Day-ahead Hourly Electricity Load Modeling by Functional Regression

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Abstract

Short-term load forecasting is important for power system generation planning and operation. For unit commitment and dispatch processes to incorporate uncertainty, a short-term load model must not only provide accurate load predictions but also enable the generation of reasonable probabilistic scenarios or uncertainty sets. This paper proposes a temporal and weather conditional epi-splines based load model (TWE) using functional approximation. TWE models the dependence of load on time and weather separately by functional approximation using epi-splines, conditional on season and area, in each segment of similar weather days. Load data are transformed from various day types to a specified reference day type among similar weather days in the same season and area, in order to enrich the data for capturing the non-weather dependent load pattern. In an instance derived from an Independent System Operator in the U.S., TWE not only provides accurate hourly load prediction and narrow bands of prediction errors, but also yields serial correlations among forecast hourly load values within a day that are similar to those of actual hourly load.

Keywords: Short-term load model, Forecasting, Day-ahead scenario, Epi-splines

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1. Introduction

Electricity load forecasting significantly influences planning and operation of power systems. Different models are used for long-term [1], medium-term [2], and short-term forecasting [3, 4]. Short-term load forecasting (STLF) applies to horizons from an hour to a day ahead, and forms the basis for unit commitment, economic dispatch, maintenance plans for generators, and electricity price forecasting for utilities and independent system operators [4, 5, 6]. Because of its essential role in power systems operations, inaccurate load forecasting can cause high operational and generation cost, equipment failure, or even system blackout. As emphasis on efficient and robust scheduling of thermal generators with uncertain load prediction increases, stochastic programming and robust optimization have been extensively studied for use in the operation of power systems [7, 8, 9, 10, 11, 12, 13, 14]. In the context of stochastic/robust unit commitment or economic dispatch by utilities or independent system operators, a regional short-term load model must provide not only accurate hour-by-hour point forecasts but also intervals and correlations to generate reasonable probabilistic scenario trajectories or ranges from which to form uncertainty sets.

A broad literature on STLF has developed for power systems of various scales. This paper focuses on regional short-term load forecasting. At the regional level, in addition to the nonstationarity in mean and variance as well as seasonal patterns, a number of external factors such as weather, time, economic activity, and social habits complicate the STLF problem with nonlinearity. Various modeling approaches have been proposed to address these difficulties. Most of the short-term load modeling approaches can be categorized as either classical statistical methods or machine learning (ML) methods. The former often presume that load is a function of several explanatory variables; e.g., previous load values, temperature and humidity, and then estimate parameters of the specified function. In contrast, ML methods do not restrict themselves to specified functions. The ML methods include artificial neural networks [15, 16] and support vector machines [17, 18, 19]. See [4, 20, 21] for broad reviews of ML methods. Hybrids of statistical and ML methods have also been applied recently [22, 23, 24, 25]. Similar day methods are often used by system operators and utilities to forecast future load and wind energy production [26, 27]. In these methods, historical days with weather and day type similar to the target day are identified, and the actual load of a similar day is taken as a forecast. Such methods are often

embedded in a more complicated load model, as in [16], because the methods do not sufficiently capture complex load features if used alone. Regarding smaller scale power systems including residential and commercial areas, as well as the building level, hybrids of multiple regression, time series, and ML based methods are often used [28, 29, 30].

Among statistical methods, time series and regression methods are widely used to build short-term load models. The autoregressive moving average (ARMA) model is one of the most frequently used time series methods [31, 32, 33, 34, 35, 36]. In addition to a Box-Jenkins time series model, Uri improved the short-term load prediction accuracy by taking into account price and weather change [31]. Amjady identified different ARMA load models for hot days and cold days while considering effects from weekdays and weekends on daily load patterns, as well as the temperature effect on load in [32]. Huang and Shih presented a modified univariate ARMA short-term load model by considering a non-Gaussian process in [33]. Taylor first established a univariate time series load model which considered within-day and within-week seasonality by using exponential smoothing in [34]. He then improved the load prediction by accounting for the intraday, intraweek, and intrayear effects of short-term load in Britain and France without external factors [35]. Liu et al. improved upon the previous work by estimating the effects of temperature, hour-of-day and type-of-day on load level using nonparametric regression, and employed an univariate ARMA model to model the load residuals caused by the nonparametric regression [36]. In general, time series methods predict well for the immediate future, but they may suffer from imprecision in the multi-step ahead predictions because of the accumulated prediction errors.

