NUMERICAL SIMULATION FOR THE DESIGN ANALYSIS OF KINEMATIC STIRLING ENGINES

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Keywords: Thermal model; Stirling engine; CHP; simulation; thermodynamics

Abstract

The Stirling engine is a closed-cycle regenerative system that presents good theoretical properties. These include a high thermodynamic efficiency, low emissions levels thanks to a controlled external heat source, and multi-fuel capability among others. However, the performance of actual prototypes largely differs from the mentioned theoretical potential. Actual engine prototypes present low electrical power outputs and high energy losses. These are mainly attributed to the complex interaction between the different components of the engine, and the challenging heat transfer and fluid dynamics requirements. Furthermore, the integration of the engine into decentralized energy systems such as the Combined Heat and Power systems (CHP) entails additional complications. These has increased the need for engineering tools that could assess design improvements, considering a broader range of parameters that would influence the engine performance when integrated within overall systems. Following this trend, the current work aimed to implement an analysis that could integrate the thermodynamics, and the thermal and mechanical interactions that influence the performance of kinematic Stirling engines. In particular for their use in Combined Heat and Power systems.

The mentioned analysis was applied for the study of an engine prototype that presented very low experimental performance. The numerical methodology was selected for the identification of possible causes that limited the performance. This analysis is based on a second order Stirling engine model that was previously developed and validated. The simulation allowed to evaluate the effect that different design and operational parameters have on the engine performance, and consequently different performance curves were obtained. These curves allowed to identify ranges for the charged pressure, temperature ratio, heat exchangers dimensions, crank phase angle and crank mechanical effectiveness, where the engine performance was improved. In addition, the curves also permitted to recognise ranges where the design parameters could drastically reduce the brake power and efficiency. The results also showed that the design of the engine is affected by the conditions imposed by the CHP interactions, and that the engine could reach a brake power closer to 832 W with a corresponding brake efficiency of 26% when the adequate design parameters were considered. On the other hand, the performance could also be very low; as the reported in experimental tests, with brake power measurements ranging 52-120 W.

43 Nomenclature

A	area (m ²)
A _o	external wet area of the tube (m ²)
C _f	non-dimensional friction coefficient
C _{fd}	form drag coefficient
C _{sf}	skin friction coefficient
C _p	constant pressure specific heat (J/kg K)
C _{pwater}	constant pressure specific heat for inlet water (J/kg K)
C _v	constant volume specific heat (J/kg K)
d	diameter (m)
d _{hy}	hydraulic diameter (m)
E	crank mechanism effectiveness
f	friction factor coefficient
freq	engine frequency (Hz)
F _R	view factor
h	convective heat transfer coefficient (W/m ² K)
h _r	radiation heat transfer coefficient (W/m ² K)
h _{water}	water film heat transfer coefficient (W/m ² K)
k	thermal conductivity (W/mK)
K	piston to displacer swept volume ratio
K _w	thermal conductivity (W/mK)
L	length (m)
m	mass (kg)
n	number of flow resistance layers
m _{water}	mass flow of the inlet water (kg/s)
M	total mass of the working gas (kg)
NTU	Number of transfer units
P	pressure level (Pa)
P _{ch}	engine charging pressure (bar)
P _{br}	engine brake power (W)
Q	heat transfer rate (W)
Q _{ht}	total heating requirement for the engine (W)
Q _{kt}	total cooling requirement for the engine (W)
Q _{lk}	heat loss due to internal conduction (W)
Q _{lsh}	heat loss due to shuttle conduction (W)
Q _{lossr}	heat loss due to regenerator inefficiency (W)
R	gas constant (J/kg K)
R _{ci}	conductive thermal resistance for tubes wall (K/W)
R _{fi}	fouling thermal resistance inside the tubes (K/W)
R _{fo}	fouling thermal resistance outside the tubes (K/W)
R _{hi}	convective thermal resistance inside the tubes (K/W)
t	time (s)
T	temperature (K)
T _{fM}	measured flame temperature (K)
T _{ratio}	cold to heat temperature ratio
T _{wi}	temperature at the internal wall of the tubes (K)
T _{wo}	temperature at the outer wall of the tubes (K)
T _{water_in}	inlet temperature of the water (K)
v	mean velocity (m/s)
V	volume (m ³)
V _{de}	total dead volume (m ³)
W	work flow per cycle (J/cycle)
W _i	engine indicated work (J/cycle)
W _s	engine shaft work (J/cycle)
W _{ploss}	energy loss due to pressure drop (J/cycle)
W-	engine forced work (J/cycle)
X	dead volume ratio

Acronyms

ACM	Aspen Custom Modeller
CHP	Combined Heat and Power
SE	Stirling Engine

Subscripts

b	buffer space
c	compression space
d	displacer
e	expansion space
f	flame
h	heater space
hous	regenerator housing space
i	inside section
k	cooler space
M	measured values
o	outside section
r	regenerator space
w	wall
whe	heater wall
wk	cooler wall

Superscripts

+	positive variation
-	negative variation

Greek symbols

α _s	surface absorptivity
γ	adiabatic constant
η _b	brake efficiency
η _m	mechanical efficiency
η _t	thermal efficiency
σ	Stefan-Boltzmann constant (W/m ² K ⁴)
ε	regenerator effectiveness
ρ	fluid density (kg/m ³)
θ	Crank rotational angle (rad)
μ	viscosity (kg/m s)

1. Introduction

The Stirling engine is a closed-cycle regenerative system that presents good theoretical capacities, which include a high thermodynamic efficiency, low emissions levels thanks to a controlled external heat source, and multi-fuel capability among others [1,2]. These could place the engine in a good position to address the efficiency and fuel flexible requirements for actual energy technologies [1,3,4]. However, the performance of actual prototypes largely differs from the mentioned theoretical potential as reported by Thomas[5] and Klemes [6]. Actual engine prototypes present low electrical power outputs and large energy losses. These are mainly attributed to the challenging mechanical interaction, the complicated heat transfer, and the complex fluid dynamics through the different engine components [7]. These increased the need for engineering tools, such as numerical simulation, that could asses design improvements on the different components in order to increase the engine performance.

