Supplier Wholesale Pricing for A Retail Chain: Implications of Centralized vs. Decentralized Retailing and Procurement under Quantity Competition

Dinçer Konur

Engineering Management and Systems Engineering, Missouri University of Science and Technology

Joseph Geunes

Industrial and Systems Engineering, University of Florida

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Abstract

We consider pricing decisions for a supplier who sells a product via a retail chain with localized retail stores throughout a region. The retail chain can influence the competition for channel profit between its retail stores and the supplier via its procurement strategy. Retail store orders may be horizontally decentralized or centrally managed by the retail chain, depending on the chain's ordering strategy. In the case of decentralization at the retail stage, the chain may prefer to coordinate procurement from the supplier to achieve better pricing terms. We model this problem as a Stackelberg game between the supplier and the retail chain and its stores, under joint ownership of the retail chain. When the retail stores are horizontally decentralized, they engage in quantity competition in the regional market. Given the supplier's pricing decisions, we analyze the retail chain's procurement strategy and store order quantity decisions. Then, the the store order quantities are used to solve the supplier's wholesale price setting problem. These analyses then determine the equilibria of the Stackelberg game between the supplier and retail chain under the leaderships of either party. Our results indicate that the retail chain will have a first mover's advantage, while the supplier might in certain cases gain a first mover's disadvantage. Furthermore, the profit-maximizing strategy for the channel may in some cases require the supplier's leadership, while in other cases, the retail chain's leadership maximizes channel profit.

Keywords: Pricing, Competing Retailers, Horizontal Centralization, Joint procurement

1 Introduction

A supplier's wholesale pricing strategy represents one of the most crucial decisions influencing the profitability and efficiency of a supply chain for a product or service. This paper studies a supplier's pricing problem for a good sold to multiple retail stores, who are jointly owned by a retail chain (chain store) from a *vertically decentralized* perspective, and discusses the implications from a *vertically centralized* perspective. In particular, under a vertically decentralized approach, parties at different stages of a distribution channel make decisions based on local objectives only. While the channel we

study is vertically decentralized, we consider different retail and procurement strategies. In particular, we consider two different retail strategies: centralized and decentralized retailing; for decentralized retailing, we consider two different procurement strategies: individual and joint procurement.

Under centralized retailing, the retail chain centralizes store orders to maximize the total profit at the retail stage by determining individual store order quantities and jointly procuring the total order quantity from the supplier on behalf of the stores. However, such centralization can be impractical (see, e.g., Lee and Billington, 1993, Chang and Harrington, 1998, 2000, 2002, 2003); therefore, we also consider decentralized retailing as an alternative retail strategy for the chain. When the retail chain adopts decentralized retailing, i.e., the retail stage is horizontally decentralized, each store determines its order quantity independently of other stores. This case corresponds to a Cournot Oligopoly among the retail stores, which assumes that the stores' market prices are determined by the total supply in the market.¹

In the case of decentralized retailing, we assume that the retail chain can procure store orders using one of two procurement strategies: individual and joint procurement. Under individual procurement, the stores directly place orders with the supplier; thus, the stores' unit purchase costs from the supplier are based on their individual order sizes. On the other hand, if the retail chain adopts joint procurement, the chain purchases the aggregated order from the supplier, and each store's unit purchase cost depends on the aggregate quantity ordered. Joint procurement for decentralized stores can be achieved in practice through distribution centers owned by the retail chain, or by outsourcing cross-docking operations (see, e.g., Waller et al., 2006). If stores are decentralized and joint procurement is assumed, the stores are able to be more competitive in the end customer market.

Figure 1 illustrates the channel under three distinct retail and procurement strategies that can be adopted by the retail chain. We note that the authors observed a similar channel setting in the structure and operation of an air-conditioning products supply channel in Florida, where a supplier manufactures air conditioning products and sells to local retail stores, who are under joint ownership of a retail chain in Florida. It is worthwhile to note that our analysis encompasses not only a retail chain setting, but also the scenario faced by a distributor, who does not own the retail stores it serves but holds a strong degree of channel power because of its relationship with the supplier and the fragmented nature of the localized retailers (see, e.g., Anupindi and Bassok, 1999). Another scenario that may be represented within our study corresponds to the case in which a manufacturer is faced with a choice between alternative retail

¹When competition over homogeneous products exists (i.e., under low product differentiation), the market can often be represented using Cournot competition, as quantity decisions will be of greater interest than pricing decisions. Hence, because we assume that the retail stores sell the same product, a Cournot competition assumption readily applies to our study.

channels: a set of individual local retailers and a retail chain with centralized control of local retail stores. Specifically, this scenario is studied in Shao et al. (2011) for transshipment decisions. In this paper, our focus is on the pricing decisions within such a system.

Figure 1: Illustrations of the Retailing and Procurement Strategies

This paper contributes to the literature by modeling a supplier's wholesale pricing decision for a set of competitive retail stores whose ordering processes can be controlled by a central decision maker (as in the case of a retail chain) under different procurement strategies. In our analysis, we allow an arbitrary number of non-identical retail stores. As the channel in this study is assumed to be vertically decentralized, the problem we study corresponds to a Stackelberg game between the supplier and the retail chain; and, we consider two scenarios for this game: (i) when the supplier acts as the leader by setting a wholesale price, and the retail chain and its stores act as the followers by determining a retail and procurement strategy, as well as how much each store will supply to the market (or, equivalently, order quantities from the supplier) under the selected strategy, and (ii) when the retail chain acts as the leader by determining the retail and procurement strategy and the supplier acts as the follower by setting a wholesale price given the retail chain's selected retail and procurement strategy. We use backward induction to determine the Stackelberg-Nash equilibrium outcomes of the resulting games. Specifically, we first characterize the quantity decisions at the retail stage given the supplier's pricing decisions, and then use this solution to solve the supplier's wholesale price setting problem.

Our analysis leads to the following conclusions. The retail chain will have a first mover's advantage, i.e., if it leads the channel, the profit at the retail stage is higher than in the case in which the supplier is the channel leader. However, channel-wide profits are not necessarily maximized under retail chain's leadership. On the other hand, the supplier may have a first mover's advantage or

disadvantage. In particular, when the supplier leads, centralized retailing is the retail chain's best response for any given supplier pricing scheme. Because the supplier's wholesale price decisions when assuming retail stage decentralization will be suboptimal if the retail chain adopts centralized retailing, the supplier sets his/her wholesale price assuming centralized retailing. Therefore, centralized retailing is the Stackelberg-Nash equilibrium outcome under the supplier's leadership. However, depending on the degree of economies or diseconomies of scale, it might be the case that centralization is not in the best interest of the supplier; that is, a supplier may be able to achieve substantial savings if it is able to dictate the retail chain's retail and procurement strategy or if the retail chain leads. Furthermore, it is possible that the supplier's leadership is not in the best interest of the retail chain or the entire channel. These observations indicate that an effective coordination mechanism may require restrictions on retail and procurement strategies.

The rest of this paper is organized as follows. A review of related studies in the literature is given in Section 2. Section 3 defines the problem setting and our solution approach. In Section 4, for a given wholesale pricing scheme, we analyze store order quantity decisions under different retail and procurement strategies, and characterize the retail chain's best strategy. Furthermore, a comparison of the different strategies is provided. In Section 5, we formulate the supplier's pricing problem under different retail chain strategies. Section 6 documents insights on channel coordination resulting from the analysis. An extension of the models to the case of multi-period decision making with procurement in excess of sales levels is discussed briefly in Section 7. We conclude in Section 8 with final remarks and potential future research directions.

2 Literature Review

In the supply chain literature, a supplier's pricing strategy in a two-echelon channel is often considered within the context of *channel coordination*. We refer the reader to the reviews by Tsay et al. (2000), Cachon (2003), and Sarmah et al. (2006) for broad classes of pricing problems under different problem settings and coordination mechanisms. While most of these studies recognize a supplier's pricing decision as a Stackelberg game, they do not consider different retail and procurement strategies at the retail echelon. We note that, beyond Stackelberg games, various cooperative, bargaining, and coalition games between a supplier and retailers have been analyzed in the literature. For literature on game-theoretic analyses of these problems, one may refer to Kohli and Park (1989), Abad (1994), Iyer and Padmanabhan (2005), Nagarajan and Sošić (2008), Sarmah et al. (2006), Xie and Wei (2009), and the references therein.

The channel we study is assumed to be vertically decentralized. Decentralized decision making

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4 often prevails in situations involving a dominant party (Toptal and Çetinkaya, 2006), when there is
5 competition at the supplier level (Moorthy, 1988), and in other situations, such as supply chains with
6 multiple inventory sites (Lee and Billington, 1993, Lee and Whang, 1999). Vertical decentralization
7 is preferred for markets with low product differentiation (Iyer, 1998). Furthermore, the decentralized
8 decision setting serves as a benchmark for comparing the effectiveness of coordination mechanisms that
9 can increase overall channel profits. As previously noted, although the channel is vertically decentralized,
10 we study centralized and decentralized retailing strategies.

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16 Horizontally centralized and decentralized control in multi-unit firms has been previously discussed
17 in the literature (see, e.g., Chang and Harrington, 1998, 2000, 2002, 2003). Centralized retailing occurs
18 when individual retail stores' decisions are controlled by a single firm, as would occur in the case of a
19 retail chain (Shao et al., 2011) or when a distributor orders on behalf of multiple retailers (Anupindi
20 and Bassok, 1999). Through centralized retailing, as noted by Shao et al. (2011), the retail stage can
21 benefit from reduced retailer competition. Centralized control of retailers has been analyzed in different
22 forms in the supply chain management literature (Dong and Rudi, 2004, Yang and Zhou, 2006, Shao
23 et al., 2011). Dong and Rudi (2004) consider a manufacturer and multiple, centralized, identical-cost
24 retailers with a focus on transshipment and wholesale price decisions. Yang and Zhou (2006) consider
25 the case when two retailers unite and behave in a centralized manner. Shao et al. (2011) consider a
26 manufacturer's wholesale price decision for two identical retailers who can be centralized or decentralized.
27 The centralization of the retail stage we apply in this paper is similar to the aforementioned literature.

