Magnetic phase shift reconstruction for uniformly magnetized nanowires

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Abstract

A new analytical model is developed for the magnetic phase shift of uniformly magnetized nanowires with ideal cylindrical geometry. The model is applied to experimental data from off-axis electron holography measurements of the phase shift of CoFeB nanowires, and the saturation induction of a selected wire, as well as its radius, aspect ratio, position and orientation, are determined by fitting the model parameters. The saturation induction value of 1.7 T of the CoFeB nanowire is found to be similar, to within the measurement error, to values reported in the literature.

Keywords: Electron Holography; Magnetic Nanowire; Magnetic Phase Reconstruction; Phase Shift Model.

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1. Introduction

One-dimensional (1D) nanomaterials have attracted considerable attention in the past two decades due to their potential to miniaturize electronic devices. Among them, ferromagnetic nanowires (NWs) are of particular interest because of their unique physical properties and possible applications in magnetic recording [1, 2], spintronics [3], microwave materials [4], and sensor devices [5, 6]. Moreover, single phase, or uniform composition electrodeposited magnetic NWs are desirable candidates for high-density storage media [7] due to their low fabrication cost. Arrays of ferromagnetic NWs can be made by electrodeposition into a nanoporous alumina membrane [8]; by adjusting the composition and geometric parameters of the array it is possible to alter the magnetic properties of the NWs [9, 10]. The macroscopic properties of the NW arrays are highly dependent upon the uniformity of the individual NW diameter, composition, and crystal structure [8, 11]. Understanding the effects of these geometric parameters on the magnetic remanence and the effective magnetic anisotropy of individual and arrays of magnetic NWs, is crucial to their design for a given application [8].

Conventional transmission electron microscopy (TEM) imaging modes provide powerful modalities for characterizing the microstructure of materials. Typically, however, only intensity information is recorded and the information about the phase of the electron wave after it has traveled through the specimen is lost. Electron holography (EH) is a unique technique that can be used to determine both the amplitude and phase of the electron wave via electron interference [12]. This phase shift is related to the component of the magnetic induction in the plane perpendicular to the electron beam and to the electrostatic potential of the specimen. Therefore, quantitative information about magnetic and electrical fields in

and around the material may be obtained with nanometer scale spatial resolution.

With off-axis EH one can study the remanent state of nanoscale magnetic materials. Combined with the conventional capabilities of scanning transmission electron microscopy (STEM) for the characterization of composition and nanostructure, the magnetic remanent state of individual nanoscale magnets can be determined. EH analysis has already provided some insight into the magnetic properties of individual ferromagnetic NWs (FMNWs), including their magnetic anisotropy, interface anisotropy, and multilayer coupling as a function of their size, shape and structure [13–17]. By using EH, and in particular the off-axis variant of EH, the details of the in-plane magnetization of the specimen, its amplitude and direction, can be depicted by adding contours to the phase image of the specimen [15].

In this paper, we report experimental EH measurements of the phase shift of CoFeB NWs as well as a new analytical model of the magnetic phase shift, assuming uniformly magnetized NWs of ideal cylindrical geometry. Using this model, we can extract quantitatively the saturation induction of the wire by fitting the model parameters to the experimentally measured phase shifts. We begin with a detailed derivation of the magnetic phase shift. The experimental phase shifts are then compared to the model predictions.

2. Analytical Model Derivation

The phase change of the electron wave passing through the specimen with respect to the undisturbed portion that passes through vacuum is related to the electrostatic potential in the specimen, which is typically taken to be its mean inner potential (MIP). The MIP of a material is determined by its composition

and density. In addition to the MIP, any in-plane magnetic field integrated along the incident beam direction will cause an additional phase change. For a thin and weakly diffracting specimen (i.e., when dynamical diffraction can be neglected) [18], the phase shift in one dimension can be expressed as [19]:

$$\varphi(x) = C_E \int dz V(x, z) - \iint dx dz B_{\perp}(x, z), \quad (1)$$

where z lies along the electron trajectory, C_E depends on the microscope accelerating voltage, V is the mean inner potential (MIP) and $B_{\perp}$ the component of the magnetic induction normal to the electron trajectory.

