Operational planning of combined heat and power plants through genetic algorithms for mixed 0-1 nonlinear programming

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Abstract

This paper is concerned with short-term (up to 24 hours) operational planning in combined heat and power plants for district energy applications. In such applications, heat and power demands fluctuate on an hourly basis due to changing weather conditions, time-of-day factors and consumer requirements. Plant energy efficiency is highly dependent on ambient temperature and operating load since equipment efficiencies are nonlinear functions of these parameters. In operational planning strategies, nonlinear equipment characteristics are seldom taken in to account, resulting in plants being operated at sub-par efficiencies. In order to operate plants at highest possible efficiencies, scheduling strategies which take in to account nonlinear equipment characteristics need to be developed. For such strategies, a mixed 0-1 nonlinear programming formulation is proposed. The problem is nonconvex and hence global optimality conditions are unknown. Classical techniques like branch-and-bound may not produce integer feasible solutions, may cut off the global optima and have an exponential increase in CPU time for a linear increase in planning horizon size. As an alternative, a solution method through genetic algorithms is proposed in which genetic search is applied only on 0-1 variables and gradient search is applied on continuous variables. The proposed method is a nonlinear extension of the one originally developed by Sakawa et al [Sakawa M, Kato K, Ushiro S. Operational planning of district heating and cooling plants through genetic algorithms for mixed 0-1 linear programming. *European Journal of Operational Research* 2002;137(3):677-687]. Numerical experiments show the proposed genetic algorithm method is more consistent in finding integer feasible solutions, finds solutions with lower optimality gaps and has reasonable CPU time as compared to branch-and-bound. From an application perspective, the proposed scheduling strategy results in 5-11% increase in plant energy efficiency.

Keywords

CHP, Energy efficiency, Thermodynamics, Scheduling, MINLP, Genetic algorithms

Nomenclature

GT: Gas turbine

DB: Duct burner

HRSG: Heat recovery steam generator

B1: High pressure steam boiler

B2: Low pressure steam boiler

B3: Low pressure steam boiler

ST1: High pressure steam turbine

ST2: Low pressure steam turbine

DS1: High pressure desuperheater

DS2: Low pressure desuperheater

S1: High pressure district steam supply

S2: Low pressure district steam supply

I: Index set of equipment [GT,DB,B1,B2,B3,ST1,DS1,ST2,DS2]

A: Ambient temperature (K)

E: Electric power (kW)

F: Fuel power (kW)

Q: Steam power (kW)

η : Combustion efficiency (dimensionless)

ε : Heat transfer efficiency (dimensionless)

ϕ : Start/stop energy (kWh/switch)

δ : On-off status (0 or 1)

t: Given hour

T: Planning horizon size (hours)

P(t): Hourly problem

P(t,T): Multi-period problem

CV: Continuous variable

BV: Binary variable

Z: Objective function

LB: Lower bound on objective function value

NLIC: Nonlinear inequality constraint

NLEC: Nonlinear equality constraint

LIC: Linear inequality constraint

VB: Variable bound

PR: Present schedule

BB: Branch-and-bound schedule

GA: Genetic algorithm schedule

1. Introduction

Combined heat and power (CHP) plants are designed to utilize residual heat from electric power generation cycles, thereby boosting energy efficiency to levels much higher than those achieved by power-only plants. CHP plants are widely used in chemical, paper/pulp and petroleum industries since these applications have simultaneous need for process heat and electric power. District energy is another application where CHP plants are widely used. Residential and commercial districts require heat for space conditioning and hot water and electricity for lighting and appliances. CHP plants cater to both needs simultaneously in an energy efficient way.

In district energy applications, heat and power demands fluctuate on an hourly basis due to changing weather conditions, time-of-day factors and consumer requirements, making it important to keep adjusting equipment loads within the plant. Operational planning (or scheduling) involves skillful determination of equipment loads such that certain objective functions are optimized subject to energy demand satisfaction and safe equipment load constraints. Most times objective functions are related to financial profits based on purchase prices of different fuels and selling prices of heat and power. Such cost optimization problems are modeled as mixed integer linear programming (MILP) problems and the resulting optimum solutions help in running plants profitably. But from a thermodynamic perspective, plant efficiency is highly dependent on ambient temperature and operating load since equipment efficiencies are nonlinear functions of these parameters. In operational planning strategies, nonlinear equipment characteristics are seldom taken in to account, resulting in plants being operated at sub-par efficiencies. It is highly likely that even though plants are operated cost optimally (reason being the relative differences in cost coefficients of the objective function), from a thermodynamic perspective the plants are operated inefficiently (it is possible to inefficiently use large quantities of cheaper fuels and yet be financially profiting). Hence, from a theoretical perspective it becomes interesting to investigate possibilities of operating plants solely on the basis of optimizing energy efficiency. In order to operate plants at highest possible efficiencies, scheduling strategies which take in to account nonlinear equipment characteristics must be developed. For such scheduling problems, this paper proposes a mixed integer nonlinear programming (MINLP) methodology.

The MINLP framework is one of the richest modeling paradigms applicable to scheduling problems in many different types of energy plants. But computational solution techniques are difficult to implement and there is no guarantee of finding global optimal solutions. For ease of implementation, integer aspects can be relaxed and linear approximations can be made. These will help in producing quick solutions, but they might not reflect physical realities. For example, the permissible operating range for medium to large-sized steam boilers is either 0% load (off status) or between 20 and 100% load. While formulating an operational planning problem, if the load (decision variable) is assumed to be continuous between the lower and upper bounds of 0 and 100%, then it could lead to solutions in which the optimum load is between 0 and 20%, which might not be acceptable to the decision maker/plant operator (in automated plants the feedback control systems will reject such I/O signals). On the other hand, if the lower bound is set to 20%, then the boiler will never turn off, resulting in wasted energy. The accurate way to model this situation is to have a 0-1 (on-off) variable to represent the boiler status, and include load range constraints in the problem. This gives the problem its integer feature. Further, boiler efficiency varies quadratically between 20 and 100% loads, giving the problem its nonlinear feature. Assuming constant or linearly varying efficiency characteristics could result in an underestimate or overestimate of optimal solutions. Hence, when developing operational planning strategies for CHP plants, the problems are inherently MINLP problems or more specifically mixed 0-1 nonlinear programming problems since only two integers (0 and 1) are involved. The goal of this paper is to not compromise on the integer and nonlinear aspects of the modeling procedure. These features are kept intact at all times.

The common approach to solve mixed integer programming (MIP) problems is the branch-and-bound method. It is known to cut off the global optima from the search process for nonconvex MINLP problems and more importantly suffers from the curse of dimensionality, wherein there is an exponential increase in computer processing time for a linear increase in problem size. In the last two decades genetic algorithms have made a foray in to solving many types of mathematical optimization problems. Although they can outperform deterministic algorithms for small-scale unconstrained test problems, they prove to be very ineffective when dealing with large-scale constrained problems which are often seen in real life. But when augmented with the capabilities of a deterministic algorithm, a hybrid genetic algorithm can prove to be extremely versatile in many complex optimization scenarios. Sakawa et al [1-3] showed in a series of papers how a hybrid genetic algorithm can outperform branch-and-bound for mixed 0-1 linear programming, MILP and nonlinear 0-1 programming (nonlinearities were approximated by piecewise linearities) problems. This paper shows the same for mixed 0-1 nonlinear programming (without linear approximations) problems. From an application perspective, this paper shows that MINLP models can be successfully applied to improve operational energy efficiency of CHP plants.

2. Literature Review

Apart from operational planning in energy systems, MIP problems are widely encountered in unit commitment/economic dispatch of thermal generators and industrial/chemical process scheduling to name just a few related engineering application domains. Interestingly, even though the problems originate in different domains, they share a great deal of similarity when it comes to modeling methodologies and computational solution techniques. Over the last five decades, significant successes have been achieved by academics around the world working in these research areas. Most research efforts can be broadly classified to be on two fronts. The first is on accurate model development to reflect the physical reality of the system under consideration. The second is towards the development of computational solution techniques seeking to solve the mathematical problems to global optimality. The following literature review is not meant to be a comprehensive account of all advances in these research areas. It reviews the most relevant papers in the context of the current work and also a few noteworthy papers which have been instrumental in guiding all research over the past years. Comprehensive reviews conducted by Saravanan et al [4], Xia and Elaiw [5], Trespalacios and Grossmann [6] and Floudas and Gounaris [7] give a chronologically accurate account of all research advances.

Cavaliere et al [8] were one of the foremost to propose the use of MIP models for operational planning in thermal power plants. They highlighted the difficulties associated with operating thermal equipment at very low loads and therefore recommended the use of 0-1 variables to represent equipment on-off status. But they assumed boiler efficiencies to be constant in the permissible operating range, thereby making their model a mixed 0-1 linear programming one which was subsequently solved by combinatorial enumeration of all possible solutions. Based on these mixed 0-1 guidelines, Logendran and Oudheusden [9] developed a model for a bi-fuel thermal power plant and found optimal solutions using the IBM/MIP solver package.

Chisman [10] was one of the foremost to deal with the problem involving multiple power plants connected to a single electric grid. For this he proposed the use of mixed 0-1 linear programming where the 0-1 variables indicated plant status rather than equipment status. The solution to the problem was obtained using a Fortran optimization routine. Gardner and Rogers [11] provided theoretical guidelines to include heat dispatch in electric dispatch problems. The resulting model for the joint planning of heat and power was a MILP model and was solved using GAMS/OSL. Makkonen and Lahdelma [12] proposed nonconvex models for such joint planning problems. The nonconvexities were converted to convexities by augmenting the model with additional constraints involving 0-1 variables. The resulting mixed 0-1 linear programming model was solved by a customized branch-and-bound algorithm incorporating an improved simplex method which exploits the special basis structure of the problems to carry out fast inversions. Rong and Lahdelma [13] developed an envelope-based branch-and-bound technique using piecewise linear functions which performed 785 times faster than the CPLEX/MIP solver. Rong et al [14] proposed a dynamic programming (DP) algorithm based on linear relaxation of 0-1 variables and obtained solutions with low optimality gaps.

Viswanathan and Grossmann [15] developed an outer-approximation algorithm for nonconvex MINLP problems. The algorithm uses a MILP master problem that linearizes nonconvex constraints and implements a penalty function to improve solution quality. This algorithm is available as GAMS/DICOPT++. Quesada and Grossmann [16] developed a branch-and-bound algorithm for convex mixed 0-1 nonlinear programming problems. They pointed out that most computational effort is in the combinatorial search for 0-1 variables rather than in the gradient search for continuous variables. So they proposed that the nodal nonlinear programming (NLP) problems need not be solved to optimality. This led to a 50% reduction in the number of tree nodes to be evaluated using branch-and-bound. Iyer and Grossmann [17] used

MILP formulations for operational planning of utility plants. They developed a shortest path algorithm which guarantees global optimality and has a linear increase in CPU time for a linear increase in planning horizon size. Mitra et al [18] developed a MILP model for scheduling of a large-scale CHP plant and solved it to global optimality using GAMS/CPLEX.

