An efficient optimization of well placement and control for a geothermal prospect under geological uncertainty

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Abstract

This study applies an efficient optimization technique based on a multivariate adaptive regression spline (MARS) technique to determine the optimal design and engineering of a potential geothermal production operation at a prospect near Superstition Mountain in Southern California, USA. The faster MARS-based statistical model is used as a surrogate for higher-fidelity physical models within the intensive optimization process. Its use allows for the exploration of the impacts of specific engineering design parameters in the context of geologic uncertainty as a means to both understand and maximize profitability of the production operation. The MARS model is initially developed from a training dataset generated by a finite set of computationally complex hydrothermal models applied to the prospect. Its application reveals that the optimal engineering design variables can differ considerably assuming different choices of hydrothermal flow properties, which, in turn, indicates the importance of reducing the uncertainty of key geologic properties. The major uncertainty sources in the natural-system are identified and ranked first by an efficient MARS-enabled total order sensitivity quantification, which is then used to assist evaluating the effect of geological uncertainties on optimized results. At the Southern California prospect, this parameter sensitivity analysis suggests that groundwater circulation through high permeable structures, rather than heat conduction through impermeable granite, is the primary heat transfer method during geothermal extraction. Reservoir histories simulated using optimal parameters with different constraints are analyzed and compared to investigate the longevity and maximum profit of the geothermal resources. The comparison shows that the longevity and profit are very likely to be overestimated by optimizations without

appropriate constraints on natural conditions. In addition to geothermal energy production, this optimization approach can also be used to manage other geologic resource operations, such as hydrocarbon production or CO₂ sequestration, under uncertain reservoir conditions.

Keywords: geothermal; surrogate; optimization; uncertainty; sensitivity

1. Introduction

Reinjection of geothermal fluids into geothermal reservoirs has been demonstrated as an essential practice for increasing the productive lifetime of reservoirs and recovery of thermal energy. Re-injection helps to maintain pressure in the geothermal reservoirs, slow down production declines in response to pressure drawdown [1], and, as a result, extent the period of time over which useful thermal energy can be recovered. The development and management of geothermal fields is complicated and expensive and maximum potential geothermal energy recovery depends on optimal well location and operation [2, 3]. Simulation-based optimization methods can address these problems by utilizing production and economic models to evolve favorable production designs and strategies through the minimization of one or more quantitative physical or economic objective functions. Often, these approaches may require a large number of intensive forward model simulations, which may quickly become impractical because of their substantial computational burden. As an alternative, a simpler and approximate “surrogate model” may be constructed as a means to provide a faster simulation of the physical system, which may potentially benefit an optimization or sensitivity-based

analysis that would otherwise requires thousands or more of iterative geothermal production simulations. Surrogate or “response surface” models that relate input variables to output responses are developed through the use of statistical models that are fitted by training datasets generated by a finite set of more complex physical simulation models. Surrogate-based optimization approaches have been extensively studied and advanced in the past decade in various application fields [4-11]. Widely used surrogate model techniques in hydrology include polynomial regression, kriging, radial basis functions, sparse grid interpolation, support vector machines, and artificial neural networks [12-14]. Here, we consider the Multivariate Adaptive Regression Spline (MARS) technique as developed by Friedman [15] more than two decades ago and routinely used in automatic engineering design [16]. MARS is a nonparametric regression technique that adaptively develops local models in local regions for flexible regression modeling of high dimensional data. Each local model is represented by a basis function and an associated coefficient to be determined. Comparative studies have shown that MARS is superior to other high dimensional regression methods (e.g. polynomials) in accuracy and reduction in computational cost of fitting process [4,17].

Optimal development and management of a geothermal reservoir will call for an accurate understanding of reservoir behavior under both natural and engineered conditions. However, for geothermal optimization problems, there are a variety of uncertainties associated with the rock properties and structural features of the formation that may significantly affect the optimized results. Assessment of these effects on optimal well placement and control will assist development and management of a geothermal reservoir.

This study couples a complex hydrothermal simulation model and a MARS-based surrogate model to investigate the effects of geological uncertainties (fault size, geological unit permeability) on optimal well placement and control (re-injection well location, production rate) in a geothermal prospect near Superstition Mountain in Southern California, USA. Comparative optimization cases are implemented using prior and posterior probability distributions of geological parameters, which are adapted from a previous study on a MARS-based Bayesian inversion [18] to represent maximal and reduced geological uncertainties respectively. To evaluate the influence of uncertainties of individual geological properties on optimal results, additional optimization experiments are designed and conducted by sequentially fixing the uncertain geological parameters during optimizing process.