The phase shift from the above equation has both the in-plane magnetic field and the MIP contribution. To extract just the magnetic contribution of the phase, we tilt the specimen by $\pm 30^\circ$ and switch on the objective lens at each angle to apply a magnetic field parallel to the beam direction [15]. Tilting the specimen with the same angle in two opposite directions generates two equal in-plane magnetic fields with opposite directions. By taking the difference of the total phase shifts from the two holograms, one obtains twice the magnetic contribution of the specimen to the phase shift, while subtracting the phase change due to the electrostatic potential. In the remainder of this section, we derive a model for this magnetic phase shift.

[Figure 1 about here.]

Consider a circular cylinder of radius R and length $t = 2d$, uniformly magnetized along its length axis which we take to lie along the cartesian x -axis. We would like to determine the magnetic phase shift profile across the cylinder, along the y -direction, taking into account the demagnetization field surrounding the (finite) cylinder. The saturation induction is given by $B_0 = \mu_0 M_0$, with M_0 the

saturation magnetization. The magnetic phase shift for this case can be derived in Fourier space as [20]:

$$\varphi_m(k_x, k_y) = \frac{i\pi B_0}{\phi_0} \frac{D(k_x, k_y, 0)}{k_\perp^2} (\hat{\mathbf{m}} \times \mathbf{k})|_z. \quad (2)$$

where $\hat{\mathbf{m}}$ is the unit vector along the magnetization direction, in this case $(1, 0, 0)$, ϕ_0 is the flux quantum, and $k_\perp^2 = k_x^2 + k_y^2$; the reference frame is illustrated in Fig. 1, where the dimensions of the cylinder have been normalized by the radius R , resulting in a unit radius and a length $t = 2\tau$, with $\tau = t/R$. The shape amplitude, $D(\mathbf{k})$, for the circular cylinder can be written in cylindrical coordinates $\mathbf{k} = (k_p, \theta, k_z)$ with $k_p^2 = k_y^2 + k_z^2$ as:

$$D(\mathbf{k}) = \frac{4\pi R}{k_p k_x} J_1(R k_p) \sin(d k_x), \quad (3)$$

where $J_1(x)$ is a regular Bessel function of the first kind. Taking a Fourier slice for $k_z = 0$, we obtain:

$$D(k_x, k_y, 0) = \frac{4\pi R}{|k_y| k_x} J_1(R |k_y|) \sin(d k_x). \quad (4)$$

This results in the following explicit form for the magnetic phase shift in 2D Fourier space:

$$\varphi_m(k_x, k_y) = \frac{4i\pi^2 R B_0}{\phi_0} \text{sign}(k_y) \frac{J_1(R |k_y|) \sin(d k_x)}{(k_x^2 + k_y^2) k_x}, \quad (5)$$

where $\text{sign}(x)$ returns the sign (+1 or -1) of its argument x . The real space phase shift is then given by the inverse 2D Fourier transform, which can be written as:

$$\varphi_m(x, y) = \frac{i R B_0}{\phi_0} \int_{-\infty}^{+\infty} dk_y \text{sign}(k_y) J_1(R |k_y|) e^{ik_y y} \int_{-\infty}^{+\infty} dk_x \frac{\sin(d k_x)}{k_x (k_x^2 + k_y^2)} e^{ik_x x} \quad (6)$$

We scale Fourier space by [the wire radius](#) R , and introduce the dimensionless parameters $\bar{x} = x/R$, $\bar{y} = y/R$, $K_x = Rk_x$, $K_y = Rk_y$, and the cylinder aspect ratio $\tau = d/R$ (see also Fig. 1):

$$\varphi_m(\bar{x}, \bar{y}) = \frac{iR^2 B_0}{\phi_0} \int_{-\infty}^{+\infty} dK_y \text{sign}(K_y) J_1(|K_y|) e^{iK_y \bar{y}} \int_{-\infty}^{+\infty} dK_x \frac{\sin(\tau K_x)}{K_x(K_x^2 + K_y^2)} e^{iK_x \bar{x}}. \quad (7)$$