Simulation based analysis allows studying the influence of different parameters in the engine performance. The effect of operational variables like working gas temperatures, working fluid[8], engine frequency, compression ratio, and pressure levels have been presented by Strauss [9], Timouni[10,11],Tew [12], Garcia[13], Paul[14],Cheng [15] and others. In addition, the influence of design variables such as heat exchangers configurations[16–18], crank mechanism geometry[15,19,20], and regenerator materials[21,22]have been also reported. The studies have guided the development of Stirling engine prototypes. Some examples are the engines developed by Batmatz[23], Karabulut[24], and Cinar [25]. However, most of the prototypes still present high investments costs and much variation in power outputs and efficiency [5,6]. These problems have decreased the reliability of the engine and thus limited its successful commercialization. For these reasons, there is still a need for the development of engines that should present higher efficiency levels, reliability on the operation, and fuel flexibility. Furthermore, the increased importance of decentralized power generation[26,27] requires that the engine should also be easy to integrate within the recent small scale combined heat and power systems (CHP).

The mentioned studies were based on different type of thermodynamic models, which could be categorized according to their degree of complexity into zeroth order, first order, second order, third order, and multidimensional analysis[28]. The majority of these models focused on the thermodynamics of the gas circuit inside the engine, and the study of the engine as a standalone system. These exposed important aspects of the engine performance, but the integration of the engine into overall systems, such as the CHP plants, was not completely considered. Few integration studies presented some approaches. For example the works of Lombardi[29], Sato [30], Entchev [31], Pålsson[32], and Li [33]. However, these are limited by the use of empirical and first order models that do not reflect the thermodynamics of the engine. For these reasons the present work aims to implement an analysis that could integrate thermodynamics, thermal, and mechanical interactions that would guide the design of Stirling engines suitable for CHP applications. This approach would permit to design the different components of the Stirling engine considering the conditions imposed by the overall CHP system.

The simulation study is part of a project that is developing a micro-scale CHP-SE system suitable for biomass combustion being carried at the Energy Technology department at the Royal Institute of Technology (KTH). The main component of the system is a Stirling engine prototype built by the

Genoa Stirling Company[34]. The prototype was tested on an experimental CHP rig, and the engine showed low performance during the experimental tests [35]. Therefore, the detailed numerical analysis of the engine prototype was applied to determine the effect that the different geometrical and operational parameters could have on the prototype. Different curves of the engine performance that guided to the proposal of design improvements were obtained.

Although the case study concentrated on the analysis of a specific prototype, the importance of integrate the thermodynamic-thermal-mechanical interactions for the design assessment of Stirling engines was probed, and thus could be generalized for the design assessment of new prototypes.

2. Technology Overview

The development of the Stirling engine technology dates back to 1816, with the first patent registered by Robert Stirling[36]. During the 19th century the engine had a slow evolution with few working prototypes. One of the most successful was a 45 HP engine installed to drive the machinery of the Dundee Foundry in 1843[37]. This engine presented a higher efficiency than the steam engines of that time. However, difficulties on the air vessels caused its replacement [37]. The constructional problems, and material failures characterized the Stirling engines of the 19th century, and thus relegated their development to small units used for driving fans and similar light duties[38].

A technological revolution for the Stirling engines began around 1937, when the Phillips research laboratories started the development of small units capable to generate 16 W of electricity[39]. The Phillips research laboratories brought the engine to a high state of technological development from 1937 to 1953[40]. Different prototypes were developed, these included small units like the 16 W single displacer engine, medium size units 500 W with rhombic drive mechanism, double acting 6 kW engines, and large units like a 90 HP built in 1948[39]. In addition, Phillips established research cooperation with prestigious companies and research institutes, such as General Motors, NASA, US Army, Ford, among others[41]. However, the difficulties for participate in a market dominated by the internal combustion engines relegated the commercial success of the engine.

The renewed interest for Stirling engines could be attributed to the actual energy scenario. The increased need to reduce the fossil fuel consumption have emphasized the fuel flexibility and low emissions advantages of Stirling engines[1]. This has turned the interest on the engine, but as part of decentralized systems such as solar concentrators and small scale combined heat and power plants [7]. In this sense, different prototypes suitable for CHP applications have been recently developed. Some examples correspond to the prototypes developed by Sato[30], Nishiyama[42], Biederman[43], Zeiler[44], Li[33] and Obara [45]. In addition, some field tests were reported by Thomas[5] and Klemes [6].

The following table compares the reported characteristics of the mentioned prototypes

Table 1: Reported values for efficiency of SE developed for CHP applications

Engine Description	Power Output (kW)	Fuel	Efficiency (%)			State of Development
			Electrical	Thermal	Combined	
Stirling Danmark SM5A [5]	9	NG/Biogas	20-22	64	84-86	Commercial prototypes
Solo-V 161 [5,6]	2-9	NG/LPG/Solar	22-24	65-75	92-96	Commercial prototypes.

						Cleanergy
WhisperGen-AC [6]	0.75-0.85	NG	11.6 -12.6	81-84	94-96	Commercial prototypes
Stirling Danmark SM3D [5,43]	35	Biomass	10-12	73	85	Commercial prototypes
SE prototype. Obara [45]	1.5	Biomass	13	59	72	Research
Small Scale CHP. Sato [30]	55	Wood/LPG	--	--	--	Demonstration plant
Alpha type prototype. Zeiler [44]	3	Wood chips	23.5	--	--	Research
Beta type prototype. Li [33]	3.4	High temperature waste gases	--	--	--	Research

Table 1 reflects the actual situation of Stirling engines, there are few commercial prototypes suitable for CHP applications. These prototypes present acceptable efficiencies, but due to the prototype stage the commercial prices differ between companies. Some reports presented prices ranging from 1125-3000 \$/kW[5,46].

3. Simulation study

This section describes the simulation analysis implemented for the design assessment of a kinematic Stirling engine. The main objective is to implement a mathematical model that would permit to identify the main parameters that affect the performance of a Stirling engine, when used for CHP applications. This would permit to obtain guidelines for the design improvement

3.1. Description of the mathematical model

The simulation was based on a second order Stirling engine model developed and validated in a previous work [47]. The model considers, sinusoidal volume variations, adiabatic expansion and compression spaces, limited heat transfer through the heat exchangers, mechanical and thermal losses, and evaluates the forced work during the cycle. The set of algebraic differential equations that describe the model are included in Appendix B. These equations were grouped in the blocks shown in Figure 1. The detailed derivation of these equations can be found in Araoz [47,48].