Adjiman et al [19] proposed a deterministic algorithm based on branch-and-bound for finding global optimal solutions to nonconvex MINLP problems. But this method was limited to cases where integer variables appear only in linear or mixed-bilinear forms and the algorithm was tested out on small-scale problems. Leyffer [20] developed a branch-and-bound algorithm for MINLP problems in which the NLP problem at each tree node is not solved to optimality. The algorithm implements an early branching rule which helps in simultaneous execution of the tree search and the nodal NLP solution. This reduces the CPU time of the classical branch-and-bound by a factor of 3. This algorithm is currently available as TOMLAB/MINLP. Pruitt et al [21] developed a comprehensive nonconvex MINLP modeling technique for distributed generation systems. They showed the difficulties in solving such problems to global optimality, and subsequently proposed a convex underestimation and linearization technique to solve the problem in order to use them for practical real-world applications. They used TOMLAB/MINLP and showed it to outperform BARON, Couenne, MINOTAUR-BnB, MINOTAUR-QPD and Bonmin. Pruitt et al [22] also evaluated the shortfalls of MIP models. They found that MILP models tend to overestimate optimum solutions as compared to MINLP models. Deep et al [23] developed a real coded genetic algorithm for MIP problems. This algorithm performs well for small-scale test problems and is currently available in the MATLAB/Global Optimization Toolbox. But it is not very versatile for large-scale real-world problems since it does not permit nonlinear equality constraints.

Savola et al [24] developed a MINLP model for different steam pressure levels and multi-period planning horizons and found that such models can be solved fairly accurately by the local solver GAMS/DICOPT. Tveit et al [25] analyzed a situation where heat and power generation are decoupled using thermal storage technologies to offset loads during peak production periods. The model was nonlinear and nonconvex but could be solved using GAMS/SBB. Kjeldsen and Chiarandini [26] developed heuristic approaches to the unit commitment problem with cogeneration plants. Theirs was a MILP model and the proposed heuristics were able to find solutions with 1-2% optimality gaps. Bosman et al [27] modeled the operation of microCHP generators as integer linear programming (ILP) problems. They proposed a local search using DP and benchmarked the results using AIMMS/CPLEX. The DP solution had a 5% loss in objective function value for a 99% speed-up in CPU time.

The literature review helps in identifying two open research problems. First, in the context of operational planning in energy systems, there hasn't been a lot of attention given towards modeling the fundamental thermodynamic characteristics of energy conversion equipment within a CHP plant. These are nonlinear characteristics and need to be accounted for if a scheduling model is to be accurate. Second, since there isn't a theoretical proof of global optimality conditions for general nonconvex MINLP problems, there is room for improvement in developing new mathematical techniques which can produce more optimal solutions and reduce CPU time. This paper makes an effort to address these two open research problems. In the modeling aspect, it preserves all nonlinearities and nonconvexities of the plant under consideration. In the computational solution aspect, it merges the search capabilities of a stochastic technique and a deterministic technique.

3. Scenario Description

The scenario presented here is that of a CHP plant at a location in the Northeast US. The scenario is interesting to analyze because of its complex energy flow network and the presence of several thermodynamic equipment commonly seen in CHP plants. This leads to the creation of several linear and nonlinear energy balance equations and equipment characteristic equations, thereby producing a non-trivial problem to analyze. The methodology used to model the equations is based on the first law of thermodynamics. It can be readily adopted to model other CHP plants with similar configurations. In such cases, there will be changes in the terms and equations of the model so they reflect the configuration of that plant, but the underlying principle of the first law of thermodynamics will still govern the modeling principles. Hence the modelling methodology is universally applicable.

3.1 Plant Structure

Fig. 1 is a diagrammatic representation of the plant structure. The plant has a gas turbine (GT) with its exhaust being passed through a heat recovery steam generator (HRSG) equipped with a duct burner (DB). At 15 °C, the turbine has a nominal rating of 10,000 kW, the burner 20,900 kW and the HRSG 40,500 kW. Electricity is also produced by a 2,000 kW high pressure steam turbine (ST1) and a 4,000 kW low pressure steam turbine (ST2). Steam is also produced by a 50,245 kW high pressure steam boiler (B1) and two 45,719 kW low pressure steam boilers (B2,B3). Plant steam is in three headers at 4,309 kPa/393 °C, 1,379 kPa/241 °C and 139 kPa/135 °C. Two desuperheater (DS1,DS2), each of 100,000 kW capacity help in regulating steam flows between the headers. District steam is supplied at two pressures; high pressure steam (S1) at 1,379 kPa/241 °C and low pressure steam (S2) at 139 kPa/135 °C. The supply lines go to a nearby steam distribution building where the pressures get further regulated before actual distribution. The gas turbine and duct burner operate on natural gas whereas the boilers can be operated on natural gas or diesel oil.

Fig. 1. Plant Structure

3.2 Equipment Modeling

Equipment modeling involves the use of thermodynamic performance test data to develop polynomial equations which relate inputs, outputs and efficiencies. Here, combustion characteristics of the gas turbine and boilers, heat transfer characteristics of the HRSG and mass/energy balance characteristics of the steam turbines are determined using least squares estimation on measured data.

The maximum gas turbine output varies with the ambient temperature and is given by Eqn. (1). The turbine is not allowed to operate below 50% load. For a fixed ambient temperature, turbine thermal efficiency varies quadratically between 50% and 100% load. Eqn. (2) gives the variation in efficiency with ambient temperature and turbine output. Fig. 2 graphically describes Eqn. (1)-(2).

$$E_{GT}^{\max} = -15,392 + 238.86A - 0.5232A^2 \quad (1)$$

$$\eta_{GT} = 0.4761 - 2.334 \times 10^{-3}A + 3.251 \times 10^{-7}E_{GT} + 8.14 \times 10^{-8}AE_{GT} + 3.638 \times 10^{-6}A^2 - 3.468 \times 10^{-10}E_{GT}^2 \quad (2)$$

The maximum permissible gas input to the duct burner varies with ambient temperature and gas turbine output. Eqn. (3) gives this relation. The minimum burner firing rate is 10% for any given temperature and turbine output. Ambient temperature, turbine output and burner input affect the heat transfer efficiency of HRSG as given by Eqn. (4). Table 1 is the joint characteristics of the turbine, burner and HRSG.

$$F_{DB}^{\max} = 42,620 - 96.66A + 0.6062E_{GT} \quad (3)$$

$$\epsilon_{HRSG} = -0.103 + 2.7 \times 10^{-3}A + 6.6538 \times 10^{-8}E_{GT} + 1.2532 \times 10^{-5}F_{DB} \quad (4)$$

Eqn. (5)-(8) are combustion efficiency characteristics of the boilers for different fuel modes. The efficiencies are quadratic functions of the boiler outputs. The boilers are not allowed to operate below 20% load. Fig. 3 graphically describes Eqn. (5)-(8).

$$\eta_{B1}^{\text{gas}} = 0.7922 + 1.727 \times 10^{-6}Q_{B1} - 1.9737 \times 10^{-11}Q_{B1}^2 \quad (5)$$

$$\eta_{B1}^{\text{oil}} = 0.7632 + 4.3797 \times 10^{-6}Q_{B1} - 5.005 \times 10^{-11}Q_{B1}^2 \quad (6)$$

$$\eta_{B2,B3}^{\text{gas}} = 0.8042 + 1.8851 \times 10^{-6}Q_{B2,B3} - 2.3517 \times 10^{-11}Q_{B2,B3}^2 \quad (7)$$

$$\eta_{B2,B3}^{\text{oil}} = 0.7662 + 4.8378 \times 10^{-6}Q_{B2,B3} - 6.1068 \times 10^{-11}Q_{B2,B3}^2 \quad (8)$$

Eqn. (9)-(10) relate the steam flows and electric outputs of the steam turbines. The steam turbines are not allowed to operate below 20% load and Q_{ST1} and Q_{ST2} are by default set to zero below that load. Fig. 4 graphically describes Eqn. (9)-(10).

$$Q_{ST1}=15,520+10.06E_{ST1} \quad (9)$$

$$Q_{ST2}=5,051+12.35E_{ST2} \quad (10)$$

Fig. 2. Gas Turbine Characteristics

Table 1 Gas Turbine, Duct Burner and HRSG Characteristics

A	255	261	266	272	278	283	289	294	300	305	311
GT load: 100%, DB firing rate: 100%											
E_{GT}	11,486	11,312	11,106	10,868	10,597	10,294	9,959	9,592	9,192	8,760	8,295
η_{GT}	0.3141	0.3143	0.3142	0.3136	0.3125	0.3108	0.3084	0.3053	0.3014	0.2966	0.2909
F_{DB}	24,898	24,256	23,594	22,913	22,212	21,491	20,751	19,991	19,212	18,413	17,594
ε_{HRSG}	0.8993	0.9062	0.9129	0.9194	0.9256	0.9315	0.9372	0.9427	0.9479	0.9528	0.9575
Q_{HRSG}	44,951	44,342	43,668	42,932	42,138	41,287	40,382	39,422	38,409	37,339	36,211
GT load: 100%, DB firing rate: 10%											
E_{GT}	11,486	11,312	11,106	10,868	10,597	10,294	9,959	9,592	9,192	8,760	8,295
η_{GT}	0.3141	0.3143	0.3142	0.3136	0.3125	0.3108	0.3084	0.3053	0.3014	0.2966	0.2909
F_{DB}	2,490	2,426	2,359	2,291	2,221	2,149	2,075	1,999	1,921	1,841	1,759
ε_{HRSG}	0.6185	0.6327	0.6468	0.6609	0.6750	0.6891	0.7032	0.7172	0.7312	0.7452	0.7591
Q_{HRSG}	17,055	17,145	17,204	17,235	17,238	17,215	17,165	17,089	16,986	16,853	16,686
GT load: 50%, DB firing rate: 100%											
E_{GT}	5,743	5,656	5,553	5,434	5,299	5,147	4,980	4,796	4,596	4,380	4,148
η_{GT}	0.2271	0.2257	0.2240	0.2222	0.2202	0.2180	0.2155	0.2128	0.2097	0.2063	0.2025
F_{DB}	21,417	20,827	20,228	19,619	19,000	18,371	17,732	17,084	16,426	15,758	15,080
ε_{HRSG}	0.8553	0.8629	0.8704	0.8777	0.8850	0.8921	0.8991	0.9059	0.9127	0.9193	0.9258
Q_{HRSG}	35,032	34,719	34,345	33,911	33,415	32,857	32,237	31,552	30,801	29,980	29,086
GT load: 50%, DB firing rate: 10%											
E_{GT}	5,743	5,656	5,553	5,434	5,299	5,147	4,980	4,796	4,596	4,380	4,148
η_{GT}	0.2271	0.2257	0.2240	0.2222	0.2202	0.2180	0.2155	0.2128	0.2097	0.2063	0.2025
F_{DB}	2,142	2,083	2,023	1,962	1,900	1,837	1,773	1,708	1,643	1,576	1,508
ε_{HRSG}	0.6137	0.6280	0.6422	0.6565	0.6707	0.6849	0.6991	0.7132	0.7274	0.7415	0.7557
Q_{HRSG}	13,308	13,496	13,651	13,771	13,855	13,902	13,909	13,875	13,795	13,667	13,486

Fig. 3. Boiler Characteristics

Fig. 4. Steam Turbine Characteristics

4. Problem Formulation

Dotzauer [28] and Pendersen et al [29] have developed accurate techniques to forecast energy demands in district energy systems. Given a forecast of the ambient temperature, electric demand (E_{demd}) and steam demands ($Q_{S1 \text{ demd}}$, $Q_{S2 \text{ demd}}$) for time t , the operational planning problem of minimizing total plant fuel consumption subject to energy demands, energy balance and equipment load range constraints is given by Problem P(t).

