Interval Optimization based Operating Strategy for Gas-Electricity Integrated Energy Systems Considering Demand Response and Wind Uncertainty

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Abstract

In the United States, natural gas-fired generators gained increasing popularity in recent years due to the low fuel cost and emission, as well as the proven large gas reserves. Consequently, the highly interdependency between the gas and electricity networks is needed to be considered in the system operation. To improve the overall system operation and optimize the energy flow, an interval optimization based coordinated operating strategy for the gas-electricity integrated energy system (IES) is proposed in this paper considering demand response and wind power uncertainty. In the proposed model, the gas and electricity infrastructures are modeled in details and their operation constraints are fully considered, wherein the nonlinear characteristics are modeled including pipeline gas flow and compressors. Then a demand response program is incorporated in the optimization model and its effects on the IES operation are investigated. Based on interval mathematics, wind power uncertainty is represented as interval numbers instead of probability distributions. A case study is performed on a six-bus electricity network with a seven-node gas network to demonstrate the

effectiveness of the proposed method; further, the IEEE 118-bus system coupling with a 14-node natural gas system is used to verify its applicability in practical bulk systems.

Keywords: Coordinated operation; demand response; gas and electricity network; integrated energy systems; interval optimization; wind power uncertainty

Nomenclature

$Q_{w,t}$	Production of gas well w at time t	$R_{j,\max}$	Maximum compression ratio of j
$Q_{w,\max}$	Maximum production of gas well w	$R_{j,\min}$	Minimum compression ratio of j
$Q_{w,\min}$	Minimum production of gas well w	$Q_{s,t}$	Capacity of storage s at time t
f_{mn}	Gas flow from node m to node n	$Q_{s,\max}$	Maximum capacity of storage s
π_m	Pressure of gas node m	$Q_{s,\min}$	Minimum capacity of storage s
C_{mn}	Flow factor of pipeline $m-n$	IR_s	Hourly inflow limit of storage s
$H_{j,t}$	Horsepower of compressor j at time t	OR_s	Hourly outflow limit of storage s
$Q_{d,t}$	Gas load d at time t	$P_{w,t}$	Wind power w at time t
$Q_{f,t}$	Gas consumption of gas-fired unit f at t	$P_{g,t}$	Generator g output at time t
$P_{d,t}$	Electric load d at time t	$P_{g,\max}$	Maximum output of generator g
$P_{c,t}$	Power consumption of compressor c	$P_{g,\min}$	Minimum output of generator g
RU_g	Ramp up limit of unit g	RD_g	Ramp down limit of unit g

$P_{l,\max}$	Power flow limit of line j	GSF	Generation shift factor
$R_{k,t}$	Spinning reserve of unit k at time t	$R_{t,\min}$	Required system reserve at time t
$Q_{r,t}$	Responsive gas load r at time t	$P_{r,t}$	Responsive electric demand r at time t
$C_{g,r,t}$	Incentive price to gas load r at time t	$K_{e,r}$	Incentive price to electricity load r at t
$K_{g,r}$	Elasticity of gas load r	$K_{e,r}$	Elasticity of electricity load r
φ_g	Proportion of gas DR participation	φ_e	Proportion of electric DR participation
$\Delta Q_{n,t}$	Unserved gas load n at time t	$\Delta P_{i,t}$	Unserved electric load i at time t

1. Introduction

In recent years, the interdependency between natural gas and electricity power energy systems are dramatically increasing with more natural gas utilized for electricity generation. In the United States, the natural gas consumption by electric power sector has increased from 32% in 2007 to 39% in 2009 [1]. Gas-fired power plants provide a linkage between natural gas and electricity networks. Compared to traditional coal-fired generators, gas-fired generators are preferred for its competitive fuel cost, lower pollutant emissions and fast response to fluctuating renewable energy [2]. In New England ISO (ISO-NE), more than 50% of electricity is now generated from natural gas, compared to only 15% in 2000, with even more growth in the use of natural gas-fired generation anticipated going forward [3]. Natural gas transmission could affect the security and the economics of power transmission. For the highly interdependency between the two energy sectors, natural gas and electricity networks are regarded as an integrated energy system (IES) [4].

Extensive research has been conducted to address the coordinated planning and operation in the gas and electricity network. In [5], a combined gas and electricity network expansion planning model is proposed to minimize gas and electricity operational cost and network expansion cost simultaneously. A co-optimization planning model is proposed in [6] considering the long-term interdependency of natural gas and electricity infrastructures under security constraints. A long-term multi-area, multi-stage model integrated expansion planning of electricity and natural gas systems are presented in [7]. As for short term economic dispatch, an operating strategy is proposed in [8] to coordinate the electricity and natural gas in Great Britain. The impact of gas network on power security and economic dispatch are investigated in [9-11]. In [9-10], integrated optimization model is proposed to incorporate the natural gas network constraints into the optimal solution of security-constrained unit commitment. [11] proposes a security constrained optimal power and natural gas flow under N-1 contingencies.

