State diagram of a perpendicular magnetic tunnel junction driven by spin transfer torque: a power dissipation approach

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The state diagram of a magnetic tunnel junction with perpendicularly magnetized electrodes in the presence of spin-transfer torques is computed in a macrospin approximation using a power dissipation model. Starting from the macrospin's energy we determine the stability of energy extremum in terms of power received and dissipated, allowing the consideration of non-conservative torques associated with spin transfer and damping. The results are shown to be in agreement with those obtained by direct integration of the Landau-Lifshitz-Gilbert-Slonczewski equation. However, the power dissipation model approach is faster and shows the reason certain magnetic states are stable, such as states that are energy maxima but are stabilized by spin transfer torque. Breaking the axial system, such as by a tilted applied field or tilted anisotropy, is shown to dramatically affect the state diagrams. Finally, the influence of a higher order uniaxial anisotropy that can stabilize a canted magnetization state is considered and the results are compared to experimental data.

I. INTRODUCTION

The development of magnetic memories using spin transfer torque (STT MRAM) has led to the use of perpendicular magnetic tunnel junctions (pMTJ) to improve the write current efficiency and maintain thermally stable magnetic states as the junction lateral size is reduced [1,2]. A pMTJ consists of two magnetic layers separated by a tunnel barrier. The magnetization direction of the reference layer is considered fixed, whereas the magnetization of the free layer can be changed with a magnetic field or an applied voltage through spin transfer torque (STT) [3-5]. For perpendicular devices in the macrospin approximation, the free layer magnetization has only two stable states either parallel (P) or antiparallel (AP) to the direction of the reference layer.

The tunnel magnetoresistance [6] (TMR) of the device is used to read the information, through the change in resistance of the junction when it switches from one configuration to the other. Since more than a decade, MgO-based tunnel junctions were shown to display a TMR ratio greater than 100% after annealing [7,8], making them interesting devices for industry.

To write efficiently information in a pMTJ, using either magnetic field or spin transfer torque, the P and AP states both have to be stable. The state diagram of a device is built by determining the magnetic state (AP or P) of the pMTJ as a function of the applied voltage and field. This provides a map of the region of junction bistability, in which either P or AP states are possible. This has been extensively studied for spin-valves [9-14] but less for MTJs [17].

The spin transfer torque in MTJs depends on the voltage. There are two contributions, the Slonczewski term [3,4], which can add or reduce the effect of the damping, and the field like torque, usually negligible in spin-valve junctions, that acts as an effective field. In contrast to spin valves, in MTJs the coefficient of the Slonczewski term does not depend in the angle formed by the magnetic moments of the reference layer and the free layer [18-20].

To analytically compute the stability of each state, the Landau-Lifschitz-Gilbert equation with two STT terms is used (Eq. 1). It describes the dynamics of the magnetization under an effective magnetic field and a bias voltage:

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{d\mathbf{M}}{dt} + \frac{a_j V}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{P}) + b_j V^2 \mathbf{M} \times \mathbf{P} \quad (1)$$

With $\mathbf{M}$ the magnetization vector of the free layer, M_s the saturation magnetization of the free layer, $\mathbf{H}_{eff}$ the effective field, α the damping coefficient of the free layer, a_j and b_j respectively the Slonczewski coefficient and the field-like torque coefficient of the spin transfer torque depending on the applied voltage, and $\mathbf{P}$ the direction of magnetization of the reference layer.

To simplify the calculation we consider a macrospin model, where the magnetization of the free layer behaves uniformly. The stable states can be found by integrating the LLGS equation for of each coupled values of the applied magnetic field and applied voltage. However, these computations are time consuming.

This paper discusses and applies a different computational method, based on Cucchiara *et al* [9,25], to generate state diagrams of pMTJs as a function of applied field and voltage that instead determines the energy extrema (minima and maxima) and considers the power dissipation associated with non-conservative torques. We will compare this method for pMTJs with a macrospin LLGS simulation for several different geometries, and then we will discuss its limitations by comparing it to experimental data.