Problem P(t)

$$\text{Minimize } Z(t) = \frac{E_{GT}(t)}{\eta_{GT}(t)} + F_{DB}(t) + \frac{Q_{B1}(t)}{\eta_{B1}(t)} + \frac{Q_{B2}(t)}{\eta_{B2}(t)} + \frac{Q_{B3}(t)}{\eta_{B3}(t)} \quad (11)$$

Subject to nonlinear inequality constraints (NLIC), nonlinear equality constraints (NLEC), linear inequality constraints (LIC) and variables bounds (VB):

$$\text{NLIC}_{1,2}(t): 0.1F_{DB}^{\max}(t)\delta_{DB}(t) \leq F_{DB}(t) \leq F_{DB}^{\max}(t)\delta_{DB}(t) \quad (12)$$

$$\text{NLIC}_3(t): Q_{B2}(t) + Q_{B3}(t) + [Q_{ST1}(t) - E_{ST1}(t)]\delta_{ST1}(t) + Q_{DS1}(t) - Q_{ST2}(t)\delta_{ST2}(t) - Q_{DS2}(t) \geq Q_{S1 \text{ demd}}(t) \quad (13)$$

$$\text{NLIC}_4(t): [Q_{ST2}(t) - E_{ST2}(t)]\delta_{ST2}(t) + Q_{DS2}(t) \geq Q_{S2 \text{ demd}}(t) \quad (14)$$

$$\text{NLEC}_1(t): \varepsilon_{\text{HRSG}}(t) = \frac{Q_{\text{HRSG}}(t)}{\frac{E_{GT}(t)}{\eta_{GT}(t)} - E_{GT}(t) + F_{DB}(t)} \quad (15)$$

$$\text{NLEC}_2(t): Q_{\text{HRSG}}(t) + Q_{B1}(t) - Q_{ST1}(t)\delta_{ST1}(t) - Q_{DS1}(t) = 0 \quad (16)$$

$$\text{LIC}_1(t): E_{GT}(t) + E_{ST1}(t) + E_{ST2}(t) \geq E_{\text{demd}}(t) \quad (17)$$

$$\text{LIC}_2(t): \delta_{DB}(t) \leq \delta_{GT}(t) \quad (18)$$

$$\text{LIC}_{3,4}(t): 0.5E_{GT}^{\max}(t)\delta_{GT}(t) \leq E_{GT}(t) \leq E_{GT}^{\max}(t)\delta_{GT}(t) \quad (19)$$

$$\text{LIC}_{5,6}(t): 0.2Q_{B1}^{\max}(t)\delta_{B1}(t) \leq Q_{B1}(t) \leq Q_{B1}^{\max}(t)\delta_{B1}(t) \quad (20)$$

$$\text{LIC}_{7,8}(t): 0.2Q_{B2}^{\max}(t)\delta_{B2}(t) \leq Q_{B2}(t) \leq Q_{B2}^{\max}(t)\delta_{B2}(t) \quad (21)$$

$$\text{LIC}_{9,10}(t): 0.2Q_{B3}^{\max}(t)\delta_{B3}(t) \leq Q_{B3}(t) \leq Q_{B3}^{\max}(t)\delta_{B3}(t) \quad (22)$$

$$\text{LIC}_{11,12}(t): 0.2E_{ST1}^{\max}(t)\delta_{ST1}(t) \leq E_{ST1}(t) \leq E_{ST1}^{\max}(t)\delta_{ST1}(t) \quad (23)$$

$$\text{LIC}_{13,14}(t): 0.01Q_{DS1}^{\max}(t)\delta_{DS1}(t) \leq Q_{DS1}(t) \leq Q_{DS1}^{\max}(t)\delta_{DS1}(t) \quad (24)$$

$$\text{LIC}_{15,16}(t): 0.2E_{ST2}^{\max}(t)\delta_{ST2}(t) \leq E_{ST2}(t) \leq E_{ST2}^{\max}(t)\delta_{ST2}(t) \quad (25)$$

$$\text{LIC}_{17,18}(t): 0.01Q_{DS2}^{\max}(t)\delta_{DS2}(t) \leq Q_{DS2}(t) \leq Q_{DS2}^{\max}(t)\delta_{DS2}(t) \quad (26)$$

$$\text{VB}(t): 0 \leq x_i(t) \leq x_i^{\max}(t) \text{ and } \delta_i(t) = \{0,1\} \text{ where } x \in [E, F, Q] \text{ and } i \in [GT, DB, B1, B2, B3, ST1, DS1, ST2, DS2] \quad (27)$$

In Problem P(t), Eqn. (11) is the expression for plant fuel consumption. Eqn. (12) is the load range constraint for the duct burner. Eqn. (13) is a composite energy balance and steam demand constraint in the 1,379 kPa/241 °C header. Eqn. (14) is a composite energy balance and steam demand constraint in the 139 kPa/135 °C header. Eqn. (15) is a composite energy balance and equipment characteristic constraint for the HRSG. Eqn. (16) is the energy balance constraint for the 4,309 kPa/393 °C header. Eqn. (17) is the electric demand constraint. Eqn. (18) is the constraint requiring the duct burner be operated only when the gas turbine is operated. Eqn. (19)-(26) are equipment load range constraints. Eqn. (27) gives the bounds for the continuous variables (CV) and binary variables (BV) in the problem.

Problem P(t) involves 10 continuous variables, 9 binary variables, 4 nonlinear inequality constraints, 2 nonlinear equality constraints and 18 linear inequality constraints. It is a small-scale mixed 0-1 nonlinear programming problem. Typically for t , 1 hour time-steps are used with the assumption of steady-state operating conditions during the hour. The goal is to have plans available for prolonged periods of operation.

But if separate Problem P(t) are solved for each hour over the planning horizon and their solutions are merged, then frequent equipment start/stop will occur leading to energy waste. To overcome this, a multi-period problem incorporating minimization of equipment start/stop is given by Problem P(t,T).

Problem P(t,T)

$$\text{Minimize } \sum_{\tau=t}^{t+T-1} \left[Z(\tau) + \sum_{i \in I} \phi_i |\delta_i(\tau) - \delta_i(\tau-1)| \right] \quad (28)$$

Subject to constraints and bounds

$$NLIC_n(\tau), NLEC_m(\tau), LIC_o(\tau) \text{ and } VB(\tau) \text{ where } n \in [1,4], m \in [1,2], o \in [1,18] \text{ and } \tau \in [t, t+T-1] \quad (29)$$

In Problem P(t,T), T is number of hours in the planning horizon and ϕ are the one-time equipment start/stop energy usage. Eqn. (28) is the expression for total plant fuel consumption over the planning horizon, which is the sum of hourly fuel consumptions and total energy used for all start/stop of equipment occurring during the horizon. The problem is subject to hourly energy demands, energy balance and equipment load range constraints as given by Eqn. (29). This is a large-scale mixed 0-1 nonlinear programming problem and the problem size depends on the number of hours in the planning horizon.

5. Proposed Genetic Algorithm

Sakawa et al [1] presented a unique genetic algorithm method to solve mixed 0-1 linear programming problems. In their method, 0-1 variables are determined by genetic algorithms and continuous variables are determined by solving linear programming problems obtained on fixing 0-1 variable values. This is a bi-level method wherein the upper level involves a single unconstrained stochastic combinatorial optimization problem and the lower level involves several constrained deterministic linear programming problems. This method is extended to solve Problem P(t,T), wherein the lower level will involve several constrained deterministic NLP problems.

5.1 Genetic Individual

Fig. 5 depicts a genetic individual for Problem P(t,T). An individual is formed by connecting sub-individuals which consist of 0-1 variables corresponding to on-off status of equipment at each hour over the planning horizon.

Fig. 5. Genetic Individual

5.2 Generation of Initial Population

In order for the evolution to proceed effectively, the initial population though generated randomly must consist only of feasible individuals. This is done by randomly generating 0-1 values for each hour in the planning horizon. The 0-1 values are then used in the corresponding Problem P(t) to reduce it to an NLP problem for the given hour. The reduced problem is solved using a sequential quadratic programming (SQP) solver. If the problem is feasible then the 0-1 values are considered in the initial population, else the procedure is repeated until a feasible individual is found. The following is the pseudocode to generate a feasible initial population.

Pseudocode for generation of initial population

Begin

for n = 1 to population size

for t = 1 to planning horizon size

 flag = 0;

do while flag < 1

 randomly generate 9 bit 0-1 string;

 use string in Problem P(t) and reduce it to NLP problem in 10 continuous variables;

 use SQP solver on NLP problem;

```

if problem is feasible then

    assign string to  $\Delta(t)$  of genetic individual n;

    flag = 1;

end if

end do while

end for

end for

End

```

5.3 Fitness Evaluation

In Problem $P(t,T)$, the term that connects subsequent hours in the planning horizon is the term associated with the total start/stop energy in the objective function. For the genetic individuals in the population, this term can be directly evaluated and does not require any solving. This makes it possible to detach subsequent hours of the planning horizon and solve each hour separately to find hourly solutions. If all hourly NLP problems are feasible, then the fitness of the individual is determined by summing the objective function values of the hourly NLP problems and adding the scalar value of the total start/stop energy term. If any hourly NLP problem is infeasible, then the individual is assigned a very high fitness value (problem is in minimization form, hence individuals with low objective function values are fitter than individuals with high objective function values) with the intention that it will die out in the evolutionary process. The novelty of this solution technique is that it eliminates the curse of dimensionality by solving for the continuous variables in each hour independent of subsequent hours and directly evaluating the term which contains binary variables spanning over the planning horizon. The following is the pseudocode for fitness evaluation.

Pseudocode for fitness evaluation

```

Begin

number of start/stop = 0;

for i = 1 to number of equipment

    for t = 2 to planning horizon size

        number of start/stop = number of start/stop +  $|\delta_i(t) - \delta_i(t-1)|$ ;

    end for

end for

total start/stop energy = [number of start/stop]x[start/stop energy];

fitness = 0;

flag = 0;

for t = 1 to planning horizon size

    use  $\Delta(t)$  in Problem  $P(t)$  and reduce it to NLP problem in 10 continuous variables;

    use SQP solver on NLP problem;

    if problem is feasible then

        fitness = fitness + objective function value;

    else if problem is infeasible then

        flag = 1;

    end if

end for

if flag is 0 then

```

fitness = fitness + total start/stop energy;

else if flag is 1 **then**

fitness = ∞ ;

end if

End

5.4 Scaling

Fitness scaling is done in order to convert the range of raw fitness values to a range more suitable for the selection procedure. Here linear scaling and ranked scaling will be used for experimentation.

Linear scaling: The raw fitness value f of an individual is converted to a scaled fitness value f_s according to the relation $f_s = af + b$. The coefficients a and b are determined based on the mean, maximum and minimum fitness in the population.

Ranked scaling: The individual with the best fitness in the population is assigned rank 1 and the individual with the worst fitness is assigned rank N . All other individuals are sorted according to their raw fitness values and assigned ranks from 2 to $N-1$.

5.5 Selection

Selection involves picking individuals to fill the mating pool to create off-springs of the next generation. Here roulette wheel selection, tournament selection and stochastic uniform selection will be used for experimentation.

Roulette wheel selection: The individuals get assigned portions of a roulette wheel proportional to their scaled fitness values. The wheel is then spun N times (once for each individual), which is to say a random number between 0 and 1 is drawn N times. Using the drawn random number and the sum of the scaled fitness values of the whole population, an appropriate individual gets picked to go in to the mating pool.