Figure 1: Block diagram for the Stirling engine model [48]

The ideal adiabatic module contains the equations that describe the thermodynamics of the working fluid inside the engine. These equations were coupled with: The internal heat transfer module, which estimates the heat transfer and pressure drop of the working fluid; the external heat transfer module that describes the thermal interactions of the engine within the heat source and the cold sink; the energy losses module that estimated the mechanical and thermal losses in the system; and the

mechanical efficiency module that evaluated the efficiency of kinematic engines by considering the forced work and the friction losses on the crank mechanism.

The described model was applied for the simulation of the Genoa Stirling engine. This is a two cylinder gamma type prototype, built for research studies by GENOA Stirling S.R.L. [34]. The main parameters for the simulation correspond to the experimental conditions of a micro scale combined heat and power system fuelled by wood pellets. The details of the experimental setup are described in Cardozo [35].

3.2. Engine Description

The main components of the engine are the crankcase, the crank mechanism with the balancing flywheel, the heat exchangers and the generator of the engine. These are shown in Figure 2. Further details of the engine are shown in Appendix C.

Figure 2: Genoa Stirling engine scheme [48]

According to the manufacturer specifications, the electrical output for the engine is close to 1 kW when air is used as working fluid, at 600 rpm rotational speed, heater temperature around 750 °C and water at 20 °C as cooling fluid [34]. Furthermore, the heat exchangers materials allows to stand higher temperatures and makes the engine suitable for coupling to systems such as a boilers based on biomass combustion. However, the engine must operate around 5000 hours to overcome the internal friction, and an initial impulse through a direct current voltage source should be given to the flywheel during this period.

The biomass Stirling engine CHP system was run for more than 5000 hours, but the experimental results showed a performance significantly below to the specified by the manufacturer. For these reasons, the simulation analysis was performed with the objective of identify the main problems that could have affected the engine performance.

3.3. Reference case simulation

The model was calibrated for the experimental conditions of the actual prototype[48]. The details of the experimental measurements can be found in Cardozo [35]. These conditions correspond to the Stirling prototype coupled to a biomass boiler and is named as reference case. The geometrical parameters necessary for the simulation are shown in Appendix A.

3.3.1. Model validation

The validity of the model was extensively studied with the GPU-3 Stirling engine [47] , and with the Genoa engine [48]. The validation for the Genoa prototype is summarized in Table 2. This table compares the simulation results for the temperatures at the hot side of the engine (T_h), the cold side of the engine (T_k), and the brake power (P_{br}), with the experimental measurements (T_{hM} , T_{kM} , P_{brM}).

Table 2: Experimental and predicted brake power for the engine [48]

Main operational inputs									
f (Hz)	5.2	5.3	5.3	5.3	5.3	5.4	5.3	5.3	5.6
T_f (K)	1388	1383	1393	1378	1383	1378	1386	1384	1377
P_{ch} (bar)	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5
Predicted and experimental temperatures									
T_h (K)	818.4	807.5	814.2	798.1	806.3	795.7	807.1	802.1	770.8
T_{hM} (K)	816.4	819.6	823.2	830.8	837.4	851.8	853.6	846.4	843.3
Error (%)	0.25	1.47	1.09	3.94	3.71	6.59	5.45	5.23	8.59
T_k (K)	321.1	321.4	321.5	321.6	321.4	321.7	321.5	321.7	322.3
T_{kM} (K)	322.4	321.8	321.6	321.6	322.4	321.8	321.7	321.6	322.1
Error (%)	0.41	0.12	0.04	0.01	0.31	0.03	0.07	0.01	0.05
Predicted and experimental brake power									
P_{brM} (W)	54.7	55.4	55.6	46.3	53.4	50.9	51.0	56.0	47.1
P_{br} (W)	53.6	52.1	53.5	50.0	52.0	50.3	51.6	51.5	46.1
Error (%)	2.06	5.98	3.81	7.94	3.02	1.14	1.31	7.82	2.12

According to these results the model capability to estimate the brake power presented errors ranging from $\pm 1.14\%$ to $\pm 8.00\%$. In the case of the temperatures at the hot side, the errors ranged from $\pm 0.25\%$ to $\pm 8.60\%$. For the temperatures at the cold side, the errors ranged from $\pm 0.05\%$ to $\pm 0.41\%$.

The errors on the estimation of the hot temperatures reflects: the difficulty to evaluate the heat transfer coefficients for the gas-solid-gas convection on the hot heat exchanger, and the thermal inertia during the measurements taken at different intervals of the engine operation. This indicates that the steady state condition was difficult to reach due to the instabilities on the combustion process as discussed in Cardozo [35].

The maximum 8% error for the for the brake power calculations corresponds to only 4W of difference. This deviation is small when compared with the limited capability that second order models have to predict the real power of Stirling engine systems[28]. The robustness of the model was further verified with the additional comparison of experimental and simulation results at different pressures and temperatures presented in Figure 3. The figure indicates that despite the mentioned errors the model calculations are valid at different pressure and temperature conditions.

Furthermore, the model was successfully validated with experimental and numerical data of another engine prototype, the GPU-3 engine [47]. Considering the mentioned reasons the model was proven acceptable for the present study.

3.3.2 Additional results

The simulation allowed to obtain additional results that described the work flow, the heat flow, the variation of mass, and temperatures along the different spaces of the engine. These were reported in detail in Araoz [48]. Table 3 presents the estimation of the different losses through the engine for the following operational conditions: $f= 5.2$ Hz, $T_f=1388$ K and $P_{ch}=12.5$ bar.

Table 3: Simulation results for the energy losses through the engine

Energy losses due to lost work	
Pressure drop heater(W)	2.1
Pressure drop cooler(W)	0.7
Pressure drop regenerator (W)	2.8
Energy losses due to forced work and friction in the crank mechanism (W)	235
Total Energy losses due to lost work (W)	240.6
Heat lost through the engine	
Losses due to internal conduction (Q_{ikr} , W)	269.8
Losses due to shuttle conduction (Q_{lsh} , W)	800.5
Regenerator losses during heating (Q_{lossr} , W)	186.2
Total heat lost (W)	1257
Additional refrigeration	
Regenerator losses during cooling (Q_{lossr} , W)	-186

From Table 3, the model attributed the main energy loss to the forced work during the cycle. This forced work occurs due to the problems in the dynamics of the crank mechanism, the effect of a not suitable buffer pressure for the cycle, and the mechanical effectiveness of the mechanism. The mentioned problems lead to periods with forced expansion and compression processes. Therefore large part of the work might be used to complete the engine cycle instead of being useful brake output. On the other hand, the pressure drops are small for this particular system. Among these, the pressure drop in the regenerator and the heater are the major contributors.