II. MODEL

We use a Lagrangian approach to the LLGS equation (eq 2) [19,20].

$$\mathcal{L} = -\frac{M_s}{\gamma_0} \dot{\varphi} \cos \theta - E_{tot}(\theta, \varphi) \quad (2)$$

The axis and angles are defined as shown in figure 1 with the polar angle θ and the azimuthal angle φ functions of time. Their time derivatives are denoted with an upper dot.

Figure 1: Definition of the axis of the model, the z -axis is the out of plane direction. θ is the angle between the out of plane axis and the magnetization $\vec{M}$. φ is the azimuthal angle.

We define two Rayleigh dissipation functions [19], to represent the non-conservative forces, namely the Gilbert damping (Eq. 3) and the Slonczewski STT (Eq. 4)

$$\mathcal{F}_\alpha = \alpha \frac{M_s}{2\gamma_0} \left(\frac{d\mathbf{m}}{dt} \right)^2 \quad (3)$$

$$\mathcal{F}_{STT} = M_s a_j V \mathbf{P} \cdot \left(\mathbf{m} \times \frac{d\mathbf{m}}{dt} \right), \quad (4)$$

with $\mathbf{m} = \mathbf{m}(\theta(t), \varphi(t))$, Using the Euler-Lagrange equations, we can express equation 1 in spherical coordinates, leading to two coupled equations. The third equation being the conservation of the norm of $\mathbf{m}$ (i.e. $|\mathbf{m}|=1$):

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial \mathcal{F}_\alpha}{\partial \dot{\theta}} + \frac{\partial \mathcal{F}_{STT}}{\partial \dot{\theta}} \quad (5)$$

To obtain the state diagram, we numerically solve equation 5 using a classical Runge-Kutta method with adaptive time steps for different applied magnetic field and bias voltage. The resulting state diagram is compared to the one obtained using a Stoner-Wohlfarth like approach, which is built up solely knowing the energy landscape.

The Stoner-Wohlfarth model [22], the minimum extrema of the energy are considered as metastable states. This model only takes into account the conservative forces. The damping torque (2nd term on the right hand side of Eq. 1) and the STT (3rd term on the RHS of Eq. 1)

are non-conservative torques, thus they can be expressed as dissipative powers. Cucchiara *et al.* used this description to compute the state diagram of spin-valves [24]. This power will stabilize the minimum (maximum) of energy when it has a negative (positive) sign.

The total energy density (E_{tot}), is composed of the demagnetization energy (E_{dem}), or shape anisotropy, the magneto crystalline (that can arise from the surface) anisotropy energy (E_{ani}) which is considered uniaxial and the Zeeman energy (E_{zee}), due to the applied field. The field like part of the STT can be expressed as a conservative torque with a field direction set by the direction of the spin-polarization (i.e. $\mathbf{P}$), with a parabolic relation to the bias voltage and linked to an energy density (E_{FLT}) that can be added to E_{tot} . See Table 1 for the associated mathematical expressions.

Energy density	Expression
E_{dem}	$\frac{\mu_0}{2} \mathbf{M} \cdot \overline{\mathbf{N}_{dem}} \cdot \mathbf{M}$
E_{ani}	$-k (\mathbf{u}_K \cdot \mathbf{m})^2$
E_{zee}	$-\mu_0 \mathbf{M} \cdot \mathbf{H}$
E_{FLT}	$-b_j V^2 \mathbf{M} \cdot \mathbf{P}$

Table 1: Expressions for the free layer magnetic energies associated with shape, magnetic anisotropy, applied field and the spin-transfer field-like torque.

with μ_0 the vacuum permeability, k the first order anisotropy constant, $\mathbf{u}_K$ the uniaxial anisotropy direction, $\mathbf{H}$ the applied field. As in [22], to obtain the dissipative power, we express the damping torque (eq.7) and the Sloncsweski torque (eq.8) as dissipation power, using the LL approach, avoiding integro-differential equations.