Tournament selection: Two or more individuals are chosen at random to play in a tournament. The tournament involves comparing the scaled fitness values. The individual with the best scaled fitness value is picked to go in to the mating pool.

Stochastic uniform selection: Individuals get assigned portions of a line according to their scaled fitness values. A selector moves over the line in equal step sizes from a randomly chosen starting point. The individuals corresponding to the portions of the line on which the selector lands are picked to go in to the mating pool.

5.6 Crossover

Crossover involves randomly choosing two individuals from the mating pool and creating off-springs of the next generation. Here one-point crossover, two-point crossover and scattered crossover will be used for experimentation.

One-point crossover: After randomly choosing two individuals for mating, a crossover point is determined, which could be at any location along the bit string of the individuals. The binary sub-strings to one side of the crossover point are exchanged between the mating individuals to create off-springs.

Two-point crossover: After randomly choosing two individuals for mating, two crossover points are determined, which are at different locations along the bit string of the individuals. The binary sub-strings to one side of both crossover points are exchanged between the mating individuals to create off-springs.

Scattered crossover: After randomly choosing two individuals for mating, a random binary string the same size of the individuals is first created. Moving bitwise in the newly created string, if the bit value is 1 then the gene corresponding to first individual is taken, and if the bit value is 0 then the gene corresponding to the second individual is taken to create an off-spring.

5.7 Mutation

Mutation involves random changes to an individual to create off-springs. Here uniform mutation and Gaussian mutation will be used for experimentation.

Uniform mutation: Some of the bits of an individual are randomly chosen for mutation with each chosen bit having a specified mutation probability. A random binary digit is then drawn from a uniform distribution. Depending on the mutation probability, the bit is either replaced or left unaffected.

Gaussian mutation: Moving bitwise along an individual, a random number drawn from a Gaussian distribution with 0 mean is added to each bit. If the new bit value is not binary, then it is clipped to the nearest binary digit.

6. Numerical Experiments

Numerical experiments are carried out in MATLAB on a desktop PC with a 3.1 GHz quad-core processor, 8 GB RAM and 64-bit Microsoft Windows 7 Professional operating system. In practice, operational planning can be done only if there are forecasts available for ambient temperature and energy demands. But this paper does not involve forecasting techniques. It verifies proof-of-concepts of the optimization aspects given forecasted temperatures and demands. For such verification experiments historical data can be used. Hence, hourly operational data for 24-hour periods are gathered for analysis. The data is from the year 2012 and is given in the appendix. Hour 1 is from 12 PM to 1 PM and Hour 24 is from 11 AM to 12 PM the following day. Total plant electric and steam output during these periods are used in energy demand constraints. Since plant steam output can be sent by either supply line to the steam distribution building, in the experimental setup the total steam demand is equally split between the two lines. Although the start/stop energy are different for the equipment, in a CHP setting when any equipment is switched on or off, it usually causes the load on some other equipment to be adjusted. Hence, for experimental purposes an average value of 10,000 kWh/switch is used for all ϕ . The experiments will indicate if there could have been a more energy efficient operational plan for the periods under consideration. In the following discussions PR, BB and GA indicate present schedule, branch-and-bound schedule and genetic algorithm schedule respectively.

TOMLAB/MINLP [30] is used for BB. The function that does the tree search is *minlpBB* and the function that solves the nodal NLP problems is the SQP solver *filterSQP*. *filterSQP* can by itself be called to solve any NLP problem. For GA, the genetic procedures on 0-1 variables are carried out using the *ga* function in MATLAB/Global Optimization Toolbox [31] and *filterSQP* is used to solve NLP problems used in determining fitness of individuals. The population size is 50 and elite preserving allows the fittest individual to proceed directly to the next generation. The evolution terminates when the fittest individual in the population is not updated for 50 successive generations or the evolution reaches a maximum of 500 generations. These parameters are determined by trial-and-error before starting the numerical experiments and are not altered during the experiments. Three combinations of scaling, selection, crossover and mutation are used in the experiments; GA' – linear scaling, roulette wheel selection, one-point crossover, uniform mutation; GA'' – linear scaling, tournament selection, two-point crossover, uniform mutation; GA''' – ranked scaling, stochastic uniform selection, scattered crossover, Gaussian mutation. Following standard practice when experimenting with genetic algorithms, 10 trials are done in all cases. Three planning horizon sizes (6, 12 and 24 hours) are considered which set up three problems; P(t,6), P(t,12) and P(t,24). Table 2 gives the number of problem variables and constraints for the three planning horizon sizes.

Table 2 Problem Dimensions

Problem	Planning Horizon	# CV	# BV	# NLIC	# NLEC	# LIC
P(t,6)	6 hours	60	54	24	12	108
P(t,12)	12 hours	120	108	48	24	216
P(t,24)	24 hours	240	216	96	48	432

6.1 Experiment 1

Experiment 1 considers the coldest 24-hour period to occur in the month of January at the plant location. This period is of interest to analyze because the steam demand on the plant is the highest during the year. At such times due to energy market dynamics at the plant location, natural gas is available only in limited quantities. So the plant uses gas only in the gas turbine and duct burner and uses diesel oil in the boilers. When setting up the experiments, the boiler efficiency equations for oil are used. The average ambient temperature over the 24-hour period is around -12 °C which results in the nominal rating of the gas turbine, duct burner and HRSG to be in the range of 11,300 kW, 24,200 kW and 44,300 kW, respectively. Precise nominal ratings are used when setting up the experiment using hourly ambient temperatures. Table A1 in the appendix shows the hourly recorded ambient temperature and total electric and steam demands. The temperature and demands are used in setting up the experiment.

Problem P(t,6) involves the first 6 hours, P(t,12) involves the first 12 hours and P(t,24) involves all 24 hours. The results of the experiment are given in Table 3. LB is the lower bound on the objective function obtained by relaxing the integrality restrictions and solving the resulting NLP problem using *filterSQP*. BB(T) is the solution by BB for planning horizon size T and BB(1) is the solution for the same planning horizon by BB but solving each hourly problem independently by ignoring continuity of equipment operation. The rest of the table is the solutions obtained by GA.

For Problem P(t,6), BB(T) produces an integer feasible solution in about 1 minute as the problem size is not very large. But solutions found in Trials 3, 7 and 10 of GA''' are more optimal than that of BB(T). This result is indicative of the nonconvexity of the problem, wherein the global minimum is unknown. The way to compare the search capabilities of the two solution techniques is to see which one produces a solution with lower objective function value. But the average CPU time for GA' is 658 seconds, for GA'' is 511 seconds and for GA''' is 528 seconds which are 8 times greater than that of BB(T). And since BB(T) solution is obtained in this case, BB(1) solution is not needed. For Problem P(t,12), BB(T) produces an integer feasible solution in about 7 minutes and BB(1) solution is not needed. But solutions found in Trials 3 and 8 of GA''' are more optimal than that of BB(T). The average CPU time for GA' is 2,277 seconds, for GA'' is 2,314 seconds and for GA''' is 2,055 seconds which are only 4 times greater than that of BB(T). The relative increase in CPU time of GA is less than that of BB(T) when the planning horizon is doubled. For Problem P(t,24), BB(T) takes about 27 minutes but reports an integer infeasible solution. This means after solving the root NLP problem in the BB tree, the addition of new integrality constraints caused the resulting nodal NLP problems

to be infeasible. Since no integer feasible solution is found, it means no upper bound to the problem is found. The upper bound in BB is an important parameter to carry out the tree search process. In this case BB had to terminate without an integer feasible solution or an upper bound. Hence, in order to have an operational plan, the hourly problems ignoring continuity of equipment operation are solved. This solution is given by BB(1) and it takes only about 7 seconds to solve 24 hourly Problem P(t) by BB. On the other hand, GA finds several unique solutions for Problem P(t,24). The solution found in Trial 6 of GA''' is the most optimum. The average processing time for GA' is 6,428 seconds, for GA'' is 6,681 seconds and for GA''' is 6,112 seconds.

Fig. 6 compares the binary variable values of PR and those obtained by BB (BB(1) from P(t,24)) and GA (Trial 6 of GA''' from P(t,24)). In the figure, grey lines mean equipment is operated. The corresponding continuous variable values (equipment loads) are given in Table A2 in the appendix. In PR all plant equipment were operated which resulted in them being at part-load conditions and also there were 3 start/stops. BB also operates all equipment and is much worse since there are 22 start/stops. This is because BB(1) ignores continuity of equipment operation for subsequent hours in the horizon. GA operates only the least number of equipment at any given time to meet the energy demands. But because of the random nature of GA, it contains 4 start/stops. Table 8 summarizes the plant performance for the three schedules being compared. PR resulted in the present plant energy efficiency to be 81.75%. BB reduces the efficiency to 80.10%. GA increases the efficiency to 86.87% since only the least number of equipment are operated, allowing them to be near full-load conditions. GA leads to a 5% increase in efficiency while still satisfying all energy demands and equipment load range constraints.

Table 3 Experiment 1 Results

	P(t,6)		P(t,12)		P(t,24)	
	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)
LB	579,515	-	1,168,312	-	2,481,018	-
BB(T)	584,934	64	1,180,521	439	x	1,662
BB(1)	-	-	-	-	2,729,126	7.13
GA'						
Trial 1	597,939	840	1,238,517	2,805	3,003,625	4,664
Trial 2	611,612	470	1,237,863	2,245	2,686,522	7,929
Trial 3	590,672	809	1,216,871	4,003	2,682,547	7,974
Trial 4	605,509	442	1,217,115	3,172	2,742,112	8,011
Trial 5	597,225	747	1,242,565	3,541	3,073,350	1,269
Trial 6	595,806	1,090	1,218,262	1,904	2,722,911	7,917
Trial 7	584,934	767	1,266,392	1,324	2,678,386	6,882
Trial 8	598,237	463	1,257,119	1,470	2,677,900	8,028
Trial 9	598,425	331	1,321,408	839	2,720,815	5,045
Trial 10	611,213	619	1,264,786	1,471	2,746,795	6,562
GA''						
Trial 1	598,425	365	1,219,189	2,798	2,827,412	5,117
Trial 2	588,109	665	1,232,659	2,161	2,745,325	7,936
Trial 3	594,684	732	1,257,027	1,591	2,741,228	5,768
Trial 4	589,302	549	1,225,380	2,647	2,775,897	6,542
Trial 5	596,055	593	1,206,666	1,110	2,793,714	6,838
Trial 6	600,292	378	1,213,086	2,466	2,893,961	3,025
Trial 7	598,425	351	1,243,059	1,925	2,710,919	7,838
Trial 8	595,979	383	1,211,801	1,926	2,719,695	7,884
Trial 9	605,034	574	1,221,472	3,297	2,756,167	7,915
Trial 10	597,225	522	1,229,302	3,216	2,679,349	7,949
GA'''						
Trial 1	597,225	433	1,195,729	1,824	2,534,219	6,272
Trial 2	601,252	513	1,186,495	2,466	2,516,414	5,019
Trial 3	583,531	629	1,174,745	2,331	2,595,231	4,281
Trial 4	589,302	549	1,186,434	2,028	2,621,353	6,579
Trial 5	590,672	317	1,201,103	1,993	2,524,330	6,696
Trial 6	590,672	411	1,242,682	2,003	2,512,828	6,530
Trial 7	582,146	772	1,186,434	1,772	2,541,763	6,550
Trial 8	589,507	654	1,174,787	2,683	2,532,972	6,829
Trial 9	591,630	452	1,199,143	1,716	2,527,719	6,468
Trial 10	580,775	547	1,192,808	1,735	2,525,340	5,897