With the integration of variable and uncertain renewable energy, the coordination of IES is facing new challenges. The uncertainties are not considered in the above model so that a small perturbation in the wind power data may lead to non-optimality or even infeasibility. Stochastic programming [8], [12-13] and robust optimization [14-17] are usually used to deal with wind uncertainties. Several works have investigated the effect of wind power uncertainty on system operation. In [8], stochastic optimization is adopted in the optimization model to deal with wind power uncertainty, in which a large number of wind forecast scenarios are generated and a scenario reduction algorithm is applied. [12-13] applied stochastic optimization to the unit commitment problem with a number of wind power

scenarios. However, stochastic optimization requires the probability distribution of wind power, which is not easy to be accurately obtained in practice. In addition, it is time consuming to generate a large number of scenarios [14]. A robust optimization approach is proposed in [15] to analyze the interdependency of the IES considering wind power uncertainty. In this work, the wind power uncertainty is actually addressed based on scenario analysis with introducing a penalty coefficient for reducing variance. [14] and [16] apply robust optimization to unit commitment problem considering wind power uncertainty. [17] proposes a look-ahead robust scheduling model for wind-thermal system considering natural gas congestion, but the constraints of the gas pipelines are considered in a simplified manner. Actually, robust optimization is usually considered to be too conservative due to the fact it always tries to find the worst-case scenario solutions which happen at a very low probability. In addition, due to the non-convex constraints of the pipeline and compressor model in the gas network, the robust optimization model for IES becomes difficult to solve.

In this paper, interval optimization [18-19] is introduced to address wind power uncertainty, wherein the wind power is represented as interval numbers. In the interval mathematics, all the uncertain information will be maintained in the solving process, which is also easy to implement in engineering applications. The interval optimization minimizes the operating cost interval rather than the worst case scenarios in robust optimization [18]. Also, it has better computational performance than stochastic optimization [19]. Furthermore, demand response has been recognized as an effective means to enhance power system operation [20-21], but few literatures considered demand response in the IES.

Therefore, this paper proposes an interval optimization based operating strategy for gas-electricity integrated energy systems considering demand response and wind uncertainty. With the objective of operating cost minimization, the multi-period power and gas flow are optimally determined. The gas and electricity networks are modeled in detail and security operation constraints are imposed. Then an incentive-based demand response program is incorporated into the proposed model and its effects on IES operation are analyzed. With the consideration of wind power uncertainty, the proposed model is solved by interval optimization. Finally, a multi-scenario case study verifies the proposed method.

The rest of this paper is organized as follows. Section 2 introduces the detailed models of the gas and electricity networks. Section 3 presents the interval optimization based operating strategy for the integrated energy system. Section 4 presents case studies to demonstrate the effectiveness of the proposed method. Finally, Section 5 concludes the paper.

2. Gas-Electricity Integrated Energy System Modeling

2.1 Natural Gas network model

The natural gas network is composed of gas well, gas pipeline, compressor, gas storage and gas loads. Natural gas is produced at gas wells and transmitted through pipelines propelled by compressors then delivered to the gas load sites. The gas storage provides a buffer to coordinate the usage of gas during multiple periods. The steady state mathematical models of each component are presented below.

2.1.1 Gas wells

Natural gas is injected from gas wells, which are commonly located at remote sites. The gas suppliers are modeled as positive gas injections at the gas well nodes. In each period, upper and lower limits are imposed on the available production of gas suppliers limited by the physical characteristics and long-term, mid-term gas contracts.

$$Q_{w,\min} \leq Q_{w,t} \leq Q_{w,\max}, w \in A_{GW} \quad (1)$$

where $Q_{w,\max}$ and $Q_{w,\min}$ are the maximum and minimum gas supply of gas well w , A_{GW} is the set containing all the gas well.

2.1.2 Gas pipeline

The gas flow through the pipeline is driven by the pressure difference between the two ends of a pipeline. Meanwhile, the physical factors, such as the length, diameter, operating temperature, altitude drop, and the friction of pipelines, also affect the gas flow.

The gas flow from node m to node n , f_{mn} (kcf/hr) is expressed as

$$f_{mn} = \text{sgn}(\pi_m, \pi_n) C_{mn} \sqrt{|\pi_m^2 - \pi_n^2|}$$

$$\text{sgn}(\pi_m, \pi_n) = \begin{cases} 1 & \pi_m \geq \pi_n \\ -1 & \pi_m < \pi_n \end{cases} \quad (2)$$

where π_m and π_n are the pressures at node m and n respectively; $\text{sgn}(\pi_m, \pi_n)$ indicates the direction of the gas flow, when it is 1, the gas flows from node m to n . C_{mn} is a constant related to the physical characteristic of each pipeline, given by

$$C_{mn} = 3.2387 \frac{T_0}{\pi_0} \sqrt{\frac{D_{mn}^5}{L_{mn} G F_{mn} Z_a T_{mn}}} \quad (3)$$

where T_0 is the standard temperature, 520° R; π_0 is the standard pressure, 14.65 psia; D_{mn} is the internal diameters of pipeline between nodes m and n , inch; G is the gas specific gravity (air = 1.0, gas = 0.6); F_{mn} is the friction factor of the pipeline; Z_a is the average gas compressibility factor; and T_{mn} is the average gas temperature. According to [22], F_{mn} varies as a function of the diameter D_{mn} ,

$$F_{mn} = \frac{0.032}{D_{mn}^{1/3}} \quad (4)$$

2.1.3 Gas compressor

During the transmission of gas in pipelines, the gas compressor stations are installed to provide pressure for the gas flow to overcome friction.