$$\mathbf{\Gamma}_{STT} = \frac{a_j V}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{P}) \quad (7)$$

$$\mathbf{\Gamma}_\alpha = \gamma_0 \frac{\alpha}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{eff}). \quad (8)$$

When the polarizer (i.e. $\mathbf{P}$) is aligned with the effective field, the spin transfer torque can either add to the damping or compensate it. From the total energy density E_{tot} , we obtain the expression of the effective field $\mathbf{H}_{eff}$, to which the precession vector is proportional. The total dissipated power P_{tot} (eq 9) is composed of two part, one representing the power dissipated by the damping (1st term on the RHS of eq. 9) and always negative and the other (2nd term on the RHS of eq. 9) the power dissipated or brought by the Sloncsweski term of the STT, which sign depends on the sign of the applied tension V

$$P_{tot} = -\gamma_0 \frac{\alpha}{M_s} \mu_0 (\mathbf{M} \cdot \mathbf{H}_{eff})^2 + \frac{a_j V}{M_s} \gamma_0 \mu_0 \mathbf{H}_{eff} \cdot (\mathbf{M} \times (\mathbf{M} \times \mathbf{P})) \quad (11)$$

The dissipative power expressions and the Rayleigh dissipation functions have the same meaning. But because Eqs.7 and 8 are written using the LL approach, avoiding integro-differential equations, and Eqs. 3 and 4 are written in the Gilbert framework, they correspond exactly to each other with the usual approximation that the damping and the Sloncsweski STT can be neglected in the expression of the rotation vector.

The competition between the two dissipated power terms will determine the stable state, as the extremum of energy stabilized will depend on the sign of the total power. A negative power means that energy is dissipated to the surroundings. With this convention, the minimum of energy is stabilized by a power with negative sign and a maximum of energy is stabilized by a positive power.

For any given applied magnetic field and voltage, at the equilibrium, the power of the non-conservative forces is zero. If it is also an extremum of the power function, we consider the sign of the power as the sign of its second derivative (curvature). This gives the same resulting state diagram as that obtained by integrating the total power on a circle close to the equilibrium position value for each $P_{tot}(H, V)$.

In the fully perpendicular case, where $\mathbf{H}$, $\mathbf{P}$ and $\mathbf{u}_K$ are along the z axis (Fig. 1), the coercive magnetic field H_c and the critical voltage V_c can be found analytically and are given by:

$$\mu_0 H_c = -b_j V^2 + (M_s - \frac{2k}{\mu M_s}) \cos \theta \quad (12)$$

$$b_j V_c^2 + \frac{a_j}{\alpha} V_c = -\mu_0 H + (M_s - \frac{2k}{\mu M_s}) \cos \theta \quad (13)$$

The direction of the reference layer magnetization is along the out-of-plane axis. In the parallel (antiparallel) state $\theta = 0$ ($\theta = \pi$).

In the following we first consider the fully perpendicular case, described previously, where the effective field axis is perpendicular to the film plane. We then break the symmetry in different ways: tilting the field, tilting the anisotropy axis, and adding a second order uniaxial anisotropy that can stabilize a canted magnetization state.

We used the following material parameters corresponding to a typical CoFeB/MgO pMTJs layer, based on the literature value closer to our experimental data. A saturation magnetization of $M_s = 0.95 \times 10^6 \text{ A} \cdot \text{m}^{-1}$ [15,16], anisotropy constant $k = 0.75 \times 10^6 \text{ J} \cdot \text{m}^{-3}$ [15,16] and damping $\alpha = 0.01$. The STT parameters are $a_j = 0.008 \text{ T} \cdot \text{V}^{-1}$ [17] for the non-conservative torque and $b_j = 0.03 \text{ T} \cdot \text{V}^{-2}$ [17] for the field like torque.

For the energy-based diagrams, we compute the extrema of energy and consider the sign of the power for each of them. For the LLGS-based diagrams, we integrate equations of motion of the magnetization for varied applied voltage for each applied field.