Fig. 6. Experiment 1 Binary Variables

	P(t,6)		P(t,12)		P(t,24)	
	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)
LB	414,073	-	883,713	-	1,813,375	-
BB(T)	417,001	52	894,646	253	x	1,316
BB(1)	-	-	-	-	2,887,093	1.66
GA'						
Best	417,001	602 (avg)	959,261	2,060 (avg)	2,031,076	7,010 (avg)
Average	434,212		991,933		2,105,993	
Worst	449,154		1,024,787		2,163,288	
# Best	1		1		1	
# ≤ BB	0		0		10	
GA''						
Best	415,316	508 (avg)	894,697	1,688 (avg)	2,036,104	6,673 (avg)
Average	424,754		927,095		2,132,776	
Worst	438,026		956,811		2,360,098	
# Best	1		1		1	
# ≤ BB	1		0		10	
GA'''						
Best	413,602	503 (avg)	896,769	1,576 (avg)	1,881,208	4,434 (avg)
Average	424,257		919,832		1,934,792	
Worst	439,527		942,418		2,008,270	
# Best	1		1		1	
# ≤ BB	1		0		10	
# Best: No. of times best soln. found, # ≤ BB: No. of soln. better than or same as BB						

6.3 Experiment 3

Experiment 3 considers the hottest 24-hour period to occur in the month of July. This period is of interest to analyze because the steam demand on the plant is the lowest during the year. The plant is run completely on gas. When setting up the experiments the boiler efficiency equations for gas are used. The average ambient temperature over the 24-hour period is around 29 °C which results in the nominal rating of the gas turbine, duct burner and HRSG to be in the range of 9,000 kW, 18,900 kW and 38,000 kW, respectively. Table A1 in the appendix shows the hourly recorded ambient temperature and total electric and steam demands.

The summary of the experiment results are given in Table 5. For all three planning horizon sizes BB(T) produces integer infeasible solutions. Hence BB(1) problems are solved. The total CPU time to solve 24 hourly Problem P(t) by BB is less than 2 seconds. GA proves to be more consistent in finding integer feasible solutions. For Problem P(t,6) GA' finds the most optimum solution once, GA'' once and GA''' twice. For Problem P(t,12) GA'' and GA''' find the most optimum solution once. For Problem P(t,24) GA''' finds the most optimum solution once. For Problem P(t,6) the average CPU time for GA' is 814 seconds, for GA'' is 412 seconds and GA''' is 433 seconds. For Problem P(t,12) the average CPU time for GA' is 2,393 seconds, for GA'' is 2,186 seconds and GA''' is 1,979 seconds. For Problem P(t,24) the average CPU time for GA' is 6,687 seconds, for GA'' is 6,148 seconds and GA''' is 6,763 seconds.

The continuous variable values corresponding to PR, BB(1) and the best GA''' are given in Table A2 in the appendix. In PR steam was produced by the HRSG and one additional boiler. But the duct burner of the HRSG was switched off after 5 hours of operation and the boiler continued operation. GA identifies that it is better to switch off the boiler and continue operation of the duct burner to increase output from the HRSG. This result is an outcome of the differences in thermodynamic characteristics of the energy conversion equipment when all are using gas as the fuel. It can be verified by examining the gas turbine and HRSG characteristics in Fig. 2 and Table 1 and the boiler characteristics in Fig. 3. When the gas turbine and HRSG are at higher loads, they jointly have better energy conversion efficiencies than any of the boilers. Table 8 summarizes the plant performance for the three schedules. In PR plant energy efficiency was 79.99% and 1 start/stop occurred. BB reduces the efficiency to 62.50% and has 65 start/stops. GA increases the efficiency by 9% to 89.32% and has 1 start/stop.

Table 5 Experiment 3 Results Summary

	P(t,6)		P(t,12)		P(t,24)	
	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)
LB	244,884	-	474,559	-	969,465	-
BB(T)	x	23	x	50	x	113
BB(1)	721,516	0.33	1,326,626	0.64	2,647,255	1.67
GA'						
Best	257,814	814 (avg)	570,801	2,393 (avg)	1,314,933	6,687 (avg)
Average	272,171		617,614		1,405,420	
Worst	293,448		716,555		1,495,818	
# Best	1		1		1	
# ≤ BB	10		10		10	
GA''						
Best	257,814	412 (avg)	505,053	2,186 (avg)	1,325,673	6,148 (avg)
Average	288,722		524,970		1,482,592	
Worst	318,632		549,803		1,791,059	
# Best	1		1		1	
# ≤ BB	10		10		10	
GA'''						
Best	257,814	433 (avg)	505,053	1,979 (avg)	1,040,469	6,763 (avg)
Average	277,562		524,576		1,062,968	
Worst	296,264		587,406		1,098,332	
# Best	2		1		1	
# ≤ BB	10		10		10	
# Best: No. of times best soln. found, # ≤ BB: No. of soln. better than or same as BB						

6.4 Experiment 4

Experiment 4 considers a 24-hour period occurring in the month of October which is during the Fall season. This period is of interest to analyze because the steam demand begins to rise as summer recedes and winter approaches. Two boilers (B1,B2) are run on gas whereas the third boiler (B3) if needed is run on oil. When setting up the experiments, the appropriate efficiency equation for each boiler is used. The average ambient temperature over the 24-hour period is around 11 °C which results in the nominal rating of the gas turbine, duct burner and HRSG to be in the range of 10,250 kW, 21,400 kW and 41,200 kW, respectively. Table A1 in the appendix shows the hourly recorded ambient temperature and total electric and steam demands.

The summary of the experiment results are given in Table 6. For Problem P(t,6), BB(T) produces an integer feasible solution in less than 2 seconds and BB(1) solution is not needed. GA'' finds the same solution as BB(T) once and GA''' finds it twice. For Problem P(t,12), BB(T) produces an integer feasible solution in less than 15 seconds and BB(1) solution is not needed. None of the GA trials find better solutions than that of BB(T). For Problem P(t,24), BB(T) takes about 23 minutes but reports an integer infeasible solution. Hence, BB(1) problems are solved and it takes less than 4 seconds to obtain the solution to 24 hourly Problem P(t) by BB. GA finds several unique solutions to Problem P(t,24) of which the best one found by GA''' is the most optimum. For Problem P(t,6) the average CPU time for GA' is 730 seconds, for GA'' is 483 seconds and GA''' is 435 seconds. For Problem P(t,12) the average CPU time for GA' is 2,872 seconds, for GA'' is 1,533 seconds and GA''' is 1,886 seconds. For Problem P(t,24) the average CPU time for GA' is 6,032 seconds, for GA'' is 6,848 seconds and GA''' is 5,156 seconds.

The continuous variable values corresponding to PR, BB(1) and the best GA''' are given in Table A2 in the appendix.. In PR steam was produced by the HRSG and mostly one additional boiler running on gas and 1 start/stop occurred in the second boiler. GA identifies that it is better to increase the output from the HRSG and reduce the usage of the boilers. Table 8 summarizes the plant performance for the three schedules. PR resulted in the present plant energy efficiency to be 78.95%. BB reduces the efficiency to 66.13% and has 50 start/stops. GA has 2 start/stops but increases the efficiency by 11% to 90.23%.

Table 6 Experiment 4 Results Summary

	P(t,6)		P(t,12)		P(t,24)	
	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)	Obj. Func. (kWh)	CPU Time (s)
LB	315,649	-	626,002	-	1,257,455	-
BB(T)	331,077	1.81	648,781	15	x	1,402
BB(1)	-	-	-	-	2,466,603	3.65
GA'						
Best	331,236	730 (avg)	718,468	2,872 (avg)	1,585,219	6,032 (avg)
Average	348,837		767,848		1,755,013	
Worst	384,993		838,918		1,934,697	
# Best	1		1		1	
# ≤ BB	0		0		10	
GA''						
Best	331,077	483 (avg)	649,354	1,533 (avg)	1,578,770	6,848 (avg)
Average	340,935		721,631		1,717,038	
Worst	362,784		784,634		1,868,481	
# Best	1		1		1	
# ≤ BB	1		0		10	
GA'''						
Best	331,077	435 (avg)	648,939	1,886 (avg)	1,325,302	5,156 (avg)
Average	354,502		680,818		1,422,951	
Worst	379,756		773,391		1,520,401	
# Best	2		1		1	
# ≤ BB	1		0		10	
# Best: No. of times best soln. found, # ≤ BB: No. of soln. better than or same as BB						

6.5 Summary

The numerical experiments show the proposed genetic algorithm method finds more optimal solutions than branch-and-bound. In many cases branch-and-bound does not find integer feasible solutions and is also seen to have an exponential increase in CPU time for a linear increase in planning horizon size. Among the genetic procedures experimented with, GA''' (ranked scaling, stochastic uniform selection, scattered crossover, Gaussian mutation) finds the most optimum solutions and has the lowest average CPU time.

Since the objective function values of the three planning horizon sizes differ in order of magnitude, the optimality gap is a more appropriate metric to compare the performance of the genetic algorithm across all experiments. Table 7 summarizes the experiment results in terms of the optimality gap and average CPU time. The optimality gap is a relative metric defined as the difference between the objective function value and the lower bound and is normalized with the objective function value. The gaps for the best, average and worst objective function values along with the average CPU time for each set of trials are calculated. The gaps for branch-and-bound solutions are also calculated for comparison. The genetic algorithm is able to achieve low optimality gaps in reasonable CPU time. In one case the best gap achieved is negative because the objective function value is lower than the lower bound which happens because of the nonconvexity of the problem. The algorithm achieves low gaps regardless of the problem size.