In the case of the thermal energy, the main losses are due to the shuttle conduction. The shuttle conduction corresponds to losses due to the oscillation of the hot displacer across a temperature gradient. These depend on the characteristics of the piston and the displacer cylinder. For instance, these losses increase with the higher thermal conductivity of the piston material, larger diameter of the displacer, and the increment of the gap space between the piston and the displacer cylinder.

The shuttle losses are followed by the internal conduction and the losses due to imperfect regeneration. The losses due to internal conduction are higher when the temperature gradient across the engine is increased. This implies that a careful selection of the operational temperatures is needed. The losses due to imperfect regeneration depend on the thermal performance of this heat exchanger, which is improved by increasing the heat transfer area and the thermal capacity of the matrix material.

It is important to mention that the thermal performance of the engine was largely validated with the data obtained during the experimental work[48]. On the other hand, the evaluation of the mechanical losses requires additional measurements that might support the model.

4. Performance Analysis

The previous section allowed to identify the following points:

- The experimental conditions of the CHP-biomass system established temperatures for the hot side of the engine that are lower than the operation temperature suggested by the manufacturer[34,35]. This reflects the complications inherent to the control of the biomass combustion, which involves the control of the flame stability, the control of the emissions, the air to fuel ratio, and the fouling among others. It must be pointed out that the detailed study of the biomass combustion is out of the scope of this paper. The simulation is limited to evaluate the influence of the hot temperature on the engine performance. A complementary study that evaluated the different combustion parameters was performed in the experimental work of Cardozo et al [49].
- The forced work during the cycle should be diminished. This implies that the dynamics for the expansion and compression processes should be improved. For this reason it is important to evaluate the mechanism synchronization at different phase angles. In addition, the influence of the mechanism effectiveness on the engine performance should be determined.
- The buffer pressure is another parameter that increases the forced work on the cycle. Therefore, it is important to find an optimal buffer pressure that minimizes forced-work periods during the operation.
- The shuttle conduction is one of the main thermal losses during the cycle. Consequently, the re-design of the displacer-cylinder space should consider: changes on the diameter of the displacer; materials with low thermal conductivity for the piston and displacer; and the reduction of the stroke length for the displacer.
- The heat exchangers determine the operational temperatures at the different spaces of the engine. For this reason, their design should determine the size that would balance the improvements of the thermal performance, but without increasing the pressure drops or dead volumes.

The above points are studied in detail in the following sections.

4.1. Analysis for the influence of the operational conditions on the engine performance

The operational parameters considered for the simulation are: the pressure charge of the engine (P_{ch}), the flame temperature reached during the combustion (T_f), and the inlet temperature of the cooling fluid (T_{water}). The variations for the mentioned parameters were introduced in the model, and the influences that these have on the brake power and brake efficiency were analysed. For instance, Figure 3 presents the results for the measured and estimated brake power at different temperatures of the working fluid inside the heater space (T_h), and different charge pressures (P_{ch}). From this figure, it can be seen that the brake power predicted by the simulation presents a linear and positive tendency for increments of the temperature at different pressure levels. These tendencies are also presented in the experimental measurements and thus support the simulation results.

Figure 3: Brake power at different pressure levels and heater temperatures

The results from Figure 3 are complemented with the numerical results obtained for the broader ranges of temperatures and charge pressures shown in Figure 4. This figure uses the non-dimensional temperature ratio (T_{ratio}) as a parameter to express the relation between the flame temperature (T_f), and the water inlet temperature (T_{water}). In Figure 4, the experimental data for three operational points [35] are marked as “♦”. The additional operational conditions were difficult to reach experimentally and thus only simulation results are presented.

Figure 4: Influence of the charged pressure and temperature ratio on the engine brake power

The curves shown in Figure 4 represent the variation of the brake power with the charged pressure at different temperature ratios. These present similar shapes and present two critical points, indicated as "1" and "2" in the curves. The brake power increases until the first critical point, after this the brake power decreases drastically. Then the brake power is nearly constant until the second critical point is reached. After this point the brake power decreases to values closer to zero, which means that the engine will not operate. The tendency is shown at different temperature ratios, and it can be observed that with higher ratios the break points are found at lower pressures. This is reflected on the experimental measurements that indicated that the temperature needed to maintain the start-up of the engine were $T_f=700\pm 10$ °C, $P_{ch}=12$ bar and $T_f=820\pm 10$ °C, $P_{ch}=15$ bar which corresponds to $T_{ratio}=0.30$ at $P_{ch}=12$ bar and $T_{ratio}=0.28$ at $P_{ch}=15$ bar. For temperature ratios larger than these limits the engine did not started.

The described behaviour could be explained by the fact that the charging pressure determines both: The buffer pressure, which is related to the increment of the forced work; and the mean pressure, which improves the power output. Consequently, it can be inferred that the increment of the charged pressure increases the engine performance, but only until the point that the forced work during the cycle becomes larger than the brake power improvement. In addition, the second brake point could represent the condition where the heat input is below to the amount required for the production of work at the higher charging pressures.

The analysis is simpler for the variations with the temperature ratio. The reduction of this ratio, which implies higher temperatures at the hot side, constantly increases the engine performance. This reduction is only limited by the thermal resistance of the material at the hot side of the engine, and the cooling requirements at the cold side.

As a conclusion, the improvement of the engine brake power requires a coordinated increment of the flame temperature and the charged pressure. In this sense, the results suggest a charged pressure close to 22 bars, with a temperature ratio closer to 0.16. This ratio correspond to flame temperatures closer to 1800 K, which for example represents the combustion of wood pellets with low humidity [50], and a water inlet temperature around 298 K. The non-coordinated selection of the mentioned parameters could diminish drastically the engine performance. For example operating at higher pressures and low temperature ratios would give very low brake powers or even no engine operation.

4.2. Analysis for the influence of geometrical parameters on the engine performance

This section presents the results for the simulation of the engine, under different geometrical conditions. It is important to mention that the variations of the geometrical parameters imply the physical modification of the prototype. This denoted budget considerations for the construction of a new prototype. For this reason this section is limited to the analysis based on numerical results.

