Title: A novel multi-model probability battery state of charge estimation approach for electric vehicles using H-infinity algorithm

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A novel multi-model probability battery state of charge estimation approach for electric vehicles using H-infinity algorithm

Abstract: Due to the strong nonlinearity and complex time-variant property of batteries, the existing state of charge (SOC) estimation approaches based on a single equivalent circuit model (ECM) cannot provide the accurate SOC for the entire discharging period. This paper aims to present a novel SOC estimation approach based on a multiple ECMs fusion method for improving the practical application performance. In the proposed approach, three battery ECMs, namely the Thevenin model, the double polarization model and the 3rd order RC model, are selected to describe the dynamic voltage of lithium-ion batteries and the genetic algorithm is then used to determine the model parameters. The linear matrix inequality-based H-infinity technique is employed to estimate the SOC from the three models and the Bayes theorem-based probability method is employed to determine the optimal weights for synthesizing the SOC_s estimated from the three models. Two types of lithium-ion batteries are used to verify the feasibility and robustness of the proposed approach. The results indicate that the proposed approach can improve the accuracy and reliability of the SOC estimation against uncertain battery materials and inaccurate initial states.

Keywords: Electric vehicles, batteries, state of charge estimation, multi-model probability, H-infinity.

1. Introduction

Lithium-ion batteries have drawn wide attention and enthusiasm of many researchers due to their high energy and power density which can meet the power demand of electric vehicles (EVs) [1, 2]. To avoid abuse of batteries in EVs, the state of charge (SOC) is recognized as a crucial parameter to be monitored and used to manage the safe and efficient operation of batteries for battery system life extension. Nevertheless, the SOC is an intrinsically implicit state of the battery that cannot be measured directly and is vulnerable to diverse types of battery materials and operating conditions [3, 4].

1.1 Review of the SoC estimation approach

With the development of battery management technologies, a large number of methods have been put forward to accurately estimate the SOC and each one has its own merits. Tab. 1 illustrates a systematical classification and comparison mainly for the existing SOC estimation methods. These methods can be roughly divided into four groups, namely look-up table methods, ampere-hour methods, model-based methods and data-driven methods. Look-up table methods are simple but not suitable for online calculation and require regular recalibration for battery open circuit voltage (OCV), electrochemical impedance spectroscopy (EIS), etc [5–8]. Ampere-hour methods are widely employed because of their ease of implementation and low computational cost. However, such open-loop methods can incur accumulative errors due to uncertain disturbances, such as ambient noises, current drift and erroneous initial values [9, 10].

Model-based methods can overcome the drawbacks associated with the above-mentioned two methods and provide the robust performance for the SOC estimation

owing to the closed-loop feedback mechanism [11–14]. Ref. [12] proposed a Thevenin battery model based battery SOC estimation through the adaptive extended Kalman filter (AEKF) algorithm, Ref. [13] employed Sigma-point Kalman filtering built a model-based battery SOC estimator, Refs. [14–16] employed Kalman filters built model-based battery SOC estimators. Results indicated that these kind of model-based methods have good robustness and accuracy [17–22]. Data-driven methods hold the unique advantage of addressing the problem of nonlinearity but require large amount of experimental data to train the model. If the data cannot reflect the comprehensive features of the battery, the SOC estimation error can be very large [23–27].

Tab. 1 Comparison of the main existing SOC estimation methods

Methods	Advantages	Disadvantages	Accuracy	Robustness
Look-up table methods [5–8]	<ul style="list-style-type: none"> • Easy implementation; • Low calculation cost • Satisfactory real-time performance. 	<ul style="list-style-type: none"> • Susceptible to uncertain factors, such as temperature, aging and driving cycles; • Requires regular calibration for battery OCV, EIS, etc.; • Requires expensive test equipment. 	Poor	Good
Ampere-hour methods [9, 10]	<ul style="list-style-type: none"> • Easy implementation; • Low calculation cost; • Excellent real-time performance. 	<ul style="list-style-type: none"> • Requires an accurate initial value; • Uses open-loop calculation and lacks the necessary correction; • Susceptible to unavoidable current drift, noisy disturbance and aging. 	Fair	Poor
Model-based methods [11–22]	<ul style="list-style-type: none"> • High accuracy; • Closed-loop control; • Good real-time performance. • Strong adaptability. 	<ul style="list-style-type: none"> • Requires a high-field battery model; • High calculation cost; • Tremendous discrepancy in their robustness and reliability. 	Good	Good
Data-driven methods [23–27]	<ul style="list-style-type: none"> • High accuracy; • High nonlinear prediction ability. 	<ul style="list-style-type: none"> • High computational complexity; • Highly dependent on the training data. 	Excellent	Poor