III. STATE DIAGRAMS

A. All perpendicular

This is a case of high symmetry. The applied field direction is out of plane, as is the magnetization of the free and reference layer. In figure 2, we define the difference between the minimum (yellow or blue dot) and the maximum value of energy (black) for $H = 0 \text{ T}$ and $V = 0 \text{ V}$ as E_0 the minimum energy barrier of the system. It is used to normalize the energy values.

Figure 2: Energy landscape normalized by E_0 as a function of θ , for $\varphi = 0$. The dashed lines represent the energy landscape for each applied field H , from $H = 3/2(M_s - 2k/\mu M_s)$, the line with larger dash, to $H = -3/2(M_s - 2k/\mu M_s)$, the line with smaller dots, in steps of $1/2(M_s - 2k/\mu M_s)$. The colored full (empty) dots represent the minimum (maximum) of energy. In yellow (blue) the parallel (antiparallel) state. In black is a local maximum.

We distinguish five energy extrema in the energy landscape (Fig.2). P (in yellow) and AP (in blue), can either be a minimum (full dot) or a maximum (empty dot) of energy. There is also a local maximum for fields less than the coercive field. The difference between the height of the local maximum and both minima represents an energy barrier that has to be overcome to switch the magnetization of the free layer.

We compared the energy based state diagram (Fig. 3a) and the LLGS direct integration model state diagram (Fig. 3b) in Fig. 3.

Figure 3: State diagram for a pMTJ with an out-of-plane applied field. (a) Energy based state diagram, in yellow (blue) the P (AP) state. The green region is the bistable (P/AP) zone. The non-hatched regions are states stabilized as minima of energy. The horizontally (vertically) hatched zone represent the zone of stability of the P (AP) state stabilized as a maximum of energy. (b) LLGS direct integration model state diagram, for the same material values, where the stability of the state P (green), AP (yellow) and P/AP (green) are computed.

The two approaches display the same stability conditions in H and V of the P and AP states and the same switching voltage (eq 13) for each transition. The energy based diagram brings additional information on the type of extremum stabilized and the corresponding switching magnetic field (eq 12). In figure 3b, the bistable area, in green, is separated in three regions, one unhatched, where the two states are both energy minima, and two hatched, where a minimum and a maximum of energy coexist.

In pMTJs, the STT coefficients a_j and b_j do not depend on the angle between the magnetization of the free and the reference layer, thus unlike spin valves [24, 25], there is no precession state possible in the all perpendicular system. The main change brought by the addition of the field-like STT is the parabolic shape of the switching magnetic field.

B. Tilted field

One way to break the cylindrical symmetry is to tilt the applied field direction away from the z axis. The tilt chosen here is 5° off the z axis. In this case, the stable states are tilted by a certain angle of the out-of-plane axis, depending linearly on the strength of the magnetic field applied. The extremum of energy are between the uniaxial axis and the direction of the applied magnetic field, for reasons of simplicity, we will still label these configurations P and AP states.

The energy based (Fig. 4a) and the LLGS (Fig. 4b) state diagrams are in agreement. The bistable area displays notches at the coercive field. They correspond to the areas where a minimum and a maximum of energy are stabilized.

Figure 4: State diagram for a pMTJ with the applied field tilted 5° from the out of plane axis. (a) Energy based state diagram, in yellow (blue) the P (AP) state. In green the bistable (P/AP) zone. The non-hatched zones are defined for states stabilized as minima of energy. The horizontally (vertically) hatched zone represent the zone of stability of the P (AP) state stabilized as a maximum of energy. (b) LLGS direct integration model state diagram, for the same material values, where the stability of the state P (yellow), AP (blue) and P/AP (green) are computed.

The energy based state diagram shows that P max (AP max) is no longer stabilized by small voltages close to the coercive field, which causes the particular shape of the bistability zone. The same behavior consistent with the shape of the bistability area present in the LLGS direct integration model diagram.

This shows that even a small tilt in the applied field can modify the state diagram considerably—which is relevant to experiment since there easily can be a small misalignment of the field and the easy magnetization axis. Consistent with the Stoner-Wohlfarth model, the coercive field value is reduced when the field is tilted.