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38 Centralized control, even in multi-unit firms such as chain stores or franchising companies, can be
39 impractical (see, e.g., Lee and Billington, 1993, Chang and Harrington, 1998, 2000, 2002, 2003). For
40 instance, one issue with centralized retailing is the stores' willingness to relinquish centralized control.
41 As noted above, another issue lies in the fact that centralization of store orders and the corresponding
42 supplier wholesale price decisions will not necessarily be in the best interest of the supplier, the retail
43 chain, or the entire channel. For instance, the authors have recently observed the following issue in the
44 Kraft Foods' supply channel in the St. Louis, MO area. Walmart is the biggest retailer for Kraft Foods'
45 products, and the Walmart distribution center in the St. Louis area controls the supply of Kraft Foods'
46 products to regional Walmart stores. However, the Kraft Foods' branch in St. Louis would benefit from
47 receiving orders directly from individual Walmart stores in the region, as this Kraft Foods' branch can
48 smooth their deliveries to the individual stores and save on trucking costs.

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59 When the retail chain adopts decentralized control of its stores, individual retail stores compete
60 with one another in the end customer market. We note that a significant amount of literature exists
61 on coordination problems under downstream (e.g., retail-level) competition in a supply chain. Ingene

and Parry (1995, 1998, 2000) study a coordination problem between a manufacturer and two retailers competing on price. Iyer (1998) studies channel coordination when retailers are engaged in price and non-price competition. Bernstein and Federgruen (2003, 2004, 2005) focus on coordinated inventory replenishment and pricing decisions in a two-echelon supply chain consisting of a single manufacturer and multiple competitive retailers. Other studies focusing on channel coordination with retailer competition consider returns policies (Padmanabhan and Png, 1997), two-part tariffs (Tsay and Agrawal, 2000), quantity discounts (Balachander and Srinivasan, 1998, Xiao and Qi, 2008), vendor managed inventory (Bernstein et al., 2006), and revenue sharing contracts (Yao et al., 2008). Additional references in this research stream can be found in Cachon (2003).

Under decentralized retailing, we consider two procurement strategies: individual procurement, where the retail chain allows its stores to order individually from the supplier, and joint procurement, where the retail chain jointly procures its stores' orders from the supplier. The setting in which decentralized retailing with joint procurement occurs is, therefore, defined similarly to the existing literature on group buying (see, e.g., Keskinocak and Savasaneril, 2008, Nagarajan et al., 2010, Chen and Roma, 2011, Zhou and Xie, 2014). However, it should be noted that when the retail stores are not under joint ownership, one would need to analyze the stability of any collaboration established for group buying. We refer the reader to Nagarajan et al. (2010) for various allocation methods discussed to stabilize a coalition for group buying. In the current study, joint procurement is defined similarly to group buying; however, it is assumed that it is the retail chain's decision whether to implement joint procurement.

Due to vertical decentralization, the decision making processes of the supplier and retail chain correspond to a Stackelberg game. In particular, when the supplier is the leader, the underlying Stackelberg game is defined similarly to the games studied by Yang and Zhou (2006), Keskinocak and Savasaneril (2008), Chen and Roma (2011), and Zhou and Xie (2014). Yang and Zhou (2006) analyze a Stackelberg game where a manufacturer leads by determining a fixed wholesale price, and two non-identical competitive retailers follow by determining their order quantities. The manufacturer's optimal pricing decision is considered when the retailers are competitive (i.e., decentralized), united (i.e., centralized), and take actions sequentially. However, joint procurement for two competing retailers is not considered in Yang and Zhou (2006). As noted previously, centralization of retailers can be impractical whereas joint procurement implementation can be relatively easier.

In our study, we consider Cournot-type quantity competition similar to Keskinocak and Savasaneril (2008). Keskinocak and Savasaneril (2008) study a supplier's pricing problem when an arbitrary number of identical retailers compete on sales volumes. They analyze the decisions of competitive and cooperative

retailers. While the competitive case corresponds to what we call decentralized retailing with individual procurement, the cooperative case is different from the centralized retailing we apply. The retailers, given a supplier's linear discount pricing function, cooperate via collaborative purchasing. However, in their study, the retailers' cooperation level is determined by the supplier, i.e., the supplier decides whether the retailers are cooperative by setting a spillover factor, which defines the additional discount a retailer achieves from the supplier as a result of other retailers' order quantities. The current study extends the setting of Keskinocak and Savasaneril (2008) to the case of non-identical retailers and considers different retail strategies along with different procurement strategies.

The decentralized retailing with joint procurement we study extends the setting of Chen and Roma (2011) and Zhou and Xie (2014), who mainly analyze group buying for two non-identical retailers, to the case of more than two retailers. In particular, while Chen and Roma (2011) assume that the supplier's pricing is fixed whether the retailers are buying from the supplier as a group or not, Zhou and Xie (2014) allow the supplier to adjust its pricing scheme depending on the retailer's level of collaborative procurement. In both of these studies, either individual or group buying is considered. In addition, we consider horizontal centralization at the retail stage. Furthermore, the studies discussed so far assume that the supplier is the leader of the Stackelberg game. In this study, we also investigate the case when the retail chain is the leader and compare the Stackelberg games under leadership of the supplier and the retail chain.

3 Problem Formulation and Methodology

Consider a supplier (manufacturer) who sells a product to an end customer market via a retail chain, who controls a set of n stores. We assume that the supplier offers linear quantity discounts. Let $c(q) = \lambda - \beta q$ be the supplier's selling price when q units are ordered, where λ is the base unit purchase cost (the supplier's base wholesale price) and β is the per unit discount offered by the supplier. The parameter β is referred to as the supplier's discount factor throughout the text. Depending on the retail chain's retail and procurement strategy, the stores' per unit purchase costs from the supplier will change. In particular, let q_i denote the order quantity for store $i \in I$, $I = \{1, 2, \dots, n\}$, and let $\vec{Q}$ denote the n -vector of q_i values. Then, the per unit purchase cost of the retail chain's stores, denoted by $c(\vec{Q})$, will

be defined as follows:²

$$c(\vec{Q}) = \begin{cases} c(q_i) = \lambda - \beta q_i & \text{under decentralized retailing with individual procurement,} \\ c(Q) = \lambda - \beta \sum_{j=1}^n q_j & \text{under decentralized retailing with joint procurement,} \\ c(Q) = \lambda - \beta \sum_{j=1}^n q_j & \text{under centralized retailing,} \end{cases} \quad (1)$$

where $Q = \sum_{j=1}^n q_j$. It should be noted that while the unit purchase cost will be decreasing according Equation (1), we would expect that the total purchase cost for any given store would not be decreasing in its order quantity; otherwise, such a store has incentive to maximize its order quantity in order to reduce its total purchase costs. In such a case, a store would order as much as possible, independent of the demand it faces in the customer market. To avoid this case, we adopt the following assumption (see also Keskinocak and Savasaneril (2008) and Chen and Roma (2011) for a similar discussion):

Assumption 1 $c(\vec{Q})q_i$ is an increasing function of $q_i \forall i \in I$.

Assumption 1 implies that any quantity purchased from the supplier will be sold in the end customer market; thus, q_i defines the quantity that store i supplies to the end customer market as well. In Section 7, we discuss the implications of relaxing this assumption.

We assume that a store's selling price in the market is affected by its market supply quantity as well as the other stores' market supply quantities. In particular, store i 's selling price as function of the market supply quantities $\vec{Q}$, denoted by $p_i(\vec{Q})$, is defined as follows:

$$p_i(\vec{Q}) = a_i - \sum_{j=1}^n b_{ij}q_j, \quad (2)$$

where $a_i \geq 0$ denotes the maximum demand for the supplier's product from store i , and $b_{ij} > 0$ is the sensitivity of store i 's customers to the availability of the product at store j . We note that $p_i(\vec{Q})$ represents the inverse demand function associated with a Cournot setting, and is widely used to model situations in which market price is determined by the quantities supplied to the market (see, e.g., Bernstein and Federgruen, 2003, Bernstein and Federgruen, 2005).

Assumption 2 (i) $b_{ii} > \sum_{j=1:j \neq i}^n b_{ij}$ and (ii) $b_{ii} > \beta \forall i \in I$.

Assumption 2(i) indicates that an increase in any store's selling price decreases the total quantity sold in the end customer market, which is intuitive and has been observed in many industries (see, Bernstein and Federgruen, 2003, Bernstein and Federgruen, 2005). Assumption 2(ii) implies that marginal revenues decrease faster than marginal costs, which bounds the order quantities (see, e.g., Keskinocak and Savasaneril, 2008).

²It is assumed that the price schedule set by the supplier is the same for each store. This is consistent with the Robinson-Patman Act, which prohibits price discrimination. Furthermore, a supplier may choose to utilize a simple pricing scheme (see, e.g., Ingene and Parry, 1995, 1998).