1.2 Contribution of the paper

Based on the analysis presented in Tab. 1, we can observe that the model-based methods have the balanced performance in the real-time SOC estimation in terms of accuracy and robustness. To achieve an accurate SOC estimation, the model-based methods require a complex ECM with high prediction precision. Nonetheless, ECMs are sensitive to the materials and SOC of the battery. It is difficult to accurately predict the dynamic characteristics of the battery using a single ECM for the entire discharging period [28]. Furthermore, it may lead to unstable estimated results.

The main contribution of this study is to propose a multi-model probability (MMP) fusion SOC estimation using H-infinity algorithm. Firstly, through the analysis of the dynamic voltage behavior of the lithium-ion battery, we have selected three typical ECMs to describe its polarization effect characteristic in its entire operation range. It is helpful to improve the globe prediction for the lithium-ion battery. Secondly, based on the comparison for the existing SOC estimation methods, the H-infinity technique is employed to develop online SOC estimators for the different ECMs. More importantly, compare with existing methods the linear matrix inequality has been employed to determine the parameter of the SOC observer more efficiently. It is noted that the H-infinity approach can improve the accuracy and robustness of the SOC estimation while the fusion process will ensure the credibility of the SOC results. Last but not least, to fuse the SOC estimations implemented by the multiple observers, the Bayes theorem-based probability method is employed to determine the optimal weights for synthesizing the SOC. The proposed method is helpful for ensuring the practical SOC estimation with

higher accuracy, reliability and redundancy.

1.3 Organization of the paper

The remainder of this paper is organized into four sections. In Section 2, the model-based SOC estimation is introduced, including the definition of SOC and the H-infinity observer for the SOC estimation. In Section 3, the necessity of fusion estimation with respect to the SOC is analyzed, and the multi-model probability fusion estimation (MMPFE) method is presented to synthesize the estimation results to improve the overall SOC estimation accuracy. Section 4 verifies the presented method under Beijing Bus driving cycles (BJDC) and Federal Urban Driving Schedule (FUDS) cycles. Conclusions are drawn in the final section.

2. Model-based SOC estimation

2.1 Definition of SoC

Battery SOC is defined as a ratio of the residual capacity to the nominal capacity. It is expressed as the following equation:

$$z(t) = z(t_0) + \frac{1}{Q_n} \int_{t_0}^t \eta_c i_L(\tau) d\tau \quad (1)$$

where $z(t)$ denotes the SOC at time t , $z(t_0)$ denotes the initial SOC value, Q_n denotes the nominal capacity of the battery, η_c denotes the Coulombic efficiency of lithium-ion batteries which will be taken as one in this study, and $i_L(\tau)$ denotes the current flowing through the battery and it is positive for charging and negative for discharging. The dynamic equation of the SOC can be rewritten as:

$$\dot{z} = \frac{\eta_c}{Q_n} i_L \quad (2)$$

2.2 Battery models

There are several battery ECMs presented in [29–32], such as the *Rint* model, *nth*-RC networks model, *RC* model, and *PNGV* model, used for battery model-based SOC estimation. According to the practical applications, the *nth*-RC networks model is the most prevalent. This type of model describes the dynamic relaxation effects of the battery through the paralleled RC networks [14]. Theoretically, the more RC pairs added to the model, the higher the accuracy of the model that can be obtained, i.e., the closer to the authentic battery the estimation can get. Nevertheless, the reality is that the performance of the models with more than three paralleled RC networks is not desirable in some aspects [29–31] and will bring high computational cost. Thus, the ECMs within three RC networks are used in this paper. Their schematic diagrams and relevant dynamic equations are shown in Tab. 2.

where U_{oc} denotes the OCV of the battery and Fig. 1 shows the relationship between battery OCV and SOC. U_t denotes the terminal voltage, R_o denotes the ohmic resistance, U_{Di} denotes the i -th polarization voltage cross the i -th polarization resistance and C_{Di} denotes the i -th polarization capacitor.