Regression on various explanatory variables is another major direction of short-term load modeling. Usually, regression models for load forecasting include weather conditions, day types, holidays, economic conditions, and social habits. The early literature considered only impacts of temperature and holidays on load [37, 38]. Haida and Muto applied transformations to include load changes in recent days and seasons, as well as annual load growth in a linear regression model [39]. Charytoniuk et al. considered load forecast as a local average of observed past loads within the local neighborhood and the specific weights on the loads defined by a multivariate product kernel [40]. Aldo et al. in [5] classified daily load curves by functional clustering, and then developed a family of functional linear regression models on the basis of obtained groups. Hong presented a multiple linear regression model

that considered load as a polynomial function of temperature for each hour in each day type in each month [4]. Black also applied multiple linear regression to examine the influence of weather on load in [41], but focused on summer weekdays in the region served by the Independent System Operator of New England (ISO-NE). In the most recent published results of competitive energy forecasting, Charlton and Singleton in [42] refined the multiple linear regression model of [4], and achieved small prediction errors and weighted root mean square error (WRMSE) simultaneously by taking into account multiple weather stations, day-of-season effects and smoother temperature forecasts. Fan and Hyndman proposed a semi-parametric additive model [43], which applied cubic splines to estimate the relationship between load and temperature as well as previous loads. Most of the multiple regression methods mentioned here achieved encouraging point predictions by establishing a load model for each hour (or each half hour) in a day but, with the exception of [43], did not take into account or assess the distributions of hourly forecasts.

Although a number of load backcasting studies have presented encouraging matches to actual loads using actual weather records as input, the associated load models may not be satisfactory for predicting a day-long trajectory of hourly load or for constructing probabilistic load scenarios or uncertainty sets for day-ahead unit commitment. Even if the model accurately captures the effects of actual weather on electricity demand, the imperfect day-ahead weather forecast could distort the corresponding load forecast. To ensure that the load forecast error distribution would reflect the imprecision in both weather forecast and the modeled relationship between weather and load, models were constructed based on the day-ahead weather forecast that is available to utilities and independent system operators [44, 45]. These models formed the basis for probabilistic scenarios as daylong trajectories of hourly loads [46]. Similarly to [44, 45], we propose a regional day-ahead 24-hour load prediction model based on the next day hourly weather forecast, but we assess the hourly non-weather dependent load component by means of an error-minimizing procedure instead of using average hourly load as in [46]. Due to its improved accuracy, the resulting load forecast is more appropriate for creating practical load scenarios.

The main contribution of this paper is a short-term load model that starts from a weather forecast and uses functional approximation rather than ordinary regression to model the complex nonlinearity between weather (i.e., temperature and dew point temperature) forecast and electricity consump-

tion levels in multiple geographic zones. By capturing the temporal load patterns in segments of enriched data, it not only accurately predicts hourly load values but produces serial correlations among them that are similar to those of actual hourly load trajectories. The model is supported by segmenting the input data according to seasonal calendar effects on electricity consumption and by enriching the data in each segment to enhance the prediction capability. Calendar months are grouped into seasons according to monthly load-temperature patterns using k -means clustering. Hourly load data for a specified day type are enriched using linear transformation among different day types. In a case study derived from ISO-NE, when applied to historical weather forecast data the proposed short-term load model leads to a narrower and less skewed distribution of prediction errors for each zone in each season, compared to two recent regression based short-term load models [4, 42].

The rest of the paper is organized as follows. After an overview of the data processing and modeling steps, Section 2 develops a weather conditional short-term load model by applying an epi-spline approximation technique. To demonstrate its regional application, Section 3 describes the partitioning of data into several segments according to calendar effects (day types and seasons), and enrichment by transformation to a reference day type. The numerical results in Section 4 indicate that our model captures the complex nonlinear relationship between electricity consumption level and weather conditions, and attains narrower and less skewed distributions of prediction errors, compared to those of two benchmark models. Section 5 concludes this paper.

2. Day-ahead hourly load model

In this paper, day-ahead hourly load models are developed for multiple zones within a region based on their corresponding weather forecasts. The temporal relationship between weather forecast and electricity consumption changes during the year because of its use for heating in winter and cooling in summer, as well as diurnal lighting patterns. As discussed in [4, 32, 36], the accuracy of load prediction could be improved if calendar month or season effects are considered in a load model. In this paper, calendar months with similar load-temperature patterns are grouped together to form an artificial season. In the same artificial season, days are further divided into three weather segments according to daily average temperature to enforce the sim-

150 ilarity in the same segment. Due to the limited load data records in each
 151 segment, and the similarity among daily load sequences in the same weather
 152 segment in the same season, load data are enriched by transforming vari-
 153 ous day types to the specified reference day type, to enhance the prediction
 154 capability. In addition to temperature, humidity and temporal dependence
 155 are other important factors to predict the 24-hour sequence of load values in
 156 the next day. The data analysis and transformation steps are detailed in the
 157 context of a case study in Section 3.

158 Motivated by the power of epi-splines to derive term and volatility struc-
 159 tures associated with financial markets [47], and their successful applications
 160 in numerous areas [48], in this paper epi-splines are applied separately in each
 161 segment of a season for each zone to model the complex nonlinear temporal
 162 dependence of load and effects of weather on load. Section 2.1 introduces
 163 epi-splines. Section 2.2 proposes a short-term load model based on weather
 164 forecast and temporal dependence.