2. MARS-based optimization framework

The MARS-based optimization framework consists of several steps that include: Conceptual design and parametric definition of the hydrothermal flow system of interest, including ranges in uncertain geological properties and operational parameters to be optimized; Development of physical hydrothermal flow models for this system; Construction and validation of a MARS surrogate model through the generation and processing of training data drawn from these steps; and Application of the MARS surrogate models in the optimization process to minimize objective functions. As illustrated in Figure 1, these steps involve the MARS-based optimization proceeds as follows:

1. Conceptual Design of the Surrogate: The conceptual model design leads to a series of M uncertain parameters with associated ranges or probability distribution functions

(PDFs), including unknown geological properties and operational parameters to be included in surrogate model. Using a Latin Hypercube (LH) method [19], these are sampled N times to yield a set of N training sample vectors. In this study all parameters are assumed to have a uniform-type of PDF.

2. Training Data Generation: These N sample vectors, each with M components, are used as inputs to develop N hydrothermal flow models for the system. Here, we utilize the NUFT (Nonisothermal Unsaturated-saturated Flow and Transport) model [20] that considers both initialization (run 1 million year to steady state) and production simulation (run to 1000 years of extraction from the initial natural state). The results of each simulation are used to construct individual model responses, in this case, representing an evaluation of the objective function to be minimized in the optimization process.

3. MARS Model Development: In MARS algorithm, local models are adaptively developed in local regions for flexible regression modeling of high dimensional data. The model can be written as $\hat{f}(\mathbf{x}) = \sum_{i=1}^k a_i B_i(\mathbf{x})$, where $\mathbf{x} \in \mathcal{R}^m$, and $\mathcal{R}^m$ is the m -dimensional space. k and a_i are the number and coefficients of associated basis functions $B_i(\mathbf{x}) = \prod_{j=1}^{J_i} [S_{ji} \cdot (x_{v(j,i)} - t_{ji})]_+$, $i = 1, 2, 3, \dots$, where $(\cdot)_+ = \max(0, \cdot)$, J_i is the interaction order of basis B_i , that is, the number of variables included in the basis function, $S_{ji} = \pm 1$ is the sign indicators, $v(j,i)$ is the index of the design variable x which is split on knots t_{ji} . a_i and $B_i(\mathbf{x})$ can be evaluated after the number of locations of knots is adaptively chosen based on the response function changes. The N pairs of sample input vector versus response (objective function) are used to construct a MARS model, as shown in shaded portion of Figure 1. A MARS model

that is fitted well does not necessarily mean it is good for prediction due to potential issues with over-fitting. Following the Leave-One-Out Cross-Validation (LOOCV) method [21], a MARS model is constructed N times, each time leaving out one of the input samples from training dataset, and using the omitted sample to test the model. The errors between output of NUFT and MARS models with these N inputs are used to evaluate the accuracy of the MARS model. Sobol' total order sensitivity indices [22-23], which represent indicators of uncertainty contribution of each input variable to the response, can also be efficiently calculated using the MARS model.

4. MARS-Enabled Optimization: The validated MARS models are coupled with an optimization algorithm to efficiently search for the minimum objective function, along with the corresponding optimal values of input parameters. Specifically, we use Bound Optimization by Quadratic Approximation (BOBYQA), a powerful derivative-free minimization algorithm developed by Powell [24], to drive this bound- constrained nonlinear optimization problem. The most attractive feature of this solver is that it seeks the least value of a function of several input variables without requiring any derivatives of the objective function.

The optimization framework is written in Python to drive hydrothermal NUFT model simulations and incorporate necessary algorithms related to LH sampling, MARS approximation, cross-validation, Sobol' sensitivity, and BOBYQA from the **Problem Solving environment for Uncertainty Analysis and Design Exploration (PSUADE)** package [25].

3. Optimization design for Superstition Mountain geothermal prospect

As shown in Figure 2a, the Superstition Mountain Geothermal prospect (SMG) is located near the Salton Sea in Southern California, USA. Geothermal and related geological characterization data for this effort have been drawn from three nearby exploratory boreholes drilled by the U.S. Navy geothermal program [26-27] and other related data and references summarized in [28].

3.1. Hydrothermal model development

As described in [30, 31], the underlying geological structure in the vicinity of the SMG has been conceptualized from an Earthvision[®] geologic model based upon data and interpretations summarized in [27, 28]. As illustrated in Figure 2b, the SMG is bounded by Superstition Mountain Fault (SMF), a faulted and fractured granitic basement, and four sandstone and alluvial layers of varied composition and hydraulic permeability. Previous studies of the prospect [26-31] suggest that strong hydrothermal communication exists (and has existed) between deeper high temperatures zones underlying the granite and the generally cooler shallow aquifers. Ground surface observations indicate a confined zone of higher temperatures surrounding the prospect [26-27], evidence of hydrothermal alteration products in surface rocks [29], and borehole temperature profiles indicative of localized hydrothermal groundwater circulation [26-27, 30-31] through low permeable granite zone. A recent numerical model was developed to simulate natural steady state hydrothermal conditions at the prospect through inclusion of a hypothesized vertical conjugate fault (CF; shown in dashed yellow in Figure 2b) normal to SMF and extending to northeast through the NAFEC-3 borehole [18, 30-31]. This model is able to predict steady-state temperatures that closely match temperature logs in the three boreholes. The core model domain shown in Figure 2b is enclosed by a larger far field