Tab. 2 The Schematic diagrams and dynamic equations of ECMs

Models	Schematic diagrams	Dynamic equations
Thevenin model		$\begin{cases} \dot{U}_D = -\frac{U_D}{R_D C_D} + \frac{i_L}{C_D} \\ U_t = U_{oc} - i_L R_o - U_D \end{cases}$
DP model		$\begin{cases} \dot{U}_{D1} = -\frac{U_{D1}}{R_{D1} C_{D1}} + \frac{i_L}{C_{D1}} \\ \dot{U}_{D2} = -\frac{U_{D2}}{R_{D2} C_{D2}} + \frac{i_L}{C_{D2}} \\ U_t = U_{oc} - i_L R_o - U_{D1} - U_{D2} \end{cases}$
3 rd -RC model		$\begin{cases} \dot{U}_{D1} = -\frac{U_{D1}}{R_{D1} C_{D1}} + \frac{i_L}{C_{D1}} \\ \dot{U}_{D2} = -\frac{U_{D2}}{R_{D2} C_{D2}} + \frac{i_L}{C_{D2}} \\ \dot{U}_{D3} = -\frac{U_{D3}}{R_{D3} C_{D3}} + \frac{i_L}{C_{D3}} \\ U_t = U_{oc} - i_L R_o - U_{D1} - U_{D2} - U_{D3} \end{cases}$

2.3 Linear matrix inequality -based H -infinity observer technique for SOC estimation

In terms of the Thevenin battery model in Tab. 2, we can obtain its state space equation

as follows:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u + \mathbf{G}\Phi \\ y = U_{oc}(SoC(t)) + U_D + \mathbf{D}u + \mathbf{H}\Phi \end{cases} \quad (3)$$

where the system matrices are

$$\begin{cases} \mathbf{A} = \begin{bmatrix} -1/C_D R_D & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1/C_D & 1/C_{Nom} \end{bmatrix} \\ \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \end{cases} \quad (4)$$

116 and $x = [U_D \ SoC]^T$ denotes the state vector that needs to be estimated, $y = U_t$ denotes the
 117 observable value, $u = i_L$ denotes the system input, and $\Phi = [\omega \ v \ \xi]^T$ denotes the noise
 118 vector including the process noise ω , v and measuring noise ξ .

119 To obtain the control gain of the observer through the linear matrix inequality (LMI)
 120 method, the observation equation should be expressed in matrix form as well. Due to the
 121 nonlinear relationship between OCV and SOC, the Nernst model is deployed to describe
 122 the OCV behavior by Eq. (5).

$$U_{oc} = K_0 + K_1 z + K_2/z + K_3 \ln z + K_4 \ln(1-z) \quad (5)$$

123 where z denotes the SOC of the battery. Hence, the observation equation can be
 124 formulated as:

$$y = \mathbf{C}x + \mathbf{D}u + \mathbf{H}\phi = \begin{bmatrix} 1 & \frac{dU_{oc}}{dz} \end{bmatrix} x + \mathbf{D}u + \mathbf{H}\phi \quad (6)$$

125 and $\frac{dU_{oc}}{dz} = K_1 - K_2/z^2 + K_3/z - K_4/(1-z)$. It is worth noting that the term $U_{oc}(SoC(t))$
 126 should not be regarded as the model error. The U_{oc} is the function with respect to the
 127 battery SOC. This is because if it is used in that manner, the observation equation will be:

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + \mathbf{D}u + U_{oc}(t) + \mathbf{H}\Phi \quad (7)$$

128 This system is not observable because the rank of the matrix $[\mathbf{C} \ \mathbf{CA}]^T$ is $1 < 2$; thus, the
 129 state observer is not in existence to estimate the system states.