Table 7 Algorithm Benchmarks

		P(t,6)		P(t,12)		P(t,24)	
		Opt. Gap (%)	Avg. CPU Time (s)	Opt. Gap (%)	Avg. CPU Time (s)	Opt. Gap (%)	Avg. CPU Time (s)
Experiment 1							
BB(T)/BB(1)		0.93	-	1.03	-	9.09	-
GA'	Best	0.93	658	3.99	2,277	7.35	6,428
	Average	3.28		6.39		10.55	
	Worst	5.25		11.59		19.27	
GA''	Best	1.46	511	3.18	2,314	7.40	6,681
	Average	2.82		4.70		10.25	
	Worst	4.22		7.06		14.27	
GA'''	Best	0.22	528	0.55	2,055	1.27	6,112
	Average	1.72		2.15		2.45	
	Worst	3.62		5.98		5.35	
Experiment 2							
BB(T)/BB(1)		0.70	-	1.22	-	37.19	-
GA'	Best	0.70	602	7.88	2,060	10.72	7,010
	Average	4.64		10.91		13.89	
	Worst	7.81		13.77		16.18	
GA''	Best	0.30	508	1.23	1,688	10.94	6,673
	Average	2.51		4.68		14.98	
	Worst	5.47		7.64		23.17	
GA'''	Best	-0.11	503	1.46	1,576	3.61	4,434
	Average	2.40		3.93		6.28	
	Worst	5.79		6.23		9.70	
Experiment 3							
BB(T)/BB(1)		66.06	-	64.23	-	63.38	-
GA'	Best	5.02	814	16.86	2,393	26.27	6,687
	Average	10.03		23.16		31.02	
	Worst	16.55		33.77		35.19	
GA''	Best	5.02	412	6.04	2,186	26.87	6,148
	Average	15.18		9.60		34.61	
	Worst	23.15		13.69		45.87	
GA'''	Best	5.02	433	6.04	1,979	6.82	6,763
	Average	11.77		9.53		8.80	
	Worst	17.34		19.21		11.73	
Experiment 4							
BB(T)/BB(1)		4.66	-	3.51	-	49.02	-
GA'	Best	4.71	730	12.87	2,872	20.68	6,032
	Average	9.51		18.47		28.35	
	Worst	18.01		25.38		35.01	
GA''	Best	4.66	483	3.60	1,533	20.35	6,848
	Average	7.42		13.25		26.77	
	Worst	12.99		20.22		32.70	
GA'''	Best	4.66	435	3.53	1,886	5.12	5,156
	Average	10.96		8.05		11.63	
	Worst	16.88		19.06		17.29	

To make a final comparison across the three problem sizes, the best gaps of GA''' from the four experiments are averaged. The average best gaps are 2.44, 2.89 and 4.20% for the three problem sizes. Similarly, the average CPU time of GA''' from the four experiments are also averaged. These are calculated to be 475, 1,874 and 5,616 seconds. Fig. 7 plots these averaged best gaps and twice averaged CPU times for GA'''. The figure indicates that on average the best optimality gap achieved by the proposed genetic algorithm method does not depend to a great extent on the problem size whereas the CPU time increases linearly with the problem size.

Altogether, 360 trials were performed for the genetic algorithm. The trials involved variations in number of problem variables and constraints, variations in coefficients of objective function terms, variations in constraint vector values and variations in genetic procedures of scaling, selection, crossover and mutation. The numerous trials and wide variations in experimental conditions are statistically significant to draw inferences about the consistency and robustness of the proposed genetic algorithm. It can be inferred that the proposed genetic algorithm has strong search capabilities for mixed 0-1 nonlinear programming problems. It evaluates several more integer feasible solutions in the search

process as compared to branch-and-bound which in many cases did not come across a single integer feasible solution. For large-scale nonconvex problems where global optimality is unknown and near optimal solutions are sufficient to make operational planning decisions, the proposed genetic algorithm method proves to be more versatile than branch-and-bound.

Table 8 gives the plant thermal performance for the 24-hour planning horizons. The proposed scheduling strategy increased plant energy efficiency by 5, 10, 9 and 11% in the four experimental studies. Although the genetic algorithm can cause random start/stops of equipment, it should be noted that operational planning strategies in medium to large-scale CHP plants are most likely to involve a human-in-the-loop strategy rather than a closed loop feedback system. The decision maker/plant operator can use the genetic algorithm solution to aid in the decision making process rather than implementing it as is.

Fig. 7. Genetic Algorithm Performance

Table 8 Plant Thermal Performance

	Fuel Input (kWh)	Energy Output (kWh)	# Start/Stop	Efficiency (%)
Experiment 1				
PR	2,669,958	2,182,678	3	81.75
BB	2,729,126	2,186,039	22	80.10
GA	2,512,828	2,182,858	4	86.87
Experiment 2				
PR	2,111,167	1,663,116	1	78.78
BB	2,887,093	1,973,349	47	68.35
GA	1,881,208	1,663,116	4	88.41
Experiment 3				
PR	1,133,596	906,721	1	79.99
BB	2,647,255	1,654,448	65	62.50
GA	1,040,469	929,341	1	89.32
Experiment 4				
PR	1,512,457	1,194,071	1	78.95
BB	2,466,603	1,631,129	50	66.13
GA	1,325,302	1,195,781	2	90.23

7. Conclusion

This paper presented a mixed 0-1 nonlinear programming formulation for operational planning in CHP plants. The formulation took into account nonlinear variations in equipment performance characteristics with changes in ambient temperature and operating load. The resulting problem was nonconvex and global optimality conditions were unknown. A genetic algorithm method was proposed and numerical experiments showed it was more reliable than branch-and-bound in finding integer feasible solutions and found more optimal solutions within reasonable CPU time. For the plant under consideration, the proposed scheduling strategy increased energy efficiency by 5-11%.

Most operational planning strategies for CHP plants tend to use the MILP modeling paradigm. But the fundamental thermodynamic characteristics of energy conversion equipment are nonlinear. This makes the planning problems inherently MINLP problems. In order for planning models to agree with physical and thermodynamic realities, the nonlinear features of the model need to be kept intact during the computational solution phase. This paper showed that it is not necessary to apply linearization techniques to planning models in order to obtain practical solutions within reasonable time.

The planning model presented here was seeking to minimize total plant energy usage based on equipment efficiency characteristics rather than minimizing total cost based on purchase prices of fuels and selling prices of heat and power. This was done since if the problem is modeled as a cost optimization problem, then the relative differences in the cost coefficients of the objective function will influence the optimum solutions and the significance of using nonlinear efficiency characteristics in the model will get overshadowed.

Formulations of operational planning problems in plants like the one in this paper have their grounding in the first law of thermodynamics. They involve energy or cost objective functions and energy balance, energy demand and equipment load range constraints. The model in this paper involved a nonlinear objective function and nonlinear inequality, nonlinear equality and linear inequality constraints, thereby making it a general purpose approach. Hence, the methodology can easily be applied in plants with equal or lesser complicated energy flow networks.

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References

- [1] Sakawa M, Kato K, Ushiro S. Operational planning of district heating and cooling plants through genetic algorithms for mixed 0-1 linear programming. *European Journal of Operational Research* 2002;137(3):677-687.
- [2] Sakawa M, Kato K, Ushiro S, Inaoka M. Operational planning of district heating and cooling plants using genetic algorithms for mixed integer programming. *Applied Soft Computing* 2001;1(2):139-150.
- [3] Sakawa M, Kato K, Ushiro S. Operation planning of district heating and cooling plants through genetic algorithms for nonlinear 0-1 programming. *Computers and Mathematics with Applications* 2001;42(10-11):1365-1378.
- [4] Saravanan B, Das S, Sikri S, Kothari D. A solution to the unit commitment problem-a review. *Frontiers in Energy* 2013;7(2):223-236.
- [5] Xia X, Elaiw A. Optimal dynamic economic dispatch of generation: A review. *Electric Power Systems Research* 2010;80(8):975-986.
- [6] Trespalacios F, Grossmann I. Review of mixed-integer nonlinear and generalized disjunctive programming methods. *Chemie Ingenieur Technik* 2014;86(7):991-1012.
- [7] Floudas C, Gounaris C. A review of recent advances in global optimization. *Journal of Global Optimization* 2009;45(1):3-38.
- [8] Cavalieri F, Roversi A, Ruggeri R. Use of mixed integer programming to investigate optimum running conditions for a thermal power station and possible extension to capacity. *Operational Research Quarterly* 1971;22(3):221-236.
- [9] Logendran R, Oudheusden D. Optimal operation for a bi-fuel thermal power plant by mathematical programming. *European Journal of Operational Research* 1981;8(3):226-234.
- [10] Chisman J. Optimal electric power load-dispatching. *Computers and Operations Research* 1979;6(1):53-58.
- [11] Gardner D, Rogers J. Joint planning of combined heat and power and electric power systems: An efficient model formulation. *European Journal of Operational Research* 1997;102(1):58-72.
- [12] Makkonen S, Lahdelma R. Non-convex power plant modeling in energy optimization. *European Journal of Operational Research* 2006;171(3):1113-1126.
- [13] Rong A, Lahdelma R. An efficient envelope-based branch-and-bound algorithm for non-convex combined heat and power production planning. *European Journal of Operational Research* 2007;183(1):412-431.
- [14] Rong A, Hakonen H, Lahdelma R. A dynamic regrouping based sequential dynamic programming algorithm for unit commitment of combined heat and power systems. *Energy Conversion and Management* 2009;50(4):1108-1115.
- [15] Viswanathan J, Grossmann I. A combined penalty function and outer-approximation method for MINLP optimization. *Computers and Chemical Engineering* 1990;14(7):769-782.
- [16] Quesada I, Grossmann I. An LP/NLP based branch-and-bound algorithm for convex MINLP optimization problems. *Computers and Chemical Engineering* 1992;16(10-11):937-947.
- [17] Iyer R, Grossmann I. Optimal multiperiod operational planning for utility systems. *Computers and Chemical Engineering* 1997;21(8):787-800.
- [18] Mitra S, Sun L, Grossmann I. Optimal scheduling of industrial combined heat and power plants under time-sensitive electricity prices. *Energy* 2013;54:194-211.

- [19] Adjiman C, Androulakis I, Floudas C. Global optimization of mixed-integer nonlinear problems. *American Institute of Chemical Engineers Journal* 2000;46(9):1769-1797.
- [20] Leyffer S. Integrating SQP and branch-and-bound for mixed integer nonlinear programming. *Computational Optimization and Applications* 2001;18(3):295-309.
- [21] Pruitt K, Leyffer S, Newman A, Braun R. A mixed-integer nonlinear program for the optimal design and dispatch of distributed generation systems. *Optimization and Engineering* 2014;15(1):167-197.
- [22] Pruitt K, Braun R, Newman A. Evaluating shortfalls in mixed-integer programming approaches for the optimal design and dispatch of distributed generation systems. *Applied Energy* 2013;102:386-398.
- [23] Deep K, Singh K, Kansal M, Mohan C. A real coded genetic algorithm for solving integer and mixed integer optimization problems. *Applied Mathematics and Computation* 2009;212(2):505-518.
- [24] Savola T, Tveit T, Fogelholm C. A MINLP model including the pressure levels and multiperiods for CHP process optimisation. *Applied Thermal Engineering* 2007;27(11-12):1857-1867.
- [25] Tveit T, Savola T, Gebremedhin A, Fogelholm C. Multi-period MINLP model for optimising operation and structural changes to CHP plants in district heating networks with long-term thermal storage. *Energy Conversion and Management* 2009;50(3):639-647.
- [26] Kjeldsen N, Chiarandini M. Heuristic solutions to the long-term unit commitment problem with cogeneration plants. *Computers and Operations Research* 2012;39(2):269-282.
- [27] Bosman M, Bakker V, Molderink, Hurink J, Smit G. Planning the production of a fleet of domestic combined heat and power generators. *European Journal of Operational Research* 2012;216(1):140-151.
- [28] Dotzauer E. Simple model for prediction of loads in district-heating systems. *Applied Energy* 2002;73(3-4):277-284.
- [29] Pedersen L, Stang J, Ulseth R. Load prediction method for heat and electricity demand in buildings for the purpose of planning for mixed energy distribution systems. *Energy and Buildings* 2008;40(7):1124-1134.
- [30] Holmstrom K, Goran A, Edvall M. User's Guide for TOMLAB/MINLP. TOMLAB Optimization.
- [31] Mathworks Inc. Matlab Global Optimization Toolbox User's Guide.