165 2.1. Approximation by epi-spline

166 An epi-spline approximates a twice-continuous differentiable function $f(r)$
 167 over an interval $[0, \varrho]$ (note that any interval $[\alpha, \beta]$ can be transformed to such
 168 an interval). The whole interval $[0, \varrho]$ is partitioned into K equal subintervals
 169 $\mathcal{A}_k = (r_{k-1}, r_k]$ where $r_0 = 0$ and $r_k = r_{k-1} + \delta$, $k = 1, \dots, K$. By applying
 170 a piece-wise constant function

$$a(r) = a_k, r \in \mathcal{A}_k \quad (1)$$

171 to approximate the second derivative $f''(r)$ over each subinterval $\mathcal{A}_k$, the first
 172 derivative $f'(r)$, $r \in \mathcal{A}_k$, is approximated by

$$s'(r) = w_0 + \int_0^r a(\eta) d\eta = w_0 + \delta \sum_{i=1}^{k-1} a_i + (r - r_{k-1})a_k. \quad (2)$$

Furthermore, the function $f(r)$, $r \in \mathcal{A}_k$, is approximated by integrating

$s'(r)$:

$$\begin{aligned} s(r) &= v_0 + \int_0^r s'(\eta) d\eta = v_0 + \sum_{i=1}^{k-1} \int_{r_{i-1}}^{r_i} s'(\eta) d\eta + \int_{r_{k-1}}^r s'(\eta) d\eta \\ &= v_0 + w_0 r + \delta \sum_{i=1}^{k-1} \left(r - r_{i-1} + \frac{\delta}{2} \right) a_i + \frac{1}{2} (r - r_{k-1})^2 a_k \end{aligned} \quad (3)$$

where v_0 and w_0 are constants. In particular, if $r = r_k$, (3) can be simplified as:

$$s(r_k) = v_0 + r_k w_0 + \delta^2 \sum_{i=1}^k \left(k - i + \frac{1}{2} \right) a_i. \quad (4)$$

Supposing that the influence of a weather variable, such as temperature, on load can be described as a continuous differentiable function of time, an epi-spline can be used to model this effect by following (4). The following sections present a short-term load model that considers weather influences and temporal effects on electricity demand.

2.2. Epi-splines based load model

Throughout this paper, let $\mathcal{M}$ denote the set of day segments in a given season, $\mathcal{J}_m$ the set of days of a given segment $m \in \mathcal{M}$ in a given season in training set, and $\mathcal{H} = \{1, 2, \dots, 24\}$ the set of hours in a day. Let each hour in a day $j \in \mathcal{J}_m$ be divided into γ equal subintervals, with $\gamma = 1/\delta$. Let $l_{j,m}^h$ be the actual load in hour $h \in \mathcal{H}$ in day $j \in \mathcal{J}_m$ in segment $m \in \mathcal{M}$, and $t_{j,m}^h$ and $d_{j,m}^h$ be the corresponding temperature and dew point temperature forecasts from the day ahead, respectively.

2.2.1. Temporal and weather conditional epi-splines (TWE) based load model

The impacts from temperature and dew point temperature on electricity consumption for $h \in \mathcal{H}$ are approximated by epi-splines $s_{t,m}(h)$ and $s_{d,m}(h)$, respectively, by following (4). Because these weather variables may have different magnitudes of impact on electricity consumption in a specified season, the pre-specified constants μ_t and μ_d are applied to weight such impacts. The model proposed in this section estimates the non-weather dependent load component by including an additional epi-spline in an error-minimization model. Because the non-weather dependent load component usually changes

197 with time, the additional epi-spline, denoted by $s_m(h)$, is a function of time
 198 only for a specified segment $m \in \mathcal{M}$. The TWE model is given by:

$$l_{j,m}^h = s_m(h) + \mu_t s_{m,t}(h) t_{j,m}^h + \mu_d s_{d,m}(h) d_{j,m}^h + e_{j,m}^h, \forall h \in \mathcal{H}, \forall j \in \mathcal{J}_m \quad (5)$$

199 where $e_{j,m}^h$ represents the fitted model error in hour h of day j in segment
 200 m . The related parameters of the load model are identified by minimizing
 201 $\|e_{j,m}^h\|_1$, in a linear program as follows:

$$\min_{\mathbf{v}_0, \mathbf{w}_0, \mathbf{a}, \mathbf{b}, \mathbf{c}} \sum_{j \in \mathcal{J}_m} \sum_{h=1}^{24} \epsilon_{j,m}^h \quad (6)$$

$$\begin{aligned} \text{s.t. } \epsilon_{j,m}^h &\geq l_{j,m}^h - \left(v_{0,m}^h + h w_{0,m}^h + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) a_{i,m} \right) \\ &\quad - \mu_t \left(v_{0,m}^t + h w_{0,m}^t + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) b_{i,m} \right) t_{j,m}^h \\ &\quad - \mu_d \left(v_{0,m}^d + h w_{0,m}^d + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) c_{i,m} \right) d_{j,m}^h, \\ &\forall j \in \mathcal{J}_m, h \in \mathcal{H} \end{aligned} \quad (7)$$

$$\begin{aligned} \epsilon_{j,m}^h &\geq -l_{j,m}^h + \left(v_{0,m}^h + h w_{0,m}^h + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) a_{i,m} \right) \\ &\quad + \mu_t \left(v_{0,m}^t + h w_{0,m}^t + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) b_{i,m} \right) t_{j,m}^h \\ &\quad + \mu_d \left(v_{0,m}^d + h w_{0,m}^d + \frac{1}{\gamma^2} \sum_{i=1}^{\gamma h} (\gamma h - i + \frac{1}{2}) c_{i,m} \right) d_{j,m}^h, \\ &\forall j \in \mathcal{J}_m, h \in \mathcal{H} \end{aligned} \quad (8)$$

$$\epsilon_{j,m}^h \geq 0, \forall j \in \mathcal{J}_m, \forall h \in \mathcal{H} \quad (9)$$