domain (Figure 3a) to alleviate boundary effects on the hydrothermal flow in targeted production area, which is adapted as the domain of the hydrothermal models in this study.

As shown in Figures 2 and 3, the X-axis of the core domain is parallel to the CF plane, and extends northeast from Superstition Mountain (and the SMF plane) by a distance of 6.5 km. The core domain is 1.5 km wide, transverse to the CF and parallel to Superstition Mountain (and the SMF plane). The center of the left boundary intersects the CF plane at right angle. The model domain extends 3.2 km from the ground surface, and is discretized by 100 m grid space vertically. The horizontal grid space increases gradually from 100 m in core region to 8 km in farthest field to save computational efforts. Representation of geological structures in the model is shown in the vertical cross section parallel with the CF plane in Figure 3b. Here, it can be seen how the geologic units are sequenced from bottom to top as low permeability granite zone overlain by the sandstone (Ti) and alluvial sediment layers (Tp2, Tp1, Qb) in a dipping orientation (Figure 3b). The 100 m thick CF plane is normal to the left boundary, which lies parallel to Superstition Mountain and the SMF plane (Figure 3c). Pressure and temperature conditions are specified along the top boundary to represent the average atmospheric conditions. Similarly, fixed pressure and high temperature conditions are specified along the bottom boundary to represent the geothermal heat source and allow for buoyancy-driven groundwater flow to enter through the bottom of domain. Lateral pressure boundary conditions were specified to induce a small hydraulic gradient parallel with the X-axis of the domain, consistent with local water table observations; Y faces of the domain are maintained as no-flux boundaries. This configuration of boundary conditions will support a hypothesized groundwater circulation pathway in which hot water migrates

vertically into the domain through faulted granite at the bottom, flows laterally through the permeable sandstone unit (Ti), and exits laterally through the right end of the domain.

Each hydrothermal model realization considered in the training process (Figure 1) will consist of two sequential hydrothermal simulations; the first is an initialization (steady state) model and the second, a corresponding production (pumping) model. The initialization model considers a particular geologic configuration (parameters and geologic properties) and runs a simulation out to a steady state at one million years, representing an undisturbed natural condition as might be seen today, largely as described in [30, 31]. The results in this model are then used as initial values for the subsequent production (pumping) model in which simulations are run for another thousand years.

3.2. Production-injection design

Four out of 15 important and uncertain hydrothermal parameters (Table 1) were identified as critical factors influencing temperature distribution in target production zone [18]. These include vertical extent and length of the conjugate fault (CF), its intrinsic hydraulic permeability, and the permeability of the Ti sandstone unit. The uncertainties of these parameters were reduced by an efficient MARS-based Bayesian inference using temperatures observed along the three boreholes (shown in Figure 4) and quantified by the resulting posterior distribution [18, 30, 31] (Figure 5). It is notable that the optimal CF height is peculiarly prone to be 3200 m, at its upper limit, in order to best match the observations, strongly suggesting that the vertical CF plane penetrates the entire impermeable granite zone in order to permit hydraulic contact with the permeable Ti formation. Although the CF height could take on other values with a much smaller probability, it will be fixed as 3200 m and, thus, excluded from further optimization steps.

The CF length, and log-permeability of the CF and Ti units are considered randomly distributed within the uniform ranges defined by values indicated by red dotted line in Figure 5. The corresponding lower and upper bounds for the three key parameters are 800 m and 1600 m, -13.3 to -12.8 (m^2), and -13.8 to 13.3 (m^2), respectively. As shown, the posterior distribution of each parameter, as examined in [18], is reduced to a quarter of the original one; this serves to better constrain the optimization in the next step.

Given a specified production well location, the optimization process seeks to locate two injection wells and determine the production well pumping rate to achieve maximum net profit from a potential SMG geothermal operation. The design considerations for the production-injection scheme are shown in Figure 6. The single production well is fixed at $X=1.3$ km along the central X-axis. The locations of two symmetric injection wells are determined by their common radial distance R from the production well and associated azimuth angle α (clockwise and counterclockwise) away from the central X-axis. R is allowed to vary between 100 to 700 m while α may range between 0 and 90° . The ranges of R and α lead to a semicircular area of possible locations for the two injection wells (Figure 6). The injection wells are restricted to lie on the down gradient side of the production well because of the presence of the impermeable granite formation on the up gradient side. Besides, the left boundary across Superstition Mountain may act as natural injection source by groundwater recharging when pressure declines around the production well.