130 Taking the original system Eq. (3) and Eq. (6) into account, the observer can be

131 constructed as:

$$\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}u + \mathbf{L}(y - \hat{y}) \quad (8)$$

$$\hat{y} = \mathbf{C}\hat{x} + \mathbf{D}u \quad (9)$$

132 The corresponding error system is:

$$e_x = x - \hat{x} \quad (10)$$

$$\dot{e}_x = (\mathbf{A} - \mathbf{L}\mathbf{C})e_x + (\mathbf{G} - \mathbf{L}\mathbf{H})\phi \quad (11)$$

$$e_y = \mathbf{C}e_x + \mathbf{H}\phi \quad (12)$$

133 If the error system is asymptotically stable, the states in the observer will approximate the
 134 real system states, and the SOC can be effectively estimated. The objective of establishing
 135 the H-infinity observer is: for a given attenuation level $\gamma > 0$, the system Eqs. (11) and (12)
 136 are stable, and the following inequality is satisfied under the zero initial condition.

$$\|e_x\| < \gamma \|\phi\| \quad (13)$$

137 where $\|\bullet\|$ denotes the norm of terms. According to the bounded real theorem in robust
 138 control theory [33, 34], the error system is asymptotically stable and Eq. (13) meets the
 139 zero initial condition if there exist matrices $\mathbf{P} = \mathbf{P}^T > 0$ and $\mathbf{X}$ with proper dimensions,
 140 giving rise to the following LMI being feasible [19]:

$$\begin{bmatrix} \mathbf{A}^T \mathbf{P} - \mathbf{C}^T \mathbf{X}^T + \mathbf{P}\mathbf{A} - \mathbf{X}\mathbf{C} + \mathbf{I} & \mathbf{P}\mathbf{G} - \mathbf{X}\mathbf{H} \\ (\mathbf{P}\mathbf{G} - \mathbf{X}\mathbf{H})^T & -\gamma^2 \mathbf{I} \end{bmatrix} < 0 \quad (14)$$

141 where $\mathbf{X} = \mathbf{P}\mathbf{L}$ and $\mathbf{I}$ denotes the unit matrix with appropriate dimensions. The gain $\mathbf{L}$ can
 142 be calculated by $\mathbf{P}^{-1}\mathbf{X}$. Through the MATLAB LMI toolbox, the term γ and matrices $\mathbf{P}$ and
 143 $\mathbf{X}$ can be computed through the **mincx** solver and **feasp** solver, respectively.

144 3. Multi-model probability SOC fusion estimation

3.1 Comparison of SOC estimation among multiple models

Although the SOC estimation methods based on a single ECM have been extensively studied in recent years and have achieved the estimation accuracy to a certain extent, the accuracy and reliability can still be improved. This can be illustrated by Fig. 2.

In Fig. 2, the SOC were estimated by LMI-based H-infinity state observers with the *Thevenin* model, *DP* model and 3rd-RC model. The SOC results estimated from the *Thevenin* model are the closest to the SOC reference as shown in segment A; whereas the SOC results from the 3rd-RC model provides the best SOC estimation in segment B. Furthermore, the SOC results from the 3rd order RC model holds the highest accuracy of the SOC estimation in the entire discharge period, they are not the best SOC estimation in each part of the entire discharge period. Hence, multi-model probability SOC fusion estimation is presented to address the problem and improve the reliability and accuracy of the SOC estimation.

3.2 Multi-model probability SOC fusion estimation

3.2.1 Scheme of the fusion estimation

According to the aforementioned analysis, it is necessary to provide fusion SOC estimation from the three ECMs for BMS. In fact, the fusion estimation technique has been well developed in satellite positioning, maneuver target tracking, and fault detection [35–37]. Especially Ref. [38] presented a novel battery open circuit voltage deterministic method for lithium-ion battery cell. Result showed that the fusion estimation technique has good robust and redundancy. In this study, we have proposed a multi-model probability (MMP) fusion SOC estimation using H-infinity algorithm. The primary architectures of

fusion estimation are centralized, distributed and hybrid ones. Due to the low computational burden and high efficiency, the distributed architecture is the most appropriate for the SOC fusion estimation scheme. The systematic block of the multi-model fusion estimation is described in Fig. 3.