The gas flow from node m to node n through the compressor j , f_{mn} is expressed as

$$f_{mn} = \text{sgn}(\pi_m, \pi_n) \frac{H_j}{k_{j2} - k_{j1} \left[\frac{\max(\pi_m, \pi_n)}{\min(\pi_m, \pi_n)} \right]^\alpha} \quad (5)$$

where k_{j1} , k_{j2} , and α are empirical parameters related to the compressor properties, H_j represents the power of compressor j , subject to the physical bound of the compressor.

$$H_{j,\min} \leq H_j \leq H_{j,\max} \quad (6)$$

where $H_{j,\max}$ and $H_{j,\min}$ are the maximum and minimum allowed pressure of the compressor.

The compression ratio between the outlet node and inlet node is subject to the following constraint:

$$R_{j,\min} \leq \frac{\max(\pi_m, \pi_n)}{\min(\pi_m, \pi_n)} \leq R_{j,\max} \quad (7)$$

where $R_{j,\max}$, and $R_{j,\min}$ are the maximum and minimum allowed compressor ratio.

The gas compressor must consume horsepower H_j to produce pressure. If the compressor node is coupled with an electricity node, the power will be supplied by the electricity network. In this case, H_j is regarded as an electricity load and will be addressed in the power flow. Otherwise, the compressor will consume natural gas directly from gas flow to provide H_j , expressed as

$$Q_c(H_j) = c_j + b_j H_j + a_j H_j^2 \quad (8)$$

where a_j , b_j , and c_j are the coefficients of the gas consumption of the compressor j .

2.1.4 Gas storage

Gas storage facilities provide a buffer to coordinate the gas flow during multi-period horizon. The gas storage level, gas withdrawal and injection amount are subject to the capacity of the storage and inflow and out-flow rates limit.

$$Q_{s,\min} \leq Q_{s,t} \leq Q_{s,\max} \quad (9)$$

$$-IR_s \leq dQ_{s,t} = (Q_{s,t} - Q_{s,t-1}) \leq OR_s \quad (10)$$

where $Q_{s,\max}$ and $Q_{s,\min}$ are the maximum and minimum operating storage capacity. IR_s and OR_s are the inflow and outflow rate limit of the storage.

2.1.5 Gas load

The natural gas load includes residential, commercial and industrial loads. The gas-fired generators are taking an increasing share of the overall gas demand. The gas load could be regarded as negative gas injections at the gas load nodes, denoted as $Q_{d,t}, d \in A_{GD}$, A_{GD} is the set of gas load.

2.1.6 Gas flow nodal balance

At each node in the gas network, the total natural gas flow injection to a node is equal to zero:

$$Q_{inj} = I_{nw} \cdot Q_w - I_{ns} \cdot dQ_s - I_{nf} \cdot Q_f - I_{nd} \cdot Q_d - I_{np} f - I_{nc} F_c(H_c) = -\Delta Q \quad (11)$$

where I_{nw} , I_{ns} , I_{nf} , I_{nr} , I_{np} and I_{nc} are the incidence matrices of gas wells, storages, gas-fired units, gas load, pipe lines and compressors, respectively. ΔQ is the unserved gas load. Note that each incidence matrix I has a dimension of (number of nodes) by (number of components), while each gas quantity matrix Q has a dimension of (number of components) by (time horizon).

2.2 Electricity network model

A DC power flow model is adopted in this paper to represent the power flow in electricity network. In the electricity sector, the operation constraints are provided as follows.

1) Power flow nodal injection

$$P_{inj} = I_{tw} \cdot P_w + I_{tg} \cdot P_g - I_{td} \cdot P_d - I_{tc} \cdot P_c + \Delta P \quad (12)$$

where P_w is wind output, P_g is the thermal generator output (including gas-fired generators), P_d is the electrical load, and P_c is the power consumption of the gas compressor. ΔP is the potential load shedding.

2) Power generation constraints: the output power of a thermal generator is kept within its physical limits

$$P_{g,\min} \leq P_{g,t} \leq P_{g,\max} \quad (13)$$

where $P_{g,\max}$ and $P_{g,\min}$ are the maximum and minimum power generation of the thermal unit g .

3) Ramping up and down constraints: the ramping up and down rates of a thermal generator are subjected to its physical limits

$$P_{g,t} - P_{g,t-1} \leq RU_g \quad (14)$$

$$P_{g,t-1} - P_{g,t} \leq RD_i \quad (15)$$

where RU_g and RD_g are the maximum ramping up and down rates of the thermal unit g .