Both injection and production wells are perforated with a length of 100 m at the 800 m depth so that injection or production is operated in the more permeable CF or Ti formation. The production rate is to be optimized within the range from 1 to 50 kg/s. To

assure reservoir pressure support, the bottom hole pressures (BHP) of injection wells are fixed as 0.5 MPa less than the background pressure provided by the initialization model, meaning that the reservoir pressure around injection wells are allowed to decline at most by 0.5 MPa. As a result, the injection wells act as artesian production wells initially, and are automatically converted to injection wells once the pressures around injection wells fall below the specified BHP. Optimal combination of the six parameters, including the geologic properties and the three operational parameters associated with the injection well placement and control, are to be determined within their bounds to minimize the objective function defined in next subsection.

3.3. Objective function

The objective for the optimization of production-injection design and control is to maximize the net profit value (NPV) after a specified time of operation (e.g., 50 years). The net profit value represents the value of net energy production minus costs associated with water production and injection, defined specifically as

$$NPV = (CNTP \times \$/MWh_t) - (CWP \times \$/p_{wat}) - (CWI \times \$/i_{wat})(1)$$

where $CNTP$, CWP , CWI represent the cumulative net thermal energy production, cumulative water production and cumulative injection respectively, which are obtained from outputs of NUFT hydrothermal model simulations (well flux, energy, pressure, temperature, etc.). The quantity $\$/MWh_t$ represents the price of thermal energy per thermal megawatt-hour (MWh_t), which is converted from electricity price per electricity megawatt-hour (MWh_e). The quantities $\$/p_{wat}$ and $\$/i_{wat}$ are the costs of water production and reinjection per ton respectively. The electricity price, cost of production

and injection, and the conversion factor from MW_{he} to MW_{ht} can be easily modified in Eq.(1) to accommodate market variation and technique improvements.

For the training phase, the six uncertain input parameters, with their ranges, will constitute a six-dimensional data space for LH sampling. A series of 1,000 hydrothermal models (including sequential initialization and production steps) are developed with NUFT. The objective function (NPV) for each model is calculated as a time series from the transient NUFT production phase model outputs. Following the procedure in Figure 1, MARS models are then fitted and validated using the training dataset that consists of 1000 pairs of input vectors (six components) versus response functions (objective functions evaluated at a series of time steps). The MARS model response surfaces are then used as fast approximations of the objective functions during the optimizing process.

4. Results and discussion

4.1. Optimizations under varying parameter space dimensionality

The quality of the MARS models is measured by comparison of the responses to the NUFT model using the 1000 input samples. As shown in Figure 7, the R-square of 1000 scatter points determined by objective function values at 50 years of operation made by the NUFT and MARS models is 0.987 and 0.975 for fitting and validation, respectively, indicating that the well-fitted MARS model has a good predictive ability. To investigate how the uncertainty of each individual parameter affects optimized results, six optimization cases defined by sequentially fixing parameter values are performed by coupled execution of the BOBYQA optimizer and the MARS model. The ranges or the fixed values of the six parameters for Cases 1-6, as well as the corresponding results, are

listed in Table 2. Note additional optimizations with other MARS models in different operational times can be readily executed with BOBYQA if needed.

As shown in Table 2, Case 1 searches the full 6-D parameter space to find the minimal objective function (i.e. maximal NPV). This case represents a scenario in which the injection well placement and production well pumping rate are determined under best available reservoir knowledge from Bayesian inversion using the temperature observations [18]. The optimal location of the two injection wells is found as $R = 473$ m and $\alpha = 74^\circ$ (Figure 6), and the optimal production well pumping rate is 30.7 kg/s. The associated three optimal reservoir properties are also obtained, among which the log permeability values of the CF and Ti units are optimized as -12.92 and -13.33 (m^2) respectively, very close to their upper bounds of -12.8 and -13.3 (m^2). It makes sense since the CF and Ti units form the groundwater circulation pathways, and the optimal values are expected to be as high as possible. The fault length is optimized as 1472 m, larger than 1200 m, which is the minimum value for the CF unit to be physically connected with the Ti unit.

