1 Dryout Avoidance Control for Multi-Evaporator Vapor 2 Compression Cycle Cooling

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10 Abstract

11 Two-phase cooling systems (e.g., pump-loops, vapor compression cycles) present
12 a promising approach to the cooling of modern high power electronic systems. Such
13 systems can achieve high coefficient of heat transfer but may incur dangerously
14 high temperature when the imposed heat flux exceeds the critical heat flux (CHF).
15 This paper presents a new control strategy that provides robust CHF avoidance
16 in two-phase cooling for multiple evaporators under dynamic heat loads. This
17 approach operates near the CHF to allow reduced coolant flow rates and greater
18 system efficiency compared with the typical, and more conservative, open-loop or
19 worst-case control strategies. The control architecture consists of two loops: an
20 outer loop to determine evaporator mass flow rate demand, and an inner loop
21 to supply and distribute the coolant using system actuators. Each control loop
22 is implemented with feedforward based on static models as well as proportional-
23 integral (PI) feedback controllers, and the decentralized approach allows scalable
24 control design without the need for high fidelity nonlinear dynamic models. Simu-
25 lations and corresponding experimental controller validations were conducted using
26 a three-evaporator vapor compression cycle (VCC) testbed with transient imposed
27 heat flux. The full closed-loop system is shown to be able to operate near CHF
28 while avoiding CHF under transient heat flux, demonstrating both efficiency and
29 robustness.

30 *Keywords:*

31 vapor compression cycle (VCC), critical heat flux (CHF), control, thermal
32 management, electronics cooling

Nomenclature

A_{cs}	cross-sectional area (m ²)
A_s	surface area (m ²)
A_v	valve area opening (%)
A,B,	dynamic system state-
C,D	space matrices
c_p	specific heat (J/kg-K)
D_h	hydraulic diameter (m)
EEV	electronic expansion valve
G	mass flux (kg/m ² -s)
h	enthalpy (J/kg)
H	heat transfer coefficient (W/m ² -K)
k_v	valve area discharge coefficient (m ² /%)
L	length (m)
K	control gain
$\dot{m}$	mass flow rate (kg/s)
N	total number of evaporators, EEV
P	pressure (Pa)
q	heat input (W)
s	complex frequency (continuous)
T	temperature (K)
t_p	time period (s)
u	control input vector
VCC	vapor compression cycle
V_m	compressor volume (m ³)
w	disturbance input vector
x	flow quality
x	state variable vector
y	output variable for feedback

y output variable vector

Greek

α	scaling factor
η_v	volumetric efficiency
ρ	density (kg/m ³)
Θ	superheat (K)
ω	compressor frequency (Hz)
ω_b	bandwidth (rad/s)

Subscripts

a	heated accumulator
c	condenser
cs	cross-sectional
$crit$	critical
d	accumulator ambient disturbance
e	evaporator
end	evaporator end
f	saturated liquid
g	saturated vapor
i	inlet
I	integral
m	compressor
n	evaporator, valve number
o	outlet
P	proportional
sat	saturated
set	setpoint
t	total
v	valve
w	wall

1. Introduction

As high-power electronics continue to be used on an industrial scale in data centers, all-electric vehicles (AEV), and buildings, thermal management technology must keep pace to provide adequate and efficient cooling [1]. For example, data centers alone accounted for an estimated 2% of U.S. electricity usage in 2010, almost half of which was spent on infrastructure and cooling [2]. Energy efficiency must be addressed in the design of new cooling systems, in addition to the challenges posed by the increasing heat flux and the large heat-load transients of electronics.

Conventional single-phase heat removal techniques such as forced-air cooling are impractical for managing new, high heat flux devices, due to low fluid thermal conductivity and heat capacity. Alternatively, two-phase (evaporative) cooling provides a large boiling heat transfer coefficient and heat capacity via the latent heat of vaporization, and therefore is an attractive thermal management solution for high heat flux applications. However, there are several technical challenges that must be addressed to ensure the efficiency and performance of two-phase cooling systems. For instance, high evaporator heat fluxes present the risk of critical heat flux (CHF) or boiling crisis, where wall dryout causes a sudden reduction in heat transfer coefficient and dramatic rise in wall temperature [3, 4]. Since CHF can lead to device failure or burnout, two-phase cooling systems must be constrained to operate with wetted, two-phase flow at evaporator outlets. This constraint frequently results in overfeeding the evaporators with excess coolant and wasting pump or compressor power in the process. Additionally, electronics with varying heat flux often require “dynamic thermal management” (DTM) [5, 6], necessitating transient analysis of the cooling system as well mitigation strategies for heat load disturbances, such as active control for disturbance rejection.

Current strategies for CHF avoidance in active two-phase cooling of electronics include heat-spreaders or cold-plates to enhance heat transfer and lower heat flux [7, 8, 9, 10, 11, 12, 13, 14], setting low evaporator outlet quality setpoints to maintain a large CHF safety factor [7, 11, 12, 13, 14], and CHF avoidance control using wall temperature feedback proposed by our group [15, 16, 17, 18]. Several experimental electronics cooling studies that implement these techniques are summarized in Table 1, with representative

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test section conditions reported. Taken together, these studies demonstrate the diverse conditions and challenges associated with two-phase electronics cooling.

Most of the past experimental research on two-phase cooling cycles for electronics applications has focused on prototype development and parametric performance evaluation. A central issue that remains for both pumped-loops and VCC for electronics cooling is CHF avoidance, which provides a challenging operating constraint. Using a large heat spreader [8] may not be suitable for electronics systems with size or weight constraints. Overfeeding the evaporators with refrigerant can provide a large CHF safety factor [7, 11, 12, 13], but it increases power consumption by pumps or compressors to supply the desired flow rate. Furthermore, there lacks systematic design approach to address the inherent nonlinearity of CHF.

In this paper, we present a control-theoretic approach to dryout-avoidance. We first determine the target evaporator outlet quality based on the estimated critical quality using the evaporator wall temperature measurements. An intuitive two-loop control structure is used to achieve the target outlet quality. The outer loop generates mass flow rate demand for each evaporator for the given heat load and end wall temperature. The inner loop provides mass flow rate supply and distribution using the system actuators (compressor, EEV, accumulator heat). The steps for this approach are displayed in Figure 1. We first use a simplified nonlinear dynamic model to tune the controller parameters for performance and robustness. Performance is characterized in terms of the speed of response (bandwidth) and level of disturbance rejection (gain). Robustness is expressed in terms of gain and phase margins. The resulting controller is then applied to the experimental three-evaporator VCC system in our laboratory to assess the actual performance of our approach. The preliminary version of the two-loop strategy has been presented in [20].

The main contribution compared to past CHF avoidance controllers [15, 16, 17, 18] is the inner-loop, outer-loop control structure where evaporator outlet quality is regulated instead of wall temperature. The outlet quality is more directly linked to CHF thru the critical quality, and the controller is formulated to maintain a fixed critical quality margin by throttling mass flow rate. Additionally, the proposed proportional-integral (PI) control architecture extends to multiple evaporators, implements feedforward, and offers improved stability over the control in [15, 16]. Finally, this approach does not require the detailed model information used in multivariate CHF avoid-

Table 1: Brief description of several electronics cooling experiments with summary of test section conditions. Values are approximate. Ranges are reported for parametric studies, and baseline values are reported for proof-of-concept and transient studies.

	Description	Fluid	Heat Flux (W/cm ²)	Mass Flux (kg/m ² -s)	Outlet Quality
Hannemann et al. [7]	pumped-loop with 16x cold plate evaporators	R-134a	6.5	17	0.3
Trutassanawin et al. [8]	mini-VCC using microchannel cold plate with copper block heat sink/spreader	R-134a	36-76	10-23	1
Mongia et al. [9]	micro-VCC for notebook cooling; cold plate evaporator followed by low heat flux superheater	isobutane	3	50	0.89
Lee and Mudawar [10]	microchannel cooling with both direct VCC and indirect two- loop VCC-pumped systems	R-134a, HFE-7100	16-86, 32-525	127-654, 1330-13400	0.49-1, -0.1
Thome et al. [11, 12, 13, 14]	microevaporators; pumped-loop, VCC, and hybrid operation	R-134a	35	200	0.3
Wen et al. [15, 16, 17, 18, 19]	VCC with 3 parallel evaporators and heated accumulator	R-134a	8	31	0.9

Fig. 1: Flow chart describing two-loop control approach in two-phase, multi-evaporator electronics cooling systems.

ance controllers [17, 18], therefore it is more applicable for cases where model
information is lacking

2. System Description and Experimental Testbed

The experimental testbed for model and control validation is a direct-expansion VCC consisting of three parallel evaporators, three selectable variable-speed compressors of different sizes, three electronic expansion valves (EEVs), and a heated accumulator tank [21]. A schematic of this multi-evaporator system is shown in Figure 2. The compressors are Maneurop[®] single-cylinder reciprocating compressors of different sizes, where the speed ω (Hz, rev/s) is controlled by variable-frequency drives (VFD). Under normal operation, only the smallest of the compressors is used (swept volume 30.23 cm³/rev). Each evaporator consists of 2.06 cm inner-diameter copper tube with a 28 cm long, 1.27 cm diameter cartridge heater insert, forming a vertical flow channel with annular geometry. Direct-current (DC) power supplies provide up to 2 kW of resistive heating to each cartridge, and a thermocouple is soldered to the end of each cartridge to measure wall temperature. The evaporators feed into a heated accumulator tank with an electric band heater supplying the heat. Refrigerant R-134a is used as the primary fluid. The condenser is a fluid-fluid plate heat exchanger, with chilled water with fixed subcooling temperature serving as the secondary fluid. Pressure and temperature sensors are positioned at key points throughout the cycle to provide thermodynamic state information. Mass flow sensors are placed before each EEV to measure individual evaporator mass flow rates. Additionally, a mass flow sensor is placed immediately after the condenser to provide the total mass flow rate measurement. The measurement uncertainties for the respective sensors are given in Table 2. Data acquisition and control uses National In-

Fig. 2: Schematic of a multiple-evaporator, direct expansion vapor compression system for imposed heat flux cooling.