4) Power transmission constraints: each transmission line in the electricity network has a maximum capacity. GSF is the generation shift factor with a dimension of (number of lines) by (number of buses), while P_{Lmax} has a dimension of (number of lines) by (time horizon).

$$-P_{L\max} \leq GSF \cdot P_{inj} \leq P_{L\max} \quad (16)$$

5) Spinning reserve constraints: spinning reserve is needed to maintain the balance between generation and demand at all time. Traditionally, spinning reserve is usually equal to the capacity of the largest generator or a certain percentage of the peak load to address load forecasting errors. For deterministic model, additional spinning reserve is required for the wind power uncertainty. However, in this work, wind power uncertainty is implicitly modeled as uncertainty constraints rather than additional spinning reserve. This consideration is similar to stochastic programming models considering wind uncertainty. Here, wind power uncertainty is represented by uncertainty bounds (i.e., interval numbers) and modeled in the power balancing constraint in (12) based on interval mathematics. If we “discretize” a wind uncertainty interval as multiple specific wind power scenarios, there should be multiple corresponding optimal solutions. The bound of such solutions will be the output bound of the optimization. Thus, the uncertainties of wind forecasts are taken into account implicitly through the interval numbers of wind power [26-27] in the power balancing constraint (12). Therefore, the spinning reserve in this model only needs to address the load forecasting errors in a conventional way. Here, 10% of the maximum load is set as the spinning reserve requirement to address the load uncertainty.

$$\sum_k R_{k,t} \geq R_{t,\min} \quad (17)$$

where $R_{k,t}$ is the spinning reserve of thermal unit k at time t , and $R_{t,\min}$ is the reserve requirement of the system at time t .

It can be seen that the gas-fired generators serve as the power source in electricity network and natural gas load in gas network meanwhile. So gas-fired generators are the components that link the two sectors together. The model of gas-fired generators is represented by a quadratic function of output power with respect to the fuel consumption. This is given as follows:

$$P_{g,i}(Q_{ng,i}) = k_{2,i}Q_{ng,i}^2 + k_{1,i}Q_{ng,i} + k_{0,i} \quad \forall i \in N_{NG} \quad (18)$$

where N_{NG} is the set of gas network nodes; $k_{2,i}$, $k_{1,i}$, and $k_{0,i}$ are the fuel consumption coefficients of the gas-fired generator i ; and $Q_{ng,i}$ is the amount of natural gas supplied to the gas-fired generator i .

2.3 Incentive demand response program

To evaluate the effects of demand response of residential consumers on electricity and natural gas demand, two incentive demand response programs are designed as linear functions with respect to the compensation price provided by utilities. At different nodes, the prices for demand response are different. The incentive prices for gas and electricity loads at each node are taken as the decision

variables in the optimization model, which could provide a reference for the utilities. The response from the consumers is modeled as a linear function of the compensation price. When the compensation is high, more consumers are willing to participate into the demand response.

$$Q_r = f(C_{g,r}) = K_{g,r} \cdot C_{g,r} \quad (18)$$

$$P_r = f(C_{e,r}) = K_{e,r} \cdot C_{e,r} \quad (19)$$

$$0 \leq Q_r \leq Q_d \cdot \varphi_g, r \in A_{GDR}, d = L_g(r) \quad (20)$$

$$0 \leq P_r \leq P_d \cdot \varphi_e, r \in A_{EDR}, d = L_e(r) \quad (21)$$

where Q_r and P_r are the reduced gas and electric load under the incentive price of $C_{g,r}$ and $C_{e,r}$, respectively. $K_{g,r}$ and $K_{e,r}$ are the corresponding load elasticity. A_{GDR} and A_{EDR} are the set of gas and electric load that participate in demand response programs; $L_g(r)$ and $L_e(r)$ are the corresponding gas and electric load index of the r th DR participant. φ_g and φ_e are the proportion of the gas and electricity load that signed the demand response contract with the utility, assumed to be 10%. In this paper, our focus is to analyze the effects of demand response on the coordinated operation of IES, the models of demand response are designed to be linear. Actually, more accurate models could be adopted but may also need to decompose to piecewise linear functions when the optimization model is solved. With the interactions of demand response, the nodal balance equations of electricity and gas networks are rewritten as

$$Q_{inj} = I_{nw} \cdot Q_w - I_{ns} \cdot dQ_s - I_{nf} \cdot Q_f - I_{nd} \cdot Q_d + I_{nr} \cdot Q_r - I_{np} \cdot f - I_{nc} \cdot F(H_c) = -\Delta Q \quad (22)$$

$$P_{inj} = I_{pw} \cdot P_w + I_{pg} \cdot P_g - I_{pd} \cdot P_d + I_{pr} \cdot P_r - I_{pc} \cdot P_c - I_{pb} \cdot P_{DR} = -\Delta P \quad (23)$$

It is worth noting that the utility could implement both electricity and natural gas demand response coordinately to achieve the overall economic operation of IES.