C. Tilted axis of anisotropy

Another way to break the symmetry is to consider an anisotropy axis tilted away from the z axis. Here we have chosen a tilt angle of 1° with the z axis. In figure 5, the energy barrier E_0 is the difference of energy between the minimum state (yellow or blue dot) and the maximum of energy (yellow or blue circle), for $H = 0$ T and $V = 0$ V.

Figure 5: Energy landscape, normalized by E_0 in function of θ for a MTJ with an anisotropy axis tilted of 5° from the out-of-plane axis. Energy landscape as a function of the angle between the magnetization of the free and the reference layer. The dashed lines represent the energy landscape for each applied field H , from $H = 3/2(M_s - 2k/\mu M_s)$, the line with larger dash, to $H = -3/2(M_s - 2k/\mu M_s)$ the line with smaller dots, by steps of $1/2(M_s - 2k/\mu M_s)$. The colored full (empty) dots represent the minimum (maximum) of energy. In yellow (blue) the parallel (antiparallel) state.

The stable states are canted further from the out-of-plane axis because of the demagnetization field; the tilt of the magnetization in the ground state is greater than 5° .

The diagrams for energy based (Fig. 6a) and LLGS direct integration model (Fig. 6b) were computed and are shown in Fig. 6.

Figure 6 : State diagram for a MTJ with an anisotropy axis tilted of 5° from the out-of-plane axis, the applied field along this axis. (a) Energy based state diagram, in yellow (blue) the P (AP) state. The bistable (P/AP) zone is shown in green. The non-hatched zones are defined for states stabilized as minima of energy. The horizontally (vertically) hatched zone represent the zone of stability of the P (AP) state stabilized as a maximum of energy. The black dot ($H = -0.22\text{mT}$, $V = -1\text{mT}$) is used as a reference to study the stability of the P state in this branch. (b) LLGS direct integration model state diagram, for the same material parameters, where the stability of the state P (yellow), AP (blue) and P/AP (green) are computed.

The notches close to the coercive field for the magnetic field and voltage condition of stabilizing a minimum and its opposite maximum correspond in both models. However, in the energy based diagram, the area where AP min is stable displays vertical branches along the coercive field values, toward the stability area of P min, and the same occurs with P min toward the AP min area whereas these braches do not appear on the LLGS direct integration .

We used the LLGS direct integration model to compute the behavior of the magnetization in one of the vertical branches. For $H = -0.22\text{ T}$ and $V = -1\text{ V}$, starting from the P minimum state (green dot), figure 7 shows that the magnetization escapes after a few precession to the AP minimum (blue dot). The energy landscape (Figure 7 inset), for these applied magnetic fields and voltages, shows the shallowness of the P minimum energy well compared to the AP minimum energy well

Figure 7 : (a) In black, the trajectory of the magnetization of the free layer of pMTJ with a tilted anisotropy axis (5°), $H = -0.22T$, $V = -1V$, starting from the P min state (green) and ending in the AP min state (blue) (b) same magnetization trajectory in the corresponding energy landscape (in black), from the P min state (green) to the APmin state. In red is the position of the metastable state of the non-conservative system.

Both minima are stabilized but not the maxima in between. A small power torque on the magnetization can disturb the shallower state. A closer inspection of the magnetization trajectory in the energy landscape (Figure 7 b) shows that the magnetization trajectory does not revolve around the position of Pmin given in the energy landscape. Due to the breaking of the cylindrical symmetry, the torques are not in compensating each other at the energy extremum position, but on the slope of the energy landscape, thus the stable state considered in the state diagram is not state in which the non-conservative torques compensate each other.

In such a case, to obtain the stability of the states, one should integrate the trajectory of the magnetization around the position of compensation of the non-torques. We aimed to present the power dissipation model as a model fast and easy to compute for state diagram in system with various geometries, thus this calculation is not part of our study. This is the limit of the power dissipation model in the cases where the cylindrical symmetry is broken.