The K_x integration results in:

$$\int_{-\infty}^{+\infty} dK_x \frac{\sin(\tau K_x)}{K_x(K_x^2 + K_y^2)} e^{iK_x \bar{x}} = \frac{\pi}{K_y^2} \begin{cases} e^{-|K_y| \bar{x}} \sinh(|K_y| \tau) & (\bar{x} > \tau) \\ 1 - e^{-|K_y| \tau} \cosh(|K_y| \bar{x}) & (\bar{x} < \tau) \end{cases}, \quad (8)$$

so that the phase shift becomes:

$$\varphi_m(\bar{x}, \bar{y}) = -\frac{2\pi R^2 B_0}{\phi_0} \int_0^{+\infty} dK_y \frac{\text{sign}(K_y)}{K_y^2} J_1(|K_y|) \times \begin{cases} e^{-|K_y| \bar{x}} \sinh(|K_y| \tau) \\ 1 - e^{-|K_y| \tau} \cosh(|K_y| \bar{x}) \end{cases} \sin(K_y \bar{y}). \quad (9)$$

where the top line is valid for $\bar{x} > \tau$, and the bottom line for $\bar{x} \leq \tau$.

The K_y integral can be performed analytically by writing the Bessel function $J_1(|K_y|)$ in its standard series expansion and integrating term by term; the resulting integrals can be expressed as hypergeometric functions of the type ${}_3F_2[\dots]$, which, in turn, can be expanded into combinations of rational and logarithmic functions, leading to the following analytical expression for the magnetic phase shift of a finite, uniformly magnetized circular cylinder:

$$\varphi_m(\bar{x}, \bar{y}, \tau) = \Re \left[\begin{cases} \kappa(\bar{x}, i\bar{y}, \tau) - \frac{\pi}{2} & 0 \leq |\bar{x}| < \tau; \bar{y} \leq -1 \\ \kappa(\bar{x}, i\bar{y}, \tau) + \bar{y} \sqrt{1 - \bar{y}^2} + \arcsin \bar{y} & 0 \leq |\bar{x}| < \tau; -1 < \bar{y} < 1 \\ \kappa(\bar{x}, i\bar{y}, \tau) + \frac{\pi}{2} & 0 \leq |\bar{x}| < \tau; 1 \leq \bar{y} \\ \kappa(\bar{x}, i\bar{y}, \tau) & \tau \leq |\bar{x}| \end{cases} \right] \quad (10)$$

where $\Re$ stands for the real part, and the function κ is defined by (using dummy arguments a , b , and c):

$$\begin{aligned}\kappa(a, b, c) \equiv & -\frac{i}{4} [g(a+b+c) + g(-a+b+c) - g(a-b+c) - g(a+b-c) \\ & + \ln \left(\frac{h(a+b-c)h(a-b+c)}{h(a+b+c)h(-a+b+c)} \right) \\ & + \ln \left(\frac{a+b-c}{a-b-c} \right) + \ln \left(\frac{a-b+c}{a+b+c} \right)]\end{aligned}\quad (11)$$

with

$$g(x) \equiv x^2 \left(1 - \sqrt{1 + \frac{1}{x^2}} \right); \quad (12)$$

$$h(x) \equiv 1 + \sqrt{1 + \frac{1}{x^2}}. \quad (13)$$

[Figure 2 about here.]

Fig. 2 shows the normalized phase shift profile $\bar{\varphi}_m(0, \bar{y})$ across the wire at the position $\bar{x} = 0$ (center plane of the wire) for a number of wire aspect ratios. The normalized phase shift is defined as:

$$\bar{\varphi}_m = \frac{\phi_0}{\pi R^2 B_0} \varphi_m. \quad (14)$$

[Figure 3 about here.]