Case 2 assumes CF length is already determined as 1200 m, and Case 3 further fixes the CF log permeability as -13.16 (m^2), both of which have the highest probability from their posterior PDFs (Figure 2b, 2c). Interestingly, the optimized results between Case 2 and 3 are almost the same and optimal azimuth α in both cases increase slightly to 90° from 74° in Case 1. This behavior can be explained by the low sensitivity of the CF length ($\text{SI} < 0.05$) and insensitivity of CF permeability ($\text{SI} < 0.01$) for the objective function (Table 2). As expected, the associated maximal NPVs of Case 2 and 3 are reduced to 130.0 M\$ and 127.1 M\$ from 145.8 M\$ in Case 1, since optimization of Case

3 (4-D) is conducted in a subspace of Case 2 (5-D), which in turn is conducted in a subspace of Case 1 (6-D).

Case 4 (3-D) represents an optimization scenario given full knowledge of the SMG site, with all the three geological properties fixed at values with maximal probability from their posterior PDFs. The NPV is substantially decreased to 74.8 from 127.1 M\$, and optimal injection-production well distance is shortened to 407 m by 66 m from Case 3. This is reasonable since Ti permeability is moderately sensitive (SI =0.21).

The comparison of Case 1-4 reveals that reduced uncertainty of CF doesn't influence the optimized results. It makes sense since the height of the CF unit is fixed at 3200 m, and its log permeability is higher than -13.3 (m^2), which assures that a high permeable plane penetrates entire impermeable granite. Since the Ti unit serves as the other permeable flow pathway, its log permeability ($\leq -13.3 \text{ m}^2$), therefore, becomes relatively important for efficient geothermal extraction. Note both injection and production wells are perforated in the Ti formation.

Case 5 (2-D) and 6 (1-D) is designed to compare the optimized production well rate assuming known azimuth ($\alpha=45^\circ$) and injection location ($R=600 \text{ m}$, $\alpha=45^\circ$) respectively. The optimized pumping rate remains almost the same between Cases 5 and 6, which is unexpected considering that the well distance R is sensitive (SI=0.3). The visualized response surface for Case 5 shows the contour of the objective function (i.e., negative NPV) with respect to R and rate in Figure 8. It is seen that the minimum area includes R from 400 to 600 m around the rate of 29 kg/s, explaining the similar optimized rate (28.9 and 29 kg/s) between Case 5 (optimal $R=420 \text{ m}$) and 6 (constant $R=600 \text{ m}$).

Figure 9 depicts the searching history of the minimal objective function for each of the six cases. It is seen that the number of MARS model evaluations required to satisfy the convergence criteria (10^{-6}) is 297, 388, 396, 359, 490, and 302 for case 1-6. These hundreds of model evaluations can be completed in seconds, while a single simulation of production NUFT model costs as long as five hours. As a result, this MARS-enabled approach can enable the otherwise computationally intensive procedure to be completed efficiently in the simulation-based optimization. In addition, Figure 8 shows multiple local minimum objective functions existing on a 2-D response surface in Case 5, and the global minimum is successfully found using the BOBYQA optimizer.

4.2. Comparison of optimized results in two sets of parameter ranges

To investigate how optimized results could be affected by the degree of uncertainty in natural hydrogeological conditions, an additional set of MARS surrogate simulations is developed following the procedure in Figure 1 using the information prior to Bayesian inversion (Chen et al., 2014), and the corresponding optimization (denoted as Case 0) is compared to Case 1, which is better constrained by posterior knowledge. The ranges of CF length, CF permeability, and Ti permeability used in Case 0 are three times wider than those in Case 1, and the corresponding optimized results also differ substantially between both cases (Table 3). Parameter sensitivities, which are considered indicators of uncertainty contribution of each input parameter to the response, are visualized in Figure 10 for the MARS simulations used in both cases. Compared to Case 0, the sensitivities of CF length, and CF and Ti permeabilities in Case 1 are reduced substantially, while those of azimuth α , well distance R , and production rate increase moderately. These results are expected, since the uncertainties (ranges) of the three hydrogeological parameters are

greatly reduced from Case 0 to Case 1, and hence the other three well placement and control parameters become relatively more important accordingly.

In Case 0, the optimal log-permeabilities of CF and Ti are -12.07 and -13.05 (m^2) respectively, near the upper bound of their ranges, but beyond the ranges in Case 1, which are narrowed by Bayesian inference using field data. This finding not only confirms that a higher permeability of CF and Ti is always preferable for thermal production, but also suggests the necessity of reducing uncertainties of reservoir properties prior to optimization. A better constrained natural system in Case 1 can lead to a much different optimal design and operation from those in Case 0. As shown in Table 3, the optimal well distance is shortened by 130 m, and the optimal pumping rate is increased from 19.9 kg/s in Case 0 to 30.7 kg/s in Case 1. Although the azimuth is also increased greatly by 52° , its variation can be neglected for heat production profit since it is insensitive in both cases (Sensitivity Index, $\text{SI} < 0.01$). The maximal NPV is increased substantially from 145.8 M\$ in Case 1 to 241.9 M\$ in Case 0. However, this higher NPV in Case 0 cannot be obtained **in practice** since the associated optimal CF and Ti permeability is beyond the more credible ranges in Case 1.