3. Optimization Model for IES Considering Demand Response

3.1 Deterministic optimization model for IES coordinated operation

The total operating costs of the IES include two parts: the costs of electricity network consisting of the generation costs of non-gas power generators, the penalty for unserved power load and costs for electricity demand response; and the costs of the natural gas network consisting of costs of gas production, cost of compressors, and penalty for unserved gas load and cost for gas load demand response. It should be noted that the gas-fired generators are considered as a type of natural-gas load in the gas network. The generation cost of gas-fired generators mainly comes from the fuel cost, which is

considered in the production and operation costs of gas network. The objective function of the optimal operation strategy in IES is given by

$$\min_{\mathbf{X}} J(\mathbf{X}) = \sum_t \left(\sum_g f_g(P_{g,t}) + \lambda_g \sum_c Q_c(H_{c,t}) + \sum_d \lambda_g Q_{d,t} + \sum_i \lambda_{ul} \Delta P_{i,t} + \sum_n \lambda_{ug} \Delta Q_{n,t} + \sum_r C_{g,r} Q_r + \sum_r C_{e,r} P_r \right) \quad (24)$$

where $\mathbf{X}$ is the control variable set: $\mathbf{X} = \{\mathbf{P}_g, \boldsymbol{\pi}, \mathbf{H}, C_{g,r}, C_{e,r}\}$, including power output of each thermal unit, pressure of each node, horsepower of each compressor, and price of demand response.

Subject to

Gas network constraints (1), (2), (5)-(10), (18), (20), (22)

Electricity network constraints (13)-(17), (19), (21), (23)

where $f_g(P_{g,t})$ is the cost function of all thermal generators. $\Delta P_{i,t}$ is the amount of unserved electricity load; $Q_{d,t}$ is the residential gas demand; $\Delta Q_{n,t}$ is the unserved amount of gas load. λ_{ul} and λ_{ug} are the penalty of unserved electricity and gas load, 1,000 \$/MWh, and 200\$/kcf, λ_g is the price for producing each kcf gas, 6.23 \$/kcf. The last term is the total cost for paying the demand response program.

3.2 Interval optimization model for IES coordinated operation

3.2.1 Interval based nonlinear optimization

As one of the effective alternatives to address uncertainties, interval analysis was firstly proposed by Moore [28]. The only available information is lower and upper bounds for inexact parameters. Then, interval analysis was extended to interval mathematical programming by Huang [29]. The interval mathematics based optimization is able to address the uncertainties by interval numbers without requirement about accurate probability distribution information. It optimizes the output bounds regarding given input intervals with acceptable computational time. It has already been applied in the boundary estimation of power flow calculation with parameter uncertainties [19]. In this paper, interval optimization is introduced to solve the optimal operation problem in gas-electricity IES.

The general mathematic formulation of a nonlinear optimization based on interval analysis is expressed by

$$\begin{aligned} & \min_{\mathbf{X}} f(\mathbf{X}, \mathbf{U}) \\ \text{s.t. } & g_i(\mathbf{X}, \mathbf{U}) \geq b_i^l = [b_i^-, b_i^+], \quad i \in L, \quad \mathbf{X} \in \Omega^n \\ & \mathbf{U} \in \mathbf{U}^l = [\mathbf{U}^-, \mathbf{U}^+], \quad U_i \in U_i^l = [U_i^-, U_i^+], \quad i \in Q \end{aligned} \quad (25)$$

where $\mathbf{X}$ is an n-dimensional vector of the decision variables, $\mathbf{U}$ is the q-dimensional uncertain vector represented by interval numbers, b_i^l is the interval of the i^{th} constraint.

To evaluate the optimal values, an order relation of interval numbers is introduced to compare two intervals [23], in which an index of interval possibility degree is used to indicate the possibility of interval $A \leq B$. Then the uncertain constraints can be converted to deterministic constraints

$$P(g_i^l(\mathbf{X}) \leq b_i^l) \geq \lambda_i \quad (26)$$

where $\lambda_i \in [0, 1]$, which is a predefined possibility, and $g_i^l(\mathbf{X}) = [g_i^-(\mathbf{X}), g_i^+(\mathbf{X})]$. Different from interval based linear optimization, this model has to be solved by two optimization process.

$$g_i^-(\mathbf{X}) = \min_U g_i(\mathbf{X}, U), \quad g_i^+(\mathbf{X}) = \max_U g_i(\mathbf{X}, U) \quad (27)$$

Thus, all the uncertainty constraints have been transformed to deterministic constraints.

Next step, the objective function is also needed to be transformed to a deterministic objective function. For the uncertain objective function $f(\mathbf{X}, U)$, let $f^l(\mathbf{X}) = [f^-(\mathbf{X}), f^+(\mathbf{X})]$ be represented by the mean and width of the interval as $\langle f^m(\mathbf{X}), f^w(\mathbf{X}) \rangle$, where

$$\begin{aligned} f^m(\mathbf{X}) &= \frac{f^-(\mathbf{X}) + f^+(\mathbf{X})}{2} \\ f^w(\mathbf{X}) &= \frac{f^+(\mathbf{X}) - f^-(\mathbf{X})}{2} \end{aligned} \quad (28)$$

The mean value indicates the expected optimal value and the width denotes the uncertainty level of the optimal solution. In the context of uncertainty, we are trying to find a solution with minimum mean as well as the width. Then the uncertain objective function can be transformed to a deterministic multi-objective function that minimize both the mean and width of the interval objective value,

$$\min_{\mathbf{X}} (f^m(\mathbf{X}), f^w(\mathbf{X})) \quad (29)$$

Similarly, two optimization processes are applied to obtain $f^-(\mathbf{X})$ and $f^+(\mathbf{X})$

$$f^-(\mathbf{X}) = \min_U f(\mathbf{X}, U), \quad f^+(\mathbf{X}) = \max_U f(\mathbf{X}, U) \quad (30)$$