Note that the profile converges to limiting values of $\pm\pi/2$ for large wire aspect ratios, since the function κ tends to zero in the limit of $\tau \rightarrow \infty$. The wire edges correspond to the points $\bar{y} = \pm 1$. For short wires, the end faces are relatively close to the center of the wire and the demagnetization field reduces the total phase shift. The normalized phase shift $\bar{\varphi}_m$ at the point $(\bar{x}, \bar{y}) = (0, 1)$ is shown as a function of the aspect ratio τ in Fig. 3 (note the logarithmic scale for τ).

Fig. 4 shows contour plots of the magnetic phase shift for four different cylinder aspect ratios. Note the different color scale for each plot, reflecting the fact that shorter cylinders have a smaller maximum phase shift.

[Figure 4 about here.]

[Figure 5 about here.]

The maximum phase jump across the wire can be computed analytically as a function of the wire aspect ratio by selecting the point $(\bar{x}, \bar{y}) = (0, 1)$ and multiplying the resulting phase shift by 2; the total phase jump $\Delta\varphi_m$ from one edge of the wire to the opposite edge at the center plane of the wire is then given by:

$$\Delta\varphi_m = \frac{\pi R^2 B_0}{\phi_0} f(\tau), \quad (15)$$

where (see Fig. 5(a))

$$\begin{aligned} f(\tau) \equiv & \pi + 4\tau - \sqrt{\frac{\tau}{2}}(3\tau + \sqrt{4 + \tau^2})\sqrt{\sqrt{4 + \tau^2} - \tau} \\ & - 2 \arctan \left(\sqrt{\frac{1}{2} \sqrt{1 + \frac{4}{\tau^2}} - \frac{1}{2}} \right) \end{aligned} \quad (16)$$

The limit of $f(\tau)$ for $\tau \rightarrow \infty$ is equal to π . The magnetic induction can then be extracted from the phase shift by inverting eq. 15.

The slope of the phase shift can be computed by taking the derivative with respect to $\bar{y}$ of the integral in eq. (9), setting $(\bar{x}, \bar{y}) = (0, 0)$, and defining:

$$g(\tau) \equiv \int_0^\infty dK_y \frac{2}{K_y} J_1(K_y) (1 - e^{-K_y \tau}) = 2(\tau + 1) - 2\sqrt{\tau^2 + 1}. \quad (17)$$

This results in the following expression for the phase shift slope at the origin:

$$\frac{d\varphi_m(0, 0; \tau)}{dy} = \frac{\pi R B_0}{\phi_0} g(\tau). \quad (18)$$

The function $g(\tau)$ is shown in Fig. 5(b); its limiting value for large τ is equal to 2.

[Figure 6 about here.]

Finally, the normalized phase shift gradient is shown as a function of aspect ratio τ and position $(0, \bar{y})$ in Fig. 6.

$$\bar{\varphi}'_m = \frac{\phi_0}{\pi R^2 B_0} \frac{d\varphi_m}{d\bar{y}}. \quad (19)$$

In the limit of $\tau \rightarrow \infty$ (i.e., a very long straight wire with uniform diameter), the function κ vanishes, and the phase shift profile is described by:

$$\varphi_m(\bar{y}; \tau = \infty) = \frac{-\pi R^2 B_0}{\phi_0} \begin{cases} -\frac{\pi}{2} & (y < -1) \\ \bar{y}\sqrt{1-\bar{y}^2} + \arcsin(\bar{y}) & (-1 < y < 1) \\ \frac{\pi}{2} & (y > 1) \end{cases} \quad (20)$$

The phase jump for the infinite wire is then given by:

$$\Delta\varphi_m = \frac{\pi^2 R^2 B_0}{\phi_0} \quad (21)$$

which, in turn, means that the magnetic induction for an infinite wire can be extracted from:

$$B_0 = \frac{\phi_0 \Delta\varphi_m}{\pi^2 R^2} = 209.735 \frac{\Delta\varphi_m}{R^2}, \quad (22)$$

where the flux quantum, ϕ_0 , equals 2070 T nm^2 .