In addition to the purchase costs, each store is subject to operating costs, and we let $w_i > 0$ denote the unit operating cost of store i . Observe that w_i can be interpreted as a unit shipping cost from store i to the end customer market (and stores located in different areas will therefore have different w_i values) or simply a unit operating (e.g., material handling or storage) cost of store i (when stores utilize different technologies, this may also lead to different w_i values). Considering the practice of store orders being shipped to a distribution warehouse and then to individual stores, w_i can be interpreted as $w_i = w + r_i$, where w would denote a shipping cost from the supplier to the retail chain's distribution warehouse and r_i would denote the shipping cost from the warehouse to store i (plus the shipping cost from store i to the market in cases where customer orders are delivered). The total operating cost of store i then amounts to $w_i q_i$. We assume that stores are non-identical; thus, $w_{i_1} \neq w_{i_2}$ for $i_1 \neq i_2, i_1, i_2 \in I$.

The profit of store i equals its revenues gained from supplying the market, less purchase and operating costs. Then, considering the above discussion, any store i 's profit function reads as

$$\Pi_i(\vec{Q}) = p_i(\vec{Q})q_i - c(\vec{Q})q_i - w_i q_i. \quad (3)$$

The supplier must deliver the entire order for each store; thus, the supplier's production quantity equals the sum of the stores' order quantities. As each store's order the quantity will be supplied to the end customer market, the supplier's production quantity is $Q = \sum_{j=1}^n q_j$. The supplier incurs production costs, which are a function of Q ; hence, the supplier's profit equals total revenue gained from selling to the stores (or the retail chain) less production costs. The supplier's objective is to maximize its profit by determining its base wholesale price λ . The supplier's profit as a function of λ , $\Pi_S(\lambda)$, then reads

$$\Pi_S(\lambda) = \sum_{i=1}^n \left(c(\vec{Q}(\lambda))q_i(\lambda) \right) - f(Q(\lambda)), \quad (4)$$

where $\vec{Q}(\lambda)$ is the n -vector of $q_i(\lambda)$ values, which denotes store i 's order quantity for the given base wholesale price λ , and $Q(\lambda) = \sum_{i=1}^n q_i(\lambda)$. The first term in Equation (4) defines the supplier's revenues gained from selling the product to the stores and $f(Q(\lambda))$ represents the production-related cost function. It is assumed that $f(q)$ is a continuous and differentiable function of production quantity q . In particular, $f(q)$ is a nondecreasing function since production cost is expected to increase with production quantity. Further characteristics of $f(q)$ will be discussed in more detail when we analyze the supplier's problem.

Next, we first focus on solving for the stores' order quantities given the retail chain's retail and procurement strategy and the supplier's base wholesale price. Then, the supplier's base wholesale price setting problem is analyzed. Table 1 summarizes the notation used throughout this paper. Additional notation will be defined as needed.

Table 1: Notation

<i>Indices</i>	
i	index for stores, $i \in I = \{1, 2, \dots, n\}$
s	index for retail and procurement strategies, $s \in \{1, 2, 3\}$
<i>Retail Stage Parameters and Functions</i>	
q_i	supply (order) quantity of store i
$\vec{Q}$	n -vector of q_i values
Q	total quantity supplied (ordered), $Q = \sum_{i=1}^n q_i$
a_i	store i 's maximum demand
b_{ij}	decrease in store i 's demand by store j 's per unit supply
$p_i(\vec{Q})$	store i 's market price as a function of $\vec{Q}$, $p_i(\vec{Q}) = a_i - \sum_{j=1}^n b_{ij}q_j$
$c(\vec{Q})$	store i 's per unit purchase cost as a function of $\vec{Q}$
w_i	per-unit operating cost of store i
$\Pi_i(\vec{Q})$	store i 's profit as a function of $\vec{Q}$
<i>Retailing Stage Solutions</i>	
$\Pi_i^s(\vec{Q})$	store i 's profit as a function of any $\vec{Q}$ under retailing strategy s
q_i^s	store i 's supply (order) quantity decision under retailing strategy s
$\vec{Q}^s$	n -vector of q_i^s values
$Q^{(s)}$	total quantity supplied (ordered) under retail chain strategy s , $Q^{(s)} = \sum_{i=1}^n q_i^s$
<i>Supply Stage Parameters and Functions</i>	
λ	supplier's base selling price (wholesale price)
β	supplier's discount factor
$f(q)$	supplier's production cost function as a function of production quantity q
$\Pi_S(\lambda)$	supplier's profit function as a function of λ
$q_i(\lambda)$	store i 's order quantity as a function of λ

4 Retail Stage

The retail chain may adopt one of three different retail and procurement strategies indexed by s : decentralized retailing with individual procurement ($s = 1$), decentralized retailing with joint procurement ($s = 2$), and centralized retailing ($s = 3$). In this section, given the supplier's pricing schedule (i.e., given λ and β values), we provide a solution method for the stores' order quantity decisions under each of the retail chain's distinct retail and procurement strategies. A comparison among different retail and procurement strategies is also documented.

4.1 Decentralized Retailing with Individual Procurement

When the retail stage is decentralized, the retail chain allows stores to independently determine order quantities. With individual procurement, each store places orders directly with the supplier. The retail chain's role is to ship store order quantities received from the supplier's facility (or the manufacturer's plant) to the stores. A warehouse, owned by the retail chain, may be used for distribution of the product

to the stores. Decentralized ordering with individual procurement represents the case where each store operates as single decision maker. We note that similar definitions of decentralization of a retail chain or stage are given in the literature (see, e.g., Chang and Harrington, 1998, 2000, 2002, 2003, Shao et al., 2011). Furthermore, this case represents the scenario when the supplier sells its product through a set of individual and independent retailers, which are not under joint ownership of a chain store. That is, an indirect channel is used by the supplier to market the product of interest through decentralized retailers. Horizontal decentralization at the retail stage has been analyzed in the literature (see, e.g., Dong and Rudi, 2004, Yang and Zhou, 2006, Shao et al., 2011).

Under decentralized retailing with individual procurement, each store's objective is to maximize its profit by determining its market supply quantity (equivalently, the order quantity from the supplier), which is affected by the supply quantity decisions of all other stores and the resulting selling price in the market. Considering Equation (2), store decisions correspond to a Cournot oligopoly and our aim is to analyze the Nash equilibrium solution of this game. Due to individual procurement, each store's purchase cost is only affected by its own individual order quantity. In particular, Equation (3) under decentralized retailing with individual procurement reads as

$$\Pi_i^1(\vec{Q}) = \left(a_i - \sum_{j=1}^n b_{ij}q_j \right) q_i - (\lambda - \beta q_i)q_i - w_i q_i. \quad (5)$$

The first term of Equation (5) denotes store i 's total revenue, the second term is the total purchase cost with discounted unit procurement cost, and the last term represents the operating costs.

Note that $\Pi_i^1(\vec{Q})$ is strictly concave in q_i when $b_{ii} > \beta$ since $\partial^2 \Pi_i^1(\vec{Q}) / \partial (q_i)^2 = -2(b_{ii} - \beta)$. Furthermore, any store i will not supply the market against a negative price, which implies that any store i 's supply quantity has an upper bound (one can state that $q_i \leq a_i / b_{ii} \forall i \in I$). Together with non-negativity constraints, $q_i \geq 0 \forall i \in I$, it follows that K_i , the strategy set of store i , is convex and compact such that $K_i = [0, a_i / b_{ii}]$. Therefore, we have the following proposition (Nash, 1951).

Proposition 1 *There exists a unique $\vec{Q}^1 = [q_1^1, q_2^1, \dots, q_n^1]$ such that*

$$\Pi_i^1(q_i^1, \vec{Q}^{1[-i]}) \geq \Pi_i^1(q_i, \vec{Q}^{1[-i]}) \quad \forall q_i \geq 0 \quad \forall i \in I,$$

where $\vec{Q}^{1[-i]} = [q_1^1, q_2^1, \dots, q_{i-1}^1, q_{i+1}^1, \dots, q_n^1]$.

Specifically, it follows from Equation (5) that if $q_i^1 > 0$, the unique Nash equilibrium solution must satisfy the following condition:

$$a_i - \sum_{j=1}^n b_{ij}q_j^1 - \lambda - (b_{ii} - 2\beta)q_i^1 - w_i = 0. \quad (6)$$

Furthermore, it can be shown that $\vec{Q}^1 \in R_+^n$ solves the stores' game under decentralized retailing with individual procurement if and only if it solves the following asymmetric linear variational inequality problem (VIP):

$$(\mathbf{M}^1 \vec{Q}^1 - \mathbf{z})(\vec{Q} - \vec{Q}^1) \geq 0, \forall \vec{Q} \in \mathcal{K}, \quad (7)$$

where $\mathbf{M}^1$ is an $n \times n$ matrix defined using Equation (6), that is, $M_{ij}^1 = b_{ij} \forall i \neq j$ and $M_{ii}^1 = 2(b_{ii} - \beta)$, $\mathbf{z}$ is an n -vector such that $z_i = (a_i - w_i - \lambda)$, and $\mathcal{K} = K_1 \times K_2 \times \dots \times K_n$ is the nonempty, convex and compact subset of R^n (see, e.g., Gabay and Moulin, 1980, Friesz et al., 1983, Harker, 1984, Nagurney, 1999, Miller et al., 1996).

Now, suppose that the market capacity for each store is sufficiently large, such that $q_i^1 > 0 \forall i \in I$. Then the next corollary follows from Equation (6).

Corollary 1 *Suppose that $q_i^1 > 0, \forall i \in I$. Then*

- $\sum_{i=1}^n (a_i - w_i - \lambda) = \sum_{i=1}^n \hat{b}_i q_i^1$, where $\hat{b}_i = b_{ii} + \sum_{j=1}^n b_{ij} - 2\beta, \forall i \in I$, and
- $\sum_{i=1}^n \Pi_i^1(\vec{Q}^1) = \sum_{i=1}^n (b_{ii} - \beta)(q_i^1)^2$.