For this deterministic objective function and constraints, a weighting factor β could be applied to solve this multi-objective optimization model, expressed as

$$\begin{aligned} \min_{\mathbf{X}} f(\mathbf{X}) &= (1 - \beta)f^m(\mathbf{X}) + \beta f^w(\mathbf{X}) \\ \text{s.t.} \quad &(26), \quad \mathbf{X} \in \Omega^n \end{aligned} \quad (31)$$

The above model can be solved by a two-stage optimization. The upper stage is to search the optimal decision variables. According to the decision variables from the upper stage, the intervals of objective functions and constraints are calculated at the lower stage.

3.2.2 Interval optimization model for IES coordinated operation

With the consideration of wind power uncertainty, the interval mathematics is applied to the above deterministic optimization model. The wind power uncertainty is represented by interval numbers, defined by the upper and lower bounds of wind power forecasts.

$$W = \{P_{w,t} | P_{w,t}^- \leq P_{w,t} \leq P_{w,t}^+, \forall t \in T\} \quad (32)$$

Eq. (32) represents the interval numbers of wind power for each time interval during the study period T . The interval numbers are the upper and lower wind power uncertainty bounds that are obtained by wind power forecasting. The wind power forecasting techniques can be found in [30-32], which proposed statistic methods for determining the interval numbers of wind power prediction. In this paper, we assume that the wind power uncertainty bounds are already obtained based on existing wind power forecasting techniques.

In interval optimization, with regard to the interval input of wind power, the objective value $J(X, P_{wind})$, i.e. the operating cost of the systems is obtained in the form of intervals, denoted as $\Psi = [J(X)^-, J(X)^+]$,

$$J(X)^- = \min_{P_w \in W} J(X, P_w), \quad J(X)^+ = \max_{P_w \in W} J(X, P_w) \quad (33)$$

The objective function of IES optimal operation in (24) and the constraints in terms of P_w are represented in the interval form, thus we have the interval based optimization model of IES optimal operation.

$$\min_X J(X, P_{wind}) = \sum_t \left(\sum_g f_g(P_{g,t}) + \lambda_g \sum_c Q_c(H_{c,t}) + \sum_d \lambda_g Q_{d,t} + \sum_i \lambda_{ui} \Delta P_{i,t} + \sum_n \lambda_{ug} \Delta Q_{n,t} + \sum_r C_{g,r} Q_r + \sum_r C_{e,r} P_r \right) \quad (34)$$

$$P_w \in W \quad \Delta P_i = [P_w^-, P_w^+] + P_{g,i} - P_{d,i} - P_{c,i} + P_{r,i} + \Delta P_i \quad (35)$$

The constraints in the deterministic model (24) will be modified in terms of interval numbers to include the uncertainty wind power interval. This interval based IES coordinated operation can be solved according to the procedure described in subsection 3.2.1.

It should be noted that, in this work, the ramping limits of thermal and gas-fired generators (14)-(15) and the capacity limits of gas storage (10) are involved in the multi-period constraints. The dimension of the optimization problem increases substantially due to the existing coupling between different sub-periods of time. The optimization model is implemented in MATLAB with YALMIP and BONMIN solver on a PC with Intel Core i7 3.00 GHz CPU and 8 GB RAM.

4. Case Study

The effectiveness of the proposed method was evaluated on two systems: a six-bus electricity network with seven-node natural gas network and the IEEE 118-bus with 14-node gas network.

4.1 Six-bus electricity network with seven-node natural gas network

A small IES consisting of a six-bus electricity network and a coupled seven-node gas network is depicted in Fig. 1. In the electricity network, three gas-fired generators are located at node 1, 2 and 6 respectively; three electricity loads are at node 3, 4 and 5; a 70 MW wind turbine (WT) is installed on node 3. In the gas network, two gas wells are at node 6 and 7 respectively, two residential gas loads are at node 1 and 3; and a compressor is installed on the pipeline between node 2 and 4. A gas storage is located at gas node 1. The two networks are coupled at three gas-fired generators, corresponding to gas load 1, 3, and 5. The detailed parameter data can be found in [10]. The wind power forecast data and its 20% uncertain bounds are shown in Fig. 2. The multipliers of total electricity and gas load are shown in Fig. 3. The scheduling horizon is 24 hours. The penalty for electricity load shedding is 1,000 \$/MW and 200 \$/kcf for gas load shedding.

Fig.1 Six-bus electricity network coupled with a seven-node gas network

Fig. 2 Wind power forecast data with 20% uncertain interval

Fig. 3 Base electricity load and residential gas load

4.1.1 Deterministic IES model

First, the base case with wind forecast and base load data (named Case 0) is solved using the deterministic IES model. The results of power generation scheduling are shown in Fig. 4, and the pressure at each node of the gas network is shown in Table 1. The gas production of gas wells are shown in Fig. 5. Fig. 6 shows the gas volume in the gas storage during the scheduling horizon. The storage level at the end of the day will be equal to that at the start of the day. No electricity load or natural gas load is shed. The output of unit 2 and 3 is very low since they are too expensive. By checking other results such as power flow and gas flow results, all the values are within corresponding security constraints of system operation.

