A new method to determine magnetic properties of the unsaturated-magnetized rotor of a novel gyro

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Abstract: A new method is proposed to determine magnetic properties of the unsaturated-magnetized, small and irregular shaped rotor of a novel gyro. The method is based on finite-element analysis and the measurements of the magnetic flux density distribution, determining magnetic parameters by comparing the magnetic flux intensity distribution differences between the modeling results under different parameters and the measured ones. Experiment on a N30 Grade NdFeB magnet shows that its residual magnetic flux density is 1.10 ± 0.01 T, and coercive field strength is 801 ± 3 kA/m, which are consistent with the given parameters of the material. The method was applied to determine the magnetic properties of the rotor of the gyro, and the magnetic properties acquired were used to predict the open-loop gyro precession frequency. The predicted precession frequency should be larger than 12.9 Hz, which is close to the experimental result 13.5 Hz. The result proves that the method is accurate in estimating the magnetic properties of the rotor of the gyro.

Keywords: magnetic properties; unsaturated-magnetized magnet; magnetic field distribution; FEM; rotational gyro

1. Introduction

Gyroscopes are widely used in navigation systems [1]. In recent years, MEMS vibration gyroscopes are boosting for **their** compact and low-cost [2], but **their performances are hard to** improve due to theoretical reasons, **with currently reported** [3] random bias drift about 0.01 °/h. Rotational gyroscopes have the ability to achieve higher precision (the bias drift of ESG reached 0.0001 °/h by the 1970s) [4, 5], while existing rotational gyros are bulky and extremely expensive [6]. In this paper, a novel rotational gyro is discussed, which is small and has the potential to achieve higher precision relative to the MEMS gyros [7].

The rotor of the gyro has a disc whose outer radius is 5 mm, inner radius is 1.5 mm and thickness is 1 mm. It is made of 2J85 Grade FeCrCo material, which is a permanent magnetic material easy to machine and sufficient to meet the magnetic requirements of the gyro. The rotor is deliberately unsaturated magnetized to achieve a weak magnetic field, which makes it easier for the rotor to start rotating. However,

this makes the magnetic parameters of the gyro rotor unknown and difficult to determine.

Magnetic parameters of the disc affect the axial driving torque and the radial constraint torque imposed on the rotor. The axial driving torque limits the maximum rotating speed of the rotor, which determines the significance of the gyroscopic effect. The radial constraint torque leads to signal couplings between different detection axes, thus its accurate estimation is critical for the gyro's signal decoupling. So, relatively precise determination of magnetic properties of the rotor is a prerequisite for the gyro analysis and signal processing.

The usual means of permanent magnet inspection can be classified as material testing or product testing. The former, by measuring the hysteresis curve, gets relatively complete technical parameters of a magnet, in order to determine the quality of a magnetic material [8-12]. This method is generally destructive and cannot be applied to the measurement of a magnetized magnet [13]. The latter, by measuring the open flux, the surface magnetic field distribution or magnetic moment of the product magnet, determine whether the magnet is identical or sufficiently close to a standard magnet. Flux meter, Gauss meter or Helmholtz coil & flux meter are generally used in the product testing. Product testing is simple and non-destructive, so it is widely used in the test of magnetized magnets [13]. The product testing methods can be applied to the measurement of an unsaturated-magnetized magnet, by comparing the target to a series of standard magnets; however, the production of unsaturated-magnetized standard magnets with known parameters is very difficult.

In this work, a new method is proposed to acquire magnetic properties of unsaturated-magnetized magnets based on magnetic field distribution and finite-element analysis (FEA). The method was proven successful in determining the magnetic parameters of a saturated-magnetized magnet (N30 Grade NdFeB). The method was applied to determine the magnetic parameters of the rotor of the gyro, which is unsaturated magnetized. And it is proven accurate by successfully predicting the open-loop precession frequency of the gyro using the magnetic parameters obtained through the method.

2. Theory and method description

A permanent magnet generates a magnetic field in the surrounding space, the distribution of which has certain features, associated with the magnetic properties of the magnet. According to Maxwell's equations, the three dimensional static magnetic field can be solved using [14]

$$\begin{cases} \nabla \times H(x, y, z) = J(x, y, z) \\ \nabla \cdot B(x, y, z) = 0 \end{cases}, \quad (1)$$

where $H(x, y, z)$ is the magnetic field strength, $B(x, y, z)$ is the magnetic flux density, and $J(x, y, z)$ is the conduction current density. For permanent magnet models, the conduction current density is zero. The equations are as follows.

$$\begin{cases} \nabla \times H(x, y, z) = 0 \\ \nabla \cdot B(x, y, z) = 0 \end{cases}. \quad (2)$$

For a permanent magnet, the following constitutive relationship between the magnetic flux density and magnetic field is applied:

$$B = \mu_0 \mu_r H + \mu_0 M_p, \quad (3)$$

where μ_0 is the permeability of vacuum, μ_r is the relative permeability, and M_p is the permanent magnetization [14, 15].