IV. SECOND ORDER UNIAXIAL ANISOTROPY

The anisotropy energy density can be written to take into account terms of higher order, where $k_1(k_2)$ is the first (second) order coefficient of anisotropy energy density.

$$E_k = -k_1 \left(\mathbf{u}_K \cdot \frac{\mathbf{M}}{M_s} \right)^2 - k_2 \left(\mathbf{u}_K \cdot \frac{\mathbf{M}}{M_s} \right)^4$$

The influence of the second order term was reported in recent experiments and it could result from the atomic structure at the interface and strains due to lattice mismatch [26,27].

When k_1 and k_2 are of opposite signs, and $0.5 < -k_2 \left(k_1 - \mu \frac{M_s^2}{2} \right)^{-1} < 1$, the ground states are canted states, CP and CAP, with the magnetization at an angle of θ_c . This angle is given by the relative contribution of the anisotropy constants: $\cos^2 \theta_c = - \left(k_1 - \mu \frac{M_s^2}{2} \right) (2 k_2)^{-1}$.

As the symmetry of the system is not broken, these canted state positions are not dependent on the azimuthal angle (ϕ). They form “easy-cone” states around the out of plane axis, where the energy of the system is independent of ϕ [28,29].

In this case we investigated the effect of a second order anisotropy constant $k_2 = 9.55 \cdot 10^5 \text{ J} \cdot \text{m}^{-3}$ [27]. The energy landscape without field or voltage contribution is described as a “sombbrero” (Fig. 9). In figure 7, the energy barrier E_0 is the difference of energy between the minimum canted state (red dot) and the maximum of energy (yellow or blue circle), ($\theta = \pi/2$) for $H = 0 \text{ T}$ and $V = 0 \text{ V}$.

Figure 8: Energy normalized by E_0 vs. angle in the case of a second order anisotropy with no applied voltage ($V=0$). The dashed lines represent the energy landscape for each applied field H , from $H = 3/2(M_s - 2k_1/\mu M_s - 4k_2/\mu M_s)$, the line with larger dash, to $H = -3/2(M_s - 2k/\mu M_s - 4k_2/\mu M_s)$ the line with smaller dots, by steps of $1/2(M_s - 2k/\mu M_s - 4k_2/\mu M_s)$. The colored dots (circles) represent the minimum (maximum) of energy in yellow (blue) the parallel (antiparallel) state. In red the CP and CAP stable minimum. In black, the local maximum states.

For these parameters, the ground states are canted states (Fig. 9a). This is also found by integrating the LLGS equation (Fig. 9b). By solving the equation, we find that stable canted states are independent of the azimuthal angle.

Figure 9: State diagram for a pMTJ with second order anisotropy $k_2 = 9.55 \cdot 10^5 \text{ J} \cdot \text{m}^{-3}$. (a) Energy based state diagram, in yellow (blue) the P (AP) state, in white the area where they are not stabilized. The horizontally (vertically) hatched zone represent the zone of stability of the P (AP) state stabilized as a maximum of energy. The non-hatched zones are defined for states stabilized as minima of energy. The red hatched zones are the stability area of the CAP (left to right hatches) and CP states (right to left hatches). (b) LLGS direct integration model state diagram, for the same material parameters, where the stability of the state P (yellow), AP (blue) and P/AP (green) are computed.

In this context, the energy based diagram show that the ground states of energy are the canted states CAP and CP (Fig. 10 a) for values of magnetic field and voltage equal to 0, these states are static but independent of φ . For other voltage values we observe a steady oscillary state around the z axis. Outside of the coercive fields, and between the switching fields, none of the energy extremum states of the energy are given as stabilized by the power. By contrast, in the LLGS direct integration model the canted states are stabilized for all applied magnetic fields between the switching voltages, but the P and AP states are stabilized for higher voltages values.

By plotting the trajectory of the magnetization versus time for a field larger than the coercive field and between the switching voltages, we compared the two diagrams. For these conditions, we can see that the magnetization settles into a canted state (Fig. 10).