202 Clearly, $\epsilon_{j,m}^h = |e_{j,m}^h|$ in (6) - (9). Once the model parameters $\mathbf{v}_0$, $\mathbf{w}_0$, $\mathbf{a}$,
 203 $\mathbf{b}$, and $\mathbf{c}$ are obtained, a day-ahead hourly load forecast for a specified day in
 204 the same segment is computed by applying formula (10) to the corresponding

205 temperature and dew point temperature forecast,

$$\hat{l}_{j,m}^h = s_m(h) + \mu_t s_{t,m}(h) t_{j,m}^h + \mu_d s_{d,m}(h) d_{j,m}^h, \forall h \in \mathcal{H}, \forall j \in \mathcal{J}'_m \quad (10)$$

206 where $\mathcal{J}'_m$ represents the set of days in segment $m \in \mathcal{M}$ in test set.

207 Parameters in the load model (5) can also be estimated by minimizing
 208 $\|e_{j,m}^h\|_2$ or $\|e_{j,m}^h\|_\infty$. The minimization of the fitted errors under L_2 norm
 209 results in a quadratic program but minimizing $\|e_{j,m}^h\|_\infty$, the maximum fitted
 210 error over the training set, leads to a linear program. The resulting linear
 211 program is modified by replacing $\epsilon_{j,m}^h$ with single decision variable ϵ in (7)
 212 - (9), and $\min_{v_0, w_0, a, b, c} \epsilon$ in (6). This paper presents load forecast results from

213 adopting the L_1 norm only, because it achieved greater accuracy in our tests.

214 An alternative approach to modeling the non-weather dependent load is to
 215 replace $s_m(h)$ by corresponding average load in hour h , as in [45]. However,
 216 TWE outperforms that method according to prediction accuracy and its
 217 stability.

218 3. Data analysis and processing

219 Our short-term load model, TWE, is applied to historical hourly load and
 220 corresponding weather forecast data in ISO-NE. Because the relationships
 221 between load and predictor variables; e.g., weather conditions, may gradually
 222 change over time, TWE is trained on a limited training period which ranged
 223 from April 2009 to March 2011, and validated on a test data set covering
 224 April 2011 to March 2012.

225 To improve the load prediction accuracy, seasons and segments of simi-
 226 lar weather days in the same season are identified in Sections 3.1 and 3.2.
 227 Section 3.3 describes enrichment of load data to enhance the load prediction
 228 capability for the specified reference day. Section 3.4 discusses the other
 229 important factors considered in the load model.

230 3.1. Division of seasons

231 ISO-NE includes eight zones in six states. Among these zones, Con-
 232 necticut (CT) accounts for about 25%, the greatest share of electricity con-
 233 sumption in ISO-NE; North Central Massachusetts (NE) claims about 20%,
 234 West Central Massachusetts (WC) 14%, Southeast Massachusetts (SE) 12%,
 235 Maine (ME) 9%, New Hampshire (NH) 9%, Rhode Island (RI) 7%, and Ver-
 236 mont (VT) 4% of the load, respectively. Figure 1 presents hourly load and

237 temperature variation ranges of these zones from April 2009 to March 2011.
 238 According to Fig. 1, zones CT and NE account for nearly half of the load
 239 in ISO-NE, and have a wider range than the other zones. The data analysis
 240 results are illustrated for CT because it has the largest and most variable
 241 loads. All eight zones in ISO-NE have similar temperature ranges.

Figure 1: Hourly zonal load and temperature profiles in ISO-NE (Apr. 2009 - Mar. 2011)

242 To investigate which months can be grouped in the same season, scatter
 243 plots of electricity consumption level vs. temperature forecast are plotted in
 244 Fig. 2 for zone CT. The trend for each month is estimated by using simple
 245 linear regression of the actual load on the forecast temperature in the training
 246 set for each zone. The other zones have the similar trend observations to
 247 those in CT. In this paper, calendar months with similar linear trends of
 248 electricity consumption based on temperature are grouped together to form
 249 seasons. The temperature-based seasons of spring, summer, fall, and winter,
 250 are identified by applying k -means clustering on the slopes of the regression
 251 lines. Although electricity consumption may be more accurately captured
 252 by a nonlinear function of temperature in each month, the linear trends
 253 appear sufficient for clustering similar months. In Fig. 2, it appears that
 254 electricity consumption will quickly increase as temperature rises in summer
 255 (June through September), while decreasing with increasing temperature in
 256 winter (December through March).