131 strument LabVIEW software system. The system is sampled at 5 Hz, which
 132 is fast enough to measure all dominant transients with good fidelity. System
 133 parameters such as geometry, physical properties, and empirical coefficients
 134 are included in Table A.12 in the Appendix.

135 Since the cycle is designed for high, transient heat-flux removal, refriger-
 136 ant exits the evaporators in the two-phase regime. The heated accumulator
 137 tank serves to separate the vapor and liquid phases in the suction line, and
 138 alter the balance of circulating vapor by boiling the collected saturated liq-
 139 uid using external band-heaters. The cycle thermodynamics are shown in

Table 2: Testbed sensors and measurement uncertainties

variable	device	uncertainty
temperature	type-T thermocouple	$\pm 0.2\text{K}$
absolute pressure	transducer	$\pm 0.3\%$ full scale
mass flow rate	Coriolis	$\pm 1\%$

Fig. 3: Pressure-enthalpy diagram of a multiple-evaporator vapor compression system for imposed heat flux cooling. Quality lines are shown for the two-phase region [17].

Figure 3 [17]. Alternative methods are available for post-evaporator heating, including internal heat exchangers [12] and secondary low heat flux evaporators [9, 12].

A major challenge for the control of our multi-evaporator VCC cooling system is the coupling between the parallel evaporators. For example, a poorly-designed control may allocate the majority of the available coolant to a single evaporator in response to a large heating disturbance, leaving too little coolant for the remaining evaporators. Such operation could easily result in evaporator burnout. Therefore, the supply and demand of refrigerant flow rates must be carefully coordinated for all of the evaporators at once. As shown in Figure 1, we use a multi-loop control strategy, where an outer loop determines the mass flow rate demands based on evaporator heat loads and wall temperatures, and an inner loop coordinates with the system actuators (compressor, EEV's, accumulator) to provide the total mass flow rate and distribute it appropriately. The objective of the outer loop control is to use mass flow rate setpoints to maintain a desired distance from CHF in terms of the difference between evaporator exit quality and the critical vapor quality, and to reject heat load disturbances. The objectives of the inner loop are to quickly track mass flow rate references, reject model uncertainty and disturbances, and stabilize the system pressure. The controllers con-

Fig. 4: Diagram showing outer loop control structure. Evaporator conditions (heat load $q_{e,n}$, superheat $\Theta_{w,end,n}$) generate mass flow rate demand $\dot{m}_{n,set}$ to maintain desired quality difference from CHF Δx_{crit} .

160 sist of a static model feedforward to provide baseline setpoints/inputs and
 161 proportional-integral (PI) feedback control for disturbance rejection. Figures
 162 4 and 5 show the control block diagrams for of the two-loop control strategy.

163 3. Outer Loop Control for CHF Avoidance

164 The objective of the outer loop control is to maintain the evaporator out-
 165 let quality at a setpoint sufficiently below the critical quality, and to provide
 166 CHF avoidance by rejecting heat load disturbances. This is accomplished
 167 by regulating the mass flow rate setpoints for the evaporators using a feed-
 168 forward/feedback controller. The feedforward portion of the mass flow rate
 169 setpoint is generated based on the desired outlet quality, the current heat
 170 input, and static energy balance. The feedback portion is based on the es-
 171 timated critical quality using the evaporator wall temperature to provide
 172 robust CHF avoidance and heat load disturbance rejection.

173 3.1. Outer Loop Feedforward Control

174 The feedforward provides an initial mass flow rate setpoint for the evap-
 175 orator by applying static energy balance to achieve a desired outlet quality.
 176 The two-phase vapor quality x as a function of enthalpy h is:

$$x = \frac{h - h_f}{h_{fg}} \quad (1)$$

177 where h_f is the saturated liquid enthalpy, and h_{fg} is the fluid enthalpy of
 178 vaporization. The static energy balance equation for the evaporator is given

Fig. 5: Diagram showing inner loop control structure. Total demand flow rate $\sum_{n=1}^N \dot{m}_{n,set}$ is provided by the compressor speed ω , and distributed using EEV openings $A_{v,n}$. The EEV are also used to stabilize differential pressure ΔP , the compressor and accumulator heat input are used to stabilize evaporator pressure P_e .

by:

$$q_e = \dot{m}_e (h_{e,o} - h_{e,i}) = \dot{m}_e h_{fg} (x_{e,o} - x_{e,i}) \quad (2)$$

where q_e is the evaporator heat input, and $h_{e,i}$ and $h_{e,o}$ are inlet and outlet enthalpies. Making the substitution $x_{e,o} = x_{crit} - \Delta x_{crit,set}$ and solving Eq. (2) for mass flow rate yields the evaporator feedforward law in terms of the current heat load, the critical quality, and desired quality difference from CHF $\Delta x_{crit,set}$:

$$\dot{m}_{e,set} = \frac{q_e}{h_{fg} (x_{crit} - \Delta x_{crit,set} - x_{e,i})} \quad (3)$$

Constant critical qualities near total-dryout are assumed for the three evaporators in the presented experimental vapor compression cooling cycle. Critical qualities near 1.00 are reasonable estimates due to the very low channel mass flux during three-evaporator operation ($\sim 30 \text{ kg/m}^2\text{-s}$). For model linearization and control validation simulations, critical qualities of 1.00, 0.98, and 0.96 are used in evaporators one, two, and three respectively to study how variations in critical quality impact the closed-loop performance. For experimental validation, we assume an initial critical quality of 0.98 for all three evaporators.

3.2. Critical Quality Estimation

CHF avoidance control depends on an accurate prediction or estimate of CHF in order to maintain sub-CHF conditions at all times. For dryout

avoidance control, it is more convenient to represent CHF in terms of critical quality and use equation (3) to calculate the necessary evaporator mass flow rate. The critical quality may be obtained from CHF by applying energy balance to the heated channel:

$$x_{crit} = \frac{q''_{CHF} A_{s,e}}{\dot{m}_e h_{fg}} + x_{e,i} \quad (4)$$

CHF prediction typically uses correlations grounded in physical principles, and with parameters identified from experimental data. For example, Katto and Ohno developed a widely-used correlation for CHF prediction in single-channel vertical boiling tubes [22], which was further updated for boiling refrigerants (R-134a and R-245fa) in microchannels in [23]. Such correlations provide useful initial CHF estimates, but ultimately have limited accuracy. Prediction error of 10-50% is normal even within the applicable range of a CHF correlation. Therefore, high heat flux cooling cycles are typically limited to conservative operation with large safety factors.

Given the limited accuracy of CHF prediction, the feedforward mass flow rate setpoints for the evaporators are augmented with feedback controllers that utilize online estimation of the critical quality. Near dryout, the evaporator wall end may experience partial dryout or transition boiling. The heat transfer coefficient at the evaporator wall end will then be a function of the critical quality and the present outlet quality. An empirical correlation for the evaporator wall end heat transfer coefficient as has been derived using testbed data [24]:

$$H_{end} = c_1 e^{(c_2 \Delta x_{crit})} \quad (5)$$

Constants c_1 and c_2 are fitted parameters (Table A.12). The exponent in Eq. (5) captures the sharp decrease in heat transfer coefficient as Δx_{crit} approaches zero. This heat transfer coefficient is in turn used in the lumped dynamic energy balance for the evaporator wall end:

$$M_{w,e} \dot{\Theta}_{w,end} = -H_{end} A_{s,e} \Theta_{w,end} + q_e \quad (6)$$

where $M_{w,e} = V_{w,e} \rho_{w,e} c_{p,w,e}$ is the thermal mass of the evaporator wall and $\Theta_{w,end} = T_{w,end} - T_{sat}$ is the local wall superheat at the end of the evaporator. The heater is assumed to be a long, steel cylinder with volume, density, an specific heat given in the Appendix (Table A.12). This model is validated

226 for the experimental testbed in [17]. Solving (6) for Δx_{crit} , we get

$$\Delta x_{crit} = \frac{1}{c_2} \ln \left(\frac{q_e - M_{w,e} \dot{\Theta}_{w,end}}{c_1 A_{s,e} \Theta_{w,end}} \right) \quad (7)$$

227 Calculating the derivative $\dot{\Theta}_{w,end}$ typically requires some amount of filtering,
 228 as any measurement noise will be amplified. One approach is to simply
 229 neglect the transient term, which yields a static estimation for Δx_{crit} [17]:

$$\Delta x_{crit} = \frac{1}{c_2} \ln \left(\frac{q_e}{c_1 A_s \Theta_{w,end}} \right) \quad (8)$$

230 Alternatively, the derivative term may be removed by dividing equation (6)
 231 by $\Theta_{w,end}$ and integrating over a finite period t_p .

$$\Delta x_{crit} = \frac{1}{c_2} \ln \left(\frac{\int_0^{t_p} \frac{q_e(t)}{\Theta_{w,end}(t)} dt - M_{w,e} \ln \left[\frac{\Theta_{w,end}(t_p)}{\Theta_{w,end}(0)} \right]}{c_1 A_{s,e} t_p} \right) \quad (9)$$

232 The integrator in Eq. (9) functions as a low-pass filter of the measurement
 233 signal, hence t_p must be chosen to balance noise sensitivity against measure-
 234 ment bandwidth. For the VCC experiments, the integrator is implemented
 235 using the trapezoidal method, and $t_p = 10$ seconds (using past 50 samples at
 236 5 Hz) provides adequate filtering.