4.2.1 Influence of the crank mechanism and phase angle on the engine performance

The crank mechanism determines the dynamics of the piston-displacer motion during the engine operation. In addition, this mechanism transfers the energy produced during the cycle into the shaft rotation that produces the electricity in the generator. The analysis considered two parameters that characterize the crank mechanism: the crank phase angle (θ), and the mechanical effectiveness (E). The crank phase angle determines the phase difference between the piston and displacer of the engine. Consequently, determines the dynamic of the volume variations. On the other hand, the effectiveness of the crank mechanism represents the ratio of the actual torque to the ideal torque that would be present if there were no friction losses [51]. Hence, determines the losses on the shaft. The analysis considered the numerical simulation over a wide range of mechanical effectiveness and phase angle values. The results for the calculation of the brake power and brake efficiency are shown in Figure 5 and Figure 6 respectively. The simulation considered the operating conditions suggested in the previous section, $P_{ch}= 22$ bars and $T_{ratio}= 0.16$.

Figure 5: Numerical results for the brake power under different phase angles and mechanism effectiveness
 $P_{ch}=22 \text{ bar}$ $T_{ratio}=0.16$

Figure 6: Numerical results for the brake efficiency under different phase angles and mechanism effectiveness
 $P_{ch}=22 \text{ bar}$ $T_{ratio}=0.16$

According to the results of Figure 5 and Figure 6, the effectiveness of the mechanism has a large influence on the brake power and brake efficiency. The region with low effectiveness presents values much below to the results that correspond to values closer to ideal effectiveness. Furthermore, the mechanism effectiveness determines the upper limits for the brake power and efficiency over the different phase angles studied. For example, mechanical effectiveness between $E=0.1$ and $E=0.3$ are limited to brake power values in the order of $P_{br}=130 \text{ W}$ and brake efficiencies around $\eta_b=0.05$. On the other hand, effectiveness between $E=0.7$ and $E=0.9$ presents values for the brake power and efficiency around $P_{br}=500 \text{ W}$ and $\eta_b=0.25$. These values confirm the importance that the mechanism effectiveness has for the performance of Stirling engines, which may explain the low performance of the actual prototype. However, additional experimental measurements that determine the losses of the mechanism are necessary to support the analysis.

The influence of the crank phase angle can also be noticed in Figure 5 and Figure 6. According to these figures, the curves for the brake power and brake efficiency have parabolic shapes with the presence of maximum points. This shape is presented for brake power and brake efficiency curves at different mechanism effectiveness. This can be better noticed in Figure 7, where the brake power and brake efficiency are plotted considering a fixed mechanical effectiveness $E=0.9$.

The presence of the maximum reflects the point where a good synchronization of the piston-displacer motion is obtained. This implies that the mechanism guarantees an expansion at high temperatures and compression at lower temperatures. In addition, the presence of forced compression and expansion processes are minimized. It is also important to notice that the brake power and efficiency that correspond to the design phase angle (90°) are slightly below to the values that correspond to the maximum point located closer to 80° .

Figure 7: Numerical results for the variation of the brake power and brake efficiency with crank phase angles
 $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$

4.2.2. Analysis for the influence of the heat exchangers on the engine performance

The analysis of the heat exchangers evaluates the adequate sizing of these elements. In the case of Stirling engines the size of the heat exchangers could have two different effects. The first is a positive increment of the engine efficiency thanks to larger heat transfer areas. The second is a negative reduction of the brake power due to large dead volume on the engine. Therefore, the sizing implies a balance between the mentioned effects to increase the overall engine performance.

4.2.2.1. Heater and cooler analysis

The heater and cooler are compact tubular heat exchangers. For this analysis, the length and diameter of the pipes were chosen as design parameters. These were varied within a range of values closer to the dimensions of the reference prototype, and the engine performance was evaluated at these different values.

Figure 8 presents the results for the influence of the heater sizing on the brake power of the engine. According to this figure the results for the brake power of the engine can be categorized in different areas. The largest values correspond to the region limited by the tube diameters of $d_h=0.003$ m to $d_h=0.006$ m, and tube lengths of $L_h=0.16$ m to $L_h=0.22$ m. The brake power decrease for diameters larger than $d_h=0.006$ m. In addition tube lengths below $L_h=0.12$ m correspond to the lowest values of the brake power. These results reflect the benefit of increasing the heat transfer area, but until the limit where the increment of the dead volume and pressure drop starts to diminish the net power output.

Figure 8: Numerical results for the variation of the brake power with the heater sizing.
 $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$

In order to complement the analysis, the influence that the heater sizing has on the brake efficiency is shown in Figure 9. From this figure, three different regions for the brake efficiency are identified. The area that corresponds to higher brake efficiencies is limited by the heater diameters between $d_{ih}=0.003$ m to $d_{ih}=0.006$ m. The figure also shows that contrary to the brake power case, the brake efficiency decreases with the increment of the heater length. This indicates that the negative influence of the dead volume and pressure drop is higher than the benefit of the increased heat transfer area.

Figure 9: Numerical results for the variation of brake efficiency with the heater sizing.
 $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$

The variations for the engine brake power and brake efficiency with the cooler sizing are presented in Figure 10 and Figure 11 respectively.

Figure 10: Numerical results for the variation of brake power with the cooler sizing.
 $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$

Figure 11: Numerical results for the variation of brake efficiency with the cooler sizing.
 $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$

From Figure 10 and Figure 11 the following points can be observed:

- The surfaces from Figure 10 and Figure 11 present similar shapes. This indicates that the effects for the variation of the diameter and length are similar on both the brake power and efficiency of the engine. For example, increasing the cooler diameter and the length of the tubes diminishes both the efficiency and brake power.

- The negative influence that the increment of the tube length has on the engine performance is higher at greater diameters. However, this influence is almost negligible at diameters closer to $d_{ik}=0.003$ m. This reflects that the design of the cooler is very sensitive to the reduction of the dead volumes, being of less influence the increment of the heat transfer area. This result reflects the consequence of higher heat transfer coefficients at the cooler side.
- The presence of maximums points for brake power and brake efficiency are observed closer to $d_{ik}=0.003$ m. These maximums represent the points where the optimum balance between dead volume and heat transfer is reached. For larger diameters an oversizing of the cooler can be observed.
- The variation of the engine performance with changes on the cooler size is lower compared to the influence that the heater sizing presented in the previous section.

In general, the heater and cooler analyses permitted to identify their effect on the brake power and brake efficiency for the engine design. This is further discussed with the results presented in Figure 12 and Figure 13.