4.3. Production-injection history analysis

Hydrothermal NUFT model simulations using the optimal parameter sets from Case 0 and 1 have been simulated to 1000 years (denoted as Simulation 0 and 1), and the histories of the first 200 and 150 years are shown in Figures 11 and 12 respectively. In Simulation 0 and 1, the BHP of both injection wells in the production models are fixed at 5.30 and 5.24 MPa, respectively, 0.5 MPa lower than the background pressure from the according initialization models. This initial local lower pressure leads to immediate water

production from the injection wells at the beginning of the production phase. As shown in Figures 11a and 12a, the extraction rates are 36.95 and 15.61 kg/s, respectively, initially, and decrease as the pressure gradient declines. After 70 years of operation in both simulations, the extraction is converted to injection and the rate keeps increasing to maintain the constant BHP. The cumulative net profit from injection wells are calculated by subtracting cost of extraction or injection operation from the revenue of extracted energy, and is maximized in 70 years (Figure 11b and 12b). Figures 11c and 12c show that the BHP of production well drops dramatically from 6.16 and 5.81 MPa to 5.27 and 4.89 MPa, respectively, in one year and then falls gradually to 5.08 and 4.63 MPa in 150 years due to constant extraction at 19.9 and 30.7 kg/s. The extracted water temperature decreases smoothly from initial 160 and 134 °C to 142 and 94 °C, respectively, after 150 years of operation in Simulation 0 and 1 (Figure 11e and 12e). The cumulative net profit of the production well, which is calculated by subtracting cost of production from the revenue of extracted energy, increases during the entire operation process (Figure 11d and 12d). The history of cumulative net profit values of both production and injection wells, as defined by NPV in Eq. (1), are shown in Figure 11f and 12f for both simulations. Although NPV in 50 years is used as the objective to be maximized in the optimization Case 0 and 1, NPV reaches peak in 170 and 130 years, respectively, and falls sharply in each case after each one is reached. It seems that the optimal parameter set from Case 0 can lead to a better longevity of the geothermal field and a larger amount of energy than Case 1. The 50-year NPV values simulated by the MARS models for Case 0 and 1 are 241.9 and 145.8 M\$, respectively. When rerun with the NUFT model, the 50-year NPV values are 265.7 and 159.5 M\$, respectively, demonstrating a reasonable accuracy of

constructed MARS models and the optimization method. The comparison of Simulation 0 and 1 indicates that the longevity and profit are very likely to be overestimated by optimizations without appropriate constraints on natural conditions.

It should be noted that the fixed BHP value determines the initial production time of the injection wells. The higher the BHP is, the shorter the production time and the lower the revenue will be. While BHP is fixed as a value 0.5MPa lower than background pressure in previous production simulations, an additional production model (denoted as Simulation 2) is constructed using the optimal parameters from Case 1, but with a decrement of 0.4 MPa from the background pressure. In other words, the fixed BHP in Simulation 2 is 0.1 MPa higher than that in Simulation 1. The resulted histories of Simulation 2 are shown in Figure 13 for comparison with those of Simulation 1 in Figure 12. As expected, the initial pumping rate of the injectors is reduced to 9.07 kg/s, and the time of shifting extracting to injecting operation is shortened to 45 years, when the cumulative revenue from the injection wells reaches the maximum. In contrast to injection wells, the histories of pressure and temperature at production well differ only slightly between Simulation 1 and 2, as well as the resulting revenue from production well. The net cumulative revenue, which is the sum of cumulative revenue from injection and production wells, reach its maximal value of 200 M\$ in 90 years, 40 years earlier and 65 M\$ less than the corresponding time and profit in Simulation 1. The comparisons between Simulation 1 and 2 suggest that a lower fixed BHP of injection wells can extend the longevity and increase the profit of a geothermal field, though the reservoir pressure must be carefully controlled in an appropriate range to assure geomechanical stability.

5. Conclusions

The results of this study indicate that coupling multivariate adaptive regression spline (MARS) model and bound optimization by quadratic approximation (BOBYQA) provides an efficient way of reducing computational demand of simulation-based optimization process. Especially in large-scale fields like Superstition Mountain Geothermal prospect, the computational cost of doing an exhaustive search using high-fidelity hydrothermal models is beyond feasible limits, and the proposed MARS-based framework was successfully implemented and a suite of optimization cases under various parameter uncertainty were executed efficiently. The major conclusions are as follows:

- The developed optimization framework is highly efficient. A MARS-based optimization case entailing hundreds of model runs could be completed in seconds, while a single high-fidelity model costs hours using the same computing facility.
- Given all the available knowledge obtained from both geophysical characterization and Bayesian inversion for Superstition Mountain Geothermal prospect (SMG), the optimal production rate is 30.7 kg/s and distance between production and injection well is 473 m in order to maximize the net profit after 50 years of potential geothermal extraction.
- Given different knowledge (uncertainty of input parameters) of the site, the optimized results could be significantly different. Sensitivity analysis could be applied to identify the key parameters where efforts should be focused to reduce the uncertainty. Both geophysical characterization (geological structure, borehole logs) and numerical method (Bayesian inversion) can be used to constrain the optimization.