237 The deviation variable $\delta (\Delta x_{crit}) = \Delta x_{crit} - \Delta x_{crit,0}$ is introduced for use
 238 in the critical quality feedback control, where $\Delta x_{crit,0}$ is a critical quality
 239 setpoint. Equation (8) may be used to determine this setpoint in terms of
 240 nominal steady-state heat load $q_{e,0}$ and wall superheat $\Theta_{w,end,0}$, or alterna-
 241 tively, it may be tuned to provide safety margin for transient heat loads
 242 (section 5.5.1). Hence, the critical quality error feedback is given by

$$\delta (\Delta x_{crit}) = \frac{1}{c_2} \left[\ln \left(\frac{\int_0^{t_p} \frac{q_e(t)}{\Theta_{w,end}(t)} dt - M_{w,e} \ln \left[\frac{\Theta_{w,end}(t_p)}{\Theta_{w,end}(0)} \right]}{t_p} \right) - \ln \left(\frac{q_{e,0}}{\Theta_{w,end,0}} \right) \right] \quad (10)$$

243 3.3. Feedback Control of Evaporator Critical Quality

244 The feedback control augments the evaporator static feedforward (3) by
 245 providing transient performance and model-error rejection. It uses critical

246 quality feedback $\delta(\Delta x_{crit})$ in (7) to modify the evaporator mass flow rate
 247 setpoint with a PI controller:

$$\delta \dot{m}_{set} = - \left(K_{P,e} + \frac{K_{I,e}}{s} \right) \delta(\Delta x_{crit}) \quad (11)$$

248 where $K_{P,e}$ and $K_{I,e}$ are the evaporator proportional and integral gains,
 249 respectively. The controller gains are tuned using linearized SISO system
 250 models (see section 5), treating coupling with other inputs and outputs as
 251 disturbances. Higher gain generally improves performance and disturbance
 252 rejection, as the evaporator critical quality difference is a monotonically in-
 253 creasing function of mass flow rate. However, high-gain comes at the cost
 254 of increased control action and noise sensitivity, and therefore a conservative
 255 proportional gain $K_{P,e} = 0.012$ is selected. The integral gain is tuned to
 256 $K_{I,e} = 6.0 \times 10^{-4}$ to yield a closed-loop, SISO bandwidth of 0.0045 rad/s with a
 257 conservative phase-margin of 70° . The integral feedback provides robustness
 258 with respect to critical quality model error, but it can result in windup of the
 259 mass flow rate if the wall superheat setpoint is very small (large, infeasible
 260 heat transfer coefficient). To avoid violation of system mass flow constraints,
 261 anti-windup saturation limits of ± 1 g/s are imposed on the integral feedback.
 262 Validation of the outer loop evaporator feedforward/feedback control coupled
 263 with the inner loop VCC mass flow rate and pressure control are presented
 264 in section 5.5. The complete control law for the evaporator mass flow rate
 265 setpoint is given as:

$$\dot{m}_{set} = \frac{q_e}{h_{fg}(x_{crit} - \Delta x_{crit,set} - x_{e,i})} - \left(K_{P,e} + \frac{K_{I,e}}{s} \right) \delta(\Delta x_{crit}) \quad (12)$$

266 4. Inner Loop Mass Flow Rate Control

267 The inner loop control is used to track the mass flow rate setpoints pro-
 268 vided by the outer loop, to stabilize other system variables such as pressure,
 269 and to reject disturbances. A decentralized approach is used, where an inde-
 270 pendent controller is designed for each actuator in the multi-evaporator VCC
 271 (EEVs, compressor, heated accumulator). Similar to the outer loop design,
 272 these controllers use a feedforward component combined with a PI feedback
 273 for improved transient response and disturbance rejection.

274 4.1. Accumulator Control

275 To maintain cycle equilibrium, the rate of evaporation in the heated ac-
 276 cumulator must be matched with the rate of saturated liquid leaving the
 277 primary evaporators. The effective quality x_{eff} of the combined flow leaving
 278 the evaporators is calculated using static energy balance:

$$x_{\text{eff}} = x_{e,i} + \frac{q_{e,t}}{h_{fg}\dot{m}_t} \quad (13)$$

279 Applying static energy balance again to the the accumulator gives the feed-
 280 forward accumulator heat input:

$$q_{a,0} = \dot{m}_t h_{fg}(1 - x_{\text{eff}}) \quad (14)$$

281 Because the evaporator and accumulator dynamics are neglected in the
 282 derivation of the accumulator feedforward control law, a PI feedback control
 283 is included to improve transient performance and disturbance rejection. The
 284 accumulator feedback control is used to track to the desired suction-line
 285 pressure $P_{e,\text{set}}$, and to reject heating disturbances that arise from changes in
 286 evaporator heat or the ambient environment:

$$\delta q_a = - \left(K_{P,a} + \frac{K_{I,a}}{s} \right) \delta P_e \quad (15)$$

287 4.2. Compressor Control

288 The compressor provides mass flow rate supply to meet the demand set
 289 by the evaporators. Mass flow rate for the variable speed compressor is given
 290 by

$$\dot{m}_t = \omega V_m \rho_g \eta_v \quad (16)$$

291 where ω is the compressor speed (Hz), V_m is the compressor volume, ρ_g is
 292 the saturated vapor density in the suction-line, and η_v is the volumetric
 293 efficiency. The feedforward compressor speed ω_0 is determined using the
 294 inverse-mapping:

$$\omega_0 = \frac{\dot{m}_{t,\text{set}}}{V_m \rho_g \eta_v} \quad (17)$$

295 where $\dot{m}_{t,\text{set}} = \sum_{n=1}^N \dot{m}_{v,n,\text{set}}$ is the total mass flow rate setpoint.

296 Because the compressor dynamics is at a much faster time-scale than the
 297 heat exchanger dynamics, the inverse-mapped compressor control has excel-
 298 lent transient response. However, it is sensitive to model error in the volu-
 299 metric efficiency term η_v . A PI feedback control is implemented to reduce the

300 effect of this model error. The output used for feedback is $y_m = \dot{m}_t + \alpha_m P_e$,
 301 where the pressure term is included to improve the regulation of suction-line
 302 pressure, and $\alpha_m = -10^{-7}$ is a scaling term to account for difference in units
 303 and balance pressure regulation vs. mass flow rate control:

$$\delta\omega = - \left(K_{P,m} + \frac{K_{I,m}}{s} \right) \delta y_m \quad (18)$$

304 4.3. Valve Control

305 The electronic expansion valves perform the important function of dis-
 306 tributing mass flow rate between the multiple evaporators. The feedforward
 307 control is determined from an inverse-mapping based on the orifice equation:

$$A_{v,n,0} = \frac{\dot{m}_{set,n}}{k_{v,n} \sqrt{\rho_v \Delta P}} \quad (19)$$

308 where $A_{v,n}$ are the valve percent openings, ρ_v is the liquid density at the valve
 309 inlet, $\Delta P = P_c - P_e$ is the differential pressure across the valves, and $k_{v,n}$ are
 310 experimentally determined discharge coefficients. Similar to the compressor,
 311 the EEV feedforward has excellent transient response since the valve dynam-
 312 ics is much faster than the heat exchanger dynamics. The feedforward is
 313 sensitive to model-error in the discharge coefficients $k_{v,n}$, as well as changes
 314 in the differential pressure. To reduce this sensitivity, a PI control is added
 315 using output variable $y_{v,n} = \dot{m}_{v,n} + \alpha_v \Delta P$, where $\alpha_v = -10^{-8}$ scales the
 316 differential pressure with the mass flow rate control:

$$\delta A_{v,n} = - \left(K_{P,v} + \frac{K_{I,v}}{s} \right) \delta y_{v,n} \quad (20)$$

317 5. Controller Tuning and Validation using Linearized Model

318 We tune the controller gains based on the nonlinear model of the three-
 319 evaporator VCC testbed (Appendix A). There are several nonlinear model
 320 elements, particularly the two-phase heat transfer coefficient, the compres-
 321 sor, the expansion valves, and to a lesser extent the fluid property functions.
 322 The evaporator heat transfer coefficient is dramatically reduced when CHF
 323 occurs, and changes rapidly in the transition regime between wetted boiling
 324 and dryout. Because of these nonlinear elements, the system dynamics can
 325 change considerably depending on the initial conditions. Therefore, the con-
 326 troller performance is analyzed by evaluating the linearized system dynamics

Table 3: Input array for model linearization

A_vk_v	0.147	0.160	0.173	mm ²
ω	50	60	70	Hz
q_a	100	300	500	W

at a wide range of operating points. Previously, we had applied this approach in the analysis and design of robust controllers for single evaporator VCC [18].