257 3.2. Days segmentation

258 A short-term load model could be built for each season. However, Fig. 2
 259 illustrates that the temperature varies in a wide range even in the same

Figure 2: Scatter plot of load vs. temperature for each month in the training set, CT (Apr. 2009 - Mar. 2011)

260 season. It is expected that the forecasting accuracy of a load model could be
 261 improved if days with similar temperature ranges in the same season were
 262 grouped together, as their daily load patterns could be similar. In this paper,
 263 all days in a season are divided into 3 segments: low, moderate, and high
 264 temperature days. A day is classified to one of the three segments according
 265 to the shortest L_2 distance between the day's average temperature and the
 266 three centroids identified by k -means clustering on the set of mean daily
 267 temperatures. Each of the resulting three segments covers all day types in a
 268 week. Because k -means clustering is an unsupervised learning method, it is
 269 appropriate to verify the clustering results by investigating the similarity in
 270 the same cluster as well as the differences among clusters. Fig. 3 displays the
 271 average daily load and temperature trajectories for each segment in summer.
 272 The widths of the corresponding 95% confidence intervals are smaller than
 273 200 MWh for the load trajectories, and smaller than 2 F for the temperature
 274 trajectories. The average daily temperature trajectory of each segment of
 275 days can be easily differentiated from the others. Moreover, the average daily
 276 load trajectories of the three segments differ significantly. Similar results are
 277 observed in the other zones for each season. Heating degree days (HDD) and
 278 cooling degree days (CDD) were also investigated as means to distinguish
 279 seasons and segment days. Because they resulted in the same seasons and
 280 similar segments to those discussed above, we did not include HDD/CDD in
 281 the short-term load model.

282 3.3. Load data transformation

283 Many short-term load modeling approaches [4, 32, 36, 43] identify an
 284 hourly load model for each day type but they require a large amount of
 285 historical data. A model that yields more reliable predictions may be built
 286 by using only recent data that better reflect current and near-future economic
 287 and demographic conditions; however, the quantity of such data is limited.
 288 This section presents a way to explore daily load pattern in a limited data
 289 set.

290 Average hourly load sequences for each day of the week in the moderate
 291 temperature segment are plotted for each season in Fig. 4, which shows that
 292 the load pattern in each day type in the same season tends to be similar,
 293 although the load levels in weekend days are lower than those in weekdays.
 294 Based on this similarity in load shapes, we enrich the data in each segment
 295 by applying a transformation among day types for each season. Here we
 296 detail the data transformation procedure briefly mentioned in [45].

Figure 3: Average daily load and temperature trajectories for three segments in summer, CT (Apr. 2009 - Mar. 2011)

297 Let $\mathcal{D}$ be the set of days of a specified day type in a segment within a
 298 season, and $\mathcal{D}'$ be the set of days of another day type in the same segment
 299 of the same season. Suppose $t_\omega \in \mathbb{R}^{24}$ is a sequence of hourly temperature
 300 forecast values in a day $\omega \in \mathcal{D}$, and let $\psi_o : \mathbb{R}^{24} \rightarrow \mathbb{R}^{24}$ be an operator that
 301 maps t_ω to the corresponding sequence of hourly load $l_\omega \in \mathbb{R}^{24}$:

$$l_\omega = \psi_o(t_\omega) + e_\omega, \forall \omega \in \mathcal{D} \quad (11)$$

302 so that $\|e_\omega\|$ is minimized over $\mathcal{D}$. Then

$$\tilde{l}_{\omega'} = \psi_o(t_{\omega'}), \forall \omega' \in \mathcal{D}' \quad (12)$$

303 simulates a possible sequence of hourly loads $\tilde{l}_{\omega'}$, if the same sequence of
 304 temperature forecasts of $\omega' \in \mathcal{D}'$ were applied to $\mathcal{D}$. Once the simulated
 305 possible sequences of hourly load for each day in day type set $\mathcal{D}'$ are obtained,
 306 a transformation between $\{\tilde{l}_{\omega'}\}$ and $\{l_{\omega'}\}$ can be found.

307 There are many possible approaches to transforming $l_{\omega'}$ to $\tilde{l}_{\omega'}$, but for
 308 easily transforming $\tilde{l}_{\omega'}$ back to $l_{\omega'}$ for the purpose of prediction in the original
 309 scale, the transform function is restricted to be linear:

$$\tilde{l}_{\omega'} = \Lambda l_{\omega'} + \zeta_{\omega'}, \forall \omega' \in \mathcal{D}' \quad (13)$$

where Λ is a matrix obtained by minimizing the error vectors $\{\zeta_{\omega'}\}$ under a specified norm; i.e., $\|\cdot\|_{L_1}$. In addition, the transformation matrix is further restricted to be diagonal to avoid singularity when computing Λ^{-1} . Although any day in a week can be viewed as a reference day type, we use Wednesday as the reference day-type to minimize the influences of weekends.

Figure 4: Average daily load sequence in each day type in moderate temperature segment in Apr. 2009 - Mar. 2011, CT

By enriching data for a specified data type, more diverse weather forecast records and their corresponding loads enhance the model prediction reliability for the specified day type. Once load forecasts for the specified day-type are generated, load forecasts for the other day types can be generated according to the linear transformation Λ^{-1} . The linear transformations on load values between the specified day type and the other day types have certain transformation errors, which are accounted for in the prediction errors.

