 - \frac{3}{(\frac{1}{25} \sin 4\pi t - \frac{13}{30} e^{-t} + \frac{3}{5}) h_w T_r} x_3 + y \\ \dot{q} = (\frac{1}{50} \sin 4\pi t + \frac{21}{100} e^{-t} - \frac{3}{20}) \frac{\dot{\omega}}{2\pi} + (\frac{2}{25} \sin 4\pi t + \frac{4}{5} e^{-t} + \frac{7}{5}) \dot{y} + (\frac{1}{25} \sin 4\pi t - \frac{13}{30} e^{-t} + \frac{3}{5}) \dot{h} \\ \quad + ((2\pi \cos(4\pi t))/25 - 21/(100e^t)) \frac{\omega}{2\pi} + ((8\pi \cos(4\pi t))/25 - 4/(5e^{-t})) y \\ \quad + (13/(30e^t) + (4\pi \cos(4\pi t))/25) h \\ \dot{h} = ((\dot{m}_t + (9/(10e^{-t}) - (2\pi \sin(4\pi t))/5) \frac{\omega}{2\pi} - (\frac{1}{10} \cos 4\pi t + \frac{9}{10} e^{-t} - \frac{7}{10}) \frac{\dot{\omega}}{2\pi} \\ \quad + (2/e^{-t} - (4\pi \sin(4\pi t))/5) y - (\frac{1}{5} \cos 4\pi t + 2e^{-t} + \frac{8}{5}) \dot{y}) (\frac{4}{25} \cos 4\pi t - \frac{19}{10} e^{-t} + \frac{17}{10}) \\ \quad - (19/(10e^{-t}) - (16\pi \sin(4\pi t))/25) (m_t - (\frac{1}{10} \cos 4\pi t + \frac{9}{10} e^{-t} - \frac{7}{10}) \frac{\omega}{2\pi} \\ \quad - (\frac{1}{5} \cos 4\pi t + 2e^{-t} + \frac{8}{5}) y) / (\frac{4}{25} \cos 4\pi t - \frac{19}{10} e^{-t} + \frac{17}{10})^2 \\ \dot{y} = \frac{1}{T_y} (u - y) \\ \dot{\delta} = \omega_0 (\omega - 1) \\ \dot{\omega} = \frac{1}{T_{ab}} (m_t - P_e - D(\omega - 1)) \\ \dot{E}'_q = -\frac{\omega_0}{T_d} \frac{x_{d\Sigma}}{x'_{d\Sigma}} E'_q + \frac{\omega_0}{T_d} \frac{x_{d\Sigma} - x'_{d\Sigma}}{x'_{d\Sigma}} V_s \cos \delta + \frac{\omega_0}{T_d} E_f \end{array} \right. \quad (23)$$

The Hamiltonian mathematical model of the HEGS in the transient of sudden load

increasing is

$$\dot{X} = [J(x) + S(x) - R(x)] \frac{\partial H}{\partial x} + g(x)u, \quad (24)$$

where $\dot{X} = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{q}, \dot{h}, \dot{y}, \dot{\delta}, \dot{\omega}, \dot{E}'_q]$. H is the Hamiltonian function of the HEGS,

$$\begin{aligned} H &= H_1 + H_2 \\ &= (T_y y (2b_0 x_1 + 2b_1 x_2 + 2b_2 x_3 + b_3 y - 2a_0 b_3 x_1 - 2a_1 b_3 x_2 - 2a_2 b_3 x_3)) / 2 + \frac{T_{ab}}{2} \omega^2. \quad (25) \\ &\quad + \frac{V_s^2}{2} \frac{x_{q\Sigma} - x_{d\Sigma}}{x_{q\Sigma} x_{d\Sigma}} \cos^2 \delta + \frac{V_s^2}{2x_{q\Sigma}} + \frac{1}{2x'_{d\Sigma} x_{d\Sigma} X_f} (X_{ad} V_s \cos \delta - x_{d\Sigma} \frac{X_f}{X_{ad}} E'_q)^2 \end{aligned}$$

3 Numerical experiments

3.1 Experiment parameters of the HEGS

In this paper, we use the data of a real hydropower plant in China to investigate the dynamic characteristics and the detailed information of energy changing of the HEGS. Moreover, the PID parameters are $k_p = 5$, $k_i = 2.5$ and $k_d = 1.5$, and the basic parameters of the HEGS are shown in Tab. 2 [10-11, 23-24].

Tab. 2. Basic parameters of the HEGS

Basic	$x'_{d\Sigma}$	$x_{q\Sigma}$	T_d	T_y	V_s	X_{ad}	X_f	T_{ab}	D	T_r	h_w
parameters											
Values	0.6775	0.9975	5.4	0.5	1	0.97	1.29	8.999	5	0.8	2

3.2 Numerical experiments and dynamic analyzing of the Hamiltonian system

In this section, the numerical experiments of the established Hamiltonian system for the transient of sudden load increasing are carried out, and the corresponding numerical results are shown in Figure. 4 and Figure 5.