Table A1 Problem Data

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Experiment 1																								
A (K)	264.25	264.35	264.46	265.02	263.99	263.32	262.80	262.24	261.78	261.28	260.72	260.50	260.20	259.97	259.41	259.22	258.54	258.37	258.22	258.23	258.51	259.56	261.30	263.54
E _{demt} (kW)	8,893	8,850	8,996	9,007	9,097	9,261	8,400	8,311	8,250	8,161	8,103	7,946	7,926	7,899	8,599	8,599	8,675	8,742	9,198	9,966	9,995	10,031	9,044	10,227
Q _{S1 demt} (kW)	39,167	38,698	38,366	37,925	38,340	38,726	39,415	39,038	39,317	39,593	39,531	39,577	39,945	40,092	39,630	40,687	41,952	43,250	44,933	47,142	47,156	45,257	44,825	42,688
Q _{S2 demt} (kW)	39,167	38,698	38,366	37,925	38,340	38,726	39,415	39,038	39,317	39,593	39,531	39,577	39,945	40,092	39,630	40,687	41,952	43,250	44,933	47,142	47,156	45,257	44,825	42,688
Experiment 2																								
A (K)	276.39	277.97	279.00	280.07	277.11	274.77	273.18	272.07	271.26	270.57	269.78	269.31	269.15	268.77	268.82	268.84	269.05	269.23	269.54	269.52	270.34	272.40	274.31	275.67
E _{demt} (kW)	10,178	10,131	10,106	10,023	10,124	10,176	13,574	14,651	14,881	14,962	14,738	14,016	13,517	13,189	12,843	12,546	12,552	12,858	13,451	14,250	14,796	14,614	14,432	14,290
Q _{S1 demt} (kW)	27,359	26,567	25,381	25,491	25,914	27,831	27,213	27,932	28,678	29,275	29,559	29,497	28,725	28,014	27,994	27,176	26,804	28,467	29,265	31,467	31,511	30,329	28,721	26,941
Q _{S2 demt} (kW)	27,359	26,567	25,381	25,491	25,914	27,831	27,213	27,932	28,678	29,275	29,559	29,497	28,725	28,014	27,994	27,176	26,804	28,467	29,265	31,467	31,511	30,329	28,721	26,941
Experiment 3																								
A (K)	306.71	308.46	308.81	309.15	308.57	308.29	307.17	305.56	303.75	301.19	299.23	298.21	297.08	296.23	295.29	294.77	294.15	295.26	297.23	299.20	301.01	302.60	304.66	305.98
E _{demt} (kW)	10,449	10,089	9,949	10,372	10,292	10,214	10,161	10,287	10,268	10,478	10,507	10,590	10,614	10,636	10,462	10,813	10,581	11,087	11,265	11,287	11,278	11,267	11,073	11,046
Q _{S1 demt} (kW)	15,670	14,339	17,115	15,113	13,268	12,889	12,489	12,318	12,079	12,271	12,291	12,187	12,231	12,745	12,761	12,376	12,380	13,068	13,993	14,861	14,800	14,913	14,597	15,076
Q _{S2 demt} (kW)	15,670	14,339	17,115	15,113	13,268	12,889	12,489	12,318	12,079	12,271	12,291	12,187	12,231	12,745	12,761	12,376	12,380	13,068	13,993	14,861	14,800	14,913	14,597	15,076
Experiment 4																								
A (K)	284.74	286.60	287.78	288.39	287.72	286.21	284.49	283.58	282.97	282.38	281.88	281.99	281.78	281.33	280.99	281.13	281.30	281.38	281.43	281.84	283.27	285.18	287.58	289.88
E _{demt} (kW)	13,266	12,922	12,611	12,465	12,432	12,635	12,846	12,933	12,949	13,062	13,117	13,002	12,370	12,002	11,728	11,494	11,641	12,155	12,522	12,459	12,759	12,829	12,600	12,146
Q _{S1 demt} (kW)	24,652	18,390	17,069	16,747	16,659	16,920	17,610	17,497	17,719	18,826	19,016	18,631	18,321	17,798	17,687	17,592	17,772	18,579	21,003	22,476	21,468	19,193	18,395	16,542
Q _{S2 demt} (kW)	24,652	18,390	17,069	16,747	16,659	16,920	17,610	17,497	17,719	18,826	19,016	18,631	18,321	17,798	17,687	17,592	17,772	18,579	21,003	22,476	21,468	19,193	18,395	16,542

Table A2 Continuous Variables

Hour		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Experiment 1																									
E _{GT} (kW)	PR	7,509	7,435	7,487	7,709	7,813	7,954	7,899	7,809	7,749	7,661	7,604	7,446	7,428	7,401	7,274	7,254	7,363	7,341	7,660	8,109	8,218	8,255	7,089	8,290
	BB	6,059	5,737	7,286	5,582	5,601	5,879	6,045	5,762	5,641	6,180	6,136	5,980	6,216	6,303	6,610	6,944	7,866	9,167	11,334	8,800	8,826	7,453	6,820	9,086
	GA	8,493	8,450	8,596	9,007	9,097	9,261	8,400	8,311	8,250	8,161	8,103	7,946	7,926	7,899	8,599	8,599	8,675	8,921	9,198	9,966	9,995	10,031	9,044	10,227
F _{DB} (kW)	PR	9,222	10,018	10,152	9,996	9,963	9,950	9,876	9,873	9,987	9,684	9,832	9,875	10,199	10,541	10,742	11,147	11,762	12,246	12,555	11,183	11,081	11,374	8,290	4,420
	BB	20,750	20,546	17,813	19,728	20,452	20,731	20,883	20,765	13,536	20,870	20,849	21,065	21,237	21,313	20,728	21,773	22,399	23,203	24,531	22,994	22,982	22,049	21,497	22,654
	GA	22,226	22,191	22,268	22,463	22,617	22,781	22,310	22,310	22,318	22,311	22,331	22,257	22,274	22,280	22,758	22,777	22,889	23,054	23,236	23,701	23,691	23,612	22,845	23,346
Q _{msc} (kW)	PR	22,409	23,254	23,453	23,475	23,441	23,479	23,322	23,218	23,272	22,832	22,917	22,831	23,159	23,509	23,596	24,027	24,771	25,305	25,912	24,641	24,620	25,060	20,870	17,803
	BB	35,449	34,792	32,723	33,555	34,457	35,115	35,466	34,905	25,473	35,447	35,309	35,400	35,882	36,062	35,531	37,311	38,995	41,181	44,548	40,587	40,618	38,229	37,012	40,799
	GA	39,810	39,736	39,968	40,597	40,793	41,087	39,724	39,601	39,519	39,390	39,317	39,057	39,035	38,997	40,204	40,212	40,368	40,781	41,238	42,457	42,487	42,488	40,844	42,562
Q _{S1} (kW)	PR	15,477	15,646	15,542	15,564	15,636	16,024	16,101	16,157	16,206	16,571	16,681	16,712	16,749	16,927	17,017	17,124	17,902	18,800	19,574	18,187	18,105	19,114	27,832	21,052
	BB	0	0	0	0	0	0	0	0	10,049	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	GA	38,924	38,061	37,165	35,252	35,887	36,364	39,107	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{S2} (kW)	PR	22,645	21,302	21,042	20,417	20,803	21,048	21,337	20,968	21,278	21,536	21,398	21,460	21,608	21,532	21,342	22,234	22,769	23,472	24,296	28,105	28,322	26,822	24,283	26,620
	BB	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,144	9,144	9,144	9,144	9,144	33,670
Q _{S3} (kW)	PR	19,186	18,610	18,204	17,691	18,084	18,208	18,570	18,235	18,378	18,747	18,564	18,650	18,872	18,715	18,631	19,335	19,775	20,325	21,622	25,208	25,040	21,293	18,621	21,838
	BB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,144	9,144	9,144	9,144	9,144	0
	GA	0	0	0	0	0	0	0	38,474	39,114	39,795	39,744	40,096	40,855	41,188	39,056	41,162	43,536	45,719	39,485	42,683	42,681	38,882	39,663	9,144
E _{ST1} (kW)	PR	401	433	451	452	451	492	501	501	500	500	500	499	498	500	501	473	502	400	400	400	0	0	0	0
	BB	1,981	1,916	1,710	617	1,783	1,948	1,554	858	0	1,981	1,967	400	833	796	1,989	1,655	809	400	400	1,166	1,169	0	1,424	1,142
	GA	400	400	400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{DS1} (kW)	PR	18,330	19,021	18,938	18,976	19,016	19,038	18,868	18,810	18,925	18,853	19,052	18,989	19,371	19,905	20,063	20,596	22,393	23,538	25,942	23,284	42,725	44,174	48,702	38,856
	BB	0	0	0	11,833	1,000	0	4,309	10,750	35,522	0	0	15,856	11,978	12,537	0	5,141	15,336	21,637	25,004	13,336	13,339	38,229	7,167	13,794
	GA	59,190	58,253	57,588	75,850	76,681	77,452	78,831	39,601	39,519	39,390	39,317	39,057	39,035	38,997	40,204	40,212	40,368	40,781	41,238	42,457	42,487	42,488	40,844	42,562
E _{ST2} (kW)	PR	983	982	1,058	845	832	816	0	0	0	0	0	0	0	0	825	845	840	900	1,138	1,456	1,776	1,776	1,955	1,937
	BB	853	1,198	0	2,808	1,712	1,434	800	1,690	2,608	0	0	1,566	877	800	0	0	0	0	0	0	0	2,578	800	0
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{DS2} (kW)	PR	22,964	22,499	21,306	23,277	23,841	24,416	39,415	39,038	39,317	39,593	39,531	39,577	39,945	40,092	25,216	26,048	27,372	27,984	26,968	25,560	21,943	20,049	17,581	15,650
	BB	24,436	20,047	38,366	1,000	13,853	17,397	25,284	14,801	4,661	39,593	39,531	16,749	24,938	25,961	39,630	40,687	41,952	43,250	44,933	47,142	47,156	10,950	30,694	42,688
	GA	39,167	38,698	38,366	37,925	38,340	38,726	39,415	39,038	39,317	39,593	39,531	39,577	39,945	40,092	39,630	40,687	41,952	43,250	44,933	47,142	47,156	45,257	44,825	42,688
Experiment 2																									