3.4. Important load forecasting factors

In addition to temperature, some other factors can also influence the electricity demand level. In Fig. 5, scatter plots of hourly load and the corresponding humidity forecast, measured by dew point temperature, are plotted for different temperature ranges in summer. The electricity consumption increases with the forecast dew point temperature across days with similar temperature forecasts. Therefore, dew point temperature is included as another factor in the short-term load model in summer. Such a relationship

330 does not appear in other seasons, but other factors, such as wind speed and
 331 cloud cover, might be included if data are available.

Figure 5: Load vs. dew point temperature (humidity) forecast for hours 15 ~ 16 within specified range of temperature forecast in summer, CT

332 While weather is an important contributor to electricity consumption,
 333 the STLTF model must also account for temporal patterns. Fig. 6 illustrates
 334 how the temporal patterns of both temperature and load vary by season.
 335 These variations confirm the need for the additional epi-spline, $s_m(h)$, to
 336 approximate the temporal dependence separately in each segment of each
 337 season.

338 4. Case study

339 Hourly load data and the corresponding weather forecasts were collected
 340 from each zone in ISO-NE for the time period ranging from April 2009 to
 341 March 2012. As the previous papers have discussed, public holidays must
 342 be differentiated from regular weekdays because they have different electric-
 343 ity consumption patterns. In this paper, New Year's Day, Memorial Day,
 344 Independence Day, Labor Day, Thanksgiving, and Christmas are all consid-
 345 ered as Sunday [4]. Moreover, because of power system outages that resulted
 346 from the extreme weather conditions, days ranging from 2010-2-5 to 2010-2-
 347 7, 2011-8-28 to 2011-8-31, and 2011-10-27 to 2011-11-4 were not considered
 348 in the short-term load model. Experimentation revealed that a slight shift in

Figure 6: Average daily load and average daily temperature trajectories in Apr. 2009 - Mar. 2011, CT

Table 1: Refined seasons

Season name	Range
Spring (Spr.)	Apr.1 - May.14
Summer (Sum.)	May.15 - Sep.14
Fall (Fal.)	Sep.15 - Nov.30
Winter (Win.)	Dec.1 - Mar.31

the division points between the four seasons, as described in Table. 1, would improve the prediction accuracy of TWE.

In the model training process, weights μ_t and μ_d were set to 0.5 for summer; otherwise, μ_d was set to 0 because dew point temperature does not drive load in the other seasons. In the validation procedure, the trained model TWE was applied to obtain 24-hour load predictions for all 8 zones in ISO-NE all at once for each day, given the day-ahead 24-hour weather forecast. Model TWE was implemented in Python and Pyomo [49] on a Dell laptop with 6GB RAM.

In practice, polynomials are frequently used to depict the nonlinear relationships between load and weather conditions [50, 51, 52]. Two recent representative regression based short-term load models [4, 42] are considered as benchmarks to show the competitive performance of TWE in capturing the nonlinear relationship between temperature forecast and load, while attaining a narrower band of prediction errors. Both benchmark approaches

364 identify a load model for each hour in a day type for each month or each
 365 season. The two load models are given in (14) and (15), and called the Hong
 366 and Charlton & Singleton (C&S) models, respectively. The Hong model is
 367 given by:

$$\begin{aligned}
 l_j^h = & \beta_0 + \beta_1 T + \beta_2 D_{i(j)} H_h + \beta_3 M_{k(j)} + \beta_4 M_{k(j)}(t_j^h) + \beta_5 M_{k(j)}(t_j^h)^2 \\
 & + \beta_6 M_{k(j)}(t_j^h)^3 + \beta_7 H_h(t_j^h) + \beta_8 H_h(t_j^h)^2 + \beta_9 H_h(t_j^h)^3 + e_j^h, \\
 & \forall j \in \mathcal{J}_H, h \in \mathcal{H}
 \end{aligned} \tag{14}$$

368 where T is the load increasing trend, and $\mathcal{J}_H$ is the days set in training
 369 set. $D_{i(j)}, i(j) = 1, \dots, 7$, $H_h, h = 1, \dots, 24$ and $M_{k(j)}, k(j) = 1, \dots, 12$ are
 370 dummy variables representing 7 day types in a week, 24 hours in a day, and
 371 12 months in a year, respectively.

372 The C&S model divides a year into four seasons: spring, summer, fall
 373 and winter; and divides days into two types: weekday and weekend. For
 374 each hour of a day type in a season, it is given as follows:

$$\begin{aligned}
 l_j^h = & \alpha_1 + \alpha_2 \tau_j + \alpha_3(t_j^h) + \alpha_4(t_j^h)\tau_j + \alpha_5(t_j^h)^2 + \alpha_6(t_j^h)^2\tau_j + \alpha_7\tau_j' + \alpha_8(t_j^h)\tau_j' \\
 & + \alpha_9(t_j^h)^2\tau_j' + e_j^h, \forall j \in \mathcal{J}_{CS}, h \in \mathcal{H}
 \end{aligned} \tag{15}$$

375 where $\mathcal{J}_{CS}$ denotes the days set of a given day type in a specified season,
 376 τ_j the day number in the sequence of days, and τ_j' the day number within
 377 the specified season. The above two benchmark models, (14) and (15), were
 378 both implemented in R. Charlton and Singleton employed several other ap-
 379 proaches to improve performance of the short-term load model, including
 380 using forecasts from multiple weather stations instead of one, applying a
 381 local average technique, and smoothing the temperature forecast [42]. To
 382 compare models on the same basis, we did not incorporate these techniques
 383 to forecast ISO-NE load in the test set. However, these useful techniques
 384 could be applied to improve the accuracy of hourly load forecasting in future
 385 work.