(a) Responses of the Hamiltonian function (H_1) and the output power (P_m) for the hydro-turbine.

(b) Responses of the Hamiltonian function (H_2) and the electromagnetic power (P_e) for the generator.

(c) Response of the Hamiltonian function (H) for the HEGS.

Fig. 4. Responds of the dynamic variables of the Hamiltonian system in the transient of sudden load increasing. (a) Responses of the Hamiltonian function (H_1) and the output power (P_m) for the hydro-turbine; (b) Responses of the Hamiltonian function (H_2) and the electromagnetic power (P_e) for the generator; (c) Response of the Hamiltonian function (H) for the HEGS.

Fig. 4 shows the corresponding responses of the dynamic variables for the hydro-turbine, the generator and the HEGS in the transient of sudden load increasing, respectively. From Fig. 4(a), for the hydro-turbine, the general trend of the Hamiltonian function (H_1) and output power (P_m) is increasing. Moreover, note that the apparent fluctuation of the output power (P_m) appears due to the selection of the dynamic transfer coefficients of the hydro-turbine. Fig. 4(b) shows, for the generator, its Hamiltonian function (H_2) and electromagnetic power (P_e) are in the same tendency, which both fluctuate in the whole transient of sudden load increasing. As shown in Fig. 4(c), the Hamiltonian function (H) of the HEGS increases gradually with time t in the transient of sudden load increasing, which is caused by the increase

of the output power of the generation system. All of the above phenomena mean that the Hamiltonian function of the established system can describe the main information of dynamic behaviors of the HEGS in the transient of sudden load increasing.

Fig. 5. Change laws of dynamic parameters of the established Hamiltonian system in the transient of sudden load increasing. (a) $\omega-t$; (e) $\delta-t$; (c) $\gamma-t$; (d) E'_q-t .

To deeply reveal the dynamic characteristics of the HEGS in the transient of sudden load increasing, the change laws of dynamic parameters of the established Hamiltonian system are shown in Fig. 5. It is clearly observed that, in the transient of sudden load increasing, due to the sudden changing of the load, the guide vane

opening increases gradually with time t . Subsequently, the hydro-turbine flow and the mechanical torque of the hydro-turbine both increase. It leads to the decreases of the pressure of the penstock and the internal transient voltage. Note that the reciprocating oscillation of the hydro-turbine rotor speed and the power-angle of the generator both appear and the hydro-turbine rotor speed fluctuates slightly. The reason is that the HEGS connects with the power system in the whole transient of sudden load increasing. All of the above phenomena are identical with the corresponding theories and engineering practices, which means that the established Hamiltonian system can better reflect the essence of the nonlinearity of the HEGS in the transient of sudden load increasing.

4 Model validation

In this section, a comparison between the numerical and experimental simulations has been made to prove the validity of the proposed model of the HEGS.

First, the basic system information of the real hydropower plant in China is shown in Tab. 3 [41].

Tab. 3. Basic system information of the real hydropower plant

The installed capacity of the hydro-turbine	150MW	The nominal output power	153MW
The hydro-turbine type	HL99.5-LJ-344.2	The nominal flow	53.5m ³ /s
The nominal speed	333.3rpm	The weighted average efficiency under design head	93.77%
The nominal head	312m		

Second, in order to obtain the experimental data, an experimental platform of the HEGS in the transient of sudden load increasing is built, as shown in Fig. 6.

Fig. 6. The experimental platform of the HEGS.

As demonstrated in Fig. 6, when the reservoirs water level raises from H_1 to H_2 , the hydro-turbine flow and the guide vane opening increase, and the system runs with the increasing load [15, 18]. In this case, through collecting the information of measuring points, the experimental data are obtained. Here, we change the experimental data from the absolute value to the relative value, as shown in Tab. 4.

Tab. 4. Experimental data of the HEGS

Time (s)	0	0.71	1.43	2.14	2.86	3.57	4.29	5
P_{mc}	0.4486	0.5624	0.6864	0.7976	0.9229	0.9305	1.0402	1.2
q_c	0.4398	0.5327	0.6368	0.7303	0.8376	0.848	0.9361	1.1

* P_{mc} denotes the relative value of the measured output power; q_c is the relative value of the measured hydro-turbine flow

Finally, the fitting curve of output power P_{mc} is acquired from the experimental data of P_{mc} . Based on the above work, we make a comparison between the numerical

output power P_m and experimental output power P_{mc} in the transient of sudden load increasing, as shown in Fig. 7.

Fig. 7. A comparison between the numerical output power P_m and experimental output power P_{mc} in the transient of sudden load increasing.