E _{GT} (kW)	PR	6,036	5,993	6,028	5,933	6,016	6,079	9,370	10,423	10,493	10,467	10,251	9,521	9,019	8,701	8,350	8,218	8,285	8,606	9,078	9,860	10,411	10,306	10,167	10,061
	BB	7,970	7,859	5,547	5,478	7,804	8,009	10,534	9,846	10,118	10,118	10,058	9,390	10,599	8,526	10,614	10,261	7,860	10,665	10,577	9,751	10,256	9,990	9,693	9,423
	GA	8,978	8,931	8,906	8,823	8,924	8,976	10,815	10,866	10,903	10,934	10,969	10,989	10,996	10,766	10,422	10,196	11,000	10,992	10,979	10,980	10,944	10,851	10,761	10,695
F _{DB} (kW)	PR	0	0	0	0	0	1,931	8,386	7,384	8,399	9,341	10,469	10,920	11,343	11,306	11,531	10,991	10,852	11,820	14,255	13,951	10,776	9,656	10,126	11,574
	BB	0	0	1,901	1,887	0	0	4,645	2,229	2,253	2,253	2,264	2,228	4,769	2,181	0	0	2,138	0	4,762	2,248	2,271	2,235	2,198	2,169
	GA	21,347	21,165	21,051	20,897	21,244	21,502	22,770	22,909	23,009	23,095	23,192	23,250	23,270	23,167	22,954	22,815	23,282	23,260	23,222	23,224	23,123	22,868	22,629	22,457
Q _{UBSG} (kW)	PR	9,965	10,044	10,165	10,147	10,002	11,521	20,758	20,321	21,387	22,342	23,382	23,287	23,330	22,961	22,893	22,161	22,092	23,516	26,873	27,165	23,905	22,764	23,389	25,137
	BB	13,958	13,955	14,154	14,126	13,895	13,921	19,544	16,783	16,888	16,888	16,821	16,480	19,544	15,993	14,855	14,723	15,596	14,886	19,544	16,667	16,927	16,858	16,769	16,675
	GA	39,895	39,722	39,617	39,423	39,767	39,993	42,774	42,928	43,039	43,133	43,239	43,301	43,322	43,020	42,518	42,187	43,336	43,312	43,271	43,273	43,164	42,882	42,615	42,420
Q _{B1} (kW)	PR	22,289	21,706	20,708	20,545	20,720	21,608	13,624	14,004	14,349	15,168	15,316	15,870	15,692	15,525	15,733	15,444	15,575	16,164	13,851	16,155	19,702	19,452	17,087	11,580
	BB	0	0	10,049	10,049	0	0	0	10,049	10,049	10,049	10,049	10,049	0	10,049	0	0	10,049	0	0	10,049	10,049	10,049	10,049	10,049
	GA	16,023	14,612	12,344	12,759	13,261	16,869	14,412	16,721	18,295	19,446	19,648	18,720	16,649	15,431	15,892	0	0	0	0	0	0	0	0	0
Q _{B2} (kW)	PR	26,606	25,521	23,965	24,380	25,213	26,629	24,248	25,767	26,008	25,535	24,906	24,331	22,925	22,030	21,856	21,074	20,209	21,506	22,179	24,005	23,801	22,750	21,231	21,395
	BB	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719	45,719
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{B3} (kW)	PR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	BB	0	0	9,144	9,144	0	0	0	9,144	9,144	9,144	9,144	9,144	9,144	0	9,144	0	0	9,144	0	9,144	9,144	9,144	9,144	9,144
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	14,514	11,824	15,488	17,731	22,931	23,711	21,539	18,498	15,058
E _{ST1} (kW)	PR	1,423	1,406	1,400	1,399	1,399	1,403	1,430	1,426	1,425	1,423	1,426	1,426	1,424	1,426	1,425	1,426	1,425	1,425	1,424	1,426	1,430	1,431	1,431	1,413
	BB	0	0	764	761	0	0	400	1,025	1,035	1,035	1,029	995	400	946	0	0	907	0	400	1,013	1,039	1,033	1,024	1,014
	GA	400	400	400	400	400	400	1,959	1,986	1,998	1,998	1,970	874	435	400	400	400	753	1,066	1,672	2,000	2,000	2,000	2,000	2,000
Q _{DS1} (kW)	PR	2,419	2,085	1,270	1,100	1,131	3,492	4,474	4,460	5,882	7,671	8,830	9,288	9,173	8,618	8,775	7,741	7,809	9,824	10,875	13,458	13,699	12,304	10,562	6,985
	BB	13,958	13,955	1,000	1,000	13,895	13,921	0	1,000	1,000	1,000	1,000	1,000	0	1,000	14,855	14,723	1,000	14,886	0	1,000	1,000	1,000	1,000	1,000
	GA	36,373	34,790	32,417	32,637	33,484	37,317	21,959	24,152	25,710	26,957	27,549	37,710	40,074	38,907	38,866	22,643	20,245	17,071	10,933	7,633	7,524	7,242	6,975	6,780
E _{ST2} (kW)	PR	2,719	2,732	2,677	2,691	2,709	2,693	2,773	2,802	2,963	3,072	3,060	3,069	3,073	3,062	3,069	2,901	2,842	2,827	2,949	2,964	2,955	2,877	2,835	2,817
	BB	2,208	2,272	3,795	3,784	2,320	2,167	2,640	3,780	3,728	3,728	3,651	3,632	2,517	3,716	2,229	2,285	3,785	2,193	2,473	3,486	3,501	3,592	3,715	3,853
	GA	800	800	800	800	800	800	800	1,799	1,979	2,030	1,799	2,154	2,086	2,023	2,021	1,949	800	800	800	1,270	1,852	1,763	1,671	1,596
Q _{DS2} (kW)	PR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	BB	0	0	1,000	1,000	0	0	0	1,000	1,000	1,000	1,000	1,000	0	1,000	0	0	1,000	0	0	1,000	1,000	1,000	1,000	1,000
	GA	13,228	12,436	11,250	11,360	11,783	13,700	13,082	2,465	1,160	1,183	4,083	0	0	0	0	0	12,673	14,336	15,134	12,003	5,438	5,271	4,702	3,777
Experiment 3																									
E _{GT} (kW)	PR	8,118	8,044	8,024	8,003	8,033	8,061	8,127	8,203	8,300	8,471	8,583	8,664	8,690	8,552	8,495	8,781	8,732	8,813	8,683	8,533	8,381	8,302	8,171	8,106
	BB	7,259	5,272	5,143	7,138	5,458	6,815	6,735	6,845	6,811	5,641	5,672	7,137	5,775	5,797	7,065	5,963	7,151	7,688	7,928	6,394	9,102	8,979	7,779	7,937
	GA	7,775	7,838	8,074	7,875	8,202	8,167	8,158	8,303	8,310	8,499	8,526	8,621	8,639	8,604	8,429	8,822	8,590	9,020	9,096	8,870	8,880	8,834	8,740	8,560
F _{DB} (kW)	PR	1,727	3,337	10,187	5,931	1,704	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	BB	0	1,600	1,589	0	1,610	0	0	0	0	1,693	1,713	0	1,741	1,750	0	1,774	0	0	0	1,758	18,626	18,814	0	0
	GA	16,368	14,016	17,665	15,308	12,786	12,483	12,176	12,023	11,868	12,048	12,119	12,028	12,097	12,588	12,704	12,237	12,350	12,752	13,454	14,856	14,687	14,844	14,263	7,136
Q _{UBSG} (kW)	PR	16,516	18,202	26,105	21,056	16,490	14,784	14,794	14,801	14,811	14,835	14,844	14,856	14,845	14,775	14,733	14,835	14,803	14,856	14,845	14,824	14,797	14,793	14,774	14,768
	BB	14,402	14,688	14,571	14,383	14,861	14,197	14,131	14,162	14,105	14,886	14,870	14,162	14,910	14,909	14,049	15,014	14,067	14,375	14,532	15,450	37,582	37,881	14,610	14,699
	GA	34,015	30,928	36,104	32,722	29,488	29,067	28,624	28,433	28,168	28,381	28,404	28,288	28,336	28,907	28,926	28,498	28,502	29,266	30,293	32,139	31,999	32,260	31,526	22,589
Q _{B1} (kW)	PR	17,156	12,520	10,049	11,538	12,304	13,147	12,219	11,919																

(kW)	BB	2,012	0	0	20,793	0	21,086	21,314	0	0	0	0	21,643	0	0	0	1,947	0	0	0	0	0	20,902	0	
	GA	21,281	20,615	18,720	18,246	18,131	18,554	19,587	19,429	19,780	21,328	21,477	21,140	20,927	20,325	20,292	20,247	20,441	21,055	18,456	20,713	19,030	14,902	13,570	11,440
Q _{HB50} (kW)	PR	23,071	22,150	22,228	21,377	20,934	21,127	21,749	21,537	21,847	22,768	22,332	21,959	21,780	21,447	21,516	21,486	21,631	22,547	24,451	25,332	24,875	23,570	23,081	21,981
	BB	16,318	15,063	14,948	40,435	14,872	40,795	41,074	14,977	14,975	15,029	15,041	41,469	14,757	14,585	14,458	15,155	15,112	14,672	14,882	14,912	15,020	15,124	40,570	14,866
	GA	41,034	39,955	37,196	36,512	36,308	36,886	38,327	38,090	38,554	40,810	41,079	40,458	39,811	38,719	38,487	38,290	38,665	40,021	36,227	39,344	37,239	32,031	30,397	27,734
Q _{B1} (kW)	PR	20,513	17,851	14,962	15,086	15,264	15,688	16,533	16,527	16,638	18,006	18,845	18,414	17,859	17,152	16,858	16,581	16,881	17,736	20,759	22,856	21,309	18,000	16,822	13,952
	BB	10,049	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10,049	21,674	0	0	0	0	25,917	0	10,049
	GA	11,335	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{B2} (kW)	PR	9,144	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	BB	45,571	45,719	45,719	0	45,719	0	45,719	45,719	45,719	45,719	45,719	0	45,719	45,719	45,719	39,559	0	45,719	45,719	45,719	45,719	45,719	0	23,243
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Q _{B3} (kW)	PR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	BB	9,144	0	0	12,068	0	12,312	14,059	0	0	0	0	0	14,844	0	0	0	9,144	0	0	0	0	0	14,628	0
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,144	9,144	9,144	9,144	9,144
E _{ST1} (kW)	PR	1,559	1,556	1,555	1,532	1,523	1,525	1,525	1,525	1,525	1,525	1,525	1,525	1,523	1,520	1,524	1,524	1,526	1,528	1,520	1,529	1,525	1,526	1,524	1,524
	BB	979	0	0	0	0	0	0	0	0	0	0	0	0	0	0	863	442	0	0	0	0	1,496	0	934
	GA	1,338	2,000	2,000	1,987	1,967	2,000	2,000	2,000	2,000	1,945	1,817	2,000	2,000	2,000	2,000	2,000	2,000	1,670	1,959	2,000	2,000	1,542	1,379	1,115
Q _{D51} (kW)	PR	12,885	8,822	6,028	5,528	5,352	5,955	7,417	7,206	7,626	9,914	10,319	9,510	8,795	7,783	7,525	7,218	7,646	9,414	14,322	17,381	15,281	10,709	9,028	5,086
	BB	1,000	15,063	14,948	40,435	14,872	40,795	41,074	14,977	14,975	15,029	15,041	41,469	14,757	14,585	14,458	15,000	16,820	14,672	14,882	14,912	15,020	10,474	40,570	0
	GA	23,391	4,315	1,556	1,000	1,000	1,246	2,687	2,450	2,914	5,720	7,277	4,818	4,171	3,079	2,847	2,650	3,025	7,696	1,000	3,704	1,599	1,000	1,000	1,000
E _{ST2} (kW)	PR	1,865	1,664	1,498	1,437	1,357	1,450	1,536	1,546	1,523	1,598	1,621	1,586	1,473	1,482	1,476	1,358	1,442	1,599	1,676	1,716	1,718	1,658	1,587	1,325
	BB	4,000	3,024	3,121	2,486	3,148	2,521	2,629	3,089	3,071	2,986	2,971	2,642	3,004	3,033	3,031	4,000	4,986	800	2,797	2,680	2,771	1,158	2,571	2,075
	GA	1,727	1,175	1,059	1,030	1,023	1,046	1,107	1,097	1,116	1,214	1,230	1,196	1,169	1,123	1,113	1,105	1,121	1,192	1,405	1,535	1,446	1,246	1,192	1,147
Q _{D52} (kW)	PR	1,566	5,548	4,980	4,613	3,789	4,585	4,879	5,096	4,613	4,361	4,434	4,425	3,454	4,073	4,117	2,873	3,648	4,623	3,073	2,053	3,080	4,677	4,665	3,551
	BB	1,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,000	3,641	0	0	0	0	1,000	0	0
	GA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0