Figure 12: Numerical results for the variation of brake efficiency and brake power with heater and cooler diameters at fixed tube length. Conditions: $E=0.9$; $P_{ch}=22$ bar; $Tratio=0.16$; $L_h=0.149$ m; $L_k=0.032$ m

Figure 12 presents the variation of the brake power and brake efficiency at different diameters, but considering fixed lengths for the tubes of the cooler and heater. The figure also presents the values that correspond to the actual design diameters ($\blacklozenge$), but at the conditions specified in the description. These curves permit to identify better the diameters of the heater and cooler that correspond to the maximums points for the brake power and the brake efficiency. In the case of the heater, the maximum for the brake power is closer to $d_{ih}=0.006$ m and the maximum for the brake efficiency is closer to $d_{ih}=0.003$ m. These differ in position and thus confirm a trade-off between efficiency and power output during the diameter sizing. In the case of the cooler diameter, the increment of this diameter negatively influences the brake power and efficiency along the entire range studied.

The present study prefers the maximization of the power output because the engine would be coupled to a combined heat and power system, and thus heat losses would be recovered. Therefore, the heater diameter is suggested to be around: $d_{ih}=0.005$ m and $d_{ik}=0.003$ m. These values would guarantee a better power output but a slight reduction of the engine efficiency.

Following a similar approach, the results for the brake power and brake efficiency with the variation of the length of the tubes at the fixed diameters are presented in Figure 13. The results for the actual design values for the cooler and heater length are also identified.

Figure 13: Variation of brake efficiency and brake power with heater and cooler lengths at fixed tube diameters.
Conditions: $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$; $d_{ih}=0.005$ m; $d_{ik}=0.003$ m

Figure 13 shows that increasing the length of the heater will constantly improve the brake power of the engine. This reflects the gain of brake power due to the increased heat transfer area. However, length values larger than $L_h=0.17$ m have a negative influence on the brake efficiency. This reflects the negative effect of the dead volume and pressure drop on the engine efficiency. The high sensitivity to the changes of the heater length is also noticed.

In the case of changes for the cooler length, the increments will constantly decrease the brake power of the engine. However, this increment will improve the brake efficiency until a maximum point is reached. After this maximum the efficiency starts to decrease along with the brake power.

Considering the mentioned trade-off between the brake power and brake efficiency, and according to the criteria of improving the brake power, the tube lengths are suggested to be closer to $L_k=0.085$ m and $L_h=0.16$ m.

4.2.2.2. Regenerator Analysis

The present analysis evaluates two possible changes in the regenerator. The first correspond to the variation of the volume that contains the regenerator matrix, and the second corresponds to the selection of the matrix screen itself.

For the first case, the volume of the regenerator is modified in two ways: With a variation of the housing length and with changes on the internal diameter of the annular gap, which is shown in

Figure C1 in Appendix C. In addition, for this case it is assumed that regenerator matrix is stainless steel, with porosity fixed and equal to the prototype conditions specified in table A3 appendix A.

The influence that the changes of the regenerator length have on the engine brake power and brake efficiency is shown in Figure 14. From this figure, it can be noticed that the increment of the regenerator length (L_r) increases the engine brake power until approximately $L_r=0.145$ m, after this point the brake power reduces slightly. On the other hand, the engine efficiency reaches a maximum value around $L_r=0.095$ m, and then sharply decreases. These results reflect a compromise between the increment of the regenerator capacity and the negative increment of dead volume and pressure drop produced by the changes on the length. This reflects the different position of the maximum values for the brake power and brake efficiency. For this analysis, the brake power is preferred and the regenerator length is suggested to be around, $L_r=0.10$ m. This value will increment the power output, but the engine will not operate at the point of maximum efficiency.

Figure 14: Numerical results for the Influence of the housing diameter and length of the regenerator in the engine performance. Conditions: $E=0.9$; $P_{ch}=22$ bar; $Tratio=0.16$; $d_{ih}=0.005$ m; $d_{ik}=0.0035$ m; $L_h=0.16$ m; $L_k=0.085$ m

Figure 14 also shows the variation of the brake power and brake efficiency for the changes on the internal diameter of the annular gap. The variation of the annular gap would affect the displacer diameter and thus the dimension of the expansion volume, which will directly influence the engine performance. Therefore, the increment of the annular diameter is translated into the increasing power output shown in Figure 14. However, this increment also implies the reduction of the regenerator annular area, and therefore the reduction of its thermal storage capacity. This is observed, when the engine efficiency and brake power start to decrease, around $d_{ihous}=0.095$ m, which is the value that maximizes both the brake power and the brake efficiency.

The analysis for the regenerator must also consider the influence that the regenerator-matrix has on the engine performance. For this purpose, the brake efficiency was calculated considering different

values of the three main parameters that characterize the wire screen mesh: Matrix porosity, wire diameter and matrix material. The results are shown in Figure 15 and Figure 16.

Figure 15: Numerical results for the variation of the brake efficiency with the regenerator porosity. Conditions: $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$; $d_{ih}=0.005$ m; $d_{ik}=0.0035$ m; $L_h=0.16$ m; $L_k=0.085$ m

The wire diameter and matrix porosity define a characteristic Reynolds number and consequently the flow and heat transfer conditions of the working fluid across the regenerator, and thus have influence on the brake efficiency of the engine. This is shown in Figure 15, where a maximum value appears in each of the curves that represent the variation of the brake power with the porosity at different wire diameters. These results correspond with the correlations used for the model, which indicates two opposite effects: The increment of the heat transfer rate accompanied with an increased pressure drop, or the reduction on the pressure drop but also reduction of the heat transfer rate. Furthermore, Figure 15 shows that the maximum point for each curve is slightly displaced towards higher porosity values when the wire diameter decreases. Therefore, the selection of the matrix must consider the correspondence between both parameters: The porosity and wire diameter. In this case a wire diameter of $d_{wire}=0.114$ mm and the porosity value closer to porosity=0.8, are suggested as a possible combination that could maximize the brake efficiency. It is also important to notice that in the case of the curve that corresponds to the widest wire diameter ($d_{wire}=0.254$ mm), the maximum value for the brake efficiency is lower compared with the maximum of the other curves. These maximums started to decrease at diameters lower than $d_{wire}=0.114$ mm, which is therefore chosen as the best wire diameter for the case studied.