The measured voltage and current as the system inputs are transmitted into the paralleled filtering system to estimate the SOC by using three H-infinity state observers based on the three ECMs. Then, the estimated SOC values, $\hat{z}_1$, $\hat{z}_2$, and $\hat{z}_3$, and the estimated terminal voltages, $\hat{U}_{t1}$, $\hat{U}_{t2}$, and $\hat{U}_{t3}$, are obtained and delivered to the data center. The estimated terminal voltages are applied to calculate the weight values ω_i by using the fusion rules that represent the contributions of each model are given to the SOC estimation. The larger the value is, the more impact on the estimation results the model will offer. Finally, the fusion SOC estimation $\hat{z}_{fs}$ can be calculated through the fusion center by Eq. (15):

$$\hat{z}_{fs} = \omega_1 \hat{z}_1 + \omega_2 \hat{z}_2 + \omega_3 \hat{z}_3 \quad (15)$$

where the weight values meet the following equation.

$$\sum_{i=1}^3 \omega_i = 1 \quad (16)$$

The fusion results can be considered as the feedbacks to the filtering system used for the initial values for the next step.

3.2.2 Multi-model probability fusion SOC estimation method

As for the multi-model fusion SOC estimation, the crucial process is to determine the weight values. In this study, the MMPFE method with respect to the SOC of the battery is presented. It is expected that the probability of the selected model will be used to signify

187 the weight values and contribute to the final SOC estimation. There are many ways to
 188 define the probability of the selected model. The residual error of the terminal voltage
 189 (RETV) in Eq. (17) is generally considered a straightforward index to reflect the
 190 consistency of the ECM with the real battery. At a given time, if the RETV is large, it
 191 apparently indicates the poor quality of the ECM to describe the battery, i.e., the
 192 probability of the model to be selected is low.

$$r_i(k) = U_t(k) - \hat{U}_{ti}(k) \quad (17)$$

193 where U_t is the measured terminal voltage, $\hat{U}_{ti}$ is the estimated terminal voltage by the
 194 observer i ($i = 1, 2, \dots, N$), and k is the time to sample the data. Thus, the weight values or
 195 the probabilities of selected models can be given by the following equation:

$$\omega_i(k) = \frac{S(k) - r_i^2(k)}{(N-1)S(k)} \quad (18)$$

196 where $S(k) = \sum_{i=1}^N r_i^2(k)$ and N is the quantity of observers ($N=3$ in this study). Because
 197 of its low calculation cost, the RETV fusion estimation (RETVFE) method is convenient
 198 for approximately determining the weight values. However, the instantaneous information
 199 of the RETV is actually superficial in revealing the SOC estimation accuracy of each
 200 model. The statistical features of the RETV should be taken into account to calculate the
 201 probabilities of diverse models. Enlightened by the multi-model estimation by a Kalman
 202 filter [40], at the sampling time k and given the measured terminal voltage $U_t(k)$, the
 203 conditional probability density function of observer i is described as:

$$f(U_t(k)|p_i) = \frac{1}{(2\pi)^{q/2} Q_i^{1/2}(k)} \exp(-r_i^T(k) Q_i^{-1}(k) r_i(k) / 2) \quad (19)$$

204 where p_i denotes the parameter set of the observed system, including the state transition

matrices, the observation matrix and so forth, $Q_i(k)$ represents the variance of the RETV of observer i , and q is the number of measured values ($q=1$ in this study because only the terminal voltage information is used).

Moreover, according to the Bayes theorem, the probability:

$$\Pr(p_j|U_t(k)) = \frac{f(U_t(k)|p_j)\Pr(p_j)}{\sum_{i=1}^N f(U_t(k)|p_i)\Pr(p_i)} \quad (20)$$

where $\Pr(p_j)$ is the probability that the parameter set is determined by p_j and j is the index ($j=1, 2$ or 3). It is the conditional probability that indicates the weight values of each model's contribution to the SOC estimation accuracy. Because

$$\Pr(p_i|U_t(k-1)) = \frac{\Pr(U_t(k-1)|p_i)\Pr(p_i)}{\Pr(U_t(k-1))} \quad (21)$$