386 Because of the flexibility of epi-splines to model high nonlinearity, we
 387 expect that TWE could provide higher quality hourly load prediction than
 388 ordinary polynomial regression models do. To test this hypothesis, we mea-
 389 sure prediction quality in terms of prediction accuracy, stability of prediction

accuracy, and skewness of prediction error distributions. Similarly to [53, 42], weighted error measurements are applied to investigate the quality of short-term load prediction in this paper. Weighted mean absolute percentage error (WMAPE) is used to emphasize the prediction accuracy in the hours in which most electricity is consumed. Weights $\{\rho_h\}, h \in \mathcal{H}$, are calculated as:

$$\rho_h = \frac{\sum_{j \in \mathcal{J}'} l_j^h}{\sum_{j \in \mathcal{J}'} \sum_{h \in \mathcal{H}} l_j^h}, \forall h \in \mathcal{H} \quad (16)$$

where $\mathcal{J}'$ denotes the set of test days. Following (16), WMAPE is defined as:

$$\text{WMAPE} = \frac{\sum_{j \in \mathcal{J}'} \sum_{h \in \mathcal{H}} \rho_h |e_j^h|}{\sum_{j \in \mathcal{J}'} \sum_{h \in \mathcal{H}} \rho_h}. \quad (17)$$

Weighted root mean square error (WRMSE) is applied to investigate the stability of prediction accuracy. The WRMSE is defined similarly as in [53]:

$$\text{WRMSE} = \sqrt{\frac{\sum_{j \in \mathcal{J}'} \sum_{h \in \mathcal{H}} \rho_h (e_j^h)^2}{\sum_{j \in \mathcal{J}'} \sum_{h \in \mathcal{H}} \rho_h}}. \quad (18)$$

Densities of prediction errors are displayed to check for skewness.

Figure 7 displays WMAPEs for each zone in ISO-NE for all seasons in the test set. In spring, summer, and fall, TWE provides the most accurate predictions in nearly all of the zones. Compared to Hong and C&S models, TWE better captures load consumption seasonality and the complex non-linear relationship between load and weather in these seasons. In winter, hourly load prediction from TWE is slightly less accurate than the better one of the other two methods. The lower accuracy of TWE in winter results from its lower prediction accuracy in the high temperature segment. In contrast, both of the benchmark approaches, build a model for each hour in each month/season. C&S model even takes into account the order a day takes in a season. This strategy results in more accurate hourly load prediction in high temperature segment of winter days. Although TWE does not predict as accurately as the winner between Hong and C&S models in winter, TWE demonstrates the most accurate prediction overall.

WRMSEs for each zone are presented in Fig. 8. TWE results in smaller WRMSE in most of the zones, especially in the highest consumption zones, CT and NE. In zone NH, the WRMSE from TWE is higher than that of the

Figure 7: WMAPEs of TWE and other two benchmark models in Apr. 2011 - Mar. 2012

417 Hong model, which is mainly caused by the less accurate prediction of TWE
 418 in the high temperature segment in winter. The prediction errors of TWE in
 419 hours 10 to 21 in a warm winter day are smaller than those from the Hong
 420 model, but they are larger at night. Outside zone NH, zonal load prediction
 421 accuracy of TWE is more stable than the other two benchmark models as
 422 measured by WRMSE.

Figure 8: WRMSEs of TWE and other two benchmark models in Apr. 2011 - Mar. 2012

423 Prediction error distributions, in the form of densities fitted by kernel
 424 density estimation in R, are plotted at three-hour intervals in Fig. 9. To save
 425 space they are presented for CT only, but similar results were observed in
 426 the other load zones. Compared to TWE, the benchmark models produce
 427 flatter prediction error densities with longer and thicker tails, especially in
 428 the daytime hours 10 to 21, during which most residential, commercial, and
 429 industrial electricity demands occur. In addition, the densities of errors from
 430 Hong and C&S are more skewed to either the left or the right than those
 431 from TWE during the daytime. At night, all of the three models share very
 432 similar densities of prediction errors.

433 According to the above comparisons at zonal level, TWE generally has
 434 the best prediction accuracy in all zones, all seasons, except for winter. In
 435 contrast to Hong and C&S models, TWE also provides more stable prediction
 436 accuracy in high-load zones, and similarly stable prediction accuracy in the
 437 other zones. TWE provides less biased predictions in most zones and hours.