4.2 Decentralized Retailing with Joint Procurement

As in the previous section, under decentralized retailing, the stores are allowed to independently determine their order quantities. With joint procurement, on the other hand, each individual store places his/her order through the retail chain. The retail chain's role, therefore, is first to place its stores' aggregated orders with the supplier, and then to deliver these orders to stores from the distribution warehouse. In this case, as noted in Equation (1), each store receives the discounted price not only based on its individual order quantity, but also based on the discounts achieved by aggregating all stores' order quantities due to joint procurement.

Under decentralized retailing with joint procurement, each store's objective is to maximize its profit by determining its market supply quantity, which is affected by the market supply quantity decisions of all other stores and the resulting selling price in the end customer market, as well as the purchase cost from the supplier. Similar to the setting of Section 4.1, store decisions correspond to a Cournot oligopoly, and we analyze the Nash equilibrium solution of this game. In particular, store i 's profit under decentralized retailing with joint procurement reads

$$\Pi_i^2(\vec{Q}) = \left(a_i - \sum_{j=1}^n b_{ij} q_j \right) q_i - \left(\lambda - \beta \sum_{j=1}^n q_j \right) q_i - w_i q_i. \quad (8)$$

The first term of Equation (8) denotes store i 's total revenue, the second term is the total purchase cost (with discounted unit procurement cost based on all of the stores' order quantities), and the last term represents the operating costs.

Note that, similar to $\Pi_i^1(\vec{Q})$, $\Pi_i^2(\vec{Q})$ is strictly concave in q_i when $b_{ii} > \beta$ since $\partial^2 \Pi_i^2(\vec{Q}) / \partial (q_i)^2 = -2(b_{ii} - \beta)$. Moreover, any store i will not supply the market against a negative price under decentralized retailing with joint procurement as well, which, together with non-negativity constraints, implies convexity and compactness of any store i 's strategy set. Similar to Proposition 1, we therefore have the following proposition (Nash, 1951).

Proposition 2 *There exists a unique $\vec{Q}^2 = [q_1^2, q_2^2, \dots, q_n^2]$ such that*

$$\Pi_i^2(q_i^2, \vec{Q}^{2[-i]}) \geq \Pi_i^2(q_i, \vec{Q}^{2[-i]}) \quad \forall q_i \geq 0 \quad \forall i \in I,$$

where $\vec{Q}^{2[-i]} = [q_1^2, q_2^2, \dots, q_{i-1}^2, q_{i+1}^2, \dots, q_n^2]$.

It follows from Equation (8) that if $q_i^2 > 0$, then the unique Nash equilibrium solution must satisfy the following condition:

$$a_i - \sum_{j=1}^n (b_{ij} - \beta) q_j^2 - \lambda - (b_{ii} - \beta) q_i^2 - w_i = 0. \quad (9)$$

Furthermore, following a similar discussion as given above for decentralized retailing with individual procurement, it can be shown that $\vec{Q}^2 \in R_+^n$ solves the stores' game under decentralized retailing with joint procurement if and only if it solves the following asymmetric linear VIP:

$$(\mathbf{M}^2 \vec{Q}^2 - \mathbf{z})(\vec{Q} - \vec{Q}^2) \geq 0, \forall \vec{Q} \in \mathcal{K}, \quad (10)$$

where $\mathbf{M}^2$ is an $n \times n$ matrix defined using Equation (9), that is, $M_{ij}^2 = (b_{ij} - \beta) \quad \forall i \neq j$ and $M_{ii}^2 = 2(b_{ii} - \beta)$, and $\mathbf{z}$ and $\mathcal{K}$ are as defined previously.

Now, suppose that the market capacity for each store is sufficiently large, such that $q_i^2 > 0 \quad \forall i \in I$. Then the next corollary follows from Equation (9).

Corollary 2 *Suppose that $q_i^2 > 0, \forall i \in I$. Then*

- $\sum_{i=1}^n (a_i - w_i - \lambda) = \sum_{i=1}^n \bar{b}_i q_i^2$, where $\bar{b}_i = b_{ii} + \sum_{j=1}^n b_{ij} - (n+1)\beta, \forall i, i \in \{1, 2, \dots, n\}$, and
- $\sum_{i=1}^n \Pi_i^2(\vec{Q}^2) = \sum_{i=1}^n (b_{ii} - \beta)(q_i^2)^2$.

4.3 Centralized Retailing

While decentralized retailing decisions can be implemented because of their practical simplicity and marketing benefits, it is possible that the retail chain may achieve substantial savings under horizontally

centralized control of its stores. As noted by Shao et al. (2011), the retail chain can benefit from decreased competition. Therefore, we consider the case when the retail chain prefers centralized control of the stores.

Under centralized retailing, the retail chain decides on the market supply quantities for each store. As individual procurement for centrally controlled stores will be less beneficial for the retail chain, we assume that the retail chain adopts joint procurement. The retail chain's objective under the centralized retailing strategy is to determine each store's market supply quantity, such that the total profit of the stores is maximized. Considering Equation (3), the retail chain's problem under centralized retailing can be formulated as follows:

$$\max_{q_i \geq 0} \left\{ \sum_{i=1}^n \left(a_i - \sum_{j=1}^n b_{ij} q_j \right) q_i - \sum_{i=1}^n \left(\lambda - \beta \sum_{j=1}^n q_j \right) q_i - \sum_{i=1}^n w_i q_i \right\}. \quad (11)$$

The first term of the objective function in Equation (11) is the total revenue that the retail chain gains through its stores, the second term is the total purchase cost paid to the supplier by the retail chain, and the last term is the total store operating cost.

Note that Equation (11) is equal to the following quadratic optimization problem:

$$\max_{q_i \geq 0} \mathbf{z}^t \vec{\mathbf{Q}} - \vec{\mathbf{Q}}^t \mathbf{M}^3 \vec{\mathbf{Q}},$$

where $\mathbf{M}^3$ is an $n \times n$ matrix defined using Equation (11), that is, $M_{ij}^3 = (b_{ij} - \beta) \forall i \neq j$ and $M_{ii}^3 = (b_{ii} - \beta)$, and $\mathbf{z}$ is as previously defined. Then, one can show that $\mathbf{M}^3$ is positive definite when $b_{ii} > \sum_{j=1:j \neq i}^n b_{ij}$. This follows from the fact that any leading principal of $\mathbf{M}^3$ can be reduced to an upper triangular matrix through elementary row operations such that each diagonal component is positive when $b_{ii} > \sum_{j=1:j \neq i}^n b_{ij}$. That is, all leading principal minors of $\mathbf{M}^3$ are strictly positive. This further implies that the objective function in Equation (11) is jointly strictly concave. Therefore, we have the following proposition.

Proposition 3 *There exists a unique $\vec{\mathbf{Q}}^3 = [q_1^3, q_2^3, \dots, q_n^3]$ such that*

$$\sum_{i=1}^n \Pi_i^3(\vec{\mathbf{Q}}^3) \geq \sum_{i=1}^n \Pi_i^3(\vec{\mathbf{Q}}) \forall \vec{\mathbf{Q}} \in R_+^n.$$

Equation (11) implies that if $q_i^3 > 0$, then the optimal solution must satisfy the condition

$$a_i - \sum_{j=1}^n b_{ij} q_j^3 - \sum_{j=1}^n b_{ji} q_j^3 - \lambda + 2\beta \sum_{j=1}^n q_j^3 - w_i = 0. \quad (12)$$

Furthermore, as the non-negativity constraints are linear, it then follows that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for solving Equation (11) (see, e.g., Bazaraa et al., 2006).

It is well known that the KKT conditions can be expressed as a VIP. In particular, the KKT conditions for Equation (11) read as the following asymmetric linear VIP:

$$(\mathbf{M}^3 \vec{\mathbf{Q}}^3 - \mathbf{z})(\vec{\mathbf{Q}} - \vec{\mathbf{Q}}^3) \geq 0, \forall \vec{\mathbf{Q}} \in \mathcal{K}. \quad (13)$$

where $\mathbf{M}^3$ is defined above, and $\mathbf{z}$ and $\mathcal{K}$ are as defined previously.

Now, suppose that the market capacity for each store is sufficiently large such that $q_i^3 > 0 \forall i \in I$. Then the next corollary follows from Equation (12).

Corollary 3 *Suppose that $q_i^3 > 0, \forall i \in I$. Then*

- $\sum_{i=1}^n (a_i - w_i - \lambda) = \sum_{i=1}^n \tilde{b}_i q_i^3$, where $\tilde{b}_i = \sum_{j=1}^n (b_{ij} + b_{ji} - 2\beta)$, $\forall i \in I$, and
- $\sum_{i=1}^n \Pi_i^2(\vec{\mathbf{Q}}^3) = \sum_{i=1}^n \sum_{j=1}^n (b_{ij} - \beta) q_j^3 q_i^3$.

4.4 Comparison of Retail and Procurement Strategies

In this section, we compare the retail chain's potential retail and procurement strategies. Recall that store quantity decisions correspond to the following asymmetric linear VIP under each retail and procurement strategy s (see Equations (7), (10), and (13));

$$(\mathbf{M}^s \vec{\mathbf{Q}}^s - \mathbf{z})^T (\vec{\mathbf{Q}} - \vec{\mathbf{Q}}^s) \geq 0, \forall \vec{\mathbf{Q}} \in \mathcal{K}. \quad (14)$$

Therefore, one can use existing variational inequality methods to solve for $\mathbf{Q}^s$, and we refer the interested reader to the books by Miller et al. (1996) and Nagurney (1999), and the references therein, for algorithmic solutions for variational inequality formulations. Algorithm 1, stated below, gives a realization of a self adoptive projection method proposed by Han (2006) for solving asymmetric linear VIPs in the form of Equation (14).