5.1. Model Linearization

We select the operating points for model linearization by varying three input variables: valve opening A_vk_v , compressor speed ω , and accumulator heat input q_a . The product A_vk_v is used because the valve discharge coefficients k_v are slightly different for each evaporator line in the experimental testbed. Therefore, the mass flow distribution between the valves is made identical by selecting the same product A_vk_v and applying equation (19). For each of the three evaporators, the heat input is set to a constant 600 W. Evaporator heat is not varied for linearization since the effect of changing q_e is similar to an opposite change in q_a , with respect to evaporator outlet quality, pressure, and mass flow rate.

Low, medium, and high values are selected for each input based on system constraints. This results in 27 different input combinations and operating points for linearization. The range for the compressor speed, 45-75 Hz, yields 50, 60, and 70 Hz as the selected values. Similarly, the range for the accumulator heat input, 0-600 W, yields 100, 300, and 500 W as the selected values. Valve openings are selected to produce low, medium, and high condenser pressure, relative to the system constraints. The condenser pressure constraints are approximately 600-1500 kPa, with the low value corresponding to a loss of subcooling and the high value representing a safety limit for the testbed. The selected values for the valve openings along with the other selected inputs are given in Table 3.

After these input combinations are determined, the corresponding steady-states are found using the static system equations (Table 4) and a nonlinear equation solver. These equations assume that the evaporator outlet flow does not reach superheat ($x_{e,o} < 1$), and that the liquid reservoir in the accumulator does not dry up. The first condition is guaranteed if the system

Table 4: Static system equations for multiple-evaporator VCC

name	equation
evaporator energy balance	$q_{e,n} = \dot{m}_{e,n} h_{fg}(x_{e,o,n} - x_{e,i})$
evaporator wall-end energy balance	$q_{e,n} = H_{end,n} A_{s,e} \Theta_{w,end,n}$
EEV mass flow rate	$\dot{m}_{v,n} = k_{v,n} A_{v,n} \sqrt{\rho_v(P_c - P_e)}$
valve-evap. mass balance	$\dot{m}_{v,n} = \dot{m}_{e,n}$
evap.-acc. mass balance	$\dot{m}_a = \sum_{n=1}^N \dot{m}_{e,n}$
average evap. outlet quality	$\bar{x}_{e,o} = \sum_{n=1}^N x_{e,o,n} \dot{m}_{e,n} / \dot{m}_a$
accumulator energy balance	$q_a = \dot{m}_a h_{fg}(1 - \bar{x}_{e,o})$
acc.-cmp mass balance	$\dot{m}_a = \dot{m}_m$
compressor mass flow rate	$\dot{m}_m = \omega V_m \rho_g \eta_V$
compressor volumetric efficiency correlation	$\eta_V = -0.13 (P_c/P_e)^{0.88} + 1.015$

357 is controlled to maintain a positive Δx_{crit} setpoint, while the second condition
 358 depends on the total system charge.

359 To analyze the system at different Δx_{crit} , constant critical qualities $x_{crit} =$
 360 $\{1, 0.98, 0.96\}$ are considered for evaporators one, two, and three, respec-
 361 tively. The resulting outputs include a wide range of system pressures,
 362 mass flow rates, evaporator outlet qualities, and wall superheats (Table
 363 5). The three selected accumulator heat inputs (100, 300, 500 W) result
 364 in three distinct groupings of systems that roughly correspond to near-
 365 dryout, near partial-dryout, and fully-wetted boiling conditions ($\Delta x_{crit} \approx$
 366 0.02-0.06, 0.09-0.14, 0.15-0.20). This is reflected by the evaporator end wall
 367 superheats (Table 5).

We linearize the nonlinear model at the 27 steady-state operating points
 to obtain state space models of the following form:

$$\begin{aligned} \delta \dot{\mathbf{x}} &= \mathbf{A} \delta \mathbf{x} + \mathbf{B}_1 \delta \mathbf{u} + \mathbf{B}_2 \delta \mathbf{w} \\ \delta \mathbf{y} &= \mathbf{C} \delta \mathbf{x} + \mathbf{D}_1 \delta \mathbf{u} + \mathbf{D}_2 \delta \mathbf{w} \end{aligned} \quad (21)$$

368 where $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector, $\mathbf{w}$ is the disturbance
 369 input vector, $\mathbf{y}$ is the output vector, and $\mathbf{A}, \mathbf{B}_1, \mathbf{B}_2, \mathbf{C}, \mathbf{D}_1, \mathbf{D}_2$ are matrices
 370 describing the linearized dynamics. Variables are expressed as deviations δ
 371 since the linearization is performed about an operating point. The system
 372 inputs $\mathbf{u}$ are $A_{v,n}$, ω , and q_a ; disturbance inputs $\mathbf{w}$ are evaporator heat $q_{e,n}$,

Table 5: Selected steady-state outputs at model linearization operating points

inputs			steady-state outputs							
A_vk_v mm ²	ω Hz	q_a W	P_e kPa	P_c kPa	$\dot{m}_v$ g/s	$\dot{m}_t$ g/s	$x_{e,o}$	$\Theta_{w,1}$ K	$\Theta_{w,2}$ K	$\Theta_{w,3}$ K
0.147	50	100	245	824	3.91	11.7	0.94	6.9	10.3	15.4
	60	100	218	810	3.95	11.9	0.94	7.1	10.6	15.9
	70	100	198	801	3.99	12.0	0.94	7.3	10.9	16.2
	50	300	275	964	4.26	12.8	0.86	1.6	2.4	3.5
	60	300	245	949	4.31	12.9	0.87	1.7	2.5	3.7
	70	300	223	940	4.35	13.0	0.87	1.8	2.6	3.9
	50	500	305	1111	4.61	13.8	0.80	0.4	0.6	1.0
	60	500	273	1096	4.66	14.0	0.81	0.5	0.7	1.1
	70	500	249	1087	4.70	14.1	0.81	0.5	0.8	1.1
	50	100	236	731	3.92	11.8	0.94	7.0	10.4	15.6
0.160	60	100	209	715	3.97	11.9	0.94	7.2	10.7	16.0
	70	100	189	705	4.01	12.0	0.94	7.3	11.0	16.4
	50	300	264	853	4.28	12.8	0.87	1.6	2.4	3.6
	60	300	234	837	4.33	13.0	0.87	1.7	2.6	3.8
	70	300	212	826	4.35	13.1	0.87	1.8	2.7	4.0
	50	500	293	982	4.63	13.9	0.80	0.4	0.7	1.0
	60	500	260	966	4.68	14.1	0.81	0.5	0.7	1.1
	70	500	236	955	4.73	14.2	0.81	0.5	0.8	1.2
	50	100	229	657	3.93	11.8	0.94	7.0	10.5	15.7
	60	100	201	640	3.98	11.9	0.94	7.2	10.8	16.2
0.173	70	100	181	628	4.02	12.0	0.94	7.4	11.1	16.5
	50	300	256	765	4.29	12.8	0.87	1.6	2.4	3.7
	60	300	225	747	4.29	13.0	0.87	1.7	2.6	3.9
	70	300	203	735	4.34	13.1	0.87	1.8	2.7	4.0
	50	500	283	879	4.39	13.9	0.80	0.5	0.7	1.0
	60	500	250	861	4.68	14.1	0.81	0.5	0.8	1.1
	70	500	226	849	4.73	14.2	0.81	0.5	0.8	1.2

373 ambient disturbance heating in the suction-line q_d , and deviations from the
 374 modeled values for valve discharge coefficient $\delta k_{v,n}$, compressor volumetric
 375 efficiency $\delta \eta_v$, and critical quality $\delta x_{crit,n}$; states $\mathbf{x}$ are x_c , P_c , T_{wc} , $x_{e,n}$, P_e ,
 376 $T_{we,n}$, $T_{we,end,n}$, and T_{wa} ; and measurable outputs $\mathbf{y}$ are P_c , P_e , $T_{we,end,n}$, $\dot{m}_{v,n}$,
 377 and $q_{e,n}$, as well as the newly defined output variables for expansion valve
 378 and compressor PI feedback, $y_{v,n}$ and y_m . The 27 linearized systems are all
 379 stable with the largest eigenvalue of $\mathbf{A}$ among all of the systems at -0.0011.
 380 This result is expected, as none of the selected operating points rest directly
 381 on an unstable equilibrium, such as the CHF boundary. Moreover, the fluid
 382 momentum change in the evaporator is negligible due to the large pressure
 383 drop across the EEV's, and this has a stabilizing effect on the system flow
 384 rates.