438 The performance of TWE, Hong and C&S models are also compared at
 439 the regional level. Fig. 10 summarizes the distributions of prediction errors

Figure 9: Densities of prediction errors in summer in 2011 - 2012 for TWE and the benchmark models: zone CT

440 in the test set for each model over ISO-NE. TWE yields narrower and less
 441 skewed distributions of prediction errors, compared to the benchmark models.

Figure 10: Box plot of prediction errors of TWE and two benchmark models for ISO-NE in test set (Apr. 2011 - Mar. 2012)

442 The autocorrelation function (ACF) is commonly used to compute serial
 443 correlations of a time series, but may be distorted in this application when
 444 lags span parts of consecutive days to which different weather forecasts apply.
 445 To restrict the attention to only the serial correlations among load values in
 446 the same day, a modified ACF, called intraday ACF (InACF), is applied to
 447 measure the serial correlations. The sample InACF at lag κ for load forecast
 448 $\{\hat{l}_j^h\}_{\mathcal{J}'}$ from a specified load model is calculated as:

$$\hat{\lambda}_\kappa = \frac{\sum_{j \in \mathcal{J}'} \sum_{h=1}^{24-\kappa} (\hat{l}_j^h - \hat{\bar{l}}) (\hat{l}_j^{h+\kappa} - \hat{\bar{l}})}{\sum_{j \in \mathcal{J}'} \sum_{h=1}^{24} (\hat{l}_j^h - \hat{\bar{l}})^2}, \kappa = 1, 2, \dots, 23 \quad (19)$$

449 where $\hat{\bar{l}}$ is the average value of series $\{\hat{l}_j^h\}$.

450 The sample InACF, $\hat{\lambda}'_\kappa$, $\kappa = 1, 2, \dots, 23$, is also calculated for actual
 451 hourly load in the test set for comparison, as displayed in Fig. 11. The
 452 InACF in Fig. 11 indicates that daily load trajectories from TWE and the
 453 two benchmark models have very similar serial correlations to those from
 454 the actual load in a day. The mean absolute errors are plotted by hour in

455 Fig. 12 to check whether the forecast daily load trajectories are realistic.
 456 From Fig. 12, TWE predictions are closer to the actual trajectory in spring,
 457 summer, and fall, in hours 10 to 21. In winter, TWE produces similar daily
 458 forecast load trajectory to the one from C&S. The daily load trajectory from
 459 TWE is better than that from the Hong model in most of the hours between
 460 10 and 21, but worse in hours 17 to 19.

461 Because TWE provides more accurate predictions than the benchmark
 462 models, as well as more stable prediction accuracy and less skewed predic-
 463 tion intervals overall both at the zonal level and the ISO-NE level, it is
 464 expected that the total cost of unit commitment and economic dispatch that
 465 result from the day-ahead hourly load trajectory could satisfy the economic
 466 requirement of utilities and independent system operators. In addition, prob-
 467 abilistic scenarios for the next day's 24-hour load sequence would be more
 468 realistic, because TWE also emphasizes the temporal pattern.

Figure 11: InACF of actual and forecast daily load trajectory of ISO-NE in test set (Apr. 2011 - Mar. 2012)

469 5. Conclusion

470 In this paper, a short-term load model, TWE, is developed to use weather
 471 forecasts as input and capture temporal patterns using functional approxi-
 472 mation. Two recent representative regression short-term load models are
 473 selected to compare distributions of prediction errors. Compared to the

Figure 12: Mean absolute errors in daily load trajectory of ISO-NE in test set (Apr. 2011 - Mar. 2012)

474 benchmarks, TWE not only obtains less biased and narrower distributions
 475 of prediction errors, but also preserves similar serial correlations among fore-
 476 casted hourly load values to the actual ones. From the perspective of accuracy
 477 in each hour and relationships among hourly loads, TWE is expected to be
 478 appropriate for both providing regional day-ahead hourly load prediction for
 479 power system generation, and generating practical regional load scenarios
 480 for day-ahead stochastic unit commitment given temperature and dew point
 481 temperature forecasts. Comparing the commitment cost and expected gen-
 482 eration cost that result from the generated load scenarios to the cost based
 483 on the actual load is an important criterion for measuring the quality of
 484 scenarios, which will be addressed in future work.

485 The quality of load prediction by TWE in winter can be further improved
 486 from two aspects: including more external factors and improving the model.
 487 Prediction accuracy might be improved by considering the chilling effect from
 488 wind speed in winter and mitigating effect of sunshine in the TWE model.
 489 The additive structure of TWE allows more factors to be considered easily.
 490 In addition, the gradient boosting framework [54, 55] could be combined with
 491 TWE, so that the weather dependent load component would be better fitted
 492 with a sequence of weak weather predictors.

493 TWE can also be applied to power systems with deep penetration of
 494 behind-the-meter distributed generation, but several other factors must be

495 included in the model when solar and wind energy are the major sources of
496 distributed generation. These additional external factors include sunshine
497 hours, cloud cover, rainfall, snow, and wind speed. On the other hand,
498 modeling load in the presence of price-based demand response would require
499 a more sophisticated approach to capture the demand-price relationship.

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