and $\Pr(U_t(k-1)|p_i) = \Pr(\underline{U}_t(k-1)) = 1$, Eq. (21) can be expressed as:

$$\omega_j(k) = \Pr(p_j|U_t(k)) = \frac{f(U_t(k)|p_j)\Pr(p_j|U_t(k-1))}{\sum_{i=1}^N f(U_t(k)|p_i)\Pr(p_i|U_t(k-1))} \quad (22)$$

where the numerator is a recursive form and j represents the observer number. According to Eq. (22), the weight values of three observers can be calculated, and the real-time fusion SOC estimation can be obtained through Eq. (15).

4. Results and discussions

This section presents the results of experiments based on two types of lithium-ion batteries, the LiMn_2O_4 (LMO) battery cell and the lithium iron phosphate/graphite (LFP) cell, respectively. The batteries are operated under two different dynamic working cycles,

and the LMI-based H-infinity state observer is applied to the SOC estimation of three ECMs for each battery. Then, the MMPFE method is used to synthesize the SOC from three ECMs and its effectiveness will be verified by experimental data of two types of lithium-ion batteries.

4.1 Experimental setup

The test bench setup is shown in Fig. 4. It is made up of an Arbin BT2000 cyclor with 16 independent channels, a constant temperature and humidity chamber, a host computer and tested cells. The cyclor is used to load the programmed current or power profiles on the cell within a range of voltage, 0–5 V, and current, -100 A – 100 A, and the measuring errors of its sensors with respect to voltage and current are less than 0.05 %. The thermal chamber is used to offer a steady operation environment, avoiding the disturbance of temperature fluctuation. The ambient temperature of the cell is set to 25 °C. The host computer controls and monitors the cyclor through Ethernet cables, accepting and storing the data collected by the cyclor's sensors, such as the voltage, current and temperature.

4.2 Parameter identification

Before the parameter identification, the traditional hybrid power pulse characterization (HPPC) test and the OCV test were conducted to determine the model parameters and the relationship between the OCV–SOC [12]. The curve between the battery OCV and the SOC can be described by Eq. (5), where its parameters were identified by the least squares method and listed in Tab. 3.

241

Tab. 3 Identified parameters for Eq. (5) representing OCV-SOC curves

	Parameters				
	K_0	K_1	K_2	K_3	K_4
LMO	3.8154	0.2918	-0.0065	0.0484	-0.0069
LFP	3.3798	-0.1030	-0.0012	0.0713	-0.0287

242 For each type of battery, there are three ECMs established for the SOC estimation. The
 243 parameter identification for every model can be addressed by a genetic algorithm (GA)
 244 similar to the method used in [2, 39]. The results are displayed in Tab. 4.

245

Tab. 4 Parameter identification results of the models

Models	LMO	LFP
Thevenin model	$R_o = 1.38 \text{ m}\Omega$	$R_o = 23 \text{ m}\Omega$
	$R_D = 0.20 \text{ m}\Omega$	$R_D = 30 \text{ m}\Omega$
	$C_D = 2.81 \text{e}4 \text{F}$	$C_D = 2.66 \text{e}3 \text{F}$
DP model	$R_o = 1.3 \text{ m}\Omega$	$R_o = 21.3 \text{ m}\Omega$
	$R_{D1} = 0.1 \text{ m}\Omega$	$R_{D1} = 3.0 \text{ m}\Omega$
	$C_{D1} = 1.28 \text{e}4 \text{F}$	$C_{D1} = 5.23 \text{e}3 \text{F}$
	$R_{D2} = 2.10 \text{ m}\Omega$	$R_{D2} = 30.0 \text{ m}\Omega$
	$C_{D2} = 2.40 \text{e}4 \text{F}$	$C_{D2} = 2.43 \text{e}3 \text{F}$
3 rd -RC model	$R_o = 1.3 \text{ m}\Omega$	$R_o = 22.0 \text{ m}\Omega$
	$R_{D1} = 0.1 \text{ m}\Omega$	$R_{D1} = 1.0 \text{ m}\Omega$
	$C_{D1} = 2.53 \text{e}4 \text{F}$	$C_{D1} = 3.73 \text{e}3 \text{F}$
	$R_{D2} = 0.1 \text{ m}\Omega$	$R_{D2} = 1.0 \text{ m}\Omega$
	$C_{D2} = 2.95 \text{e}4 \text{F}$	$C_{D2} = 2.98 \text{e}3 \text{F}$
	$R_{D3} = 2.0 \text{ m}\Omega$	$R_{D3} = 30 \text{ m}\Omega$
	$C_{D3} = 2.47 \text{e}4 \text{F}$	$C_{D3} = 2.26 \text{e}3 \text{F}$