Algorithm 1 *Self-adaptive Projection method for asymmetric linear VIP given in Equation (14).*

1. Start with a $\mathbf{Q}^0 \in R^n$. Set $0 < \gamma < 2$, $\alpha_0 > 0$, $\epsilon \geq 0$, and a sequence $\{\tau_\ell\} \subseteq [0, \infty)$ with $\sum_{\ell=0}^{\infty} \tau_\ell < \infty$. Set $\ell := 0$
2. Determine $e(\mathbf{Q}_j^\ell, \alpha) = \mathbf{Q}^\ell - P_{\mathcal{K}}[\mathbf{Q}_j^\ell - \alpha(\mathbf{M}^s \mathbf{Q}_j^\ell + \mathbf{z})]$, $P_{\mathcal{K}}[\cdot]$ being the orthogonal projection from R^n onto $\mathcal{K}$. If $\|e(\mathbf{Q}_j^\ell, \alpha_\ell)\|_\infty \leq \epsilon$, stop.
3. Compute the next iterate using $\mathbf{Q}^{\ell+1} = \mathbf{Q}^\ell - \gamma(I + \alpha_\ell \mathbf{M}^s)^{-1} e(\mathbf{Q}_j^\ell, \alpha_\ell)$.
4. Choose the next parameter $\alpha_{\ell+1}$ from the interval $\frac{1}{1+\tau_\ell} \alpha_\ell \leq \alpha_{\ell+1} \leq (1 + \tau_\ell) \alpha_\ell$. Set $\ell := \ell + 1$ and go to Step 2.

Algorithm 1 can be used to determine the stores' market supply quantities and the total profit at the retail stage under each of the three distinct retailing and procurement strategies. In what follows, we first investigate the stores' market supply quantities. Then, the total retail-stage profits gained under different strategies are compared.

Let $Q^{(s)} = \sum_{i=1}^n q_i^s$, i.e., $Q^{(s)}$ denotes the stores' total order quantity from the supplier under strategy s . Corollary 4 is then implied by Corollaries 1, 2, and 3.

Corollary 4 *Suppose that $q_i^s > 0 \forall i \in I$. Then $Q^{(s)}$ decreases as λ increases and $Q^{(s)}$ increases as β increases.*

In the following corollary, we compare $Q^{(1)}$, $Q^{(2)}$, and $Q^{(3)}$.

Corollary 5 *Suppose that $q_i^1 > 0$, $q_i^2 > 0$, and $q_i^3 > 0 \forall i \in I$. Then*

- $Q^{(1)} \leq Q^{(3)} \leq Q^{(2)}$ if $b_{ij} \leq 2\beta \forall i \neq j \in I$,
- $Q^{(3)} \leq Q^{(1)} \leq Q^{(2)}$ if $b_{ij} \geq 2\beta \forall i \neq j \in I$.

Corollary 5 also follows from Corollaries 1, 2, and 3. Corollary 5 states that the stores' aggregated order quantity is maximized under decentralized retailing with joint procurement. On the other hand, the value of the supplier's discount factor determines whether centralization will increase or decrease the aggregate order quantity relative to the case of decentralized retailing with individual procurement. We next illustrate the implications of Corollary 5 using an example.

Example 1 *Consider a retail chain with two stores. Let $[a_1, a_2] = [150, 125]$, $[w_1, w_2] = [10, 15]$, $[b_{11}, b_{12}] = [2, 0.3]$, and $[b_{21}, b_{22}] = [0.3, 1.6]$. Furthermore, let the supplier's selling price be determined by $c(q) = 60 - \beta q$. Figure 2 illustrates the stores' total order quantities and profit under each retail and procurement strategy of the retail chain for varying values of the supplier's discount factor.*

We observe from Figure 2a that $Q^{(1)}$, $Q^{(2)}$, and $Q^{(3)}$ increase with β , as noted in Corollary 4. Furthermore, as implied by Corollary 5, $Q^{(3)} \leq Q^{(1)}$ for $\beta < 0.15$, whereas $Q^{(3)} > Q^{(1)}$ for $\beta > 0.15$. We note the following additional observations based on Figure 2.

- As $\beta \rightarrow 0$, $Q^{(1)} \rightarrow Q^{(2)}$ and $\sum_{i=1}^n \Pi_i^1(\vec{Q}^1) \rightarrow \sum_{i=1}^n \Pi_i^2(\vec{Q}^2)$. This result is expected considering the definitions of decentralized retailing with individual and joint procurement stated in Equations (5) and (8), respectively.
- As β increases, $Q^{(2)} - Q^{(1)}$ and $\sum_{i=1}^n \Pi_i^2(\vec{Q}^2) - \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$ increase, while $Q^{(2)} - Q^{(3)}$ and $\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) - \sum_{i=1}^n \Pi_i^2(\vec{Q}^2)$ decrease.

Figure 2: Illustration of Example 1

- We observe from Figure 2b that $\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) > \sum_{i=1}^n \Pi_i^2(\vec{Q}^2)$ and $\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) > \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$. Furthermore, we observe that $\sum_{i=1}^n \Pi_i^2(\vec{Q}^2) > \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$; nevertheless, as discussed next, this is not always the case, i.e., it is possible that joint procurement can result in total retail-stage profits less than with individual procurement for decentralized stores.

Recall that the supplier does not have control over the retail chain's retail and procurement strategies. The supplier's profit levels for a given pricing schedule will, however, be strongly affected by the retail and procurement strategy adopted by the retail chain. Specifically, as discussed in Corollary 5, the total quantity ordered from the supplier differs under each of the retailing strategies. Therefore, the supplier's preferences for retail and procurement strategies will differ according to the economies or diseconomies of scale the supplier faces. For a given pricing schedule, we summarize the supplier's best interests with respect to the retail chain's retail and procurement strategies as follows.

- When the stores are decentralized, the supplier will benefit from joint procurement if s/he is subject to economies of scale under the given pricing schedule. On the other hand, the supplier will benefit from individual procurement of the decentralized stores if s/he is subject to diseconomies of scale under the given pricing schedule.
- In case joint procurement is not possible (for instance, if the retail chain does not have a distribution warehouse), whether the supplier will benefit from decentralization depends on the level of economies or diseconomies of scale under the given pricing schedule.

We note that similar to the first point above, Chen and Roma (2011) and Zhou and Xie (2014) also remark that revenues of a supplier (manufacturer) can be higher or lower when competitive retailers jointly procure from the supplier (i.e., when they engage in group buying). Our results extend this

to the case of an arbitrary number of non-identical retailers. And, the second point further compares centralization and decentralization of the retail stage from the supplier's perspective, where it is noted that the supplier may prefer decentralization. In other settings, Anupindi and Bassok (1999) and Shao et al. (2011) also observe that a manufacturer (supplier) might be hurt by centralization of the retailers.

The above points lead to important insights on coordination and centralization of supply and retail stages. It should be noted that the channel-wide profits under different retail and procurement strategies vary depending on the supplier's economies or diseconomies of scale. Therefore, since the maximum channel-wide profits can be achieved under a specific retail and procurement strategy, a coordination policy or approach may need to constrain the retail and/or procurement strategy. In Section 6, we discuss further insights on the coordination of the channel under different channel characteristics. Next, we compare the total retail-stage profits gained under different strategies.

Recall that $\vec{Q}^1$, $\vec{Q}^2$, and $\vec{Q}^3$ denote the n -vectors of stores' market supply quantities under decentralized retailing with individual procurement, decentralized retailing with joint procurement, and centralized retailing, respectively. Observe that

$$\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) \geq \sum_{i=1}^n \Pi_i^1(\vec{Q}^1) \text{ and } \sum_{i=1}^n \Pi_i^3(\vec{Q}^3) \geq \sum_{i=1}^n \Pi_i^2(\vec{Q}^2).$$

The inequalities above simply follow from the definition of centralized retailing: under centralized retailing, the retail chain maximizes total profit as defined in Equation (11) while stores maximize their individual profits under decentralized retailing (both with individual and joint procurement). Therefore, the retail chain will prefer centralized control of its stores over decentralized control. On the other hand, in cases where complete centralization is not practical or possible, the retail chain's joint procurement may increase or decrease the profits gained at the retail stage, depending on the supplier's pricing scheme and store characteristics (a_i , b_{ij} , and w_i values). This is demonstrated in the following example, which assumes identical market price functions for the retailers.

Example 2 Consider a retail chain with two stores such that stores' market price functions are identical, i.e., $a_i = a$ and $b_{ij} = b \forall i, j \in I$ (in this case Equation (2) reads as $p(\vec{Q}) = a - b \sum_{j=1}^n q_j$). Let $a = 100$, $b = 2$, and $[w_1, w_2] = [10, 15]$. Furthermore, let the supplier's selling price be determined by $c(q) = 60 - \beta q$. Figure 3 illustrates the stores' total order quantity and profit under each retail and procurement strategy for varying values of the supplier's discount factor.

The following observations are based on Figure 3.

- As pointed out previously, $Q^{(2)} \geq Q^{(1)}$ and $Q^{(2)} \geq Q^{(3)}$. On the other hand, as can be seen in Figures 3a and 3c, it is possible to have $Q^{(1)} \geq Q^{(3)}$ or $Q^{(1)} \leq Q^{(3)}$.

Figure 3: Illustration of Example 2

- As we discussed, we observe in Figures 3b and 3d that $\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) > \sum_{i=1}^n \Pi_i^2(\vec{Q}^2)$ and $\sum_{i=1}^n \Pi_i^3(\vec{Q}^3) > \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$. On the other, it is possible that $\sum_{i=1}^n \Pi_i^2(\vec{Q}^2) > \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$ or $\sum_{i=1}^n \Pi_i^2(\vec{Q}^2) < \sum_{i=1}^n \Pi_i^1(\vec{Q}^1)$.