In terms of the phase gradient, the slope of the phase at the center $(\bar{x}, \bar{y}) = (0, 0)$ of the infinite wire is given by:

$$\frac{d\varphi_m(\bar{y}; \tau = \infty)}{d\bar{y}} = \frac{-2\pi R^2 B_0}{\phi_0} 2\sqrt{1-\bar{y}^2} \quad (23)$$

so that:

$$\frac{d\varphi_m(0, 0; \tau = \infty)}{dy} = \frac{-2\pi R^2 B_0}{\phi_0} \frac{2}{R} = -\frac{4\pi R B_0}{\phi_0} \quad (24)$$

and finally

$$B_0 = -\frac{\phi_0}{4\pi R} \frac{d\varphi_m(0, 0; \tau = \infty)}{dy} = 164.725 \frac{1}{R} \frac{d\varphi_m(0, 0; \tau = \infty)}{dy} \quad (25)$$

If the slope is measured from a phase reconstruction experiment, then the magnetic induction can be determined from the above relations.

[Figure 7 about here.]

3. Results and Discussion

Fig. 7 shows an experimental hologram of a CoFeB nanowire of approximate diameter 36 nm extending over a hole in a holey carbon grid. The nanowire was examined in its remanent state after an external magnetic field was applied perpendicular to its axis. The wire is slightly tapered towards its end and has a nearly spherical end cap, which is different from the cylindrical model described in the previous section; but given that the end cap represents only a small volume of material, it is likely that the model remains a reasonable approximation for the phase shift.

[Figure 8 about here.]

The magnetic phase shift extracted from the hologram and its negatively tilted pair is shown in Fig. 8(a), and as a fringe map representing $\cos(10\varphi_m)$ in Fig. 8(b). As shown in Fig. 8(a), the magnetic contour map of the examined nanowire has uniform magnetization exactly along the axis of the NW in an applied magnetic field perpendicular to the axis of NW. This shows that the shape anisotropy of the wire was the main controlling factor for the magnetization direction. The corresponding magnetic induction of this NW is 1.7 ± 0.3 T. This value is consistent

with what is reported for CoFeB thin films with similar composition ($0.75 - 1.7$ T, measured by vibrating-sample magnetometer (VSM) or a B-H tracer) [21, 22].

The phase shift model was used to refine seven parameters: the coordinates of the tip of the wire, the inclination of the wire with respect to the horizontal axis of the phase map, the wire radius R and aspect ratio τ , the saturation induction B_0 , and a phase offset parameter to account for the fact that the average phase shift across the experimental data set may differ from zero. Using a downhill simplex minimization routine, the root-mean-square difference between the experimental and simulated phase maps was minimized, resulting in the following parameters: the wire tip is located at $(158.7(1), 121.0(1))$ nm with respect to the lower left corner of the phase map, wire inclination angle $42.4(1)^\circ$, radius $R = 17.9(5)$ nm, aspect ratio $\tau = 87.9(5)$, saturation induction $B_0 = 1.73(2)$ T, and a phase offset parameter of $-0.30(1)$ rad; the resulting wire length amounts to $1.57(7)$ μm , so that the wire center lies outside of the field of view. [The digit between parenthesis corresponds to the standard deviation of the fitting result, i.e., the uncertainty in the last digit of each parameter, after running the fitting procedure ten times with slightly different initial conditions.](#) The cosine of the reconstructed phase map is shown in Fig. 8(d). Note that the computed phase contours agree well with those of the experimental data in Fig. 8(b). The small black dot in Fig. 8(a) and (c) indicates the fitted position of the end face of the cylindrical wire. Note that in the experimental phase map, the wire extends slightly beyond this point, reflecting the fact that the true shape of the wire is not quite a perfect cylinder.

The phase profile comparison across the wire is shown in Fig. 9; the inset shows a portion of the (rotated) phase map from Fig. 8(a), and the rectangular box indicates where the phase profile was extracted and averaged over the horizontal

dimension of the box. The continuous line corresponds to the best fit profile for the parameters described in the previous paragraph; the diamond symbols represent the experimental phase shift profile. There is satisfactory agreement between the two profiles, indicating that the uniformly magnetized circular cylinder represents a reasonable model for the magnetic nanowire.

[Figure 9 about here.]