385 5.2. Input-Output Pairing for Decoupled Control

386 The input-output pairings for the inner loop mass flow rate control are
 387 selected in order to compensate for disturbances or uncertainty in specific
 388 model parameters such as valve discharge coefficient. This results in pairings
 389 that are physically sound, for instance using valve opening to control indi-
 390 vidual mass flow rate and differential pressure. The SISO controller design
 391 for an inherently multi-input/multi-output (MIMO) system is justified if the
 392 coupling between the loops is weak. One method of assessing the strength of
 393 this coupling is the relative gain array (RGA) [25]. Given a MIMO transfer
 394 function $G(s)$, the RGA number frequency ω is defined as

$$\text{RGA number } (\mathbf{G}(j\omega)) = \| (\mathbf{G}(j\omega) - \mathbf{G}(j\omega)^T) (\mathbf{G}(j\omega)^{-1})^T \|_{\text{sum}} \quad (22)$$

395 The sum-norm is matrix 1-norm which sums the absolute value of the el-
 396 ements. If $G(j\omega)$ is diagonal, then RGA number is 0, hence it intuitively
 397 characterizes the coupling between the diagonal SISO loops. The RGA has
 398 additional useful properties: it is independent of scaling or units, large RGA
 399 elements indicate two-way interactions between inputs and outputs, and the
 400 RGA number is closely related to the condition number of the system (ratio
 401 of maximum and minimum singular values of a scaled, nonsingular plant)
 402 [26]. Consequently, large RGA elements imply stability issues with the de-
 403 centralized control scheme as well as sensitivity to uncertainty, while a large
 404 RGA number suggests that the system is ill-conditioned [26]. The RGA
 405 analysis has been used by Jain et al. to analyze thousands of possible input-
 406 output pairings for decentralized control of a VCC for air-conditioning and

Table 6: RGA for the system of linearized VCC models at 0.05 rad/s. The selected input-output pairings resulted in a nearly diagonal RGA, with low MIMO coupling at this frequency. Values shown are averaged over the 27 linearized models. The maximum deviation from the mean across all values is 0.034, indicating low model variance at this frequency.

	$A_{v,1}$	$A_{v,2}$	$A_{v,3}$	ω	q_a
$y_{v,1}$	1.073	-0.020	-0.020	-0.035	0.002
$y_{v,2}$	-0.020	1.073	-0.020	-0.035	0.002
$y_{v,3}$	-0.020	-0.020	1.073	-0.035	0.002
y_m	-0.030	-0.030	-0.030	1.176	-0.084
P_e	-0.002	-0.002	-0.002	-0.071	1.077

refrigeration [27]. Pairings with the lowest RGA numbers are best suited for decoupled SISO control. Additional physical insight is used to select the final configuration. We use RGA to evaluate the inner loop control architecture and identify any coupling issues in the multi-evaporator VCC for electronics cooling. The RGA's and RGA numbers are calculated at two frequencies for each of the 27 linearized models: 0.05 rad/s (EEV and compressor control crossover) and 0.005 rad/s (accumulator control crossover). Table 6 shows the averaged RGA for the three-evaporator system at 0.05 rad/s, and the low off-diagonal elements as well as the diagonal elements near one indicate low MIMO coupling. The mean RGA number is 0.96, indicating that all of the systems are fairly well-conditioned. The maximum RGA number is 1.16, and corresponded to the operating condition at row 7 of Table 5, while the minimum RGA number is 0.76, corresponding to the operating condition at row 21 of Table 5. These extremes represent the highest and lowest system operating pressures, respectively.

Table 7 shows the averaged RGA for the three-evaporator system at 0.005 rad/s, and the greater RGA values indicate a larger degree of coupling at this frequency. Overall, the RGA is still diagonal and fairly well-conditioned. The mean RGA number is 3.95, indicating a higher level of coupling than at 0.05 rad/s. This is due to the larger gain in pressure response at low frequencies, which physically couples the accumulator, compressor, and EEV's. The maximum RGA number is 5.43, and corresponded to the operating condition at row 7 of Table 5, while the minimum RGA number is 2.62, corresponding to the operating condition at row 21 of Table 5. As in the high-frequency case, these extremes represent the highest and lowest system operating pressures,

Table 7: RGA for the system of linearized VCC models at 0.005 rad/s. The selected input-output pairings resulted in a fairly diagonal RGA, with some two-way interaction between the compressor and accumulator. Values shown are averaged over the 27 linearized models. The maximum deviation from the mean across all values is 0.337, indicating more model variance at 0.005 rad/s relative to 0.05 rad/s.

	$A_{v,1}$	$A_{v,2}$	$A_{v,3}$	ω	q_a
$y_{v,1}$	1.219	-0.038	-0.038	-0.146	0.002
$y_{v,2}$	-0.038	1.219	-0.038	-0.146	0.002
$y_{v,3}$	-0.038	-0.038	1.219	-0.146	0.002
y_m	-0.138	-0.138	-0.138	1.863	-0.449
P_e	-0.006	-0.006	-0.006	-0.424	1.443

432 respectively.

433 The RGA analysis indicates that the selected inner loop, input-output
434 pairings provide a well-conditioned system for decentralized control, with
435 limited interactions that would cause instability near the crossover frequen-
436 cies.

437 5.3. Controller Gain Tuning

438 The PI controllers for the outer and inner loops are tuned separately using
439 the linearized SISO models. This decoupled approach is reasonable given the
440 RGA analysis at the relevant crossover frequencies, as well as the time-scale
441 separation between the outer loop evaporator dynamics (bandwidth $\omega_b \sim$
442 0.005 rad/s) and the inner loop ($\omega_b \sim 0.05$ rad/s). Controller gains (Table
443 8) are selected for desired closed-loop phase margin (PM) and bandwidth
444 ω_b , such that the controllers are robust and the inner loop is faster than the
445 outer loop to allow the mass flow rate references to be tracked successfully.
446 In general, the controllers are tuned conservatively to avoid input saturation,
447 particularly for the compressor speed (45 to 75 Hz) and accumulator heat
448 (600W max). Phase margins $\geq 90^\circ$ are chosen for the EEV and compressor
449 controllers to ensure robustness to model uncertainty.

450 5.4. Stability and Performance of Inner Loop Mass Flow Rate Control

451 The primary function of the EEV and compressor PI controllers is to
452 eliminate steady-state error that results from disturbances to the empirical
453 parameters for those components, namely the valve discharge coefficients k_v
454 and the compressor volumetric efficiency η_v . Similarly, the accumulator PI

Table 8: VCC PI control gains [20]

control	PM (deg)	ω_b (rad/s)	K_P	K_I
evaporator	70	4.5e-3	1.2e-2	6.0e-4
accumulator	55	5.0e-3	4.4e-2	4.0e-5
compressor	90	5.0e-2	4900	190
EEV	113	5.7e-2	250	50

control compensates for heating disturbances in the suction-line that may affect pressure. These heating disturbances include unmodeled ambient heat gain/losses, as well as evaporator heat load disturbances. The inner loop mass flow control is applied to the nonlinear model, and the system is linearized about the same 27 operating points as the open-loop system in order to evaluate the closed-loop stability and disturbance rejection. The largest eigenvalue for all of the inner loop linearized systems is $-7.5\text{e-}4$, indicating that closed loop stability is maintained with the mass flow rate controller.

5.4.1. Valve Discharge Coefficient Disturbance Rejection

The expansion valves are modeled using an orifice equation assuming a constant discharge coefficients $k_{v,n}$. However, due to oversizing, the typical percent opening during experiments is between 2-10%. At these small openings, the behavior of the EEV is very nonlinear, and open-loop control using the constant k_v model is likely to result in steady-state error to mass flow rate and differential pressure. The PI controllers for $y_{v,n}$ aim to use feedback to mitigate the uncertainty in $k_{v,n}$, allowing zero steady-state error in mass flow rate and differential pressure setpoints. To evaluate controller performance, Bode magnitude plots are presented showing the response from a disturbance input $\delta k_{v,n}$ to valve output $y_{v,n}$ (Figures 6, 7, 8). The Bode plots reveal that the closed-loop system has excellent disturbance rejection for all three EEV over all frequencies, reducing the gain to about 20% of the open-loop value at high-frequencies and to less than 5% of the open-loop gain at frequencies lower than 0.01 rad/s. The behavior of the EEV controller does not change significantly across the 27 different operating points.

5.4.2. Compressor Volumetric Efficiency Disturbance Rejection

Similar to the expansion valves, the compressor model is sensitive to error in the volumetric efficiency term η_v , which may lead to steady-state error in

Fig. 6: Magnitude plot for linearized systems showing closed-loop rejection of an EEV-one discharge coefficient disturbance

Fig. 7: Magnitude plot for linearized systems showing closed-loop rejection of an EEV-two discharge coefficient disturbance

Fig. 8: Magnitude plot for linearized systems showing closed-loop rejection of an EEV-three discharge coefficient disturbance

482 the total mass flow rate and evaporator pressure. Bode magnitude plots in
 483 Figure 9 show the open-loop and closed-loop gains from a $\delta\eta_v$ disturbance to
 484 the compressor output variable y_m , as well as the relative gain. While there is
 485 at least some disturbance rejection at all frequencies, the reduction in gain for
 486 the closed-loop system is not as dramatic compared to the EEV control, with
 487 less than 5% gain occurring at frequencies lower than about 0.0015 rad/s.
 488 One reason for this relative decrease in bandwidth is the evaporator pressure
 489 term in y_m , which is coupled to the slower dynamics of the accumulator
 490 tank. Still, the disturbance rejection performance of the compressor control
 491 is good, especially considering that the compressor model is less-prone to
 492 error compared with the EEV, where the small percent area openings can
 493 exhibit large, nonlinear variations in the discharge coefficients. Finally, it
 494 is noted that for the compressor there is more variation in the open-loop
 495 gains across the 27 linearized systems, but that the closed-loop gains are
 496 very similar, especially at low frequencies.