Figure 10: In grey is the sphere on which the magnetization exists. All ϕ position are equal in the canted state, represented by the dashed circle for $H = -0.15$ T and $V = 0.15$ V, The black arrow represents the final magnetization position in this computation.

V. EXPERIMENTAL DATA

To compare this model to experiment, we show a state diagram from a pMTJ fabricated by IBM. It has a lateral dimension of $100 \times 240 \text{ nm}^2$. We realized a quasi-static measurement at room temperature, using a voltage sweeps with a constant rate of 100 V/s for different applied fields.

Figure 11: Experimental state diagram obtained for a nanopillar of $100 \times 240 \text{ nm}^2$. The free layer of the pMTJ is a CoFeB (0.8 nm) grown on MgO (0.9 nm), the reference layer is formed of a SAF, composed, from the tunnel barrier, by CoPd (14 nm)/Ru(0.8 nm)/CoPd(6 nm)/CoFeB(0.8 nm).

In this experimental diagram the ratio of α/a_j is of the order of 1.4 for the AP to P transition and 3.4 for the P to AP transition and the span of switching field considered is too short to estimate b_j value. The efficiency of the torque as a function of the transition, as the angle between the reference and the free layer magnetizations, was not taken into account in this model. The magnetization at saturation of the free layer is 1.25T; at room temperature, the coercive magnetic field for $V = 0$ V is about 30 mT whereas it is estimated at 90 mT at 0 K. The state diagram is not centered in 0mT due to the emitted reference layer magnetic field not being entirely compensated.

There are three areas corresponding to the parallel and anti-parallel state, and an overlapping area of both states. The bistability area of figure 11 matches the area in figure 2, 4 and 6 where P min and AP min are stable. However, areas where a minimum and a maximum are both stabilized in the model do not seem to be stabilized in the experimental diagram.

From all the presented broken symmetry case, this diagram would be closer to the canted magnetic field; however the discrepancy of the experimental data and both models could come from two origins. First, the size of the junction could be too large to behave as a macrospin. The nucleation/propagation processes, could change the stability of the maximum, as the energy to consider is not the switching energy but the nucleation energy. Secondly, thermal fluctuations could be responsible of the discrepancy. If the limit angle of stability of a maximum stable state is a small cone of solution around the stable state, then a small perturbation could make the magnetization more attracted to the minimum of energy.

VI. CONCLUSION

In this article, we compared two ways of analyzing the LLGS model, an energy based method and a direct integration of the LLGS equations. In the LLGS direct integration model, for fixed values of H and V we computed the trajectory of the magnetization as a function of the initial magnetization configuration. In the power based model, the stable states of the energy as a function of the applied field and the field like torque are computed. The Slonczewski spin torque and the damping effect on the magnetization are expressed as dissipative powers. This model allows to take into account the effects of the voltage and the change occurring in the anisotropy are directly integrated in the energy landscape [28].

For the fully perpendicular geometry and the tilted field, the state diagrams found with both approaches are in excellent accord. The LLGS direct integration model diagram shows the stability area of the P, AP and P/AP states. Why the configuration is stabilized is found with the power dissipation model. For example, a P or AP state can be a maximum of the magnetic energy that is stabilized by spin-transfer torque. In the cases where the symmetry is broken, the bistability area is reduced close to the switching field and for the area when an energy minimum and a maximum are both stabilized.

In the case of tilted anisotropy axis shows the limits of the power dissipation model in the cases where the symmetry is broken. The non-conservative torques do not compensate each other in the extremum position of the energy landscape.

In the case of a second order anisotropy, for voltages smaller than the switching voltage values, the LLGS direct integration model shows the stability of canted states. In the energy based state diagram, these canted states are stabilized for magnetic field values smaller than the coercive field and for large values no extremum state of the energy, is seen to be stabilized.

This energy based model gives complementary information to the LLGS direct integration model, on the stability of states in a macrospin model, here for a pMTJ in different geometries. Moreover based on energy dissipation analysis, it does not demand to calculate the full magnetization trajectory and is consequently much faster.

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