Based on these observations, we note that a simple wholesale pricing schedule offered by the supplier can fail to increase channel-wide profits unless the supplier imposes constraints on the retail chain's strategy. The next section analyzes the supplier's pricing problem.

5 The Supplier's Pricing Problem

In this section, we first formulate the supplier's pricing problem as a Mathematical Programming problem with Equilibrium Conditions (MPEC) in the general retailing settings. Recall from Equation (4) that the supplier's profit function, $\Pi_S(\lambda)$, depends on the retail chain's retail and procurement strategy and the resulting store order quantities. Thus, we will reformulate the supplier's pricing problem for each retail and procurement strategy. A generic solution approach is also discussed.

For given λ and β values, it was noted in Section 4 that there exists a unique solution for stores' order quantity decisions, and $\vec{\mathbf{Q}}^s$ denotes the n -vector of stores' order quantities under the retail chain strategy s , $s \in \{1, 2, 3\}$. Given that β satisfies Assumption 2(ii), the supplier's problem of determining the base wholesale price λ for retail strategy s can be formulated as follows:

$$\max_{\lambda \geq 0} \left\{ \sum_{i=1}^n \left(c(\vec{\mathbf{Q}}^s) q_i^s \right) - f(Q^{(s)}) | (\mathbf{M}^s \vec{\mathbf{Q}}^s - \mathbf{z}) (\vec{\mathbf{Q}} - \vec{\mathbf{Q}}^s) \geq 0, \forall \vec{\mathbf{Q}} \in \mathcal{K} \right\}, \quad (15)$$

where the definitions of $\mathbf{M}^s \forall s \in \{1, 2, 3\}$, $\mathbf{z}$, and $\mathcal{K}$ are given Section 4 for each $s = \{1, 2, 3\}$. Equation (15) defines an MPEC with a nonlinear objective function, where the constraints are defined by the asymmetric linear variational inequality formulation of the stores' order quantity decisions. We note that Equation (15) corresponds to an NP-hard problem, and we refer the reader to Luo et al. (1996), Outrata et al. (1998), Scheel and Scholtes (2000), Dempe (2003), and the references therein, for further discussion on MPECs.

In the case where the retail chain's strategy is decentralized retailing with individual procurement, the supplier's pricing problem should account for this strategy. In particular, given $\beta < b_{ii}$ and market capacities that are sufficiently large for each store such that $q_i^1 > 0 \forall i \in I$, it follows from Equation (6) that the supplier's base wholesale price setting problem given in Equation (15) reads as

$$\begin{aligned} \max_{\lambda \geq 0} \quad & \sum_{i=1}^n (a_i - w_i) q_i^1 - \sum_{i=1}^n \sum_{j=1}^n b_{ij} q_j^1 q_i^1 - \sum_{i=1}^n (b_{ii} - \beta) (q_i^1)^2 \\ \text{s.t.} \quad & a_i - \sum_{j=1}^n b_{ij} q_j^1 - \lambda - (b_{ii} - 2\beta) q_i^1 - w_i = 0, \forall i \in I. \end{aligned}$$

In the case where the retail chain's strategy is decentralized retailing with joint procurement, the supplier's pricing problem should be set accordingly. In particular, given $\beta < b_{ii}$ and market capacities that are sufficiently large for each store such that $q_i^2 > 0 \forall i \in I$, it follows from Equation (9) that the supplier's base wholesale price setting problem given in Equation (15) reads as

$$\begin{aligned} \max_{\lambda \geq 0} \quad & \sum_{i=1}^n (a_i - w_i) q_i^2 - \sum_{i=1}^n \sum_{j=1}^n b_{ij} q_j^2 q_i^2 - \sum_{i=1}^n (b_{ii} - \beta) (q_i^2)^2 \\ \text{s.t.} \quad & a_i - \sum_{j=1}^n (b_{ij} - \beta) q_j^2 - \lambda - (b_{ii} - \beta) q_i^2 - w_i = 0, \forall i \in I. \end{aligned}$$

Finally, in the case where the retail chain's strategy is centralized retailing, the supplier should solve its pricing problem for centralized retail stores. In particular, given $\beta < b_{ii}$ and market capacities that are sufficiently large for each store such that $q_i^3 > 0 \forall i \in I$, it follows from Equation (12) that the supplier's base wholesale price setting problem given in Equation (15) reads as

$$\begin{aligned}
& \max_{\lambda \geq 0} \quad \sum_{i=1}^n (a_i - w_i) q_i^3 - \sum_{i=1}^n \sum_{j=1}^n (b_{ij} + b_{ji} - \beta) q_j^3 q_i^3 \\
& \text{s.t.} \quad a_i - \sum_{j=1}^n b_{ij} q_j^3 - \sum_{j=1}^n b_{ji} q_j^3 - \lambda + 2\beta \sum_{j=1}^n q_j^3 - w_i = 0, \forall i \in I.
\end{aligned}$$

The next Corollary follows from Equations (6), (9), and (12), and Corollary 4.

Corollary 6 *Given $\beta < b_{ii}$, suppose that $q_i^s > 0 \forall i \in I$ and $\forall s \in \{1, 2, 3\}$ for $0 \leq \lambda \leq \Lambda_s$. Then $\sum_{i=1}^n \left(c(\vec{Q}^s) q_i^s \right) - f(Q^{(s)})$ is concave in λ over $0 \leq \lambda \leq \Lambda_s$ if $f(q)$ is convex in q .*

Given $\beta < b_{ii} \forall i \in I$, Corollary 6 states that the supplier's base wholesale price setting problem is concave over the λ values when all stores supply the market at equilibrium for a given discount factor and $f(q)$ is convex in q . Recall that we assume that $f(q)$, the supplier's production cost function, is a nondecreasing continuous and differentiable function of q . Convex production costs are observed in practice and have been studied in the literature (see, e.g., Klein, 1961, Veinott, 1964, Eliashberg and Steinberg, 1987, Smith and Zhang, 1998). In particular, when the marginal production cost increases in the production quantity, the production cost function will be convex. This can be due to increased capacity requirements (Johnson and Montgomery, 1974) or expensive overtime requirements (Smith and Zhang, 1998, Ming-hui and Cheng-xiu, 2005). Corollary 6 further implies concavity of the supplier's base wholesale price setting problem when $f(q)$ is a linear function of the production quantity.

Corollary 6 follows from the fact that $Q^{(s)}$ and $q_i^s \forall i \in I$ are linearly decreasing with λ . It then follows that there is a λ value that maximizes the supplier's profit for any retail and procurement strategy adopted by the chain store. Therefore, one can use the first-order optimality conditions to check the optimality of such a given λ value. However, checking whether a λ value satisfies the first-order condition is not possible because an explicit formula is not available for the derivatives of q_i^s with respect to λ . Therefore, we suggest the use of a linear approximation method to find a near optimal λ value. Specifically, this method approximates the supplier's profit function by using linear functions defined over specified ranges of λ values. In creating these linear approximations, one can use Algorithm 1 to find the value of the supplier's profit function for a given λ value. The following section discusses resulting insights on channel coordination.

6 Insights on Channel Coordination

In the channel of interest in this study, the supplier sets his/her pricing decisions and the retail chain adopts a retail and procurement strategy considering the stores' reactions under each strategy. In particular, we have analyzed three different strategies: decentralized retailing with individual

procurement, decentralized retailing with joint procurement, and centralized retailing. Let C_1 , C_2 , and C_3 define the retail chain's strategy of decentralized control of its stores with individual procurement, decentralized control of its stores with joint procurement, and centralized control of its stores, respectively. This, in turn, indicates that the supplier also has three strategies: price setting for decentralized retailing with individual procurement (S_1), price setting for decentralized retailing with joint procurement (S_2), and price setting for centralized retailing (S_3).

Depending on the sequence of decisions, we might have two Stackelberg games: (i) the supplier is the leader and the retail chain is the follower and (ii) the retail chain is the leader and the supplier is the follower. In both of the Stackelberg games (i) and (ii), there are 9 possible outcomes, shown in Table 2, each of which corresponds to a pair of strategies, denoted as (S_u, C_v) $u \in \{1, 2, 3\}$ and $v \in \{1, 2, 3\}$. Let $\Pi_S(S_u, C_v)$, $\Pi_C(S_u, C_v)$, and $\Pi_T(S_u, C_v)$ denote the total profit of the supplier, retail chain, and the channel as a result of strategy selections (S_u, C_v) , respectively. In what follows, we characterize the Stackelberg-Nash equilibrium outcomes of the Stackelberg games (i) and (ii).

Table 2: Possible Outcomes of the Stackelberg Games

	Retail Chain		
	(S_1, C_1)	(S_1, C_2)	(S_1, C_3)
	(S_2, C_1)	(S_2, C_2)	(S_2, C_3)
	(S_3, C_1)	(S_3, C_2)	(S_3, C_3)

Now, suppose that the supplier is the leader, i.e., we have Stackelberg game (i) and let $(S^{(i)}, C^{(i)})$ denote its Stackelberg-Nash equilibrium outcome. As the follower, for any given value of the supplier's base wholesale price, the retail chain will prefer centralized retailing since $\Pi_C(S_u, C_3) \geq \Pi_C(S_u, C_1)$ and $\Pi_C(S_u, C_3) \geq \Pi_C(S_u, C_2) \forall u \in \{1, 2, 3\}$ as discussed in Section 4.4. This implies that $C^{(i)} = C_3$. Thus, the supplier, as the leader, will set his/her base wholesale price for centralized retailing as $\Pi_S(S_3, C_3) \geq \Pi_S(S_1, C_3)$ and $\Pi_S(S_3, C_3) \geq \Pi_S(S_2, C_3)$. That is, $S^{(i)} = S_3$ as noted in the next corollary.