The space distribution of magnetic field around a magnet can be determined through finite-element (FE) modeling based on the above theories. The strongest magnetic field at a certain distance from the surface of magnets can be measured by a gauss magnetometer, as is shown in Fig. 1. By adjusting the measuring distance, the peak magnetic flux density distribution curve can be acquired. The magnetic parameters of a magnet can be represented by this magnetic field distribution curve. And the magnetic field distribution of a parameters-known magnet can be solved by FEM. So, the parameters of the measured magnet can be determined by comparing the results of FE modeling and the measurement.

Fig. 1. Schematic diagram of the strongest magnetic field distribution measurement, (a) the 3D view and (b) the Side view.

A rough procedure of the acquisition of magnetic properties is shown in Fig. 2. First, **we build a static magnetic model**, using a set of initial parameters, and get **its flux density distribution results** through FEA. The error between the FEA results and the measurement results **is evaluated as the root mean squared error (RMSE)**. If the error of this model is smaller than last one, adjust the parameters and repeat the procedure again; if the error of this model is larger than last one, then clearly the estimated value of magnetic properties should be between the last two model parameters. The RMSEs for the FEA results are calculated according to Eq. 4.

$$err = \sqrt{\frac{\sum_{i=1}^n (S_i - M_i)^2}{n}}, \quad (4)$$

where n is the measured position point number, S_i ($i=1, 2, \dots, n$) are the simulation results, M_i ($i=1, 2, \dots, n$) are the measurement results, and err is the RMSE.

Fig. 2. Flow chart of magnetic properties acquisition.

The dichotomy is used in the choosing of parameters, in order to improve efficiency, and to reduce the parameter selection range until the desired accuracy is met. By increasing FEA meshing density and the solution precision, the accuracy of this magnetic parameter estimation method can be further improved.

3. Experiments and Results

The method was first applied to the parameters-known saturated-magnetized magnet to verify the correctness of the method itself. Then it is used to obtain magnetic properties of the unsaturated-magnetized rotor of the gyro.

3.1 Verify the method using well known magnet.

In order to test the method, a cubic saturated-magnetized N30 Grade NdFeB magnet was chosen. The parameters of N30 are well known, and it is a linear permanent magnetic material, whose properties can be set by the coercivity, H_c , and the remnant flux density, B_r , making it easy to calculate and clear to compare with the measurement results.

The measurements were carried out at room temperature, using a gauss meter which has a precision of 10 nT. During the measurement, the central axis of the sensor was kept to coincide with the magnetization direction of the magnet by adjusting the orientation of the magnet to find the strongest flux density at a certain distance. The magnet was moved to different distances along the central axis of the sensor, and the maximum flux densities there were measured.

FEA models were built according to the measurement condition, as is shown in Fig. 3. Due to the symmetry of the geometry and the magnetic field, only a quarter of the geometry needs to be modeled. And the solution domain is also reduced to 1/4, in order to improve the computing speed, and to reduce the memory consumption, which is helpful to achieve a more accurate calculation. The spherical surface of the region is set to be the Neumann boundary condition; by doing this the model needs a large region to improve precision. The two flat surfaces are set to be the Odd symmetry

condition. The distribution of the flux density near the magnet can be calculated as shown in Fig. 3.

Fig. 3. Flux density distribution near the magnet calculated through FE model.

According to the given parameters of N30, Br is between 1.08 T and 1.16 T and H_c not less than 796 kA/m [16]. We set $Br = 1.08$ T and $H_c = 796$ kA/m as the start point, and change H_c to obtain different results. The results for different H_c s while $Br = 1.08$ T were compared with the measurement results to find the optimal H_c as is shown in Fig. 4a. It is hard to determine which is optimal from the figure, especially when FEA results are close to the measurement. The RMSEs for the results of different H_c s are calculated according to Eq. 4. And the RMSEs of some FE modeling results for different H_c s are shown in Fig. 4b.

Fig. 4 (a). Measurement and FEA results of the flux density distribution of the N30 magnet and (b) the RMSEs of FEA results for different H_c s, while Br is 1.08 T.

By the same way, the RMSEs for different combinations of Br and H_c were obtained as shown in Fig. 5. The minimum of RMSEs appears in the bottom of the plot. According to FE modeling, the parameters of the magnet should be $Br=1.10$ T with the uncertainty within 0.02 T, and $H_c=801$ kA/m with the uncertainty within 5 kA/m.

Fig. 5. The RMSEs of FEA modeling results for different H_c s and Br s.