Figure 16 complements the matrix analysis with the results of the simulation considering matrix materials with different thermal conductivities: Stainless steel, copper, aluminium and monel. For these simulations the wire diameter was fixed equal to $d_{wire}=0.114$ mm. The results show that the variation of the brake efficiency with different thermal conductivities is almost negligible at the simulation conditions. This confirms the fact that the heat losses due conduction through the regenerator are very small compared with the other losses on the system. However, it is important to notice that the regenerator model is a simplified version, and consequently the effects of additional

variables such as the regenerator thermal stress, and possible corrosion must be considered to select an adequate material. These effects were experimentally reported by Gheith et al [52], and the use of copper was not recommended due to the risk of oxidation. Therefore, among the other three materials, the stainless steel and monel are preferred over the aluminium which in addition is limited by the melting temperature of this material.

Figure 16: Variation of the brake efficiency with the regenerator porosity considering different matrix materials. Conditions: $E=0.9$; $P_{ch}=22$ bar; $T_{ratio}=0.16$; $d_{ih}=0.005$ m; $d_{ik}=0.0035$ m; $L_h=0.16$ m; $L_k=0.085$ m; $d_{wire}=0.114$

4.3. Proposed Improvements

Table 3 summarizes the suggested improvements derived from the performance analysis. It is important to mention that these improvements imply many challenges for the manufacturing process, but still are useful guidelines for the engine design.

Table 4: Proposed improvements for the prototype

Main input parameters	Original design parameters	Proposed improvements
Operational conditions		
P_{ch} (bar)	12.5	22
T_{ratio}	0.21	0.16
Crank mechanism		
θ (deg)	90	80
Heater analysis		
d_{ih} (m)	0.003	0.005
L_h (m)	0.149	0.160
Cooler analysis		
d_{ik} (m)	0.003	0.003
L_k (m)	0.032	0.085
Regenerator analysis		
d_{ihous} (m)	0.078	0.095
L_r (m)	0.070	0.100
K_{wr} (W/mK)	27	27
Porosity	0.87	0.80
d_{wire} (m)	2.5×10^{-4}	1.14×10^{-4}
Experimental Results		
Mean brake power (W)	54.7	--
Numerical Results		
Brake power (W)	53.6	832

5. Conclusions

In this paper, a numerical simulation that integrated the thermodynamics, thermal and mechanical interactions that characterize Stirling engines used for CHP applications was implemented and proved useful for the design assessment. This approach allowed to identify the following points:

- There is a need to reach equilibrium conditions between the heat transferred to the engine and the heat that the internal fluid requires to complete the thermodynamic cycle. According to the numerical results an imbalance could be present when the engine is charged at elevated pressures and the temperature at the hot side remains below a required value.
- There is a large and negative influence that the forced work have on the performance of kinematic engines. The minimization of the forced work requires to determine adequate operational parameters (charged pressure), and design parameters (phase angle). The identification of these parameters requires a simulation that integrates the thermodynamic and the mechanical analysis, as implemented for this case study.
- The design of the heat exchangers requires a balance between the dead volumes and the required heat transfer areas. The oversizing would drastically reduce the brake power, and the opposite would reduce the brake efficiency.
- The curves obtained with the simulations not only allowed to identify ranges where the parameters improved the engine performance. But, also allowed to identify the ranges where the design parameters could drastically reduce the brake power and efficiency of the system. Therefore, by using these guidelines the failure risks on the design of the engine should be reduced.
- For the case study, it was found that design modifications would increase the brake power to values closer to 832 W with a corresponding brake efficiency of 26%. On the other hand, the performance could also be very low as the reported for the experimental conditions, with a brake power and efficiency around 52 W and 3% respectively.

The methodology implemented showed the importance of a simulation that integrates all the components that determine the performance of Stirling engine for CHP applications. This is especially important for small/micro scale systems, since the interactions are more relevant at these sizes.

6. Acknowledgments

The work was done with the financial support of the Swedish International Development Cooperation Agency (SIDA); the division of Heat and Power Technology, Department of Energy Technology at Royal Institute of Technology (KTH), in Sweden; and Universidad Mayor de San Simon (UMSS) in Bolivia.

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702

Appendix A: Detailed Stirling engine parameters

Table A1: Inputs for the Cooler in ACM

Variable	Value	Units	Description
d_o	0,005	m	tubes external diameter
d_i	0,003	m	tubes internal diameter
kw	14,200	W/mK	material conductivity
L	0,032	m	tubes length
num	162	--	number of tubes
sl	0,005	m	space between tubes

Table A2: Inputs for the Heater in ACM

Variable	Value	Units	Description
de	0,005	M	tubes external diameter
di	0,0031	M	tubes internal diameter
kw	14,2	W/m K	material conductivity
len	0,149	m	tubes length
num	36,0	--	number of tubes
sl	0,005	m	space between tubes

Table A3: Inputs for the Regenerator in ACM

Variable	Value	Units	Description
din	0,078	m	regenerator housing internal diameter
dout	0,107	m	regenerator housing external diameter
dwire	2,1e-004	m	wire diameter of the matrix
kwr	27,0	W/m K	thermal conductivity of the matrix material
Lr	0,07	m	length of the regenerator housing
porosity	0,87		matrix porosity

Table A4: Inputs for the expansion- compression spaces and crank mechanism

Variable	Value	Units	Description
vclc	4,4e-006	m ³	compression space clearance volume
vcle	2,6e-005	m ³	expansion space clearance volume
vswc	9,26e-005	m ³	compression space swept volume
vswe	2,5134e-004	m ³	expansion space swept volume
dispd	0,062	m	displacer diameter
displ	0,07	m	displacer length
effmek	0,8	--	mechanism effectiveness
F	10	Hz	Frequency
jgap	0,006	M	gap between cylinder displacer and wall
kpist	16,27	W/m K	piston conductivity
pbuff	1,2e+006	Pa	buffer pressure
phase	90,0	deg	phase angle advance
pmean	1,25e+006	Pa	mean operating pressure
strk	0,035	m	displacer stroke
dispd	0,062	m	displacer diameter

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Table A5: Working and Cooling Fluid Inputs in ACM

Variable	Value	Units	Description
Working fluid	Nitrogen	---	working fluid inside the engine
Cooling Fluid	Water	-----	cooling fluid through the engine cooler
T_{cooling}	298	K	Inlet temperature of the cooling fluid

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Table A6: External combustion inputs in ACM