Corollary 7 $(S^{(i)}, C^{(i)}) = (S_3, C_3)$.

Now, suppose that the retail chain is the leader, i.e., we have Stackelberg game (ii) and let $(S^{(ii)}, C^{(ii)})$ denote its Stackelberg-Nash equilibrium outcome. As the follower, for any given retailing and procurement strategy, the supplier will prefer to set his/her base wholesale price accordingly. Specifically, given retail chain's strategy is C_v , the supplier's best strategy is S_v as $\Pi_S(S_v, C_v) \geq \Pi_S(S_u, C_v)$ when $u \neq v$. This, in turn, implies that the retail chain will select his/her best strategy considering that the supplier will follow by setting the base wholesale price accordingly. Therefore, the Stackelberg-Nash equilibrium outcome of the Stackelberg game (ii) will be as given in the next corollary.

Corollary 8 $(S^{(ii)}, C^{(ii)}) = \arg \max\{\Pi_C(S_1, C_1), \Pi_C(S_2, C_2), \Pi_C(S_3, C_3)\}.$

Note that $\arg \max\{\Pi_C(S_1, C_1), \Pi_C(S_2, C_2), \Pi_C(S_3, C_3)\} \geq \Pi_C(S_3, C_3).$ The following observations are due to Corollaries 7 and 8.

- When the retail chain is the leader, centralized retailing or joint procurement in case of decentralized retailing is not guaranteed to be in the best interest of the retail chain. That is, C_1 can be preferred over C_2 and C_3 , and C_2 can be preferred over C_3 . This is because of the supplier's reaction to the retail chain's strategy while determining the base wholesale price.
- The retail chain has a first mover's advantage. That is, the retail chain cannot decrease the total profit of the retail stage if he/she is the first mover.
- When the supplier is the leader, the retail chain will centralize its stores. This follows from the fact that the supplier has no control over the retail chain's retail and procurement strategy selection, even if the supplier can affect the store quantity decisions with his/her wholesale price. Specifically, the supplier knows that the retail chain will centralize its stores for any given decision of the supplier. Therefore, the supplier sets the base wholesale price for centralized retailing.
- The supplier might have a first mover's advantage or disadvantage. In particular, each of the following three cases is possible:
 - $\Pi_S(S^{(i)}, C^{(i)}) > \Pi_S(S^{(ii)}, C^{(ii)})$: In this case, the supplier benefits from centralization of the retail stores by the retail chain. Therefore, the supplier has the advantage of being the first mover.
 - $\Pi_S(S^{(i)}, C^{(i)}) = \Pi_S(S^{(ii)}, C^{(ii)})$: This case is observed when it is best for the retail chain to centralize its stores in case he/she is the leader. Therefore, the supplier has no advantage or disadvantage by being the first mover. The same holds for the retail chain as well. That is, the retail chain has no advantage or disadvantage by leading the channel.
 - $\Pi_S(S^{(i)}, C^{(i)}) < \Pi_S(S^{(ii)}, C^{(ii)})$: In this case, the supplier does not benefit from centralized control of the retail chain's stores. Therefore, the supplier has the disadvantage of being the first mover.
- Neither $(S^{(i)}, C^{(i)})$ nor $(S^{(ii)}, C^{(ii)})$ is necessarily in the best interest of the channel.

The above three observations are illustrated in the following three examples. Throughout our numerical examples, we assume that the supplier's production cost function consists of a fixed setup cost and a

linear component in the production quantity, that is, $f(q) = \gamma + \alpha q$, where $\gamma \geq 0$ is the production setup cost and $\alpha > 0$ is the unit production cost³. Furthermore, for solving the supplier's base wholesale price setting problem, we use a linear approximation of λ with a precision of 10^{-5} (i.e., the solution deviates from the optimal value of λ by no more than 10^{-5}).

Example 3 Consider a retail chain with two stores such that $[a_1, a_2] = [150, 125]$, $[w_1, w_2] = [10, 15]$, $[b_{11}, b_{12}] = [2, 0.3]$, and $[b_{21}, b_{22}] = [0.3, 1.6]$. Furthermore, suppose that a supplier offers a discount factor $\beta = 0.5$ and the supplier's production cost function is defined by $f(q) = 100 + 25q$. Table 3 documents the channel statistics for this example.

Table 3: Channel Statistics for Example 3

	$\Pi_S(S_k, C_k)$	$\Pi_C(S_k, C_k)$	$\Pi_T(S_k, C_k)$
(S_1, C_1)	1308.2	648.9	1957.1
(S_2, C_2)	1332.2	621.9	1954.1
(S_3, C_3)	1426.1	594.7	2020.7

Recall that $(S^{(i)}, C^{(i)}) = (S_3, C_3)$ and, it follows from Corollary 8 that $(S^{(ii)}, C^{(ii)}) = (S_1, C_1)$ for Example 3 based on Table 3. As can be observed from Table 3, $\Pi_S(S_3, C_3) > \Pi_S(S_1, C_1)$ and $\Pi_C(S_3, C_3) < \Pi_C(S_1, C_1)$, which means that being the first mover is advantageous for both the supplier and the retail chain. However, the total chain's profit is maximized when the supplier leads.

Example 4 Consider the retail chain of Example 3 and suppose that a supplier offers a discount factor of $\beta = 0.1$ and the supplier's production cost function is defined by $f(q) = 200 + 5q$. Table 4 documents the channel statistics for this example.

Table 4: Channel Statistics for Example 4

	$\Pi_S(S_k, C_k)$	$\Pi_C(S_k, C_k)$	$\Pi_T(S_k, C_k)$
(S_1, C_1)	1657.8	905.6	2563.4
(S_2, C_2)	1660.5	903.2	2563.6
(S_3, C_3)	1564.0	912.0	2476.0

Again $(S^{(i)}, C^{(i)}) = (S_3, C_3)$ based on Corollary 7 and, it follows from Corollary 8 that $(S^{(ii)}, C^{(ii)}) = (S_3, C_3)$ for Example 4 based on Table 4. That is, $(S^{(i)}, C^{(i)}) = (S^{(ii)}, C^{(ii)})$; thus, being the first mover has no advantage for either party. And, unfortunately, the channel's total profit is suboptimal regardless of the leader.

³Considering Corollary 6, it should be noted that the methods discussed are valid for any increasing as well as decreasing linear production cost function.

Example 5 Consider the supplier of Example 4 and suppose that he/she sells through a retail chain with two stores such that $[a_1, a_2] = [250, 200]$, $[w_1, w_2] = [20, 16]$, $[b_{11}, b_{12}] = [1.5, 0.2]$, $[b_{21}, b_{22}] = [0.2, 3]$. Table 5 documents the channel statistics for this example.

Table 5: Channel Statistics for Example 5

	$\Pi_S(S_k, C_k)$	$\Pi_C(S_k, C_k)$	$\Pi_T(S_k, C_k)$
(S_1, C_1)	5218.8	2674.9	7893.7
(S_2, C_2)	5236.4	2669.2	7905.6
(S_3, C_3)	5132.7	2670.8	7803.5

We know that $(S^{(i)}, C^{(i)}) = (S_3, C_3)$ and one can note that $(S^{(ii)}, C^{(ii)}) = (S_1, C_1)$. It can be observed from Table 5 that both $\Pi_S(S_3, C_3) < \Pi_S(S_1, C_1)$ and $\Pi_C(S_3, C_3) < \Pi_C(S_1, C_1)$. This implies that being the first mover is not an advantage for the supplier (while it is an advantage for the retail chain, which we already know from Corollary 8). Furthermore, neither $(S^{(i)}, C^{(i)})$ nor $(S^{(ii)}, C^{(ii)})$ maximizes the channel's total profit.

The above three examples demonstrate the implications of Corollaries 7 and 8. In particular, while the retail chain is guaranteed to increase or maintain his/her profits by leading the channel compared to the case when he/she is the follower, the supplier is not guaranteed to achieve an increase in his/her profits by being the leader. In fact, it is possible that the supplier would not prefer being the first mover; hence, in such a case, the supplier might ask the retail chain to act as the first mover. Furthermore, since the retail chain will not be worse off by acting first mover, he/she would accept the supplier's offer. In terms of the channel's total profit, on the other hand, it is possible that the maximum value will not be achieved under the lead of any party. That is, neither $(S^{(i)}, C^{(i)})$ nor $(S^{(ii)}, C^{(ii)})$ is necessarily in the best interest of the channel-wide profits. Thus, a coordination mechanism to move the system to the channel-wide optimum can be constructed that will benefit both the supplier and the retail chain. Such a coordination mechanism might require constraints on retail and procurement strategies.

7 Extensions

In this section, we formulate the retail chain's problem in the absence of Assumption 1, and assume that the supplier sets not only the base wholesale price, but also the discount factor and supply capacities. Furthermore, we consider m planning periods such that planning periods are indexed by $\ell \in L = \{1, 2, \dots, m\}$. Suppose the supplier first announces his/her base wholesale price, discount factor, and supply limit in each period; then, the retail chain determines its order quantities as well as the market supply quantities to the end customer market. Let λ_ℓ , β_ℓ , and $\widehat{Q}_\ell$ be the supplier's base wholesale price,

discount factor, and supply limit in period ℓ . Moreover, let $q_{i\ell}^o$ and $q_{i\ell}^s$ denote the order quantity and market supply quantity for store i at the beginning of period ℓ , respectively.