In summary, the CoFeB nanowire examined by electron holography showed uniform remanent magnetization along its length axis for an applied magnetic field perpendicular to the axis. This indicates that the magnetization direction was mainly controlled by the shape anisotropy of the wire. For a thin film of CoFeB, electrodeposited under similar conditions as the examined nanowire, a magnetization of 1.7 T was reported [23] (measured by VSM). We derived a closed-form expression for the magnetic phase profile of an isolated uniformly magnetized cylinder and showed that the model phase calculation matched the phase profile of the CoFeB nanowire extracted by electron holography with a high degree of confidence. This illustrates the power of combining the electron holography technique with phase shift modeling for the quantitative extraction of magnetic information from holograms of magnetized nanowires.

4. Methods

The NW arrays were electrodeposited into nanoporous alumina membranes obtained by a two-step anodization technique [4, 24]. The electrodeposition of CoFeB followed the procedure developed by Ciureanu et al. [23]. The NWs were grown inside the pores by pulse-potential electrodeposition in a standard

three-electrode-cell, to produce uniformly fabricated quasi-hexagonal arrays of FMNWs [4, 8, 23]. Prior to the deposition of the NWs, an adhesion layer of Ti, with a thickness of 15 nm, and a 1 μ m thick Au layer were sputtered on the back side of the alumina membrane. The Au/Ti/Al film acted as the working electrode in the cell (cathode) and a platinum sheet was used as the counter electrode (anode) with a saturated calomel reference electrode. Individual wires were mounted on holey carbon-coated copper grids by dissolving the nanoporous membranes in either sodium hydroxide (1 M of NaOH, 6 hours) or phosphochromic acid (0.5 M H_3PO_4 + 0.2 M of H_2CrO_4 , 2 hours) [16].

The remanent magnetization of individual magnetic wires was analyzed using a Tecnai G2 STEM equipped with a field-emission source operating at 200 kV, a rotatable electrostatic biprism, and a charge coupled device (CCD) camera. The microscope objective lens was turned off while the holograms were being acquired so that the magnetic nanowires could be measured in a magnetic-field-free condition. Images were acquired using a long focal-length Lorentz lens located below the sample.

To separate the magnetic contribution from the MIP [25], we conducted an in situ magnetization reversal experiment by turning on the current of the objective lens while the sample was tilted; for each pair of holograms, the sample was tilted $\pm 20^\circ$. The orientation of the wire with respect to the tilt axis determines the final direction of the angle between the applied magnetic field and the length axis of the wire; the NW orientation with respect to the tilt axis of the holder determines the magnitude and direction of the applied magnetic field. A biprism voltage of 140 V and a magnification of 42,000 $\times$ were used in Lorentz mode with an average magnetic resolution [12] of 15 nm. Reference holograms were acquired from

empty holes in the holey carbon-coated copper grids, away from any magnetic nanowires. To provide quantitative information about the magnetic properties of the NWs, phase images were extracted from reconstructions of the holograms with the help of the “Semper” software package [26].

Acknowledgments

MDG would like to acknowledge financial support from the National Science Foundation, grant # DMR-1306296. AAZ and KLK would like to acknowledge financial support from the Natural Science and Engineering Research Counsel and technical help from B. Gormann, L. Yang, and 4DLabs. We thank Carignan (U. Montreal) for supplying the CoFeB nanowires.

References

- [1] C. Ross, H. Smith, T. Savas, M. Schattenburg, M. Farhand, M. Huang, M. Walsh, M. Abraham, R. Ram, Spin confinement by anisotropy modulation, *J. Vac. Sci. Technol. B* 17 (1999) 3168.
- [2] A. Fert, L. Piraux, Magnetic nanowires, *J. Magn. Magn. Mater.* 200 (1999) 338.
- [3] T. Shimizu, K. Aoki, Y. Tanaka, T. Terui, S. Shingubara, Preparation of ultrahigh-density magnetic nanowire arrays beyond 1 Terabit/inch² on Si substrate using anodic Aluminum Oxide template, *Jap. J. Appl. Phys.* 50 (2011) 06GE01.