Fig. 9: Magnitude plot for linearized systems showing closed-loop rejection of a compressor volumetric efficiency disturbance

5.4.3. Accumulator Ambient Heat Disturbance Rejection

For the accumulator, the impact of a disturbance heat input q_d on the evaporator pressure P_e is considered. Absolute and relative Bode magnitude plots for the open-loop and closed-loop gains from q_d to P_e are shown in Figure 10. Unlike the static models for EEV and compressor, the accumulator model uses dynamic mass and energy balance and is also coupled with the evaporator dynamics. As a result, the open-loop system has significant dynamics, with the large tank volume acting as a low-pass filter for heat load inputs and disturbances. Consequently, the disturbance rejection performance is only relevant at low frequencies, approximately less than 0.001 rad/s. At these frequencies, the closed-loop is less than 20% of the open-loop gain, indicating good disturbance rejection. Moreover, the accumulator controller is robust, with virtually no variation in the closed-loop performance across the 27 different operating points.

5.5. Stability and Performance of Outer Loop Critical Quality Control

The outer loop control, described in section 3, generates mass flow rate setpoints based on the known evaporator heat inputs and wall temperature

Fig. 10: Magnitude plot for linearized systems showing closed-loop rejection of an ambient heating disturbance

514 feedback. To evaluate the stability and performance of the outer loop control
 515 for the three-evaporator VCC, the control is incorporated into the nonlin-
 516 ear model along with the inner loop controller, and the closed-loop model
 517 is linearized about the 27 different operating points in Table 5. As with
 518 evaluating the local stability of the open-loop and inner loop systems, the
 519 Routh-Hurwitz criterion is used to determine the stability of the outer loop
 520 linearized systems. The smallest negative eigenvalue of among the 27 sys-
 521 tems is $-7.1\text{e-}4$, indicating that all of the eigenvalues have negative real parts
 522 and that the outer loop linearized systems are stable.

523 The outer loop control aims to reduce the effects of two types of distur-
 524 bances: evaporator heat input disturbances and critical quality model-error
 525 disturbances. If the closed-loop gain from an evaporator heat-load distur-
 526 bance to the critical quality difference Δx_{crit} is small, then the system will
 527 have good CHF avoidance under transient heating conditions. On the other
 528 hand, if the closed-loop gain from a critical quality disturbance (model-error)
 529 to the critical quality difference Δx_{crit} is small, then the CHF avoidance con-
 530 troller will have good robustness. This is especially important considering
 531 limited prediction capability of critical quality models, which are often ex-

perimentally based and only accurate for certain fluids under the conditions covered by the identifying data-set.

5.5.1. Evaporator Heat-Load Disturbance Rejection

Figures 11, 12, 13 show the Bode magnitude responses from evaporator heat-load disturbances $q_{e,n}$ to critical quality differences $\Delta x_{crit,n}$ for the open-loop and closed-loop systems, for each of the three evaporators. Excellent disturbance rejection is achieved across all frequencies. One reason for the fast response of the closed-loop system is the feedforward component of the outer loop control, which instantaneously changes the mass flow rate when combined with the inner loop control. The peak of the absolute magnitude for the closed-loop response is useful for determining Δx_{crit} setpoints. If the maximum expected heating disturbance is known, this gain can be used to estimate the maximum change in critical quality difference, and a Δx_{crit} setpoint can be chosen with some safety margin added such that CHF is likely to be avoided. For example, the peak gain for a 1 W disturbance is about $1e-4$ based on Figure 11, meaning that a 300 W heat load disturbance would decrease $\Delta x_{crit,1}$ by a maximum of about 0.03. Therefore, a $\Delta x_{crit,1}$ setpoint of at least 0.03 is needed if the system is to avoid CHF under evaporator-one heat load disturbances up to 300 W.

5.5.2. Evaporator Critical Quality Disturbance Rejection

Figures 14–16 show the Bode magnitude responses from error in $x_{crit,n}$ to critical quality differences $\Delta x_{crit,n}$ for the open-loop and closed-loop systems, for each of the three evaporators. Disturbances are reduced to approximately 35% of the open-loop values at frequencies less than 0.003 rad/s. This indicates that the outer loop control provides some degree of robustness to critical quality model-error at lower frequencies. The steady-state gain may be further reduced by increasing the proportional feedback gain K_e , at the expense of increasing sensitivity to measurement noise. The gain is increased to about 1.2 at frequencies near 0.1 rad/s, where the rejection of heat-load disturbances is also near the minimum. This indicates that large error in the estimated critical quality could trigger CHF during heat-load transients, and that a safety margin of at least 1.2 times the expected critical quality error should be added to $\Delta x_{crit,set}$. It is interesting to note the three distinct groupings of the frequency responses across the 27 different operating points. They correspond to the different Δx_{crit} at the linearization points, with the rejection bandwidth decreasing with decreased $\Delta x_{crit,0}$. This has the impor-

Fig. 11: Magnitude plot for linearized systems showing closed-loop rejection of evaporator-one heat-load disturbance

Fig. 12: Magnitude plot for linearized systems showing closed-loop rejection of evaporator-two heat-load disturbance

Fig. 13: Magnitude plot for linearized systems showing closed-loop rejection of evaporator-three heat-load disturbance

568 tant implication that at smaller Δx_{crit} the system is less robust, and therefore
 569 operation very close to the critical quality with small Δx_{crit} safety margin
 570 is not recommended. The decrease in bandwidth with respect to decreases
 571 in Δx_{crit} is also reflected when comparing the different evaporators. Since
 572 the critical qualities for evaporator-two and three are lowered by 0.02 and
 573 0.04 respectively, the critical quality disturbance rejection bandwidth is less
 574 in these evaporators relative to evaporator-one.

575 Note that a third type of disturbance exists as model uncertainty in the
 576 transition boiling heat transfer coefficient, i.e., error in the fitted coefficients
 577 c_1 and c_2 . This disturbance is more complicated to evaluate, as it does not
 578 directly affect x_{crit} , but it impacts the closed-loop system through the wall-
 579 end temperature feedback. Based on equations (10) and (12) for the outer
 580 loop controller, error in the coefficient c_1 would have a negligible impact on
 581 the closed-loop system, while error in the exponential coefficient c_2 would
 582 effectively change the feedback gains $K_{P,e}$ and $K_{I,e}$. More detailed analysis
 583 of the closed-loop effects of heat-transfer model error remains a topic for
 584 future study.

Fig. 14: Magnitude plot for linearized systems showing closed-loop rejection of an evaporator-one critical quality disturbance

Fig. 15: Magnitude plot for linearized systems showing closed-loop rejection of an evaporator-two critical quality disturbance

Fig. 16: Magnitude plot for linearized systems showing closed-loop rejection of an evaporator-three critical quality disturbance

5.6. Effect of Size Scaling

The control scheme presented here is designed and tested for a relatively large-scale system, and consequently scaling of this approach must be considered for smaller systems such as microelectronics. First, the dynamics prescribed by the lumped wall energy balance equation (6) are strongly influenced by the wall mass (volume and density product), surface area, and heat transfer coefficient. The ratio of wall volume to surface area leads to a first-order time constant that is roughly proportional to device length. Consequently, as the device size is reduced, the response to heating disturbances becomes faster, necessitating a higher control bandwidth. For example, the dynamics of the VCC testbed are on the order of minutes, but a micro device may have thermal dynamics on the order of milliseconds. Naturally, this increases sensing and actuator requirements, and has significant implications on cost and design feasibility. However, given an accurate evaporator model, the general approach for CHF avoidance control design is unchanged; PI gains are selected to throttle evaporator mass flow rate based on wall temperature feedback in order to reject heating disturbances and maintain a reasonable margin from the critical quality. The dynamic response of the closed-loop

603 model indicates the expected change in Δx_{crit} at various frequencies due to
604 heating disturbances, which then is used to tune the outer loop controller.
605 While this approach is simple, it relies on the time-scale separation of the in-
606 ner and outer controllers, and may fail if the evaporator dynamics are faster
607 than the inner loop or the measurement bandwidth. A possible method of
608 increasing the evaporator time-scale is to incorporate a heat-spreader, which
609 increases the evaporator thermal mass and surface area.

610 6. Nonlinear Simulation Results

611 The two-loop, combined feedforward-feedback control architecture de-
612 tailed above is first tested in nonlinear simulation of a three-evaporator VCC
613 system for CHF avoidance in the presence of large evaporator heat input dis-
614 turbances. The same nonlinear model used in the stability and performance
615 analysis is also used for simulation [17, 24]. Input saturation constraints
616 and anti-windup for the integral feedback control are also included, although
617 saturation is not reached in simulation by the choice of controller gains. An
618 initial operating point is selected to place system inputs near the middle
619 of their operable range: $P_{e,0} = 235$ kPa, $\Delta P_0 = 575$ kPa, $\Delta x_{crit,0} = 0.09$,
620 $\Theta_{w,0} = 0.5$ K, and $q_{e,0} = 600$ W. The static system (Table 4) is solved for the
621 steady-state inputs: $q_a = 260$ W, $\omega = 57$ Hz, and $A_v = [3.57, 3.34, 5.63]$ %.
622 The closed-loop system is subjected to a 10 minute, 300 W heat load in-
623 crease in a single evaporator and -150 W heat load decreases in the other
624 two evaporators. This disturbance profile tests the performance of the outer
625 loop dryout avoidance control by subjecting one evaporator to a 50% heat
626 load increase. The simultaneous heat load decreases in the other evaporators
627 help test the inner loop mass flow rate distribution control. Additionally,
628 small changes are made to the η_v and k_v coefficients used in feedforward (as
629 percentage of original values, -6.3 % and $[+9, +4, +3.5]$ %, respectively) to
630 test model error disturbance rejection.