We first state the retail stage problem. Here we assume that the stores are subject to inbound and outbound operating costs and let $w_{i\ell}^o$ and $w_{i\ell}^s$ denote store i 's unit operating cost in planning period ℓ for ordering from the supplier and supplying to the end customer market, respectively. Furthermore, the market price function is assumed to vary each planning period such that $a_{i\ell}$ and $b_{ij\ell}$ are the maximum demand at store i and sensitivity of store i 's customers to the availability of the product at store j in planning period ℓ . Finally, we assume that each store can carry inventory to the next period and let $h_{i\ell}$ denote store i 's unit inventory carrying cost for products not supplied to the end customer market by the end of period ℓ .

Under decentralized retailing with individual procurement, the Cournot oligopoly among the stores can be represented as follows:

$$\begin{aligned} \max_{\vec{q}_i^o \geq \mathbf{0}, \vec{q}_i^s \geq \mathbf{0}} \quad & \sum_{\ell=1}^m \left[\left(a_{i\ell} - \sum_{j=1}^n b_{ij\ell} q_{j\ell}^s \right) q_{i\ell}^s - (\lambda_\ell - \beta_\ell q_{i\ell}^o) q_{i\ell}^o - h_{i\ell} (q_{i\ell}^o - q_{i\ell}^s) - w_{i\ell}^o q_{i\ell}^o - w_{i\ell}^s q_{i\ell}^s \right] \quad \forall i \in I \\ \text{s.t.} \quad & q_{i0}^o = q_{i0}^s = 0 \quad \forall i \in I \\ & q_{i\ell}^s \leq \sum_{r=1}^{\ell} q_{ir}^o - \sum_{r=1}^{\ell-1} q_{ir}^s \quad \forall i \in I, \forall \ell \in L \\ & \sum_{j=1}^n q_{j\ell}^o \leq \bar{Q}_\ell \quad \forall \ell \in L, \end{aligned}$$

where each store i maximizes his/her profit over the planning horizon, equal to the sum of the profits in each planning period, where the profit in a given period equals the revenue gained from the market (the first term inside the squared brackets) minus the purchase cost (the second term inside the squared brackets) minus inventory carrying cost (the third term inside the squared brackets) minus inbound and outbound operating costs (the fourth and fifth terms inside the squared brackets, respectively). The first set of constraints requires initial inventory of 0 at each store. The second set of constraints correspond to limits on the market supply quantities for each store in every period (such that, in a planning period, a store cannot supply more than the total quantity ordered, including the current period's order, less the total quantity that has been supplied to the market up to the planning period). Finally, the last set of constraints ensures that the cumulative order quantity of the stores does not exceed the supplier's capacity for each planning period.

Note that in the above oligopoly, the stores are not only competing with their market supply quantities but also with their order quantities, as the supplier has limited capacity in each period. Furthermore, even if Assumption 1 holds for each planning period, a store's order quantity can be larger than his/her market supply quantity in the period to take advantage of the discounted price in the period. By replacing the purchase cost for store i in period ℓ , which is $(\lambda_\ell - \beta_\ell q_{i\ell}^o) q_{i\ell}^o$ under individual procurement, with $(\lambda_\ell - \beta_\ell \sum_{j=1}^n q_{j\ell}^o) q_{i\ell}^o$, one can formulate the Cournot oligopoly in case of

decentralized retailing with joint procurement. It should be noted that the constraints of the Cournot oligopoly under decentralized retailing with joint procurement will be the same as the constraints of the Cournot oligopoly under decentralized retailing with individual procurement. Finally, in the case of centralized retailing, the retail chain would maximize the total profit of the stores assuming that $(\lambda_\ell - \beta_\ell \sum_{j=1}^n q_{j\ell}^o) q_{i\ell}^o$ is the purchase cost for store i at period ℓ .

We next state the supplier's problem. Let $\tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}})$ be the equilibrium (or optimal, if centralized retailing is adopted) order quantity for store i in period ℓ given the supplier's base whole sale prices $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_m]$, discount factors $\beta = [\beta_1, \beta_2, \dots, \beta_m]$, and supply capacities $\hat{\mathbf{Q}} = [\hat{Q}_1, \hat{Q}_2, \dots, \hat{Q}_m]$. Suppose that $f_\ell(q)$ is the supplier's production cost function in planning period ℓ . Then, for decentralized retailing with individual procurement, the supplier's problem can be formulated as follows:

$$\max_{\lambda \geq 0, \beta \geq 0, \hat{\mathbf{Q}} \geq 0} \sum_{\ell=1}^m \left[\sum_{i=1}^n \left(\lambda_\ell - \beta_\ell \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}}) \right) \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}}) - f_\ell \left(\sum_{i=1}^n \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}}) \right) \right],$$

where the supplier maximizes his/her total profit, equal to the sum of the profits in each planning period, where the profit in a planning period equals the revenue gained from selling to the stores (the first component in the squared brackets) minus the production cost (the second component in the squared brackets) in each period. For decentralized retailing with joint procurement or centralized retailing, one can formulate the supplier's problem by replacing the sales revenue component $\sum_{i=1}^n \left(\lambda_\ell - \beta_\ell \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}}) \right) \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}})$ with $\sum_{i=1}^n \left(\lambda_\ell - \beta_\ell \sum_{i=1}^n \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}}) \right) \tilde{q}_{i\ell}^o(\lambda, \beta, \hat{\mathbf{Q}})$. Finally, if the supplier has production capacities $\mathbf{P} = [P_1, P_2, \dots, P_m]$ such that P_ℓ is the production capacity in period ℓ , one would add $\hat{\mathbf{Q}} \leq \mathbf{P}$ constraints.

We note the the models presented in this section are more complex than the problems analyzed in Sections 4 and 5; however, in some cases one can use the methods discussed previously in the analyses of the problems formulated in this section. In particular, if the product is perishable and lasts for one period, the problems at the retail chain can be separated for each period, which would imply that the supplier's problem is also separable for each planning period. Then, under Assumptions 1 and 2, the problems for each planning period reduce to problems analyzed in Sections 4 and 5. We pose the extended models of this section as future research questions in Section 8. Finally, it is worth noting that Corollaries 7 and 8 still hold under the settings of this section as well. Therefore, the insights of Section 6 are still applicable.

8 Conclusions and Future Research

This study modeled a supplier's pricing decisions for sales of a good to an end customer market via a set of stores who are jointly owned by a retail chain. We considered three different retail and

procurement strategies: decentralized retailing with individual procurement, decentralized retailing with joint procurement, and centralized retailing. For each strategy, we provided equivalent variational inequality formulations and a consistent solution method to determine the stores' equilibrium or optimal quantity decisions. We contribute to the literature on supplier pricing problems for competitive retailers (i.e., when the retail stage is decentralized) by considering an arbitrary number of non-identical stores, and by considering more general forms of the supplier's production cost function.

We further contribute to the literature by analyzing and comparing centralization of the retail stage with two decentralized retailing strategies. We have noted that while centralized retailing increases total retail stage profit, decentralized retailing with joint procurement increases total supply of the product at the end customer market. On the other hand, when centralized retailing is compared to decentralized retailing with individual procurement, the total supply to the end customer market can be larger or smaller. These findings imply that the supplier may enjoy or suffer from centralization/decentralization at the retail stage and joint/individual procurement for decentralized retailers, depending on whether s/he observes economies or diseconomies of scale in production. We note that similar observations are noted in Chen and Roma (2011) and Zhou and Xie (2014) about the effects of group buying on a supplier and in Anupindi and Bassok (1999) and Shao et al. (2011) about the effects of retail stage centralization.

As the channel is assumed to be vertically centralized, the sequence of actions heavily influences both the supplier's and the retail chain's strategies. In particular, we analyzed the Stackelberg game between the supplier and the retailer under both the supplier's and the retail chain's leadership. We showed that the supplier's leadership might decrease the total profit of the retail stage and the channel as well as his/her profits. That is, the supplier might have the first mover's disadvantage. On the other hand, our results conclude that the retail chain will have the first mover's advantage. Therefore, if the supplier has the first mover's disadvantage, he/she might offer the leadership to the retail chain. Nevertheless, it is possible that the channel wide profits are not necessarily maximized under the leadership of the supplier or the retail chain. Therefore, a coordination mechanism can require constraints on retail and procurement strategies.

The setting of this study serves as a benchmark for future research directions. Studying the supplier's problem with competing stores under more general market price and store cost functions remains as a future research direction. While generalized Stackelberg games have been analyzed in the literature (see, e.g., Sherali et al., 1983, Miller et al., 1991, Tobin, 1992), these Stackelberg games assume that retailers compete with each other and one of the retailers acts as a leader. Another future research direction would involve analyzing coordination mechanisms under different procurement strategies. For decentralized, competitive retailers, pricing as a coordination mechanism has been

studied in the literature (see, e.g., Ingene and Parry, 1995, 1998, 2000, Bernstein and Federgruen, 2003, Keskinocak and Savaseneril, 2008). Our study will be helpful in the analysis of coordination mechanisms for the generalized settings considered in this paper.

Another important future research direction consists of analyzing the supplier's problem when the competitive stores can be cooperative. In the absence of a retail chain, it is still possible that the competing stores act together via buying groups/coalitions/cooperations (Keskinocak and Savaseneril, 2008, Nagarajan et al., 2010, Chen and Roma, 2011, Zhou and Xie, 2014). An interesting research question, therefore, would be to analyze a supplier's discount pricing decisions when retailers can form coalitions to take advantage of a supplier's discount pricing. Furthermore, this question can be investigated under different sequences of actions. Finally, the extended models presented in Section 7 remain as future research questions.

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