- [4] L. Carignan, A. Yelon, D. Ménard, C. Caloz, Ferromagnetic nanowire metamaterials: theory and applications, *IEEE Trans. Microw. Theory Tech.* 59 (2011) 2568.
- [5] D. Reich, M. Tanase, A. Hultgren, C. Bauer, S. Chen, G. Meyer, Biological applications of multifunctional magnetic nanowires, *J. Appl. Phys.* 93, 93 (2003) 7275.
- [6] M. Tanase, D. Silevitch, A. Hultgren, L. Bauer, P. Searson, G. Meyer, D. Reich, Magnetic trapping and self-assembly of multicomponent nanowires, *J. Appl. Phys.* 91 (2002) 8549.
- [7] C. A. Ross, Patterned magnetic recording media, *Annual Rev. Mater. Sci.* 31 (2001) 203.
- [8] L. Carignan, C. Lacroix, A. Ouimet, M. Ciureanu, A. Yelon, D. Ménard, Magnetic anisotropy in arrays of Ni, CoFeB, and Ni/Cu nanowires, *J. Appl. Phys.* 102 (2007) 023905.
- [9] T. Thurn-Albrecht, J. Schotter, G. Kästle, N. Emley, T. Shibauchi, L. Krusin-Elbaum, K. Guarini, C. Black, M. Tuominen, T. Russell, Ultrahigh-density nanowire arrays grown in self-assembled diblock copolymer templates, *Science* 290 (2000) 2126.
- [10] S. Parkin, M. Hayashi, L. Thomas, Magnetic domain-wall racetrack memory, *Science* 320 (2008) 190.
- [11] M. Darques, J. Spiegel, J. De la Torre Medina, I. Huynen, L. Piroux, Ferromagnetic nanowire-loaded membranes for microwave electronics, *J. Magn. Mater.* 321 (2009) 2055.

- [12] R. E. Dunin-Borkowski, M. R. McCartney, D. J. Smith, Electron holography of nanostructured materials, *encyclopaedia of nanoscience and nanotechnology*, American Scientific Publishers 3 (2004) 41.
- [13] C. Beeli, B. Doudin, J. P. Ansermet, P. A. Stadelmann, Measurement of the remanent magnetization of single Co/Cu and Ni nanowires by off-axis TEM electron holography, *Ultramicroscopy* 67 (1997) 143.
- [14] E. Snoeck, R. E. Dunin-Borkowski, F. Dumestre, P. Renaud, C. Amiens, B. Chaudret, P. Zurcher, Quantitative magnetization measurements on nanometer ferromagnetic cobalt wires using electron holography, *Appl. Phys. Lett.* 82 (2003) 88.
- [15] R. E. Dunin-Borkowski, T. Kasama, A. Wei, M. Tripp, M. J. Hytch, E. Sneeck, R. J. Harriosn, A. Putnis, Off-axis electron holography of magnetic nanowires and chains, rings and planar arrays of magnetic nanoparticles, *Microsc. Res. Technol.* 64 (2004) 390.
- [16] A. Akhtari-Zavareh, L. Carignan, A. Yelon, D. Ménard, T. Kasama, R. Herring, R. E. Dunin-Borkowski, M. R. McCartney, K. L. Kavanagh, Off-axis electron holography of ferromagnetic multilayer nanowires., *J. Appl. Phys.* 116 (2014) 023902.
- [17] J. Cantu-Valle, I. Betancourt, J. E. Sanchez, F. Ruiz-Zepeda, M. M. Maqableh, F. Mendoza-Santoyo, B. J. H. Stadler, A. Ponce, Mapping the magnetic and crystal structure in cobalt nanowires, *J. Appl. Phys.* 118.
- [18] M. Gajdardziska-Josifovska, M. R. McCartney, W. J. de Ruijter, D. J. Smith, K. Weiss, J. M. Zuo, Accurate measurements of mean inner potential of

crystal wedges using digital electron holograms, *Ultramicroscopy* 50 (1993) 285.