631 A single simulation is performed, where the large 300 W disturbance
632 is applied to evaporator-three (Figure 17). Repeating the simulation for
633 the other two evaporators yields nearly identical results. In each of the
634 simulations the controller quickly adjusts mass flow rates in response to the
635 cooling needs, while also regulating pressure and wall temperature.

Fig. 17: Simulated closed-loop response with large heat increase applied to evaporator-three, small heat decrease applied to evaporators-one and two [20].

7. Experimental Results

7.1. Open-Loop Response

The simulation results provide the confidence to apply the control approach to our physical testbed. Open-loop tests are first conducted to determine if dryout would occur in the uncontrolled system given the selected heat load disturbance profile. Figures 18–20 show that indeed all three evaporators reach dryout about 50 seconds after the 300 W heat load increase.

7.2. Closed-Loop Response

Validation tests for multi-evaporator CHF avoidance control are conducted on the experimental testbed for each of the three heat disturbance combinations. When the large heat load increase is applied to evaporators one and two, the controller easily regulates wall superheat and system pressure (Figures 21–22). For evaporator-three, wall superheat increases more than in the other evaporators. Subtle differences in cartridge heater alignment, evaporator wall end thermocouple position, and solder quality contribute to the larger temperature gain in evaporator-three. As a result, the outer loop demands more mass flow rate, while the inner loop engages the

Fig. 18: Experimental open-loop response with large heat increase applied to evaporator-one, small heat decreases applied to evaporators-two and three [20].

Fig. 19: Experimental open-loop response with large heat increase applied to evaporator-two, small heat decreases applied to evaporators-one and three [20].

Fig. 20: Experimental open-loop response with large heat increase applied to evaporator-three, small heat decreases applied to evaporators-one and two [20].

Fig. 21: Experimental closed-loop response with large heat increase applied to evaporator-one, small heat decreases applied to evaporators-two and three [20].

Fig. 22: Experimental closed-loop response with large heat increase applied to evaporator-two, small heat decreases applied to evaporators-one and three [20].

Fig. 23: Experimental closed-loop response with large heat increase applied to evaporator-three, small heat decreases applied to evaporators-one and two [20].

Fig. 24: Estimated Δx_{crit} from closed-loop CHF avoidance experiments.

compressor, EEVs, and accumulator heater to provide this increased flow rate.

Figure 24 shows the estimated Δx_{crit} for each evaporator in each of the three closed-loop experiments. For the first two cases, very little deviation in the estimated Δx_{crit} is observed, indicating that the feedforward control provides adequate mass flow rate for the evaporators. In the third experiment, the estimated Δx_{crit} decreases significantly due to an increase in the evaporator wall end temperature. This change could be attributed to an inaccurate critical quality estimate for evaporator-three, decreasing the effectiveness of the feedforward control.

Recall that a constant $x_{crit} = 0.98$ is assumed for the experimental evaporators (section 3.1). The actual critical quality for evaporator three is close to 0.93 for the test conditions, due to slight differences in the evaporator three alignment and flow. Therefore, evaporator three operates much closer to CHF than the other two evaporators. The feedback component of the control helps mitigate this uncertainty, by increasing the mass flow rate and limiting the rise in wall temperature. However, for this high-quality, saturated, annular flow regime, x_{crit} will decrease with increased mass flow rate as the increased shearing forces draw liquid away from the wall and into the vapor core [28]. This nonlinearity decreases the linear PI control performance at higher mass flow rates, effectively reducing the controller gain. Still, the

Fig. 25: Experimental response using only feedforward outer loop control, with 300W heat increase applied to evaporator-three, 150W heat decreases applied to evaporators-one and two.

674 PI control succeeds in avoiding full dryout.

675 To demonstrate the impact of the feedback control, the experiment in
 676 Figure 23 is repeated with the temperature-based critical quality feedback
 677 removed from the outer loop. Without this feedback correction, there is a
 678 dramatic rise in wall temperature, as shown in Figure 25. This highlights
 679 the limitations of only using open-loop mass flow rate setpoints for two-
 680 phase high heat flux removal. Additionally, the estimated evaporator-three
 681 Δx_{crit} for both the feedforward-only and the combined feedforward-feedback
 682 experiments are presented in Figure 26, showing that the feedback control
 683 reduces the change in Δx_{crit} enough to avoid reaching the critical condition.

684 An important feature of the CHF avoidance control is that it does not
 685 require a highly accurate CHF correlation or heat transfer model. A constant
 686 critical quality and single wall end heat transfer model (5, 6) is used for
 687 all three evaporators, resulting in excellent CHF avoidance for evaporators
 688 one and two and adequate performance for evaporator three. However, if
 689 more detailed information is available, the CHF avoidance controller can
 690 be improved. For example, specific nonlinearities may be controlled using
 691 gain-scheduling or feedback-linearization.

Fig. 26: Comparison of the evaporator-three Δx_{crit} responses to a 300W increase in heat load, for feedforward-only and combined feedforward-feedback control.

8. Conclusions and Discussion

This paper presents a dynamic dryout avoidance strategy by using combined feedforward-feedback control. This approach extends the operable range of the system to within 0.09 of Δx_{crit} while rejecting heat load disturbances of up to 50% greater than the steady-state load. The two-loop control architecture consists of an outer loop providing evaporator dryout avoidance, and an inner loop providing mass flow rate and pressure reference tracking. The outer loop control utilizes evaporator wall temperature measurement in the estimation of Δx_{crit} , allowing robust, feedback-based CHF avoidance.

A key design feature is the explicit evaluation of the Δx_{crit} response to heat load disturbances using the dynamic VCC model. This information has significant practical application. Given the expected size of a heating disturbance, the maximum deflection in Δx_{crit} may be determined, and the Δx_{crit} setpoint can be tuned accordingly to balance safety margin and efficiency. This extends evaporator setpoint design methods to include dynamic effects as well as the static conditions. Furthermore, the design paradigm for CHF avoidance is shifted, from selecting a CHF at some level above the target heat flux to choosing an outlet quality at a level below the critical quality. This design approach allows the system to operate at conditions with higher exit qualities and greater energy efficiency.

The outer loop control has potential application in cooling systems other than VCC, such as two-phase pump-loops, where custom inner-loops can be designed based on cycle configuration to track mass flow references. However, its application is limited to evaporators with conditions similar to the

717 test case, that is, saturated boiling of refrigerant at relatively low mass flux.
 718 For other operating conditions, further experimental validation is required
 719 to determine the effectiveness of wall temperature feedback control for CHF
 720 avoidance. Additional limitations of the two-loop control strategy include
 721 evaporator scaling effects, especially as size is decreased and disturbance
 722 response time is shortened, as well as the requirement for individual evapo-
 723 rator flow rate measurement and control. Furthermore, this strategy assumes
 724 a uniform heat flux, and may need to be adjusted for evaporators with large
 725 density factors or hot spots.

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Appendix A. Nonlinear Model

The nonlinear model for the complete vapor compression cycle is derived using lumped, 1D mass and energy conservation laws applied to the dynamic system components (condenser, evaporators, accumulator), and is based on several low-order VCC models developed in the past [29, 30, 31, 32, 33, 24, 34, 35, 36, 37, 15, 17]. The model is presented here as a several systems of ordinary differential equations (ODE) in matrix-form, accompanied by tables describing the matrix elements z as well as other necessary closure relations. Static expressions used for the compressor and expansion valves. Finally, the order of the system may be reduced by one by applying mass balance to the complete cycle. This eliminates the need for one of the dynamic variables and

877 equations, for example, accumulator liquid volume $V_{a,f}$ and the accumulator
 878 mass balance. $V_{a,f}$ can then be computed algebraically using cycle mass
 879 balance, as in Table A.11.