From Fig. 7, the general changing trend of numerical output power P_m is consistent with that for experimental output power P_{mc} , and they both increase in the transient of sudden load increasing. Note that the numerical output power P_m and experimental output power P_{mc} have some deviations for $0 < t < 0.47$. There are two fundamental factors could contribute to this phenomenon. The first is that the nonlinear system has higher sensitivity to initial conditions. The second is that the selection of transfer coefficients of the hydro-turbine has subtle error at $0 < t < 0.47$. However, it's worth mentioning that the details of the change for fitting curve of output power P_{mc} cannot be better shown because of the finite number of experimental data points. It means that the fitting curve of output power P_{mc} cannot

reflect the fluctuation of output power P_{mc} , while it can indicate the trend of output power P_{mc} . Therefore, we can draw a conclusion that the proposed function is valid.

Remark 1:

It is well known that, for the previous studies, the step disturbance is often introduced into the Hamiltonian system to reveal the transient dynamic characteristics of the system. To prove the superiority of the established Hamiltonian system for the transient of sudden load increasing, we have made a comparison between a previous model [23] with the step disturbance and the novel Hamiltonian mathematical model of the HEGS.

Fig. 8. The comparison between the novel and the previous Hamiltonian system with different step disturbances ($u=0.01$, $u=0.05$, $u=0.1$) in the transient of sudden load increasing.

From Fig. 8, for the novel Hamiltonian model, when $0 \leq t \leq 5$, the Hamiltonian function of the HEGS increases with time t . It is noteworthy that, for $0 \leq t \leq 3.5$, the

fluctuation of the Hamiltonian function of the HEGS is greater than that at the range of $3.5 < t \leq 5$. For the previous Hamiltonian system, the waveform of the Hamiltonian function of the HEGS fluctuates markedly with the increase of different step disturbances. The phenomenon indicates that the dynamic characteristics of the HEGS for the previous Hamiltonian system depend on the variation of the step disturbance. Moreover, for different step disturbances, the Hamiltonian functions of the HEGS all increase rapidly in a short time, and then they decrease gradually until $t=5$. In the actual operation of the hydropower station, the Hamiltonian function of the HEGS however increases continuously with the output power of the HEGS increases in the whole transient of sudden load increasing. At the final stage of the transient of sudden load increasing, the Hamiltonian function of the HEGS changes slightly. Therefore, the established Hamiltonian function has significant advantages to reflect the dynamic characteristics of the HEGS in the transient of sudden load increasing. That is to say, the established Hamiltonian system for the transient of sudden load increasing is reasonable.

5 Conclusions

In this paper, we have introduced six dynamic transfer coefficients of the hydro-turbine into the HEGS. From the point of view of the transmission and dissipation of energy of the system, we put the HEGS to the theory frame of the generalized Hamiltonian system. Considering the elastic water-hammer impact of the penstock and the nonlinearity of the generator, a novel Hamiltonian mathematical model of the HEGS for the transient of sudden load increasing has been established

using the method of orthogonal decomposition. Moreover, the numerical and experimental simulations of the established Hamiltonian system have been carried out based on the information of a real hydropower plant.

From the simulation results, we draw the following conclusions. First, the established Hamiltonian function of the HEGS contains detailed information of energy changing of the generation system in the transient of sudden load increasing, which is appropriate. Second, based on the novel Hamiltonian system, the dynamic characteristics of the system for the transient of sudden load increasing are better described. The responses of the dynamic parameters of the system are in good agreement with the corresponding theories and engineering practices. Thus, the established Hamiltonian system is reliable. Finally, we find that the novel Hamiltonian system can be better applied to the large fluctuation process from the comparison between the novel and the previous Hamiltonian system in the transient of sudden load increasing. These methods and results will provide new ideas to the research of the stability of the hydropower station.

6 Discussion

The Hamiltonian system which is usually used to carry on the theory research, provides the theoretical guidance for actual operation of the HEGS. Specifically, the theoretical guidance of the Hamiltonian system can be described as follows. Firstly, the Hamiltonian system reflects the energy dissipation and energy exchange with the outside environment, and it can be utilized to analyze the energy flow of the HEGS. Secondly, the Hamiltonian function which is treated as the Lyapunov candidate

function under some conditions, is used to research the stability of the generation system. Finally, based on the Hamiltonian system, the Hamiltonian controller of the HEGS can be designed to improve the operational stability of the hydropower station.

In our future work, we will attempt to transform the proposed generalized Hamiltonian system to the traditional Hamiltonian system. Meanwhile, we will in-depth investigate the energy characteristics and the dynamic behaviors of the Hamiltonian system in the start-up, shut-down and load rejection transient. Moreover, the different forms of the Hamiltonian function for different transition processes will be discussed.

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