- [19] L. Reimer, *Transmission Electron Microscopy*, Springer, 1991.
- [20] M. Beleggia, Y. Zhu, Electron-optical phase shift of magnetic nanoparticles I. Basic concepts, *Philos. Mag.* 83 (2003) 1045.
- [21] Y. T. Chen, S. U. Jen, J. M. Yao, Y. D. and Wu, C. C. Lee, T. L. Tsai, Y. C. Chang, Magnetic and electrical properties of amorphous cofeb films, *J. Appl. Phys.* 99 (2006) 053701.
- [22] L. Heyne, M. Kläui, D. Backes, P. Möhrke, T. A. Moore, J. G. Kimling, O. Boulle, U. Rüdiger, H. L.J., A. Fraile Rodríguez, F. Nolting, K. Kirsch, R. Mattheis, Direct imaging of current-induced domain wall motion in cofeb structures, *J. Appl. Phys.* 103 (2008) 07D928.
- [23] M. Ciureanu, F. Bron, L. Clime, P. Ciureanu, A. Yelon, T. A. Ovari, R. W. Cochrane, F. Normandin, T. Veres, Magnetic properties of electrodeposited CoFeB thin films and nanowire arrays, *Electrochimica Acta* 50 (2005) 4487.
- [24] S. Zhao, K. Chan, A. Yelon, T. Veres, Preparation of open-through anodized aluminium oxide films with a clean method, *Nanotechnology* 18 (2007) 245304.
- [25] R. E. Dunin-Borkowski, M. R. McCartney, D. J. Smith, S. S. P. Parkin, Towards quantitative electron holography of magnetic thin films using in situ magnetization reversal, *Ultramicroscopy* 74 (1998) 61.

- [26] W. O. Saxton, T. J. Pitt, M. Horner, Digital image processing: the Semper system, *Ultramicroscopy* 4 (1979) 343.

List of Figures

1	Illustration of the cylinder geometry in a scaled cartesian reference frame (all dimensions are scaled by the cylinder radius). . . .	20
2	Normalized phase shift profile across the wire for various wire aspect ratios τ	21
3	Normalized phase shift at location $(\bar{x}, \bar{y}) = (0, 1)$ as a function of τ .	22
4	Phase contour plots for selected values of the cylinder aspect ratio τ .	23
5	Illustration of the functions $f(\tau)$ (a) and $g(\tau)$ (b).	24
6	Normalized phase shift gradient along the line $(0, \bar{y})$ as a function of τ	25
7	Experimental hologram of a CoFeB nanowire of diameter 36 nm. .	26
8	(a) Magnetic phase shift map corresponding to the hologram of Fig. 7; (b) $\cos(10\varphi_m)$; (c) result of downhill simplex root-mean-square fit of the phase shift model to the data in (a), and (d) $\cos(10\varphi_m^t)$ where φ_m^t is the best fit phase map.	27
9	Averaged vertical phase profile across the boxed area in the inset. The best fit profile closely matches the experimental profile. The vertical dashed lines indicate the extent of the nanowire.	28

Figure 1: Illustration of the cylinder geometry in a scaled cartesian reference frame (all dimensions are scaled by the cylinder radius).

Figure 2: Normalized phase shift profile across the wire for various wire aspect ratios τ .

Figure 3: Normalized phase shift at location $(\bar{x}, \bar{y}) = (0, 1)$ as a function of τ .

Figure 4: Phase contour plots for selected values of the cylinder aspect ratio τ .

Figure 5: Illustration of the functions $f(\tau)$ (a) and $g(\tau)$ (b).

Figure 6: Normalized phase shift gradient along the line $(0, \bar{y})$ as a function of τ .

Figure 7: Experimental hologram of a CoFeB nanowire of diameter 36 nm.

Figure 8: (a) Magnetic phase shift map corresponding to the hologram of Fig. 7; (b) $\cos(10\varphi_m)$; (c) result of downhill simplex root-mean-square fit of the phase shift model to the data in (a), and (d) $\cos(10\varphi_m^t)$ where φ_m^t is the best fit phase map.

Figure 9: Averaged vertical phase profile across the boxed area in the inset. The best fit profile closely matches the experimental profile. The vertical dashed lines indicate the extent of the nanowire.