246 4.3 Analysis of fusion SOC estimation results

247 4.3.1 LiMn₂O₄ battery

248 The nominal voltage of the LiMn_2O_4 battery cell is 3.7 V, and its nominal capacity is 35
249 Ah. The actual maximum available capacity is 31.4 Ah. The Beijing bus driving cycles
250 (BJDCs) test is carried out for the LiMn_2O_4 battery cell, and the current and SOC profiles
251 are shown in Fig. 5.

252 The LMI-based H_∞ state observer is utilized to estimate the SOC from three ECMs and
253 the RETVFE and MMPFE methods are used to synthesize the estimated SOC results. All
254 results are shown in Fig. 6. To illustrate the robustness of the state observer, the initial
255 SOC error is set to 50%.

256 From the zoomed picture in Fig. 6, it can be seen that the MMPFE method could
257 effectively give larger weight values to the appropriate model because, during the time
258 range of 5184 s – 5616 s, the estimated SOC results based on the Thevenin model are
259 closer to the real SOC than those from the other two models. The fusion SOC estimation
260 results are superior to those obtained by the single model or the RETVFE method.
261 Moreover, in the time range between 11880 s and 12240 s, from the zoomed figure B in
262 Fig. 6, it is apparent that the SOC results obtained from the MMPFE method shares almost
263 the same accuracy as those estimated from the 3rd-RC model in this region. The max
264 absolute error (MAE), mean estimation error (MEE) and root mean square error (RMSE)
265 of the SOC are recorded to reveal the comprehensive advantages of the algorithms. The
266 evaluation results of the SOC estimation for the LiMn_2O_4 battery cell in the five methods
267 are listed in Tab. 5. The LMI-based H_∞ state observer for the SOC estimation can
268 converge very fast within 500 seconds, leading to the overall SOC estimation errors for
269 the three models less than 2% even by setting the initial SOC of 50% away from the true

value.

Tab. 5 Comparison of the MAE, MEE and RMSE of the LiMn_2O_4 battery cell

Methods	MAE (%)	MEE (%)	RMSE (%)
Thevenin model	2.28	0.92	1.04
DP model	1.99	0.91	1.02
3 rd -RC model	1.89	0.90	1.02
RETVFE	1.94	0.89	1.00
MMPFE	1.89	0.86	0.98

Comparing the statistical results among several methods, it is demonstrated that the allocation of the weight values for the fusion SOC estimation relying on the residual errors of terminal voltages may lead to contrary outcomes, such as the MAE of the RETVFE being worse than that of the 3rd-RC model (1.94% vs. 1.89%). In contrast, the MMPFE method is superior to the other methods as this approach is able to select the suitable model in different time ranges. The MMPFE method is more accurate than the other methods based on a single ECM in most of time with the minimal MAE. Furthermore, the smallest MEE and RMSE (0.86% and 0.98%) indicate that the MMPFE method provides sufficient stability and reliability of SOC estimation.

4.3.2 LiFePO_4 battery

The LiFePO_4 battery cell is experimented with FUDS cycles on the test bench, and its nominal voltage and nominal capacity are 3.3 V and 2.3 Ah. The actual maximum available capacity is 2.19 Ah. The corresponding specific profiles are shown in Fig. 7.

Suppose that the initial SOC value is 80% with 20% offset of the true value. The LMI-based H-infinity state observers are employed to estimate the SOC from three ECMs

and five methods. All the SOC estimation results are shown in Fig. 8.