Fig. 4 Optimal output of each unit in Case 0

Fig. 5 Gas production of gas wells in Case 0

Fig. 6 Gas volume in gas storage in Case 0 during the scheduling horizon

Table 1 Pressure at each node of gas network in Case 0 for 1-24 h

π /psia	1	2	3	4	5	6	7	8	9	10	11	12
Node1	116.23	113.62	112.51	110.98	109	107	105	102.7	101	100	100	100
Node2	151.05	152.35	153.09	154.33	156	159	162	166.2	169	170	170	170
Node3	156.16	160.07	162.23	164.74	168	172	177	182.8	187	188.6	188	188
Node4	79.39	80.82	81.52	82.87	85.3	89	91	96.08	99	100	100	100
Node5	161.2	164.45	166.15	168.7	173	177	182	188.6	193	194.6	194	194
Node6	182.25	186.27	187.96	191.81	198	207	213	222.7	229	230.9	229	229
Node7	102.22	103.88	104.65	106.21	109	113	116	121.5	124	126.3	127	128
π /psia	13	14	15	16	17	18	19	20	21	22	23	24
Node1	100	100	100	100	100	100	100	100	100	100	100	100
Node2	170	170	170	170	170	170	170	170	170	170	170	170
Node3	189	190.4	190	189	188	187.01	185	184	184	185	188	192
Node4	100	100	100	100	100	100	100	100	100	100	100	100
Node5	194	195.4	195	195	194	193.03	192	191	191	192	194	196
Node6	229	230.1	230	229	229	228.14	227	226	227	227	229	231
Node7	127	126.4	127	130	132	132.11	134	134	134	134	132	131

To conduct a comparative study, several cases are designed to evaluate the interdependency between electricity and gas network. In Case 1, 2, and 3, the residential gas loads are increased by 20%, 30%, and 50% respectively. The power output of Unit 1 in the above cases are shown in Fig. 7. And the comparison results of total cost, unserved electricity and gas loads during the scheduling horizon are shown in Table 2.

Fig. 7 Power output of Unit 1 under different cases

Table 2 Comparison results of the cases

	Total cost(\$)	Total unserved electricity load(MW)	Total unserved gas load(kcf)
Case 0	1435328.02	0	0
Case 1	2197233.20	588.79	0
Case 2	2889961.51	1163.9	461.61
Case 3	5770446.32	1575.9	12578.0

Comparing Case 0 and Case 1, it can be seen that when the gas load increases, at 7th hour, the pressure different between node 1 and node 2 reach the limit. There will be not enough gas supply for the gas-fired generations, leading to large amount of electricity load shedding. When the gas load increases further, the gas load shedding also occurs. The operating cost increases dramatically with the unserved load amount due to the large penalty for energy imbalance.

4.1.2 Deterministic IES model with demand response

The incentive demand response program described in 2.3 is applied to the deterministic IES model. Based on Case 1, three cases with electricity demand response, gas demand response, and gas-electricity demand response are studied, which are denoted as Case 1-DR1, Case 1-DR2, and Case 1-DR3 respectively.

The amount of total electricity demand response of Case 1-DR1 and Case 1-DR3 are shown in Fig. 8. The optimized electricity DR prices in Case 1-DR1 are shown in Table 3. Specifically, the effects of DR in Case 1-DR1 can be clearly observed in Fig. 9.

Fig. 8 Power output of Unit 1 under different cases

Fig. 9 Effect of electricity DR in Case 1-DR1

Table 3 Electricity DR prices at each load node in Case 1-DR1

Price(\$/MW)	1	2	3	4	5	6	7	8	9	10	11	12
Load 1	23.47	0	0	0	0	5.48	41.91	42.15	42.15	54.58	54.59	55.58
Load 2	23.47	0	0	0	0	5.48	41.91	42.15	42.15	60.54	60.54	61.54
Load 3	23.47	0	0	0	0	5.48	41.91	42.15	42.15	59.41	59.41	59.41
Price(\$/MW)	13	14	15	16	17	18	19	20	21	22	23	24
Load 1	54.58	54.58	55.58	54.58	54.58	54.58	62.54	71.91	56.22	55.28	42.15	42.15
Load 2	60.54	60.54	60.54	60.54	60.54	60.54	69.36	79.75	62.35	62.35	42.15	42.15
Load 3	59.41	59.41	59.41	59.41	59.41	59.41	68.07	78.27	61.19	60.18	42.15	42.15

The results of the above three cases are compared with those of Case 1, shown in Table 4.