880 *Appendix A.1. Condenser*

$$\begin{bmatrix} z_{c,11} & z_{c,12} & 0 \\ z_{c,21} & z_{c,22} & 0 \\ 0 & 0 & z_{c,33} \end{bmatrix} \begin{bmatrix} \dot{x}_c \\ \dot{P}_c \\ \dot{T}_{w,c} \end{bmatrix} = \begin{bmatrix} \dot{m}_m - \sum_{n=1}^N \dot{m}_{v,n} \\ \dot{m}_m h_m - \sum_{n=1}^N \dot{m}_{v,n} h_v + H_c A_{s,c} (T_{w,c} - T_{r,c}) \\ H_c A_{s,c} (T_{r,c} - T_{w,c}) - H_{cool} A_{s_{cool}} (T_{w,c} - T_{cool}) \end{bmatrix} \quad (\text{A.1})$$

$z_{c,11}$	$A_{cs,c}L_c \left(\frac{\partial \rho_c}{\partial x_c} \right)$
$z_{c,12}$	$A_{cs,c}L_c \left(\frac{\partial \rho_c}{\partial P_c} \right)$
$z_{c,21}$	$A_{cs,c}L_c \left(h_c \frac{\partial \rho_c}{\partial x_c} + \rho_c \frac{\partial h_c}{\partial x_c} \right)$
$z_{c,22}$	$A_{cs,c}L_c \left(h_c \frac{\partial \rho_c}{\partial P_c} + \rho_c \frac{\partial h_c}{\partial P_c} - 1 \right)$
$z_{c,33}$	$M_{w,c} = V_{w,c} \rho_{w,c} c_{p_{w,c}}$
ρ_c	$\frac{\rho_g^2 (h_m - h_g) + \rho_g \rho_f (h_g - h_f + h_m - h_v) + \rho_f^2 (h_f - h_v)}{(\rho_g + \rho_f) (h_m - h_v)}$
$\frac{\partial \rho_c}{\partial x_c}$	$\frac{\rho_g^2 (h_m - h_g) + \rho_g \rho_f (h_g - h_f) + \rho_f^2 (h_m - h_f)}{(\rho_g + \rho_f) [h_m - 0.5 (h_g + h_f)]}$
$\frac{\partial \rho_c}{\partial P_c}$	$\frac{\partial \rho_c}{\partial \rho_g} \frac{\partial \rho_g}{\partial P_c} + \frac{\partial \rho_c}{\partial \rho_f} \frac{\partial \rho_f}{\partial P_c} + \frac{\partial \rho_c}{\partial h_g} \frac{\partial h_g}{\partial P_c} + \frac{\partial \rho_c}{\partial h_f} \frac{\partial h_f}{\partial P_c} + \frac{\partial \rho_c}{\partial h_v} \frac{\partial h_v}{\partial P_c}$
$\frac{\partial \rho_c}{\partial \rho_g}$	$\frac{h_m - h_g}{h_m - h_v} + \frac{\rho_f^2 (h_g - h_f)}{(\rho_g + \rho_f)^2 (h_m - h_v)}$
$\frac{\partial \rho_c}{\partial \rho_f}$	$\frac{h_f - h_v}{h_m - h_v} + \frac{\rho_g^2 (h_g - h_f)}{(\rho_g + \rho_f)^2 (h_m - h_v)}$
$\frac{\partial \rho_c}{\partial h_g}$	$\frac{-\rho_g^2}{(\rho_g + \rho_f) (h_m - h_v)}$
$\frac{\partial \rho_c}{\partial h_f}$	$\frac{\rho_f^2}{(\rho_g + \rho_f) (h_m - h_v)}$
$\frac{\partial \rho_c}{\partial h_v} \frac{\partial h_v}{\partial P_c}$	$\frac{\rho_g^2 (h_m - h_g) + \rho_g \rho_f (h_g - h_f) - \rho_f^2 (h_m - h_f)}{(\rho_g + \rho_f) (h_m - h_v) (2h_m - h_g - h_f)} \left(\frac{\partial h_g}{\partial P_c} + \frac{\partial h_f}{\partial P_c} \right)$
h_c	$\frac{(2x_c - 1) h_m + 0.5 (h_g + h_f)}{2x_c}$
$\frac{\partial h_c}{\partial x_c}$	$h_m - \frac{(2x_c - 1) h_m + 0.5 (h_g + h_f)}{2x_c^2}$
$\frac{\partial h_c}{\partial P_c}$	$\frac{1}{4x_c} \left(\frac{\partial h_g}{\partial P_c} + \frac{\partial h_f}{\partial P_c} \right)$

Table A.9: Matrix elements for the condenser

881 *Appendix A.2. Evaporator*

882 Note: evaporator index number n omitted

$$\begin{bmatrix} z_{e,11} & z_{e,12} & 0 & 0 \\ z_{e,21} & z_{e,22} & 0 & 0 \\ 0 & 0 & z_{e,33} & 0 \\ 0 & 0 & 0 & z_{e,44} \end{bmatrix} \begin{bmatrix} \dot{x}_e \\ \dot{P}_e \\ \dot{T}_{w,e} \\ \dot{T}_{w,e,end} \end{bmatrix} = \begin{bmatrix} \dot{m}_v - \dot{m}_e \\ \dot{m}_v h_v - \dot{m}_e h_{e,o} + H_e A_{s,e} (T_{w,e} - T_{r,e}) \\ H_e A_{s,e} (T_{r,e} - T_{w,e}) + q_e \\ H_{e,end} A_{s,e} (T_{r,e} - T_{w,e}) + q_e \end{bmatrix} \quad (\text{A.2})$$

$z_{e,11}$	$-\frac{A_{cs,e} L_e \rho_g \rho_f (\rho_f - \rho_g)}{[\rho_g + x_e (\rho_f - \rho_g)]^2}$
$z_{e,12}$	$\frac{A_{cs,e} L_e \rho_e}{\rho_g \rho_f} \left\{ (\rho_g - \rho_e x_e) \frac{\partial \rho_g}{\partial P_e} + [\rho_f - \rho_e (1 - x_e)] \frac{\partial \rho_f}{\partial P_e} \right\}$
$z_{e,21}$	$A_{cs,e} L_e \left\{ \rho_e (h_g - h_f) - \frac{h_e \rho_g \rho_f (\rho_f - \rho_g)}{[\rho_g + x_e (\rho_f - \rho_g)]^2} \right\}$
$z_{e,22}$	$A_{cs,e} L_e \left(\frac{h_e \rho_e}{\rho_g \rho_f} \left\{ (\rho_g - \rho_e x_e) \frac{\partial \rho_g}{\partial P_e} + [\rho_f - \rho_e (1 - x_e)] \frac{\partial \rho_f}{\partial P_e} \right\} + \rho_e \left[x_e \frac{\partial h_g}{\partial P_e} + (1 - x_e) \frac{\partial h_f}{\partial P_e} \right] - 1 \right)$
$z_{e,33}$	$M_{w,e} = V_{w,e} \rho_{w,e} c_{p_{w,e}}$
$z_{e,44}$	$M_{w,e} = V_{w,e} \rho_{w,e} c_{p_{w,e}}$
ρ_e	$\frac{\rho_g \rho_f}{\rho_g + x_e (\rho_f - \rho_g)}$
h_e	$h_f + x_e (h_g - h_f)$

Table A.10: Matrix elements for the evaporator

$$\begin{bmatrix} z_{a,11} & z_{a,12} & 0 \\ z_{a,21} & z_{a,22} & 0 \\ 0 & 0 & z_{a,33} \end{bmatrix} \begin{bmatrix} \dot{P}_a \\ \dot{V}_{f,a} \\ \dot{T}_{w,a} \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^N \dot{m}_{e,n} - \dot{m}_m \\ \sum_{n=1}^N \dot{m}_{e,n} h_{e,o,n} - \dot{m}_m h_g + H_a A_{s,a} (T_{w,a} - T_{r,a}) \\ H_a A_{s,a} (T_{r,a} - T_{w,a}) + q_a \end{bmatrix} \quad (\text{A.3})$$

$z_{a,11}$	$V_{g,a} \frac{\partial \rho_g}{\partial P_a} + V_{f,a} \frac{\partial \rho_f}{\partial P_a}$
$z_{a,12}$	$\rho_f - \rho_g$
$z_{a,21}$	$V_{g,a} \frac{\partial \rho_g u_g}{\partial P_a} + V_{f,a} \frac{\partial \rho_f u_f}{\partial P_a}$
$z_{a,22}$	$\rho_f u_f - \rho_g u_g$
$z_{a,33}$	$M_{w,a} = V_{w,a} \rho_{w,a} c_{p_{w,a}}$
$V_{f,a}$	$\frac{m_{total} - \rho_c A_{cs,c} L_c - \rho_e A_{cs,e} L_e - \rho_{g,a} V_a}{\rho_{f,a} - \rho_{g,a}}$
$V_{g,a}$	$V_a - V_{f,a}$

Table A.11: Matrix elements for the accumulator**Table A.12:** Parameters for VCC testbed

$A_{cs,c}$	1.66e-3	m ²	$A_{cs,e}$	2.06e-4	m ²	$A_{s,a}$	0.715	m ²
$A_{s,c}$	0.145	m ²	$A_{s,cool}$	0.145	m ²	$A_{s,e}$	1.12e-2	m ²
$c_{p,w_{a,c,e}}$	500	J/kg-K	c_1	2300	W/m ² -K	c_2	20	
$D_{h,e}$	7.9e-3	m	$k_{v,1}$	0.044	mm ² /%	$k_{v,2}$	0.048	mm ² /%
$k_{v,3}$	0.029	mm ² /%	L_c	0.483	m	L_e	0.28	m
m_t	1.5	kg	V_m	3.023e-5	m ³	V_a	4.54e-2	m ³
$V_{w,a}$	2.5e-3	m ³	$V_{w,c}$	4.84e-4	m ³	$V_{w,e}$	3.55e-5	m ³
η_V	0.67		$\rho_{w_{a,c,e}}$	8.03e3	kg/m ³	h_v	81700	J/kg