- For SMG site, sensitivity analysis shows that water circulation is the primary heat transfer method other than heat conduction through granite zone during production. Well-constrained permeability and width of the granite fault won't influence the optimization results given the fact that the determined 3200 m height of the fault guaranteed the high permeable plane to penetrate the entire granite zone to contact permeable Ti formation. The shallow Ti formation, therefore, become relatively important for an efficient geothermal production.
- History analysis of production simulations using different optimal results reveals that the longevity and profit are very likely to be overestimated by optimizations without appropriate constraints on natural conditions. Bottom hole pressure (BHP) of the injection wells is found to affect the profit substantially, indicating maintaining a balance between pressure support and thermal breakthrough is critical to the geothermal field management. While BHP is fixed in the study, it is better to be treated as an uncertain parameter to investigate its influence on thermal production systematically.
- The pumping rate of the production well optimized in this study is constant throughout the production operation, though a time-varying pumping strategy might, in fact, be a more flexible option to control reservoir pressure and maximize profit. The production-injection design could include the varying pumping rate for sequential management periods, and the proposed MARS-based optimization framework could be used to obtain them.

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Tables and Figures

Table 1. Important hydrogeological parameters used in simulation of natural hydrothermal conditions at Superstition Mountain prospect. Model domain is illustrated in Figure 2 and 3, and the simulated temperature contour is shown as background in Figure 3b, 3c.

Input parameter set	Value	In optimization
Fault height (m)	3200	fixed
Fault length (m)	1200	adjustable
Fault log permeability (m ²)	-13.16	adjustable
Ti log permeability (m ²)	-13.44	adjustable
Bottom boundary temperature (°C)	165	fixed
Granite thermal conductivity (W/m-C)	3.0	fixed
Ti thermal conductivity (W/m-C)	2.0	fixed
Granite log permeability (m ²)	-18	fixed
Fault thermal conductivity (W/m-C)	2.0	fixed
Tp1 thermal conductivity (W/m-C)	0.5	fixed
Tp2 log permeability (m ²)	-14	fixed
Tp2 thermal conductivity (W/m-C)	0.5	fixed
Tp1 log permeability (m ²)	-14	fixed
Qb thermal conductivity (W/m-C)	0.5	fixed
Qb log permeability (m ²)	-14	fixed

Table 2. Parameter set and the optimal values for optimization case 1-6. Injection well location and production well pumping rate are optimized under various uncertainties of other parameters after 50 years of operation.

Input parameter	Range	SI	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Fault length (m)	800~1600	0.03	1472	1200	1200	1200	1200	1200
Fault log perm. (m ²)	-13.3~ -12.8	0.0075	-12.82	-12.82	-13.16	-13.16	-13.16	-13.16
Ti log perm. (m ²)	-13.8~ -13.3	0.21	-13.32	-13.32	-13.32	-13.44	-13.44	-13.44
Azimuth (°) ¹	0~90	0.0082	74	90	90	82	45	45
Well distance (m) ¹	100~700	0.30	473	473	473	407	417	600
Prod. rate (kg/s)	1~50	0.75	30.7	29.6	30.4	28.7	28.9	29.0
Max NPV (M\$)			145.8	130.0	127.1	74.8	74.5	66.4
Model evaluation #			297	388	396	359	490	302

¹ Injection well locations can be determined by injection-production well distance R and azimuth α , as illustrated in Figure 5. Gray shaded values are fixed and not included for optimization.

Table 3. Range, optimal values (Opt), and Sobol’ total order sensitivity indices (SI) of the parameters for optimization case 0 and case 1, in which injection well location and production well pumping rate are optimized under prior and posterior range of the three geological parameters after 50 years of operation.

Input parameter	Case 0			Case 1		
	Range	SI	Opt	Range	SI	Opt
Fault length (m)	100~3200	0.094	1233	800~1600	0.03	1472
Fault log perm. (m ²)	-14~ -12	0.12	-12.07	-13.3~ -12.8	0.0075	-12.82
Ti log perm. (m ²)	-15~ -13	0.33	-13.05	-13.8~ -13.3	0.21	-13.32
Azimuth (°) ¹	0~90	0.0031	22	0~90	0.0082	74
Well distance (m) ¹	100~700	0.18	603	100~700	0.30	473
Prod. rate (kg/s)	1~50	0.64	19.9	1~50	0.75	30.7
Max NPV (M\$)			241.9			145.8
Model evaluation #			272			297

¹ Injection well locations can be determined by injection-production well distance R and azimuth α , as illustrated in Figure 5.