Fig. 6. The minimum RMSEs of FEA modeling results for different H_c s against permeability.

The result can be refined by **getting** a better knowledge of the relative recoil permeability of the magnet as is shown in Fig. 6. Normally, the relative recoil permeability of N30 material is between 1.05 and 1.10. According to FE

modeling, the parameters of the magnet should be $Br=1.09$ T with the uncertainty within 0.01 T, and $H_c=803$ kA/m with the uncertainty within 3 kA/m. The RMSE between the measurement and the FEA results of **these parameters** is within 0.24 uT. This indicates that the method is of high accuracy in the magnetic parameter prediction of N30 Grade NdFeB in regular shape. And the precision can be further improved by a better knowledge of the relative recoil permeability, **acquired** by measuring the magnets in the same batch for example.

3.2 Determination of magnetic properties of the rotor

The rotor of the gyro is made of 2J85 Grade FeCrCo material, which is a nonlinear permanent magnet. Different from linear permanent magnets, nonlinear permanent magnets cannot be defined simply by H_c and Br [17, 18]. In order to have a whole understanding of the material, the dynamic hysteresis loop cluster of it is obtained by measuring pieces cut from the same 2J85 rod, as demonstrated in Fig. 7. And the different status of magnetization can be described by different magnetization curves distinguished by either coercive field strength (here we still use H_c as the symbol of it) or residual magnetic flux density (Corresponds to the Br in saturation magnetization).

Fig. 7. Schematic diagram of the dynamic hysteresis loop cluster of the magnetic material of the rotor disc.

As described in the measurement of cubic NdFeB magnet, we first measured the maximum flux density at series of distances using gauss meter. FE models were then built according to the measurement condition. The magnetic properties of the rotor disc were set according to different magnetization curve with different H_c s, as shown in Fig. 7. Some of the results together with the measurement results are shown in Fig. 8.

Fig. 8. Measurements and FEA results of the distribution of the flux density of the rotor

The RMSEs of some FE modeling results for different coercive field strength were calculated according to Eq. 4 and is shown in Fig. 9.

Fig.9. The RMSEs of FEA modeling results for different H_c s

The RMSEs are almost symmetrical about $H_c=19.1$ kA/m, and the minimum of RMSEs appears in the range between 19.0 kA/m and 19.2 kA/m, so we take 19.1 kA/m as H_c . According to FE modeling, the coercive field strength equals 19.1 kA/m with the uncertainty within 0.1 kA/m, and the residual magnetic flux

density should be 0.54 T with the uncertainty within 0.003 T (when the obtained dynamic hysteresis loop cluster is supposed to be accurate). The RMSE between the measurement and the FEA results of this parameter is within 0.27 uT. It is sufficient for basic analysis of the gyro.

4. Discussion

The magnetic properties acquired above were used in the modeling of the gyro to predict the open-loop precession frequency, and that was testified by experiments.

4.1 Related knowledge of the gyro

The mechanical structure of the gyro is **mainly composed of a rotor, a stator and coils**, as is shown in Fig. 10. Alternating currents are imposed on the coils to generate a rotating magnetic field which pulls the rotor to rotate at high speed. The rotor and the detection electrodes form a differential capacitor structure. The radial tilt angles of the rotor reflect the input angular velocity, and they are measured through a differential capacitor detection circuit [19, 20].

Fig. 10. The mechanical structure of the gyro.

When the rotor tilts radially, it is constrained radially by a magnetic torque generated by the stator and the coils. When the rotation speed is high and the tilt angle is small (below 2 °), this constraint torque is equivalent to an elastic torque, which can be expressed as

$$M = -C\varepsilon, \quad (5)$$

where M is the constraint torque, ε is the radial tilt angle, and C is the elasticity coefficient. C is related to the magnetic properties of the rotor, stator structure, exciting currents and the rotating speed. This tilt situation can be modeled, and C can be calculated through FEM.

4.2 Dynamics of the rotor

According to the principle of double **degrees of freedom (DOFs)** rotational gyro, under the condition of small angular movement, when elastic torque exists on the sensitive axis, the radial angular motion of the rotor can be expressed as

$$\begin{cases} -I_x \ddot{\beta} + H \dot{\alpha} - C_x \beta = M_x^* \\ I_y \ddot{\alpha} + H \dot{\beta} + C_y \alpha = M_y^* \end{cases}, \quad (6)$$

where I_x and I_y are the rotational inertias of the rotor around the X axis and Y axis respectively; α and β are the tilt angles of the rotor about the Y axis and negative X axis respectively, as shown in Fig. 11; H is the angular momentum of the rotor about the Z axis; C_x and C_y are elasticity coefficients about the X axis and Y axis respectively; M_x^* and M_y^* are external torques about the X axis and Y axis respectively [21].