Table 4 Comparison results of the DR cases

	Total cost(\$)	Electricity DR cost(\$)	Gas DR costs(\$)	Operating costs(\$)
Case 1	2,197,233.20	-	-	2,197,233.20
Case 1-DR1	1,866,126.23	24,968	-	1,841,158.23
Case 1-DR2	1,588,197.77	-	9,459.2	1,578,739.57
Case 1-DR3	1,562,941.00	23,607	9,400.3	1,529,933.70

From the comparison in Table 4, it can be observed that the coordinated gas-electricity DR program achieve better system economy than single electricity DR or natural gas DR. The utilities will gain more profit if they implement a coordinated DR program in the IES.

4.1.3 Sensitivity analysis of incentive demand response

A sensitivity analysis of incentive DR with respect to the price elasticity of demand is performed to demonstrate the impact of DR model on system operation. To better illustrate the impact of price elasticities on the system economy, this sensitivity analysis applies the assumption that electricity DR is implemented on PL1 and the gas DR is implemented on gas load node 1. The total operation costs with respect to the price elasticity variations of the above nodes are shown in Fig. 10. The price elasticities of electricity and gas loads are denoted with K_e and K_g , respectively.

Fig. 10 Sensitivity analysis of price elasticity of demand response

It can be observed that when $K_e = 0$ and $K_g = 0$, that is, no DR is implemented, the operating cost is very high. With the increase of either or both of the elasticity K_e or K_g , the total operating cost will decrease. From the viewpoint of a utility, they expect to have more consumers participate in the incentive DR programs. Therefore, the price elasticity will be higher and the operating cost will be reduced. However, in real application, the elasticity is closely related to the willingness of the

consumers to participate in the incentive DR programs. It should be noted that the scales of price elasticities of electrical load and gas load are different because electrical load and gas load use different base units MW and kcf, respectively.

4.1.4 Interval optimization based IES model with demand response

According to the formulations and algorithms in 3.2, the interval optimization is applied to the IES model with DR. 3 different levels of wind power uncertainty are considered based on Case1. 10%, 20% and 30% wind power intervals are considered in Case I1, Case I2, and Case I3 respectively with coordinated gas-electricity DR program. Through solving the optimization model, the intervals of operating cost in the above three cases are summarized in Table 5.

Table 5 Intervals of the operating costs of IES

Uncertainty level (%)	Expected total cost(\$)	Maximum costs(\$)	Minimum costs(\$)
Case 1 ($\pm 0\%$)	1,562,941.00	1,562,941.00	1,562,941.00
Case I1 ($\pm 10\%$)	1,569,700.37	1,579,311.65	1,548,089.09
Case I2 ($\pm 20\%$)	1,563,655.12	1,598,311.05	1,540,999.18
Case I3 ($\pm 30\%$)	1,590,910.60	1,620,279.62	1,541,541.58

Form Table 5, it can be observed that with higher uncertainty level of wind power, the width of the operating cost interval is larger. The interval results will provide the decision with the information that the operating cost will fall in which interval under a certain level of wind power uncertainty. The interval power output scheduling of Unit 1 under 20% wind power uncertainty is shown in Fig. 11.

Fig. 11 Power output of Unit 1 under 20% wind power uncertainty

4.2 IEEE 118-bus system with 14-node gas network

In this section, a large gas and electricity IES consisting of a modified IEEE 118-bus system and 14-node gas network [24] is used to demonstrate the performance of the proposed method. In the modified

118-bus system, a total capacity of 1460 MW wind power is distributed on node 12, 17, 56, and 88, electricity DR program is implemented on node 3, 7, 16, 29, 40, 55, 80, 88, 95, and 112; gas demand response is implemented on 3, 5, 10, 11 and 14. The system structure is shown in Fig. 12.

Fig. 12. System configurations of the IEEE 118-bus with 14-node gas network IES

Using the interval based optimization method in this large IES, the optimal operating cost intervals at each hour under 20% wind power uncertainty are shown in Fig. 13.

Fig. 13 Operating cost intervals of IES system under 20% wind power uncertainty

The simulation is carried out using the BONMIN nonlinear optimization solver in YALMIP [25] on a PC with Intel Core i7 3.00 GHz CPU and 8 GB RAM. The computational time of the algorithm on this system is 38.056 seconds, which should satisfy the requirement of practical implementation.

5. Conclusion

In this paper, an interval optimization based operating strategy of gas-electricity IES is proposed to optimally coordinate the operations of the coupled two energy sectors considering demand response and wind power uncertainty. The contributions of this paper are summarized as follows:

- 1) The electricity and gas networks are modeled in details in purpose of coordinated operations within the security constraints of both systems.
- 2) An incentive demand response program is incorporated into the model that provides utilities with an intelligent compensation prices for electricity and gas demand response. The utility companies could coordinate the peak electricity and gas load through the optimized IES demand response.
- 3) Interval optimization is applied on the optimization model of IES coordinated operation to address wind power uncertainty.
- 4) The proposed method is verified by two case studies. The demand response program is proven to be effective in improving the operation efficiency in the IES. The interval optimization provides profit intervals with regarding to the wind power uncertainty levels for the decision makers.

The interval based optimization framework of gas-electricity IES and the demand response program is easy to implement for utilities or ISOs that supply both gas and electricity to customers. The proposed method has a promising value in engineering applications. The uncertainty of demand response can also be represented as interval numbers in the framework, which will be addressed in our future work.

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