Variable	Value	Units	Description
T_{ad}	1387	K	Flame temperature in the combustion chamber
absorp	0,70	---	absorptivity of the heater material

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Appendix B
Main equations for the Stirling engine model

Stirling engine module

Mean Pressure

$$P = \frac{MR}{\left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e} \right)}$$

Pressure variation

$$dP = \frac{-\gamma P \left(\frac{dV_c}{T_{ck}} + \frac{dV_e}{T_{he}} \right)}{\left[\frac{V_c}{T_{ck}} + \gamma \left(\frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} \right) + \frac{V_e}{T_{he}} \right]}$$

Mass of the working gas in the different spaces

$$m_c = p \left(\frac{V_c}{RT_c} \right); m_k = p \left(\frac{V_k}{RT_k} \right); m_r = p \left(\frac{V_r}{RT_r} \right); m_h = p \left(\frac{V_h}{RT_h} \right); m_e = p \left(\frac{V_e}{RT_e} \right)$$

Mass accumulation

$$dm_k = m_k \left(\frac{dP}{P} \right) d\theta; dm_r = m_r \left(\frac{dP}{P} \right) d\theta; dm_h = m_h \left(\frac{dP}{P} \right) d\theta$$

$$dm_c = \frac{PdV_c + \frac{V_c dP}{\gamma}}{RT_{ck}}; dm_e = \frac{PdV_e + \frac{V_e dP}{\gamma}}{RT_{he}}$$

Mass Flow

$$m_{ck} = -dm_c; m_{he} = dm_e; m_{kr} = m_{ck} - dm_k; m_{rh} = m_{he} + dm_h$$

Conditional Temperatures

$$\begin{aligned} \text{If } m_{ck} > 0 \quad & \text{then } T_{ck} = T_c \quad \text{else } T_{ck} = T_k \\ \text{If } m_{he} > 0 \quad & \text{then } T_{he} = T_h \quad \text{else } T_{he} = T_e \end{aligned}$$

Temperatures

$$dT_c = T_c \left(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c} \right); dT_e = T_e \left(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e} \right)$$

Energy

$$dQ_k = \frac{V_k dP_{c_v}}{R} - c_p (T_{ck} m_{ck} - T_{kr} m_{kr}); dQ_r = \frac{V_r dP_{c_v}}{R} - c_p (T_{kr} m_{kr} - T_{rh} m_{rh})$$

$$dQ_h = \frac{V_h dP_{c_v}}{R} - c_p (T_{rh} m_{rh} - T_{he} m_{he}); dW_c = PdV_c; dW_e = PdV_e$$

743 **Internal heat transfer module**

744 Heat transfer from the heater wall to the working gas

$$745 \quad Q_h = \frac{1}{R_{cih} + R_{hih} + R_{fih}} (T_{woh} - T_h)$$

746 Heat transfer from the cooler wall to the working gas

$$747 \quad Q_k = \frac{1}{R_{cik} + R_{hik} + R_{fik}} (T_{wik} - T_k)$$

748 Heat loss during the regenerator process

$$749 \quad Q_{lossr} = (1 - \varepsilon) \times Q_r$$

750 Regenerator effectiveness

$$751 \quad \varepsilon = \frac{NTU}{1 + NTU}$$

752 **External heat transfer module**

753 Heat transfer from the flame to the external wall of the heater

$$754 \quad Q_h = \frac{1}{\frac{1}{h_{rh} A_{oh}} + R_{foh}} (T_{ad} - T_{woh})$$

$$755 \quad h_{rh} = \alpha_s \sigma A_{oh} F_R (T_{ad} + T_{woh}) (T_{ad}^2 + T_{woh}^2)$$

756 Estimation of the outlet temperature of the cooling fluid

$$757 \quad T_{wok} = T_{water_in} + Q_k \left(\frac{1}{h_{ok} A_{ok}} + \frac{1}{2m_{water} C_{pwater}} \right)$$

$$758 \quad h_{ok} = \frac{1}{\frac{1}{h_{water}} + R_{fok}}$$

759 **Energy Losses**

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Pressure drop in the heat exchangers

$$761 \quad \Delta P = \frac{f}{d_{hy}} \frac{1}{2} \rho v^2 l$$

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Pressure drop in the regenerator

$$763 \quad \Delta P = C_f n \frac{\rho}{2} u^2$$

$$C_f = C_{fd} + \frac{C_{sf}}{Re}$$

$$C_{sf} = 68.556 \quad C_{fd} = 0.5274 \quad \text{wire screens}$$

$$C_{sf} = 70.035 \quad C_{fd} = 0.9307 \quad \text{metal felts}$$

Total Pumping losses

$$W_{ploss} = \int_0^{2\pi} \left(\sum_{i=1}^{i=3} \Delta P_i \times \frac{dV_e}{d\theta} \right) d\theta$$

Energy losses due to internal conduction

$$Q_{lk} = \frac{k_r A_r}{L_r} (T_{wih} - T_{wik})$$

Energy losses due to shuttle conduction

$$Q_{lsh} = 0.4 \frac{Z^2 K_{pist} D_d}{J L_d} (T_e - T_c)$$

773 Mechanical Efficiency module

774 Thermal efficiency (η_t): Ratio between indicated or thermodynamic work and total heat input

$$\eta_t = \frac{W_i}{Q_{ht}}$$

775

776 Mechanical efficiency (η_m): Ratio between shaft work and indicated work of the cycle

$$\eta_m = \frac{W_s}{W_i}$$

778 Mechanical efficiency considering the mechanism effectiveness and forced work:

$$\eta_m = E - \left(\frac{1}{E} - E \right) \frac{W_-}{W_i}$$

780 Forced work

$$W_- = \oint (P - P_b)^+ (dV)^- + \oint (P - P_b)^- (dV)^+$$

782 Brake efficiency (η_b): Ratio between the actual shaft work and the total heat input to the engine

$$\eta_b = \frac{W_s}{Q_{ht}} = \frac{W_i}{Q_{ht}} \frac{W_s}{W_i} = \eta_t \eta_m$$

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Appendix C: Stirling engine components

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Figure C1: Annular Regenerator

Annular gap that contains
the regenerator material

Displacer piston

Housing Internal
Diameter

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Figure C2: Engine Crank Mechanism

Figure C3: Compression space and Power Piston

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Figure C4: Heater Head and Displacer

Figure C5: Cooler heat exchanger

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