Fig. 11. Coordinates definition of the gyro when the rotor tilts.

In open loop situation, only elastic torque taken into consideration, the torques $M_x^* = M_y^* = 0$. The nutation is weak compared with other torques so it is proper to set $I_x \ddot{\beta} = I_y \ddot{\alpha} = 0$. Eq. 6 can be simplified as Eq. 7.

$$\begin{cases} H \dot{\alpha} - C_x \beta = 0 \\ H \dot{\beta} + C_y \alpha = 0 \end{cases}. \quad (7)$$

When $t=0$, $\alpha(0) = \alpha_0$, $\beta(0) = \beta_0$. Assuming $C_x = C_y = C$, solve Eq. 7, then we get

$$\begin{cases} \beta = \varepsilon_0 \cos(\omega_0 t + \varphi_0) \\ \alpha = \varepsilon_0 \sin(\omega_0 t + \varphi_0) \end{cases}. \quad (8)$$

It is clear the precession angles have a periodic oscillation form, where $\varepsilon_0 = \sqrt{\alpha_0^2 + \beta_0^2}$ is the angle amplitude of the oscillation, $\omega_0 = C/H$ is the angular frequency, and $\varphi_0 = \tan^{-1}(\beta_0/\alpha_0)$ is the initial phase.

So, in open loop situation, the rotor starts precession at a certain frequency after a

tilt angle shock. The angular frequency is decided by the elasticity coefficient divided by the angular momentum of the rotor. And the frequency is the angular frequency divided by 2π , shown as follows.

$$f = C/(2\pi H). \quad (9)$$

4.3 Measurement of the precession frequency

In open loop situation, there is no servo torque, so all the elastic torque comes from magnetic interaction between the rotor and the stator. Fig. 12 shows one axis output of the detection circuit after an impulse tilt angle input when the rotating speed is 15306 rpm, and the total average current in the coils is 110 mA.

Fig. 12. Output of the detection circuit for one axis.

The output signal takes the form of a damped oscillation [22], as shown in Fig. 12. The precession angle presents a rapid decay due to the damping effect; however, the attenuation does not affect the oscillation frequency. The frequency of the wave shown in Fig. 12 is measured to be 13.5 Hz.

4.4 Calculation of the precession frequency

As shown in Fig.11, FE models were built to study the relationship between the tilt angle of the rotor and the radial torque imposed on it, by setting the exciting current (110 mA) to the coils and the magnetic properties, obtained above, to the rotor. The results showed that the constraint torque is proportional to the tilt angle when the tilt angle is small (below 2 °), and the elasticity coefficient can be determined as shown in Eq. 5. As the rotor rotates at high speed, the elasticity coefficient for calculation should be an average value during a rotating cycle, which is obtained by setting the rotor to be at different rotating angles. At the highest speed, the elasticity coefficient

calculated through FEA is $1.0 \times 10^{-3} \text{ N} \cdot \text{m/rad}$. Considering the fact that the actual working speed is lower than the highest speed (the gyro is unstable at maximum speed), the calculated elastic coefficient C is greater than $1.0 \times 10^{-3} \text{ N} \cdot \text{m/rad}$.

When the rotating speed is 15306 rpm, the minimum frequency of the precession calculated using Eq. 9 is 12.9 Hz. This result is smaller than the measured frequency 13.5 Hz, but is close enough. The result testified that **the method described above** was accurate in acquiring the magnetic properties of the non-saturated magnetized rotor of the gyro.

5. Conclusions

In this work, a new method is proposed to determine the magnetic properties of the unsaturated-magnetized rotor of a novel gyro. The rotor is small, **of an irregular shape**, and intentionally unsaturated-magnetized, which makes its magnetic properties difficult to determine by usual means. The method has been confirmed by experimenting on a parameter-known NdFeB magnet, and was applied to determine the rotor's magnetic properties. **The obtained magnetic properties were** used to predict the open-loop gyro precession frequency, which is consistent with the experimental result. This result proved that the method is accurate in estimating the magnetic properties of the rotor.

The method proposed is simple to conduct, suitable for saturated and unsaturated magnetized magnet, having no requirements for a certain shape and volume. The results are determined **not only by one single value, but by the distribution of values for all points**, thus by increasing the number of measurement points, and calibrating the system errors, the estimation accuracy of the magnetic parameters can be further improved.

Acknowledgement

The authors would like to thank National Natural Basic Research program of China (No. 2012CB934104), National Science Foundation of China (No. 61071037), National Science Foundation of Heilongjiang Province of China (No. F201418), and Fundamental Research Funds for the Central Universities (No. HIT.NSRIF.2014040) for financial support.

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