The enlarged sub-Figs. A and B in Fig. 8 represent two areas, corresponding to the time range being 4392 s – 4608 s and 8568 s – 8820 s, respectively show the maximum error of the SOC estimation. In the zoomed Fig. A, the SOC_s from the Thevenin model are closer to the real SOC_s from the other models in this region; meanwhile, the MMPFE method provides better SOC estimation results than a single Thevenin model method with the global minimum SOC error of 3.00%. Moreover, in the zoomed Figure B, the MMPFE method almost provides almost the same as results as the single DP model method, which represents the best estimation level in this region. Generally, the MMPFE method benefits the SOC estimation of the LiFePO₄ battery as well. It improves the local estimation accuracy and overcomes the disadvantage of a single model. The MAE, MEE and RMSE of the SOC estimation of the five methods for the LiFePO₄ battery are shown in Tab. 6.

Tab. 6 Comparison of the MAE, MEE and RMSE of the LiFePO₄ battery

Methods	MAE (%)	MEE (%)	RMSE (%)
Thevenin model	3.55	1.83	2.06
DP model	3.23	1.60	1.86
3 rd -RC model	3.34	1.69	1.96
RETVFE	3.22	1.70	1.96
MMPFE	3.00	1.58	1.83

From the MAE, it can be summarized that the MMPFE method for the SOC is beneficial for improving the global estimation accuracy with a minimum error of 3.00%. From the statistical analysis of the SOC estimation error, the proposed MMPFE method can achieve the best estimation performance in terms of MEE (1.58%) and RMSE (1.83%),

which means that this method can obtain the globally accurate and robust results.

5. Conclusions

To improve the accuracy and reliability of the SOC estimation, a novel multi-model probability battery SOC estimation approach using the H-infinity algorithm has been proposed. The proposed approach synthesizes the SOC_s obtained from three ECMs, where the LMI-based H-infinity state observer is applied to develop the model-based SOC estimators and its effectiveness is verified by the experimental data obtained from two types of lithium-ion batteries respectively operating at two EV deriving cycles. The comparison among the SOC estimation approaches based on three single-model, the RETVFE method and the MMPFE method, the MMPFE method exhibits its prominent merits, i.e., that some local estimation errors have been rectified and the statistical errors of the SOC estimation have declined, which indicate that the accuracy and reliability of the SOC estimation has been improved against battery materials, driving cycles and inaccurate initial SOC values.

In the future, we will investigate the real-time parameter identification method to improve the online SOC estimation accuracy based on the MMPFE method.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 51507012) and Australian Research Council Linkage Grant LP110200302. The systemic experiments of the lithium-ion batteries were performed at the National Engineering Laboratory for Electric Vehicles, Beijing Institute of Technology. Any opinions expressed in this paper are solely those of the authors and do not represent those

of the sponsor.

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Figure Captions

Fig. 1 The open-circuit voltage as a function of battery SOC

Fig. 2 Comparison of SOC estimation profiles based on various models

Fig. 3 Systematic block of the multi-model fusion estimation method

Fig. 4 Schematic of the battery test bench

Fig. 5 BJDCs test of the LiMn_2O_4 battery cell: (a) current vs. time profile; (b) reference SOC vs. time profile

Fig. 6 The SOC estimation profiles and the fusion SOC estimation results for the LiMn_2O_4 battery cell

Fig. 7 FUDS test of the LPF cell: (a) current vs. time profile; (b) reference SOC vs. time profile

Fig. 8 The SOC estimation profiles and the fusion SOC estimation results for the LiFePO_4 battery cell

Fig. 1 The open-circuit voltage as a function of battery SOC

Fig. 2 Comparison of SOC estimation profiles based on various models

Fig. 3 Systematic block of the multi-model fusion estimation method

Fig. 4 Schematic of the battery test bench

Fig. 5 BJDCC test of the LMO cell: (a) current vs. time profile; (b) reference SOC vs. time profile

Fig. 6 The SOC estimation profiles and the fusion SOC estimation results for the LMO cell

Fig. 7 FUDS test of the LFP cell: (a) current vs. time profile; (b) reference SOC vs. time profile

Fig. 8 The SOC estimation profiles and the fusion SOC estimation results for the LFP cell