Figure 1. Schematic diagram of the MARS-based optimization framework. The gray shaded part shows that MARS models are trained by dataset generated from the physical models.

Figure 2. Superstition mountain geothermal prospect. (a) Location in Imperial County, California, USA (Bjornastad et al. [26]); (b) Geological model looking from the Northeast, and showing (from bottom) the granite basement, sandstone Ti, and three alluvial units Tp2, Tp1, and Qb (Figure3b). Dashed outline illustrates 3D core domain of hydrothermal models. The dashed yellow plane represents the fault plane. The three red tubes represent Naval exploratory (NAFEC) boreholes developed for the geothermal prospect. Modified from Chen et al. [18].

Figure 3. Hydrothermal model domain showing (a) Far field and core area (red shaded area); (b) vertical slice of core domain at $Y = 28\text{km}$, where the conjugate fault crosses; (c) horizontal slice of core domain at depth $Z = 800\text{m}$, where the production and injection wells are perforated. The fault height and length, and the temperature distribution correspond to the input parameter set listed in Table 1. Adapted from Chen et al. [18].

Figure 4. Measured temperature profiles along the three “NAFEC” boreholes (Tiedeman et al., 2001) [27]. The red circle marks indicate the discrete locations along the measured data curves used as observations in the Bayesian inversion process.

Figure 5. Posterior distribution of (a) fault height, (b) fault length, (c) fault permeability, (d) Ti permeability, which are inferred from Bayesian inversion by Chen et al. [18]. The red dashed line indicates 5% probability, which helps to determine the narrowed ranges of parameters used in optimizations.

Figure 6. Reinjection-production scheme design. Fault length ranges from 0.8 to 1.6 km. The production well location is fixed while the locations (radial distance R and azimuth α) of two symmetric injection wells are to be determined within semicircle gray area. Both production and injection wells are perforated 100m in length at depth of 800m. The background is the temperature contour as shown in Figure 3c.

Figure 7. Scatter plots of objective function calculated from surrogate MARS model data versus NUFT hydrothermal model from 1000 input samples for (a) MARS model fitting, and (b) MARS model cross-validation.

Figure 8. The visualized 2D response surface of objective function (-NPV) corresponding to injection-production well distance and production rate in case 5. White area indicates positive objective function, that is, negative NPV.

Figure 9. Minimum objective function (i.e., maximum NPV after 50 years of operation) searching curve for (a) Case 1 with 6 uncertain parameters, (b) Case 2 with 5 uncertain parameters, (c) Case 3 with 4 uncertain parameters, (d) Case 4 with 3 uncertain parameters, (e) Case 5 with 2 uncertain parameters, (f) Case 6 with 1 uncertain parameter. The number of model evaluations and optimal parameter values for the 6 cases are presented in Table 2.

Figure 10. Comparison of sensitivity of objective function to 6 uncertain input parameters between optimization Case 0 and Case 1, in which the 3 hydrogeological parameters are constrained by prior and posterior information of a Bayesian inversion by Chen et al. [18]. The sensitivity is measured by Sobol' total order sensitivity indices. The range and optimal value of the 6 parameters are presented in Table 3.

Figure 11. Plots of history simulated using the optimal parameter set from optimization Case 0 (denoted as Simulation 0). The left column shows (a) flux in injection wells with fixed bottom pressure, resulting extraction at the beginning, (c) production well bottom pressure with fixed rate, and (e) production well bottom temperature. The right column shows cumulative (b) revenue from injection wells, (d) revenue from production well, and (f) net profit value.

Figure 12. Plots of history simulated using the optimal parameter set from optimization Case 1 (denoted as Simulation 1). The left column shows (a) flux in injection wells with fixed bottom pressure, resulting extraction at the beginning, (c) production well bottom pressure with fixed rate, and (e) production well bottom temperature. The right column shows cumulative (b) revenue from injection wells, (d) revenue from production well, and (f) net profit value.

Figure 13. Plots of history with fixed injection well pressure 0.1 MPa higher than that in Simulation 1 (denoted as Simulation 2). The left column shows (a) flux in injection wells with fixed bottom pressure, resulting extraction at the beginning, (c) production well bottom pressure with fixed rate, and (e) production well bottom temperature. The right column shows cumulative (b) revenue from injection wells, (d) revenue from production well, and (f) net profit value.